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CONDITIONAL MONOTONOUS FUNCTIONS OVER A FINITE SET. PART I.

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1. Introduction

Let $E_k = \{0,1,\dots,k-1\}$ where k is an integer with $k \geq 2$, $P_k^{(n)}$ the set of all functions $f(x_1,x_2,\dots,x_n)$ of n variables defined whenever all the $x_i \in E_k$ and with values in E_k and $P_k = \bigcup_{n\geq 1} P_k^{(n)}$. The operation of superposition (composition) and the closure $[M]$ of a subset M of P_k are introduced in the usual manner (see e.g. [3] and [4]).

Let r an arbitrary partial order on E_k. Let $M_r^{(n)}$ the set of all $f(x_1,x_2,\dots,x_n) \in P_k^{(n)}$ satisfying the following condition:

$(a_1,a_2,\dots,a_n),(b_1,b_2,\dots,b_n) \in E_k^n$, $a_i \; r \; b_i$ $(i=1,2,\dots,n)$ implies $f(a_1,a_2,\dots,a_n) \; r \; f(b_1,b_2,\dots,b_n)$.

Let $M_r = \bigcup_{n\geq 1} M_r^{(n)}$. M_r is the set of all r-monotonous functions of P_k, because $M_r = [M_r]$ holds.

These closed classes M_r are very interesting not only with respect to the manifold applications but also with respect to the difficulty of the mathematical problems concerning these classes (e.g. the number of functions in $M_r^{(n)}$ or the problem of the existence of a finite base for M_r).

Because every partial order r is a binary relation, for h = 2 the connection between M_r and r is a special case of the concept Polρ of an arbitrary h-ary relation

$$\rho = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & & \vdots \\ a_{h1} & a_{h2} & \dots & a_{hm} \end{pmatrix} .$$

Let $Pol\rho = \bigcup_{n\geq 1} Pol^{(n)}\rho$, where $Pol^{(n)}\rho$ consists of all that functions $f \in P_k^{(n)}$, for which the row-wise application of f to n arbitrary column of ρ produces a column of ρ again.

The particularity of this paper is, that (for the first time) a weaker conception of the monotonity (the conditional monotonity) is investigated. We intend to explain the character of these weakening on the following example.

Let for $a \in E_3$ the relation $\{\vec{\tilde{a}}\}$ defined by

$E_3 \times \{a\} \cup \{(0,0), (1,1), (2,2)\}$.

We consider the set M of all functions $f \in P_3^{(n)}$, n = 1,2,..., satisfying the condition

$$\left.\begin{array}{l} \text{For all } a \in E_3 \text{ holds: If } (b_1,b_2,\dots,b_n),\ (c_1,c_2,\dots,c_n) \in E_3^n, \\ f(b_1,b_2,\dots,b_n)=a \text{ and } b_i\ \{\vec{\tilde{a}}\}\ c_i \text{ for } i = 1,2,\dots,n, \text{ then} \\ f(b_1,b_2,\dots,b_n)\ \ \{\vec{\tilde{a}}\}\ f(c_1,c_2,\dots,c_n). \end{array}\right\} \quad (1)$$

Let r_a the partial order $\begin{pmatrix} 0 & 1 & 2 & b & c \\ 0 & 1 & 2 & a & a \end{pmatrix}$, where $\{a,b,c\} = E_3$, so holds obviously $\bigcap_{a\in E_3} M_{r_a} \subseteq M$. If we consider the function g(x,y) (given table 1), so we see, that $\bigcap_{a\in E_3} M_{r_a} \subset M$ holds. The difference between $\bigcap_{a\in E_3} M_{r_a}$ and M is the particularity of the conception of the conditional monotonity.

x \ y	0	1	2
0	0	2	2
1	1	1	2
2	2	2	2

Table 1.

In addition to the definition (1) we investigate in this paper other conditions too.

Ju. I. Shurawljow was attentive to the functions with the property (1) working on the theory of noncorrect algorithms (see [5] and a paper prepared by him for Problems of Cybernetics (russian) Vol. 33). He regards the so-called correcting functions, which are such functions of $P_k^{(n)}$, which the results of the working of n algorithms on a set of m objects with respect to a measure of divergence, given from practical aspects, approximate as well as possible to an a priori given m-tuple of values of a certain predicate on this m obejcts. The fascination of this investigations is the following: If you make only few conditions on the correcting functions, so it is relatively easy in the arising voluminous set of functions of P_k to find an optimal correcting function in the sense of Shurawljow. Compared to it the in the practice relevant correcting functions are satisfying additional conditions. In the through it restricted set of functions it is more difficult to find optimal correcting functions. In particular to it you do need knowledges on the set of functions satisfying such conditions. Ju. I. Shurawljow said us certain of such conditions. Other conditions we added in result of discussions with him.

Here now our work begins. We investiagete the sets of functions which are given by certain of these conditions. In this paper we restrict us to k = 3 and to certain collections of the by Shurawljow named conditions, where essential differences to the usual monotonity here always appear. In general the set of the conditional monotonous functions do not be closed (with respect to the operation of superposition).

In this paper we investigate the with respect to the inclusion partial ordered set of the sets, which are defined by the various combinations of our conditions and show that some of these sets are equal $[\{x\}]$ (the set of the selector-funcitons). In two other papers, which will be published in Rostocker Mathematisches Kolloquium, we investigate the closure of sets of conditional monotonous functions and the clique-number of the graph defined by the threee partial orders r_o, r_1, r_2 and E_3^n.

We remark still, that we see an other application of the conditional monotonous functions in the mathematical description of votes too, if we allow the abstention from voting. We intend to facilitate by this example an interpretation of the essential contents of the by us investigated conditions. To it we consider the following situation. n persons vote in open election. On the base of their results a chairman has to give a result of the vote.

Let 0,1 resp. 2 the denotation of the results "No", "Yes" resp. "abstention from voting". A consequent chairman takes into consideration certainly the following rules. If all n persons vote with the same "a", $a \in E_3$, then he votes with "a" too. If nobody of the n persons votes with "a", $a \in E_2$, then he has to vote with "b", $b \in E_3 \setminus \{a\}$. If the chairman votes on the base of a concrete situation of vote with "a", $a \in E_2$, then he has to vote with "a" too, if in a second vote only one of the n persons changed his mind to "a". By these and similar considerations we receive the by us in the second paragraph defined conditions.

We are obliged to Ju. I. Shurawljow, who us refered by his request, to investigate the by us as conditional monotonous functions denoted types of functions of P_k, to a new type of questions in the k-valued logic P_k and to an interesting application of the k-valued logic.

2. Basic types of sets of functions

We define here some essential types of sets of functions over E_3. Let $E_3 = \{0,1,2\}$, $P_3^{(n)}$ the set of all functions $f(x_1,x_2,\dots x_n)$ of n variables defined whenever all the $x_i \in E_3$ and with values in E_3 and $P_3 = \bigcup_{n\geq 1} P_3^{(n)}$. The operation of superposition and the closure [M] of a subset M of P_3 are introduced in the usual manner (see e.g. [3]).

Let $R = \{\{0,1\},\{0,2\}, \{1,2\}\}$. We consider now for functions $f(\underline{x}) \in P_3^{(n)}$ with $\underline{x} = (x_1,x_2,\dots,x_n)$ for arbitrary integer $n\geq 1$ the following conditions:

Condition 1. $\bigwedge_{a \in E_3} f(a,a,\dots,a) = a.$

Condition 2. $\bigwedge_{M \in R\setminus\{0,1\}} \bigwedge_{\underline{\alpha} \in M^n} f(\underline{\alpha}) \in M.$

Condition 2'. $\bigwedge_{M \in R} \bigwedge_{\underline{\alpha} \in M^n} f(\underline{\alpha}) \in M.$

Definition 1. For every nonempty proper subset M of E_3 let $\underset{M}{\approx}$ the relation

$((E_3 x M) \cup \{(0,0),(1,1),(2,2)\})$ and $\underset{M}{\leq}$ the relation $((\overline{M} x M) \cup \{(0,0),(1,1),(2,2)\})$ where $\overline{M} = E_3 \setminus M$ is. If $\underline{\alpha} = (\alpha_1,\alpha_2,\ldots,\alpha_n)$, $\underline{\beta} = (\beta_1,\beta_2,\ldots,\beta_n) \in E_3^n$, then $\underline{\alpha} \underset{M}{\simeq} \underline{\beta}$ or $\underline{\alpha} \underset{M}{\leq} \underline{\beta}$ holds, iff for $i = 1,2,\ldots,n$ $\alpha_i \underset{M}{\simeq} \beta_i$ or $\alpha_i \underset{M}{\leq} \beta_i$ holds.

Example. $(1,0,0,2) \underset{\{1,2\}}{\simeq} (1,1,0,2)$ and $(1,0,0,2) \underset{\{1,2\}}{\leq} (1,1,0,2)$, $(2,0,0,1,2) \underset{\{1,2\}}{\simeq} (1,1,0,2,2)$ but $(2,0,0,1,2) \underset{\{1,2\}}{\not\leq} (1,1,0,2,2)$!
Obviously are the relations $\underset{\{a\}}{\simeq}$ and $\underset{\{a\}}{\leq}$ for all $a \in E_3$ equal.

Condition 3. $\bigwedge_{a \in E_3} \bigwedge_{\underline{\alpha},\underline{\beta} \in E_3^n} (f(\underline{\alpha}) = a \Rightarrow (\underline{\alpha} \underset{\{a\}}{\simeq} \underline{\beta} \Rightarrow f(\underline{\alpha}) \underset{\{a\}}{\simeq} f(\underline{\alpha})))$.

Condition 3'. $\bigwedge_{a \in E_3} \bigwedge_{\underline{\alpha},\underline{\beta} \in E_3^n} (\underline{\alpha} \underset{\{a\}}{\simeq} \underline{\beta} \Rightarrow f(\underline{\alpha}) \underset{\{a\}}{\simeq} f(\underline{\beta}))$.

Condition 4. $\bigwedge_{M \in R} \bigwedge_{\underline{\alpha},\underline{\beta} \in E_3^n} (f(\underline{\alpha}) \in M \Rightarrow (\underline{\alpha} \underset{M}{\simeq} \underline{\beta} \Rightarrow f(\underline{\alpha}) \underset{M}{\simeq} f(\underline{\beta})))$.

Condition 4'. $\bigwedge_{M \in R} \bigwedge_{\underline{\alpha},\underline{\beta} \in E_3^n} (\underline{\alpha} \underset{M}{\simeq} \underline{\beta} \Rightarrow f(\underline{\alpha}) \underset{M}{\simeq} f(\underline{\beta}))$.

Condition 5. $\bigwedge_{M \in R} \bigwedge_{\underline{\alpha},\underline{\beta} \in E_3^n} (f(\underline{\alpha}) \in M \Rightarrow (\underline{\alpha} \underset{M}{\leq} \underline{\beta} \Rightarrow f(\underline{\alpha}) \underset{M}{\leq} f(\underline{\beta})))$.

Condition 5'. $\bigwedge_{M \in R} \bigwedge_{\underline{\alpha},\underline{\beta} \in E_3^n} (\underline{\alpha} \underset{M}{\leq} \underline{\beta} \Rightarrow f(\underline{\alpha}) \underset{M}{\leq} f(\underline{\beta}))$.

Let K the set of all functions of P_3 satisfying the condition 1. K^s denotes for $s \in \{2,2', 3,3', 4,4', 5,5'\}$ the set of all functions of K satisfying the condition s. If $M \subseteq \{2,2', 3,3', 4,4', 5,5'\}$, then $K^M := K$, iff M is the empty set. In all other cases $K^M := \bigcap_{s \in M} K^s$.

Let finally $\mathcal{K} := \{K^M \mid M \subseteq \{2,2', 3,3', 4,4', 5,5'\}\}$ and $|\mathcal{K}|$ the cardinality of $\mathcal{K}$. Obviously $|\mathcal{K}| \leq 2^8 = 256$ holds.

In the next paragraph we'll show, that $|\mathcal{K}|$ is rather less than 2^8 and investigate the partial ordered set $(\mathcal{K},\leq)$.

3. The invesitagtion of $\mathcal{K}$

It is obvious, that the lemma 1 holds.

Theorem 3. $X \in K$ is a subalgebra (with respect to the operation of superposition) of P_3, iff $X \in \{K, K^2, K^{2'}, K^{3'}\}$.

Proof. It follows directly by their definitions, that the sets K, K^2, $K^{2'}$ and $K^{3'}$ are subalgebras of P_3. We have to show still, that $X \subset [X]$ holds for $X \in \{K^3, K^{2,3}, K^{2',3}\}$. $K^3 \supset K^{2,3} \supset K^{2',3}$ holds by tehorem 1. Therefore it is enough to find a function $f(x,y) \in K^{2',3}$ with $f(f(x,y),z) \in K^3$. $f_6(x,y)$, given by table 2, is such a function.■

Remark. We'll investigate the subalgebras, generated by K^3, $K^{2,3}$ or $K^{2',3}$, in the part II of this paper.

Theorem 4. $K^{3'} = [\{x\}]$.

Proof. The following statement holds ([1]): If $A = [A] \subseteq P_k$ and $[A \cap P_k^{(\max(k,3))}] = [A \cap P_k^{(1)}]$, then $A \subseteq [P_k^{(1)}]$.
By the tehorem 3 we know $K^{3'} = [K^{3'}]$. We have to prove still, that $f(x_1,x_2,x_3) = x_i$ $(i \in \{1,2,3\})$ holds for all $f(x_1,x_2,x_3) \in K^{3'}$. Let $f(x_1,x_2,x_3) \in K^{3'}$ and without loss of generality $f(0,1,2) = 0$.
By our conditions and the theorem 1 we know, that we receive for all $(a,b,c) \in E_3^3$ $f(a,b,c) \in \{a,b,c\}$. By lemma 5 $K^{4'} = K^{3'}$ holds.
Therefore we receive $(0,a,b) \underset{\{1,2\}}{\overset{\sim}{\rightarrow}} (0,1,2)$ and $f(0,a,b) = 0$ for all $a,b \in E_3$.
Let now $\{c,d\} = \{1,2\}$. Then $(0,c,c) \underset{\{d\}}{\overset{\sim}{\rightarrow}} (d,c,c)$, $(d,c,0) \underset{c}{\overset{\sim}{\rightarrow}} (d,c,c)$ and $f(d,c,c) = f(d,c,0) = d$ hold. Now we receive $(d,a,b) \underset{\{0,c\}}{\overset{\sim}{\rightarrow}} (d,c,0)$ and $f(d,a,b) = d$ for all $a,b \subseteq E_3$.■

Let $\rho = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$, $\rho_2 = \begin{pmatrix} 0&0&0&0 \\ 1&1&1&1 \\ 2&2&2&2 \\ 0&0&2&2 \\ 1&2&1&2 \end{pmatrix}$ $\rho_{2'} = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 1&1&1&1&1&1&1&1 \\ 2&2&2&2&2&2&2&2 \\ 0&0&0&0&1&1&1&1 \\ 0&0&2&2&0&0&2&2 \\ 1&2&1&2&1&2&1&2 \end{pmatrix}$

and $\rho_{3'} = \begin{pmatrix} 0&0&1&2 \\ 1&2&0&1 \end{pmatrix}$.

Theorem 5. $K = \mathrm{Pol}\rho$, $K^2 = \mathrm{Pol}\rho_2$, $K^{2'} = \mathrm{Pol}\rho_{2'}$ and $K^{3'} = \mathrm{Pol}\rho_{3'}$.

Proof. The first three statements are direct conclusions of our conditions. In [2] $\mathrm{Pol}\rho_{3'} = [\{x\}]$ is proved.■

Lemma 1. If $i \in \{2,3,4,5\}$, then $K^{i'} \subseteq K^{i}$. ■

Therefore we receive $|K| \leq 3^4 = 81$.

Lemma 2. $K^{4'} = K^4$,

Proof. Let $M \in R$. If $f(\underline{\alpha}) \notin M$, then $f(\underline{\alpha}) \underset{M}{\overset{\sim}{\rightarrow}} f(\underline{\beta})$ holds for every $\underline{\beta} \in E_3^n$. ■

Therefore we receive $|K| \leq 2.3^3 = 54$.
In the same way we are able to prove the

Lemma 3. $K^{5'} = K^5$.

Therefore $|K| \leq 2^2.3^2 = 36$ holds.

Lemma 4. $K^{3'} = K^{5'}$. ■

Proof. Let $\{a,b,c\} = E_3$. Then $\underset{\{a,b\}}{\leq}$ is the relation $\begin{pmatrix} c & c & a & b & c \\ a & b & a & b & c \end{pmatrix} = \rho_{a,b}$ and $\underset{\{c\}}{\overset{\sim}{\rightarrow}}$ is the relation $\begin{pmatrix} a & b & a & b & c \\ c & c & a & b & c \end{pmatrix} = \rho_c$. It is obvious that $Pol\rho_{a,b} = Pol\rho_c$ holds. Because $K^{3'} = Pol\rho_o \cap Pol\rho_1 \cap Pol\rho_2$ and $K^{5'} = Pol\rho_{1,2} \cap Pol\rho_{o,2} \cap Pol\rho_{o,1}$ hold, we receive $K^{3'} = K^{5'}$. ■

Therefore $|K| \leq 2.3^2 = 18$ holds.

Lemma 5. $K^{3'} = K^{4'}$.

Proof. Let $\{a,b,c\} = E_3$. Then $\underline{\alpha} \underset{\{a\}}{\overset{\sim}{\rightarrow}} \underline{\beta}$ holds, iff $\underline{\alpha} \underset{\{a,b\}}{\overset{\sim}{\rightarrow}} \underline{\beta}$ and $\underline{\alpha} \underset{a,c}{\overset{\sim}{\rightarrow}} \underline{\beta}$. Let $f(\underline{x}) \in K^{4'}$ and for $\underline{\alpha}, \underline{\beta} \in E_3^n$ let $\underline{\alpha} \underset{\{a\}}{\overset{\sim}{\rightarrow}} \underline{\beta}$. Then $f(\underline{\alpha}) \underset{\{a,b\}}{\overset{\sim}{\rightarrow}} f(\underline{\beta})$ and $f(\underline{\alpha}) \underset{\{a,c\}}{\overset{\sim}{\rightarrow}} f(\underline{\beta})$ hold. Therefore we receive: If $f(\underline{\alpha}) = a$, then $f(\underline{\beta}) \in \{a,b\}$ and $f(\underline{\beta}) \in \{a,c\}$, that means $f(\underline{\beta}) = a$, if $f(\underline{\alpha}) = b$, then $f(\underline{\beta}) \in \{a,b\}$ and if $f(\underline{\alpha}) = c$, then $f(\underline{\beta}) \in \{a,c\}$. That means, that $f(\underline{\alpha}) \underset{\{a\}}{\overset{\sim}{\rightarrow}} f(\underline{\beta})$ holds. Therefore $K^{4'} \subseteq K^{3'}$ holds. Let now $f(\underline{x}) \in K^{3'}$ and $\underline{\alpha} \underset{\{a,b\}}{\overset{\sim}{\rightarrow}} \underline{\beta}$. Let $\underline{\gamma} = (\gamma_1, \gamma_2, \ldots, \gamma_n)$ the following element of E_3^n. For $i=1,2,\ldots,n$ we set $\gamma_i = \alpha_i$, iff $\alpha_i = \beta_i$, and $\gamma_i = a$ else. Then $\underline{\alpha} \underset{\{a\}}{\overset{\sim}{\rightarrow}} \underline{\gamma} \underset{\{b\}}{\overset{\sim}{\rightarrow}} \underline{\beta}$ holds. If now $f(\underline{\alpha}) \in \{a,b\}$, then we receive $f(\underline{\gamma}) \in \{a,b\}$ and $f(\underline{\beta}) \in \{a,b\}$. That means, that $f(\underline{x}) \in K^4 = K^{4'}$ (lemma 2) holds. Therefore $K^{3'} \subseteq K^{4'}$ holds too. ■

Therefore $|K| \leq 3^2 = 9$ holds.

Lemma 6. $K^{3'} \subseteq K^{2'}$.

Proof. Let $f(\underline{x}) \in K^{3'} = K^{4'} = K^4$ (lemma 2 and 5), $a,b \in E_3$, $a \neq b$, $\underline{\beta} \in \{a,b\}^n$ and $\underline{\alpha} = (a,a,\ldots,a)$. Then $f(\underline{\alpha}) = a$, $\underline{\alpha} \underset{\{a,b\}}{\simeq} \frac{\beta}{2'}$ and $f(\underline{\alpha}) \underset{\{a,b\}}{\simeq} f(\underline{\beta})$ hold. Therefore we receive $f(\underline{\beta}) \in \{a,b\}$ and $f(\underline{x}) \in K^{2'}$. ■

Therefore $K \leq 7$ and $K = \{K, K^2, K^{2'}, K^3, K^{3'}, K^{2,3}, K^{2',3}\}$ hold.

By the lemma 1-6 the following inclusions are valued:

$$K^{3'} \subseteq K^{2',3} \subseteq K^{2,3} \subseteq K^3 \subseteq K \qquad (1)$$

$$K^{2,3} \subseteq K^2 \subseteq K \qquad (2)$$

$$K^{2',3} \subseteq K^{2'} \subseteq K^2 \qquad (3)$$

The functions $f_i(x,y)$ for $i = 1,2,3,4,5,6$ are given by the table 2.

x	y	$f_1(x,y)$	$f_2(x,y)$	$f_3(x,y)$	$f_4(x,y)$	$f_5(x,y)$	$f_6(x,y)$
0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2
0	1	0	2	0	1	2	1
1	0	0	0	0	1	2	2
0	2	1	0	0	1	2	2
2	0	0	0	0	1	2	2
1	2	2	1	1	1	1	1
2	1	0	1	1	1	1	2

Table 2.

By the table 3 the function g(x,y,z) is given.

z \ x, y									
x	0	1	2	0	1	0	2	1	2
y	0	1	2	1	0	2	0	2	1
0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2

Table 3.

Now the following relations, given by table 4, hold.

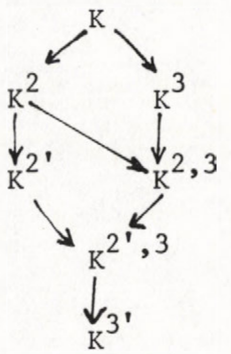
	because		because
$K \supset K^3$	$f_1 \in K \setminus K^3$	$K^{2'} \supset K^{2',3}$	$g \in K^{2'} \setminus K^{2',3}$
$K^3 \supset K^{2,3}$	$f_4 \in K^3 \setminus K^{2,3}$	$K^2 \subset K^3$	$f_2 \in K^2 \setminus K^3$
$K^{2,3} \supset K^{2',3}$	$f_5 \in K^{2,3} \setminus K^{2',3}$	$K^3 \subset K^2$	$f_4 \in K^3 \setminus K^2$
$K^{2',3} \supset K^{3'}$	$f_3 \in K^{2',3} \setminus K^{3'}$	$K^{2'} \subset K^3$	$g \in K^{2'} \setminus K^3$
$K \supset K^2$	$f_1 \in K \setminus K^2$	$K^3 \subset K^{2'}$	$f_4 \in K^3 \setminus K^{2'}$
$K^2 \supset K^{2,3}$	$f_2 \in K^2 \setminus K^{2,3}$	$K^{2'} \subset K^{2,3}$	$g \in K^{2'} \setminus K^{2,3}$
$K^2 \supset K^{2'}$	$f_2 \in K^2 \setminus K^{2'}$	$K^{2,3} \not\subset K^{2'}$	$f_5 \in K^{2,3} \setminus K^{2'}$

Table 4.

We receive therefore, together with the relations (1) - (3), the

Theorem 1. (i) $|K| = 7$

(ii) $(K, \subseteq)$ is given by mapping 1. ■



Mapping 1.

$X \to Y$ denotes for $X, Y \in K$ in this mapping , that $X \supset Y$ holds and for all $Z \in K$ with $X \supseteq Z \supseteq Y$ holds $X = Z$ or $Z = Y$, and $X \supset Y$ holds, iff an integer n with $n \geq 2$ exists with $X_1, X_2, \ldots, X_n$, $X_1 = X$, $X_n = Y$ and $X_i \to X_{i+1}$ for $i = 1, 2, \ldots, n-1$.

By the mapping 1 you are able easy to prove the

Theorem 2. $(K, \subseteq)$ is a distributive lattice. ■

R e f e r e n c e s

[1] Harnau, W., Eine Verallgemeinerung der Vertauschbarkeit in der k-wertigen Logik. EIK 12 (1976) 1/2, 33-43.

[2] Harnau, W., Über antidiagonale Relationen. Rostock 1976 (unpublished).

[3] Яблонский, С.В., Лупанов, О.Б., /редакторы/, Дискретная математика и математические вопросы кибернетики, Том 1. Москва 1974.

[4] Rosenberg, I. Über die funktonale Vollständigkeit in mehrwertigen Logiken. Praha 1970.

[5] Журавлев, Ю.И., Корректные алгебры над множествоми некорректных /эвристических/ алгоритмов. I. Кибернетика, Киев, 1977, 4, 14-21.

Ö s s z e f o g l a l ó

Feltételesen monoton függvények véges halmazon. I.

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Legyen $E_3 := (\{0,1,2\}; \leq)$ részben rendezett halmaz. Az összes feltételesen monoton $f: E_3^n \to E_3$ $n=1,2,\ldots$ függvényeknek a nyolc Zsuravljov feltétel lehetséges kombinációinak eleget tevő részhalmazai a tartalmazásra, mint részben rendezésre nézve 7 elemü disztributiv hálót alkotnak.

A feltételesen monotonitás gyengébb feltétel mint a szokásos monotonitási. A fenti részhalmazok közül egy éppen a szelektor függvények kompozicióra nézve zárt részhalmaza.

A Zsuravljov feltételek mint szavazási szabályok értelmezhetők.

Резюме

Условно монотонные функции на конечных множествах. Часть I

Густав Бурош, Клаус-Дитер Древс,
Валтер Харнау

Пусть $E_3 = (\{0,1,2\};\leq)$ частичное упорядоченное множество. Подмножество условно монотонных функций $f: E_3^n \to E_3$ $n=1,2,\ldots$ удовлетворяющее возможным сочетаниям восьми условий Журавлева составляют дистрибутивную сеть с мощностью семь.

Условное множество является более слабым условием, чем монотонность в обычном смысле. Одно из вышеуказанных подмножеств является замкнутым, - относительно композиции - подмножество селекторных функций.

Условия Журавлева могут быть представлены как правила голосования.