

ON PREDICTIVE DECONVOLUTION OF LONG-RUN
STATIONARY TIME SERIES

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ABSTRACT

Robinson's statistical minimum-delay model / or the method of predictive deconvolution / has been effectively used in seismic prospecting for oil and gas. It is used to eliminate multiple reflections from surface layers and reverberations in water layer. However, in our opinion, this model is not clear in some respects.

In this paper we try to give a new interpretation and more general condition for this model, which are possibly more suitable to practice. We also point out that, with the new conditions, the computation process based on observations is just the same as in the case of Robinson's model. In other words, the Robinson's assumptions contain some simplifications, however his computation gives practically correct results.

We also give examples for the predictive deconvolution of the new process.

§.1. ROBINSON'S MODEL

In order to fix ideas, let us consider a specific physical situation, namely the problem of seismic exploration for oil in the earth's sedimentary strata. The source is an explosion or another form of energy, which is introduced into the ground at the surface. The reflection response x_n is the seismic reflection record / time series / which is digitally recorded at the surface. The reflection coefficient sequence ξ_n is a digitized representation of the reflectivity of the earth as a function of depth. More exactly speaking, ξ_n is the reflection coefficient of the interface n , where the travel time of the input signal in

going to the interface n is $\frac{n}{2}$. As a result, knowledge of the sequences for various geographic locations on the surface allows the seismic interpreter to make contour maps of the earth's sedimentary structure at depth.

By certain assumptions / see [8] , p.457 / Robinson introduced the following equation:

$$x_n + a_1 x_{n-1} + a_2 x_{n-2} + \dots + a_p x_{n-p} = \varepsilon_n \quad n=0,1,2,\dots \quad (1)$$

where the a_c -s are unknown deterministic values, which depend on the geological structure of the observed area.

In the case of a noise appearing, the reflection response has the form

$$y_n = x_n + v_n \quad (2)$$

where v_n is a noise. Here we suppose the noise is eliminated. The predictive deconvolution problem is to compute the ε_n -s from the x_n -s. However, (1) implies a system of equations having more unknown variables than the number of equations, so it is impossible to find the ε_n -s.

Robinson proposed the statistical method as follows:

Although the model (1) is deterministic, i.e. there is no random variable in it, but the sequence ε_n may be considered as a realization of a random white noise, i.e.

$$E \varepsilon_n = 0 \quad E \varepsilon_n \varepsilon_s = \begin{cases} \sigma^2 & n=s \\ 0 & n \neq s \end{cases} \quad (3)$$

If we suppose further that

$$1 + a_1 z + a_2 z^2 + \dots + a_p z^p \neq 0 \quad \text{for } |z| \leq 1 \quad (4)$$

then x_n is a stationary autoregressive process. The coefficients a_1, a_2, \dots, a_p then can be estimated from the observations x_n -s, and the ε_n -s are estimated by

$$\hat{\varepsilon}_n = x_n + \hat{a}_1 x_{n-1} + \dots + \hat{a}_p x_{n-p} \quad (5)$$

§.2. SOME REMARKS ON ROBINSON'S MODEL

In our opinion, Robinson's model is not clear in some respects:
/a/ In the equation (1) the ξ_n -s are deterministic values, so we can consider them as special random variables such that

$$E\xi_n = \xi_n, \quad \text{var } \xi_n = 0$$

which contradicts the assumption (3) .

/b/ In practice the interfaces are not so arranged as in our assumptions, i.e. the travel time of the input signal in going to some interface is not always equal to $\frac{n}{2}$, where n is some positive integer, but may be an arbitrary real value t . Thus we can not consider the ξ_n -s as exact reflection coefficients.

/c/ We recourse to irregularity of the sequence ξ_n to make information for the earth's sedimentary structure at depth. For example, if $\xi_n = 0$, we think that perhaps there is no interface at depth n , if $\xi_n \approx 1$, we think it may be an interface between oil and gas strata... The assumption that the ξ_n -s have the same mean and variance seems not always suitable to the practice.

/d/ In his model Robinson supposed only that the sequence ξ_n is uncorrelated. / see [6] /. However, by the following example we want to show that in practice , when we have to estimate the correlation function from the sample time series, this assumption is not always sufficient for getting good estimates.

Example 1: Let our model be

$$x_n + ax_{n-1} = u_n, \quad n = 1, 2, 3, \dots$$

where $|a| < 1$, $u_n = \cos nW$ and W is an uniformly distributed random variable on $[0, 2\pi]$. Then

$$Eu_n = 0 \quad Eu_n u_s = \begin{cases} \frac{1}{2} & n = s \\ 0 & n \neq s \end{cases}$$

Thus the sequence u_n is a white noise in wide sense.

As usual, then a is estimated by

$$\hat{a} = \frac{-\frac{1}{N} \sum_{n=1}^N x_{n+1} x_n}{\frac{1}{N} \sum_{n=1}^N x_n^2}$$

Now suppose $a = 0$ then

$$\hat{a} = -\cos W - \frac{[\cos(2N+1)W - \cos W] \sin W}{\sin(2N+1)W + (2N-1)\sin W}$$

from which we can see that

$$\lim_{N \rightarrow \infty} \hat{a} = -\cos W \quad \text{a.s.}$$

Thus the estimate \hat{a} is always a random variable, its limit is also such a random variable, which has not any connection with the true value a . Hence we can not say that \hat{a} is a good estimate of a .

§.3. THE MODIFIED MODEL

In order to modify Robinson's model so that it be more suitable to practice, let us firstly consider the simplest case:

Suppose after explosion the input signal $f(t)$ propagates to the earth's crust. When acting an interface having reflection coefficient ξ it reflects to the surface with reflected wave $g(t) = \xi f(t)$. Since the elastic wave $f(t)$ represents the motion of particle about its equilibrium point, $f(t)$ always has a damped sinusoidal form / in the case of explosion it is relatively narrow with great frequency /. Now let us consider some observed value u on $g(t)$. In geophysics the arrival time of a reflected wave is usually considered as a uniformly distributed random variable / i.e. we do not know exactly when the reflected wave appears /. Thus the observed value u can also be considered as a random variable $u = g(\tau)$ where τ is a uniformly distributed random variable on some interval $[a, b]$. We have

$$\begin{aligned} Eu &= \frac{1}{b-a} \int_a^b \xi f(t) dt \approx 0 \quad / \text{ cf. Riemann lemma } / \\ Eu^2 &= \frac{\xi^2}{b-a} \int_a^b f^2(t) dt = C \xi^2 \end{aligned} \quad (6)$$

Or more exactly speaking, Eu is negligible compared with Eu^2 . although ξ is some fixed value / even in the case $\xi > 0$ /, the measured value u may be arbitrary value in $[-\xi, \xi]$. Hence u can not be considered as an approximation of ξ .

By the above reason, we propose to modify (1) by introducing the new model:

$$x_n + a_1 x_{n-1} + \dots + a_p x_{n-p} = u_n \quad n=0,1,2,\dots \quad (7)$$

Where $Eu_n = 0$, $Eu_n^2 = C\varepsilon_n^2$

For the reflection coefficients are different, Eu_n^2 can not be constant. However, by estimating the reflection coefficients from oil wells, White and Obriend /1974/ of British Petroleum, Schoenberger and Levin /1974/ of Exxon saw that the reflection coefficient sequence ε_n is like the realization of a white noise, i.e. for N large enough

$$\frac{1}{N} \sum_{n=0}^{N-1} \varepsilon_n^2 = \gamma^2 > 0 \quad (8)$$

$$\frac{1}{N} \sum_{n=0}^{N-1} \varepsilon_{n+s} \varepsilon_n \approx 0 \quad n = 1, 2, \dots$$

/ see [8] , p.490 /. By (6) and (8) we have

$$\frac{1}{N} \sum_{n=0}^{N-1} Eu_n^2 = C \frac{1}{N} \sum_{n=0}^{N-1} \varepsilon_n^2 \approx C\gamma^2 = \sigma^2 > 0 \quad (9)$$

We suppose that in the case of a complicated geological phenomenon the variables u_n -s are independent.

Briefly speaking, we suppose that the reflection response satisfies (7) and the following three conditions:

/a/ The variables u_0, u_1, u_2, \dots are independent with mean 0 and

$$E|u_n|^{2+\varepsilon_0} < K < \infty \quad \text{for some } K \text{ and } \varepsilon_0 > 0 \quad (10)$$

/b/ $\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N-1} Eu_n^2 = \sigma^2 > 0 \quad (11)$

/c/ $1 + a_1 z + \dots + a_p z^p \neq 0 \quad \text{for } |z| \leq 1 \quad (12)$

We would like to remark that the condition (10) is obvious because the variables u_n -s are uniformly bounded. For the validity of the condition (12) the reader can see in [5] and [6] .

§.4. PREDICTIVE DECONVOLUTION OF LONG-RUN STATIONARY AUTOREGRESSIVE PROCESS

Definition 1: We call the process x_n satisfying (7) , (10) , (11) and (12) a long-run stationary autoregressive process.

This notion does not occur in the standard literature on

time series analysis.

Definition 2: We call the process y_n a stationary autoregressive process corresponding to the above defined long-run stationary process x_n if y_n satisfies

$$y_n + a_1 y_{n-1} + \dots + a_p y_{n-p} = v_n \quad n = \dots -1, 0, 1, 2, \dots \quad (13)$$

where v_n is a white noise with variance σ^2 .

Theorem: Let x_n be a long-run stationary process satisfying (7), (10), (11), (12) then for $s = \dots -1, 0, 1, 2, \dots$ there exist the limits

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N-1} x_{n+s} x_n = \varphi_s \quad \text{a.s.} \quad (14)$$

where φ_s is the correlation function of the corresponding stationary process y_n .

Proof: By (12) we can take the reciprocal $B(z)$ of the Z-transform $A(z)$

$$B(z) = \frac{1}{1 + a_1 z + \dots + a_p z^p} = 1 + b_1 z + b_2 z^2 + \dots$$

and the process x_n can be written in the form

$$x_n = \sum_{s=0}^{\infty} b_s u_{n-s} \quad \text{where } u_n = 0 \quad \text{for } n < 0$$

By (10) $E u_n^2 < d$ for some $d > 0$.

Let

$$\xi_n = u_n^2 - \bar{E} u_n^2, \quad \delta_0 = \frac{\varepsilon_0}{2}$$

Using Minkowski's inequality, we have

$$\begin{aligned} (E |\xi_n|^{1+\delta_0})^{\frac{1}{1+\delta_0}} &= (E |u_n^2 - \bar{E} u_n^2|^{1+\delta_0})^{\frac{1}{1+\delta_0}} \leq \\ &\leq (E |u_n|^{2+\varepsilon_0})^{\frac{1}{1+\delta_0}} + \bar{E} u_n^2 < K^{\frac{1}{1+\delta_0}} + d \end{aligned}$$

from which

$$E |\xi_n|^{1+\delta_0} < (K^{\frac{1}{1+\delta_0}} + d)^{1+\delta_0} < \infty$$

The sequence ξ_n satisfies the conditions of Markov's theorem

/ see [2], p.287 / therefore

$$P \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N-1} \xi_n = 0$$

Thus we have

$$P \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N-1} (u_n^2 - E u_n^2) = P \lim_{N \rightarrow \infty} \left(\frac{1}{N} \sum_{n=0}^{N-1} u_n^2 - \frac{1}{N} \sum_{n=0}^{N-1} E u_n^2 \right) = 0 \quad (15)$$

By (11) and (15)

$$P \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N-1} u_n^2 = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N-1} E u_n^2 = \sigma^2$$

Since the variables u_n^2 -s are independent, by the equivalence theorem / see [3], p.263 / we have

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N-1} u_n^2 = \sigma^2 \quad \text{a.s.} \quad (16)$$

For $s = 1, 2, 3, \dots$ we can write

$$\frac{1}{N} \sum_{n=0}^{N-1} u_n u_{n+s} = \sum_{l=0}^s \frac{1}{N} \sum_{z=0}^{\lfloor \frac{N-l-1}{s+1} \rfloor} u_{zs+l+z} u_{(z+1)s+l+z} \quad (17)$$

Now consider

$$E \left(\frac{1}{N} \sum_{z=0}^M \eta_z \right)^2 = \frac{1}{N^2} \sum_{z=0}^M E u_{zs+l+z}^2 E u_{(z+1)s+l+z}^2 \leq \frac{Nd^2}{N^2} = \frac{d^2}{N} \rightarrow 0$$

where

$$M = \left\lfloor \frac{N-l-1}{s+1} \right\rfloor, \quad \eta_z = u_{zs+l+z} u_{(z+1)s+l+z}$$

Here we have used the independence of the variables u_n -s. Using Tchebychef's inequality we get

$$P \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{z=0}^M \eta_z = 0$$

We can see the variables η_z -s are independent, using the equivalence theorem again we get

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{z=0}^M \eta_z = 0 \quad \text{a.s.}$$

From which and (17) we have

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N-1} u_n u_{n+s} = 0 \quad \text{a.s.} \quad (18)$$

From (17) and (18) we have

$$\begin{aligned} \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N-1} x_{n+s} x_n &= \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N-1} \sum_{l=0}^{\infty} \sum_{r=0}^{\infty} b_l b_r u_{n+s-l} u_{n-r} = \\ &= \sum_{l=0}^{\infty} \sum_{r=0}^{\infty} b_l b_r \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N-1} u_{n+s-l} u_{n-r} = \sigma^2 \sum_{r=0}^{\infty} b_{r+s} b_r = E y_{n+s} y_n = \psi_s \quad \text{a.s.} \end{aligned}$$

Thus the proof is complete.

Remark: If we know $\psi_s, s=0,1,2,\dots,p$ then a_1, a_2, \dots, a_p are determined by Yule-Walker-type equations

$$\begin{pmatrix} \psi_0 & \psi_1 & \dots & \psi_{p-1} \\ \psi_1 & \psi_0 & \dots & \psi_{p-2} \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{p-1} & \psi_{p-2} & \dots & \psi_0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{pmatrix} = - \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_p \end{pmatrix}$$

Therefore the u_n -s are determined by

$$u_n = x_n + a_1 x_{n-1} + \dots + a_p x_{n-p} \quad (19)$$

In practice we estimate ψ_s by

$$r_s = \frac{1}{N} \sum_{n=0}^{N-1} x_{n+s} x_n$$

and then a_1, a_2, \dots, a_p are estimated by

$$R \hat{a} = -r \quad (20)$$

where

$$\hat{a} = (\hat{a}_1, \hat{a}_2, \dots, \hat{a}_p)'$$

$$r = (r_1, r_2, \dots, r_p)'$$

$$R = \begin{pmatrix} r_0 & r_1 & \dots & r_{p-1} \\ r_1 & r_0 & \dots & r_{p-2} \\ \vdots & \vdots & \ddots & \vdots \\ r_{p-1} & r_{p-2} & \dots & r_0 \end{pmatrix}$$

Therefore the estimate \hat{u}_n of u_n is obtained by

$$\hat{u}_n = x_n + \hat{a}_1 x_{n-1} + \dots + \hat{a}_p x_{n-p}$$

and we have

$$\lim_{N \rightarrow \infty} \hat{u}_n = u_n \quad \text{a.s.}$$

§.5. ON THE LIMITING DISTRIBUTION OF THE ESTIMATES

It is well known, that if x_n is a stationary autoregressive process then $a_i, i=1, \dots, p$ has a limiting normal distribution. However this is not always true for a long-run stationary process. For the sake of simplicity here we illustrate this by the following example:

Example 2: Let x_n satisfy

$$x_n + a x_{n-1} = u_n \quad |a| < 1, \quad n = 0, 1, 2, \dots$$

where the u_n -s are independent with $E u_n = 0$ and

$$E u_n^2 = \begin{cases} \sigma^2 & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

Then by (20) a is estimated by

$$\hat{a} = \left(\frac{1}{N} \sum_{n=0}^{N-1} x_n^2 \right)^{-1} \left(-\frac{1}{N} \sum_{n=0}^{N-1} x_n x_{n-1} \right)$$

Suppose $a = 0$, then $\hat{a} \equiv 0$, i.e. the distribution of \hat{a} is degenerated.

Thus, although the estimation problem is the same for both processes, but we must be careful when we want to test hypotheses for a long-run stationary process.

§.6. SOME NUMERICAL EXAMPLES

Example 3: Let x_n be a long-run stationary process defined by

$$x_n + ax_{n-1} = u_n \quad a = 0.5, n = 0, 1, 2, \dots, 499 \quad (21)$$

$$u_n = \begin{cases} 4W_n & n = 10k+l, l=0,1,2,3 \\ W_n & n = 10k+l, l=4,5,6,7 \\ 3W_n & \text{if } U_n \leq 0.5 \\ 0 & \text{if } U_n > 0.5 \end{cases} \quad (22)$$

$k = 0, 1, 2, \dots, 49$

where the W_n -s are independently, normally distributed random variables with mean 0 and variance 1, the U_n -s are independent uniformly distributed r.v.-s on $[0,1]$ and independent of W_n . Thus

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N-1} E u_n^2 = 8.6$$

From randomly generated u_n -s we can compute the x_n -s / see Table 1, here only the first 60 observations are printed /

Table 1: Long-run stationary process defined by (21) and (22)

n	x_n	n	x_n	n	x_n	n	x_n
0	2.9643	15	-1.2439	30	4.9835	45	1.1672
1	-1.6459	16	0.4143	31	0.3325	46	-0.5887
2	-1.2496	17	1.9409	32	0.2347	47	-0.9002
3	-6.9688	18	-0.9705	33	-0.0831	48	0.4501
4	3.6511	19	0.4852	34	0.3186	49	-0.2251
5	-2.1529	20	-1.2999	35	0.1936	50	-0.5336
6	1.0097	21	1.6766	36	-1.2565	51	2.0615
7	2.0500	22	-1.6783	37	1.1738	52	-1.8660
8	-1.0250	23	-2.9980	38	-0.5869	53	-10.3786
9	1.0432	24	1.4378	39	2.3421	54	3.6968
10	-4.2701	25	-1.4391	40	-0.5952	55	-3.5001
11	-0.1656	26	1.7844	41	-0.0013	56	2.3429
12	3.4760	27	-1.6509	42	-4.9674	57	-1.7414
13	-5.7481	28	0.8254	43	7.0509	58	2.7707
14	3.8189	29	-1.7480	44	-3.6282	59	-4.9872

By (20) we get the estimate $\hat{a} = 0.4680$.

Hence the estimates \hat{u}_n -s are also obtained / In table 2 only the first 30 values are printed /

Table 2: Long-run white noise defined by (22) and its estimate

	long-run white noise	estimated long-run white noise	n	long-run white noise	estimated long-run white noise
0	2.9643	2.9643	15	0.6655	0.5432
1	-0.1637	-0.2587	16	-0.2077	-0.1679
2	-2.0725	-2.0198	17	2.1481	2.1348
3	-7.5936	-7.5536	18	0.0	-0.0622
4	0.1667	0.3900	19	0.0	0.0311
5	-0.3274	-0.4444	20	-1.0573	-1.0729
6	-0.0668	0.0022	21	1.0267	1.0683
7	2.5548	2.5224	22	-0.8399	-0.8937
8	0.0	-0.0657	23	-3.8372	-3.7834
9	0.5307	0.5636	24	-0.0612	0.0348
10	-3.7485	-3.7819	25	-0.7202	-0.7662
11	-2.3006	-2.1638	26	1.0648	1.1109
12	3.3933	3.3986	27	-0.7587	-0.8159
13	-4.0101	-4.1215	28	0.0	0.0529
14	0.9449	1.1291	29	-1.3358	-1.3623

From table 1 and table 2 we can see that the condition $u_8 = 0$ / in practice it means that $\epsilon_8 = 0$, i.e. there is no interface at $n = 8$ / can not be recognized on the reflection response x_n , because $x_8 = -1.025$. However after the predictive deconvolution we get $\hat{u}_8 = -0.0657$ which is approximately 0.

Example 4: Let x_n be a long-run stationary process defined by

$$x_n + a_1 x_{n-1} + a_2 x_{n-2} = u_n \quad n = 0, 1, 2, \dots, 499 \quad (23)$$

where $a_1 = -0.7$, $a_2 = 0.49$ and

$$u_n = \begin{cases} 4v_n & n = 6k+l, l=0,1 \\ v_n & n = 6k+l, l=2,3 \\ 0 & n = 6k+l, l=4,5 \end{cases} \quad k = 0,1,\dots,83 \quad (24)$$

The v_n -s are independently , uniformly distributed random variables on $[-0.5,0.5]$. Thus

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N-1} E u_n^2 = 0.47$$

By (20) we get the estimates $\hat{a}_1 = -0.6268$, $\hat{a}_2 = 0.4475$. In table 3 the first 30 values of u_n and its estimate are printed.

Table 3: Long-run white noise defined by (24) and its estimate

	long-run	estimated		long-run	estimated
n	white noise	white noise	n	white noise	white noise
0	-1.0830	-1.0830	15	-0.2498	-0.2138
1	1.0675	0.9882	16	0.0	0.0467
2	0.1832	0.2518	17	0.0	0.0257
3	0.0092	0.0642	18	0.0507	0.0458
4	0.0	-0.0023	19	1.7568	1.7445
5	0.0	-0.0289	20	0.1240	0.2443
6	1.3899	1.3707	21	-0.0776	-0.0529
7	-0.5841	-0.4816	22	0.0	-0.0526
8	-0.1011	-0.1218	23	0.0	-0.0456
9	-0.0429	-0.0902	24	-0.1231	-0.1293
10	0.0	-0.0218	25	-1.7805	-1.7715
11	0.0	0.0097	26	0.0198	-0.0960
12	-1.3358	-1.3183	27	-0.1032	-0.1116
13	-1.0959	-1.1862	28	0.0	0.0425
14	0.1625	0.0672	29	0.0	0.0382

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Об обратном фильтре асимптотически стационарных временных рядов

Фан Данг Кау

Резюме

Статистическая модель Робинсона была эффективно использована в области сейсмического исследования ресурсов нефти и газа. По нашему мнению эта модель в некотором смысле не ясна.

В этой работе обобщая условия оригинальной модели дается новый подход к проблеме. Мы показываем, что ход численного счета, который требуется более общим условиям совпадает с оригинальными формулами Робинсона. В конце работы результаты иллюстрируются численными экспериментами.

ASZIMPTOTIKUS STATICONÁRIUS IDŐSOROK PREDIKTIV DEKONVOLUCIÓJÁRÓL

Phan Dang Cau

Összefoglaló

Robinson statisztikai modelljét hatásosan használják a gáz és olaj-kutatásokban. Azonban, véleményünk szerint ez a modell néhány szempontból nem világos. A jelen dolgozatban a Robinson modell egy új értelmezését adjuk. Bebizonyítjuk, hogy az új feltételek mellett a megfigyeléseken alapuló számítási folyamat ugyanugy történik, mint a Robinson modell esetén. Illusztrációként adunk néhány numerikus példát.