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THE RELATION BETWEEN ANTIKEYS AND M-MINIMAL COVERS IN THE RELATION SCHEMES

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A b s t r a c t

In this paper, we introduce the notion of so-called M-minimal covers for relation scheme and prove some of its properties. Basing upon these properties, a necessary and sufficient condition under which a subset X of Ω is an antikey for a relation scheme is established when the set of all keys for the relation scheme was known.

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§1. Definitions:

In this section we present some necessary definitions.

Let $S = \langle \Omega , F \rangle$ be a relation shheme and

$\mathcal{K}_s = \{ K_1, K_2, K_3, \ldots, K_m \}$ be the set of all keys for S

Let us denote:

$$H = \bigcup_{i=1}^{m} K_i = \{ a_1, a_2, \ldots, a_p \} \subseteq \Omega$$

$M = \{ 1,2,3,\ldots,m \}$ is set of all indexes for keys.

Recall that $K \subseteq \Omega$ is a key for S if:

a) $K^+ = \Omega$

b) $\nexists K' \subset K$ such that $(K')^+ = \Omega$.

The subset $K^{-1} \subset \Omega$ is called an antikey for S if:

a) $K \nsubseteq K^{-1} \quad \forall K \in \mathcal{K}_s$

b) $\forall X : (X \subseteq \Omega \,\&\, K^{-1} \subset X) \Rightarrow \exists K \in \mathcal{K}_s :$ $K \subseteq X$.

Let $\mathcal{K}_s^{-1}$ be the set of all antikeys for S .

1.1 We construct the set I_j as follows:

$$\forall a_j \in H : \; I_j = \{ i \mid a_j \in K_i \, , \; i \leq m \}, \; j \leq p \, .$$

It is obvious that:

a) $I_j \subseteq M$ and $I_j \neq \emptyset$, $\forall j \leq p$

b) $M = \bigcup_{j=1}^{p} I_j = \{ 1,2,\ldots,m \}$.

Thus I_j is the set of all indexes for keys containing a_j . For any given $a_j \in H$, the set I_j is completely determined by a_j .

Let $\mathcal{I}_M = \{ I_1, I_2, \ldots, I_p \}$. Let $\mathcal{N} \subseteq \mathcal{I}_M$.

The set $\mathcal{N}$ is said to be a M-minimal cover if $\mathcal{N}$

satisfies the following conditions:

a) $\quad M = \bigcup\limits_{I_j \in \mathcal{N}} I_j$

b) $\quad \overline{\exists}\, \mathcal{N}' \subset \mathcal{N} \quad : \quad \bigcup\limits_{I_j \in \mathcal{N}'} I_j = M$.

That means, if $\mathcal{N} \subseteq \mathcal{J}_M$ is a M-minimal cover then for all $\mathcal{N}' \subset \mathcal{N}$, we have $\bigcup\limits_{I_j \in \mathcal{N}'} I_j \subset M$.

If $\mathcal{N} \subseteq \mathcal{J}_M$ only satisfies condition (a) , we say that $\mathcal{N}$ is a M-cover.

It is easy to see that $\mathcal{J}_M$ is a M-cover and contains at least one M-minimal cover.

<u>1.2</u> We can define the notion of M-minimal cover in an another way:

Given the set $\mathcal{K}_S$ for a relation scheme $S = \langle \Omega , F \rangle$, we can determine a matrix $\mathcal{M}(\mathcal{K}_S) = (\alpha_{ij})$ having p rows and m columns as follows:

$$\alpha_{ij} = \begin{cases} 1 & \text{if } a_i \in K_j \\ o & \text{otherwise .} \end{cases}$$

We call r_i , the i-th row of matrix $\mathcal{M}(\mathcal{K}_S)$ for every $i \leqslant p$, and then

$$\mathcal{M}(\mathcal{K}_S) = \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_p \end{pmatrix}$$

Let us define:

$$r_i \leqslant r_j \iff \alpha_{il} \leqslant \alpha_{jl} \quad \text{for all } l \leq m.$$

It is obvious that

$$r_i \leqslant r_j \iff I_i \subseteq I_j \text{ .}$$

We say that

$$\mathcal{M}^{(k)} = \begin{pmatrix} r_{i_1} \\ r_{i_2} \\ \vdots \\ r_{i_k} \end{pmatrix}$$

is a submatrix of $\mathcal{M}(\mathcal{K}_s)$ and the meaning of the following notations are obvious:

$$\mathcal{M}^{(k)} \subseteq \mathcal{M}(\mathcal{K}_s) \ , \ r_{i_j} \in \mathcal{M}^{(k)} \quad \forall i_j \leq i_k \ .$$

The row vector $C\left[\mathcal{M}^{(k)}\right] = (l_1, l_2, \ldots, l_m)$ is called the characteristic vector of the submatrix $\mathcal{M}^{(k)} \subseteq \mathcal{M}(\mathcal{K}_s)$

if $l_j \in \{o, 1\}$ and $l_j = o \iff \sum_{i=i_1}^{i_k} \alpha_{ij} = o$, $1 \leq j \leq m$.

If we remove any row r_j from the matrix $\mathcal{M}^{(k)}$, then the remaining part is denoted by $\mathcal{M}^{(k)} - \{r_j\}$.

The submatrix $\mathcal{M}^{(k)} \subseteq \mathcal{M}(\mathcal{K}_s)$ is called a M-minimal cover if $\mathcal{M}^{(k)}$ satisfies the following conditions:

a) $C\left[\mathcal{M}^{(k)}\right] = (1, 1, \ldots, 1)$

b) $\bar{\exists}\, \mathcal{M}^{(k')} \subseteq \mathcal{M}^{(k)} : \ C\left[\mathcal{M}^{(k')}\right] = (1, 1, \ldots, 1)$.

If $\mathcal{M}^{(k)}$ only satisfies condition (a) then it is called a M-cover.

Let be given a relation scheme $S = \langle \Omega, F \rangle$ and the set of all its keys $\mathcal{K}_s$. Then the matrix $\mathcal{M}(\mathcal{K}_s)$ is completely determined , $\mathcal{M}(\mathcal{K}_s)$ is a M-cover , $r_i \neq o$ and $r_i \leq C\left[\mathcal{M}(\mathcal{K}_s)\right] = (1, 1, \ldots, 1)$, $\forall i \leq m$, $\mathcal{M}(\mathcal{K}_s)$ contains at least one M-minimal cover submatrix.

In the following , we will show that $S = \langle \Omega, F \rangle$ has at least one antikey , assuming that $(\emptyset \to \Omega) \notin F$.

1.3 Example:

Let $S = \langle \Omega, F \rangle$ be a relation scheme and $\mathcal{K}_s = \{K_1, K_2, K_3, K_4\}$,

where $K_1 = \{a_1, a_2\}$, $K_2 = \{a_2, a_3, a_4\}$,

$K_3 = \{a_2, a_4, a_5\}$, $K_4 = \{a_4, a_6\}$.

a) By definition 1.1 :

$$M = \{1,2,3,4\}$$

$$H = \bigcup_{i=1}^{4} K_i = \{a_1, a_2, a_3, a_4, a_5, a_6\}$$

$I_1 = \{1\}$ $\quad$ $I_2 = \{1,2,3\}$ $\quad$ $I_3 = \{2\}$

$I_4 = \{2,3,4\}$ $\quad$ $I_5 = \{3\}$ $\quad$ $I_6 = \{4\}$

$\mathcal{J}_M = \{I_1, I_2, I_3, I_4, I_5, I_6\}$ is an M-cover:

$$M = \bigcup_{i=1}^{6} I_i = \{1,2,3,4\} \quad .$$

And $\mathcal{N}_1 = \{I_1, I_4\}$ $\quad$ $\mathcal{N}_2 = \{I_2, I_4\}$

$\mathcal{N}_3 = \{I_2, I_6\}$ $\quad$ $\mathcal{N}_4 = \{I_1, I_3, I_5, I_6\}$

are M-minimal covers .

b) By definition 1.2 :

$r_1 = (1,0,0,0)$ $\quad$ $r_2 = (1,1,1,0)$ $\quad$ $r_3 = (0,1,0,0)$

$r_4 = (0,1,1,1)$ $\quad$ $r_5 = (0,0,1,0)$ $\quad$ $r_6 = (0,0,0,1)$

$$\mathcal{M}(\mathcal{K}_s) = \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$c\left[\mathfrak{M}(\mathcal{K}_s)\right] = (1,1,1,1)$$

and

$$\mathfrak{M}^{(1)} = \begin{pmatrix} r_1 \\ r_4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix} \qquad c\left[\mathfrak{M}^{(1)}\right] = (1,1,1,1)$$

$$\mathfrak{M}^{(2)} = \begin{pmatrix} r_2 \\ r_4 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix} \qquad c\left[\mathfrak{M}^{(2)}\right] = (1,1,1,1)$$

$$\mathfrak{M}^{(3)} = \begin{pmatrix} r_2 \\ r_6 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad c\left[\mathfrak{M}^{(3)}\right] = (1,1,1,1)$$

$$\mathfrak{M}^{(4)} = \begin{pmatrix} r_1 \\ r_3 \\ r_5 \\ r_6 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad c\left[\mathfrak{M}^{(4)}\right] = (1,1,1,1)$$

are M-minimal covers in matrix representation.

§2. Let

$$\mathfrak{F} = \left\{ \mathfrak{M}^{(k)} \,\middle|\, \mathfrak{M}^{(k)} \text{ is a M-minimal cover, } k \le p \right\}$$ be the set of all M-minimal covers.

Theorem 2.1

Let $r_i \in \mathfrak{M}(\mathcal{K}_s)$ be any row. Then there exists a M-minimal cover $\mathfrak{M}^{(k)} \subseteq \mathfrak{M}(\mathcal{K}_s)$ such that $r_i \in \mathfrak{M}^{(k)}$.

Proof:

Let be given any row $r_i \in \mathfrak{M}(\mathcal{K}_s)$.

1) The case: $c\left[r_i\right] = (1,1,1,\ldots,1)$ then $r_i \in \{r_i\}$.

2) The case: $c\left[r_i\right] \neq (1,1,1,\ldots,1)$

i) If $\mathfrak{M}(\mathcal{K}_s) \in \mathfrak{F}$ then $r_i \in \mathfrak{M}(\mathcal{K}_s)$.

ii) If $\mathfrak{M}(\mathcal{K}_s) \notin \mathfrak{F}$. From $c\left[\mathfrak{M}(\mathcal{K}_s)\right] = (1,1,\ldots,1)$ there exists $j \neq i$ such that $c\left[\mathfrak{M}(\mathcal{K}_s) - \{r_j\}\right] = (1,1,1,\ldots,1)$. In fact, suppose the contrary, that

$\forall j \neq i : c[\mathcal{M}(\mathcal{K}_s) - \{r_j\}] \neq (1,1,\ldots,1)$.

On the other hand: $\mathcal{M}(\mathcal{K}_s) \notin \mathcal{J}$, $c[\mathcal{M}(\mathcal{K}_s)] =$

$(1,1,\ldots,1)$, showing that $c\ [\mathcal{M}(\mathcal{K}_s) - \{r_i\}] = (1,1,\ldots 1)$.

Because $\forall j \neq i$, $c[\mathcal{M}(\mathcal{K}_s) - \{r_j\}] = (\eta_1, \eta_2, \ldots, \eta_m)$

$\neq (1,1,\ldots,1)$ there exists a column q_j such that $\eta_{q_j} = o$,

showing that $\alpha_{jq_j} = 1$ and $\alpha_{tq_j} = o$ for every $t \neq j$.

Let $j_1 \neq j_2$, $j_k \neq i$, $k = 1,2$ then $q_{j_1} \neq q_{j_2}$.

Were this false, and we have $q_{j_1} = q_{j_2}$.

Consequently $\alpha_{j_1 q_{j_1}} = \alpha_{j_2 q_{j_1}} = 1$ i.e in the

q_{j_1} - th column there are two elements equal to 1.

Hence for the submatrix

$\mathcal{M}(\mathcal{K}_s) - \{r_{j_1}\}$ we have $\eta_{q_{j_1}} = 1$, a contradiction.

Thus, for all $j \in \{1,2,\ldots,i-1,i+1,\ldots,p\}$ we have different columns $q_1, q_2, \ldots, q_{i-1}, q_{i+1}, \ldots, q_p$ such that in each column there is only one element equal to 1.

It follows that the vector r_i has the q_i-th component equal to 1. Since $c\ [\mathcal{M}(\mathcal{K}_s) - \{r_i\}] = (1,1,\ldots,1)$ then in the q_i-th column there are at least two elements equal to 1. Suppose $\alpha_{iq_i} = \alpha_{i'q_i} = 1$, $i \neq i'$.

It follows that $K_{q_{i'}} \subset K_{q_i}$. We arrive to a contradiction, (by the definition of a key.).

We have proved that there exists $j \neq i$ such that

$c[\mathcal{M}(\mathcal{K}_s) - \{r_j\}] = (1,1,\ldots,1)$.

Now, let us consider the submatrix $\mathcal{M}^{(1)} = \mathcal{M}(\mathcal{K}_s) - \{r_j\}$ $\subset \mathcal{M}(\mathcal{K}_s)$

a) If $\mathcal{M}^{(1)} \in \mathcal{J}$ then $r_i \in \mathcal{M}^{(1)}$

b) If $\mathcal{M}^{(1)} \notin \mathfrak{I}$ and because $C[\mathcal{M}^{(1)}] = (1,1,\ldots,1)$ then there exists $j_1 \neq i$ such that $C[\mathcal{M}^{(1)} - \{r_{j_1}\}] = (1,1,\ldots,1)$. Since the matrix $\mathcal{M}(\mathcal{K}_s)$ has p rows and $p < +\infty$, it follows that there exists $k > o$ such that

$$\mathcal{M}(\mathcal{K}_s) \supset \mathcal{M}^{(1)} \supset \ldots \supset \mathcal{M}^{(k)} \supset \phi$$

and $\mathcal{M}^{(l)} \notin \mathfrak{I}$, $C[\mathcal{M}^{(l)}] = (1,1,\ldots,1)$ $o \leq l \leq k-1$,

$$\mathcal{M}^{(k)} \in \mathfrak{I} \implies r_i \in \mathcal{M}^{(k)}$$

The theorem 2.1 is completely proved.

From Theorem 2.1 , we have the following corollary.

<u>Corollary 2.1</u>:

Any M-cover has a M-minimal cover.

<u>Definition</u>:

Let $\mathcal{M}^{(k)} = \begin{pmatrix} r_{i_1} \\ r_{i_2} \\ \vdots \\ r_{i_k} \end{pmatrix}$ be a M-minimal cover

(or $\mathcal{N} = \{I_{i_1}, I_{i_2}, \ldots, I_{i_k}\}$).

Then the set $Q = \{a_{i_1}, a_{i_2}, \ldots, a_{i_k}\} \subseteq H$ determined by the matrix $\mathcal{M}^{(k)}$ (or $\mathcal{N}$) is called a representative set of the set of all keys.

<u>Theorem 2.2</u>:

Let $S = \langle \Omega , F \rangle$ be a relation scheme and $\mathcal{K}_s$ be the set of all its keys. Let $Q \subseteq H$, and let $\mathcal{N} \subseteq \mathfrak{I}_M$ be the set determined by Q (or the matrix $\mathcal{M}^{(k)} \subseteq \mathcal{M}(\mathcal{K}_s)$). Then the set $\mathcal{N}$ is a M-minimal cover if and only if the set Q satisfies following conditions:

a) $\forall K \in \mathcal{K}_s \implies \exists a \in Q$ such that $a \in K$

b) $\forall Q' \subset Q \implies \exists K \in \mathcal{K}_s$ such that $\forall a \in Q' \implies a \notin K$

Proof:

Suppose that $\mathcal{N} \leq \mathcal{J}_M$ is a M-minimal cover. We need prove that the set Q satisfies both conditions (a) and (b) .

a) Since $M = \bigcup_{I_i \in \mathcal{N}} I_i$ then for all $i \in M$ there exists $I_j \in \mathcal{N}$ such that $i \in I_j \iff$ for all $K \in \mathcal{K}_s$ there exists $a \in Q$ such that $a \in K$.

b) Let Q' be any proper subset of Q . The set Q' determines $\mathcal{N}' \subset \mathcal{N}$. Then there exists $j \in M$ and $j \notin \bigcup_{I_i \in \mathcal{N}'} I_i$. Equivalently, there exists $K \in \mathcal{K}_s$ such that for every $a \in Q'$: $a \notin K$.

Conversely , let Q be a set that satisfies both conditions (a) and (b) . We have to prove that the set $\mathcal{N} \leq \mathcal{J}_M$ determined by the set Q is a M-minimal cover.

i) It is clear that $M \supseteq \bigcup_{I_i \in \mathcal{N}} I_i$. We must prove that $M \subseteq \bigcup_{I_j \in \mathcal{N}} I_j$ i.e for all $i \in M$ then $i \in \bigcup_{I_j \in \mathcal{N}} I_j$.

Since for all $K \in \mathcal{K}_s$, there exists $a \in Q$ such that $a \in K \iff$ for all $i \in M$, there exists $I_j \in \mathcal{N}$ such that $i \in I_j \iff$ for all $i \in M$, there exists $I_j \in \mathcal{N}$ such that $i \in \bigcup_{I_j \in \mathcal{N}} I_j$.

ii) Let $\mathcal{N}'$ be any proper subset of $\mathcal{N}$. The set $\mathcal{N}'$ is determined by the set Q'. It is obvious that Q' is a proper subset of Q . Hence by condition (b) there exists $K \in \mathcal{K}_s$ such that for all $a \in Q'$, $a \notin K \iff$ there exists $i \in M$ such that for all $I_j \in \mathcal{N}'$, $i \notin I_j$.

This shows that $M \supset \bigcup_{I_j \in \mathcal{N}'} I_j$.

Theorem 2.2 is completely proved.

Theorem 2.3:

Let $S = \langle \Omega, F \rangle$ be a relation scheme and $\mathcal{K}_s$ be the set of all of its keys. Let Q be any subset of H. Then the set $K^{-1} = \Omega - Q$ is an antikey for S if and only if the set $\mathcal{N}$, determined by the set Q, is a M-minimal cover .

Proof:

The only if part: Let K^{-1} be any antikey for S . We show that the set $\mathcal{N}$ determined by the set $Q = \Omega - K^{-1}$ is a M-minimal cover, i.e we must prove that the set Q satisfies the following conditions:

a) For all $K \in \mathcal{K}_s$ there exists $a \in Q$ such that $a \in K$

b) For any $Q' \subset Q$, there exists $K \in \mathcal{K}_s$ such that for all $a \in Q'$ then $a \notin K$.

Now let us show the condition (a) :
Since K^{-1} is an antikey for S , for every $K \in \mathcal{K}_s$, $K \nsubseteq K^{-1} = \Omega - Q$. Then there exists $a \in Q$ such that $a \in K$.

We remain to prove the condition (b). Let Q' be any proper subset of Q . Since $Q' \subset Q$ then $\Omega - Q \subset \Omega - Q'$, i.e $X = \Omega - Q'$ is an extension for $K^{-1} = \Omega - Q$. By the definition for antikey, there exists $K \in \mathcal{K}_s$ such that $K \subseteq X = \Omega - Q'$, i.e for all $a \in Q'$, $a \notin K$.

The if part: Suppose $Q \subseteq H$ satisfies both condition (a) and (b). Let us show $K^{-1} = \Omega - Q$ is an antikey for S, i.e we must prove that

α) For all $K \in \mathcal{K}_s$, $K \nsubseteq K^{-1}$

β) For any $X \subseteq \Omega$ is an extension+/ of K^{-1} $(K^{-1} \subset X)$, there exists $K \in \mathcal{K}_s$ such that $K \subseteq X$.

Now we prove

α) Assume the contrary that there exists $K \in \mathcal{K}_s$ such that $K \subseteq K^{-1} = \Omega - Q$, i.e for all $a \in Q$ then $a \notin K$. This contradicts to the condition (a).

Thus for all $K \in \mathcal{K}_s$ then $K \nsubseteq K^{-1}$ holds.

β) Let X be a subset of Ω such that $K^{-1} \subset X$, i.e $K^{-1} = \Omega - Q \subset X$. It follows that there exists $a \in X$ and $a \in Q$. Consider $Q' = Q - \{a\} \subset Q$. By the condition (b) there exists $K \in \mathcal{K}_s$ such that for all $a' \in Q'$ then $a' \notin K$. That means

$$K \subseteq \Omega - Q' = (\Omega - Q) \cup \{a\} \subset X \cup \{a\} = X \text{, i.e}$$

$K \subseteq X$. Thus, for all extension X of K^{-1}, there exists $K \in \mathcal{K}_s$ such that $K \subseteq X$.

The proof is complete.

From Thecrem 2.3 the following corollaries are obvious.

Corollary 2.2:

Let $S = \langle \Omega, F \rangle$ be a relation scheme and $\mathcal{K}_s$ be the set of all of its keys. Then any antikey K for S has the following form:

$$K^{-1} = \Omega - \{a_{i_1}, a_{i_2}, a_{i_3}, \dots, a_{i_k}\} .$$

Where $Q = \{a_{i_1}, a_{i_2}, a_{i_3}, \dots, a_{i_k}\} \subseteq H$ is a representative set of the set $\mathcal{K}_s$.

Corollary 2.3:

Let $S = \langle \Omega, F \rangle$ be a relation scheme, $\mathcal{K}_s$ be

+/ i.e. a superset of K^{-1}

the set of all keys for S , $\mathcal{K}_s^{-1}$ be the set of all of its antikeys, and $\mathfrak{J}$ be the set of all M-minimal covers.

Then $$|\mathfrak{J}| = |\mathcal{K}_s^{-1}|.$$

Where $|\mathfrak{J}|$ is the cardinality of $\mathfrak{J}$ and $|\mathcal{K}_s^{-1}|$ is the cardinality of $\mathcal{K}_s^{-1}$.

Theorem 2.4:

Let $S = \langle \Omega, F \rangle$ be a relation scheme and $\mathcal{K}_s$ be the set of all keys for S . The set $\{Q_i\}$ of all representative sets for $\mathcal{K}_s$ will be denoted by $\mathcal{Q}$.

Then $$H = \bigcup_{K_i \in \mathcal{K}_s} K_i = \bigcup_{Q_i \in \mathcal{Q}} Q_i .$$

Proof:

It is obvious that $\bigcup_{Q_i \in \mathcal{Q}} Q_i \subseteq H$.

We have to prove that

$$H \subseteq \bigcup_{Q_i \in \mathcal{Q}} Q_i .$$

By Theorem 2.1 , for any row $r_i \in \mathcal{M}(\mathcal{K}_s)$, there exists a M-minimal cover $\mathcal{M}^{(k)} \subseteq \mathcal{M}(\mathcal{K}_s)$ such that $r_i \in \mathcal{M}^{(k)}$. This is equivalent to say that, for all $a_j \in H$ there exists a M-minimal cover $\mathcal{N} \subseteq \mathfrak{J}_M$ such that $I_j \in \mathcal{N}$. Let Q_t be the representative set which determines $\mathcal{N}$. Obviously $a_j \in Q_t$, i.e $a_j \in \bigcup_{Q_i \in \mathcal{Q}} Q_i$, showing that $H \subseteq \bigcup_{Q_i \in \mathcal{Q}} Q_i$. The proof is complete.

Definition:

Let $S = \langle \Omega, F \rangle$ be a relation scheme. Let us denote

$$G^* = \Omega - H$$

$$\Omega_1 = \Omega - G^*$$
$$F_1 = F - G^*$$

and $S_1 = \langle \Omega_1, F_1 \rangle$.

In [3] HO THUAN and LE VAN BAO proved that the set of all keys for $S = \langle \Omega, F \rangle$ is the same as the set of all keys for $S_1 = \langle \Omega_1 , F_1 \rangle$, i.e $\mathcal{K} = \mathcal{K}_s = \mathcal{K}_{s_1}$.

Thus the set of all M-minimal covers on Ω is the same as the set of all M-minimal covers on Ω_1 .

Let us investigate this problem in more detail.

We have the following theorem:

<u>Theorem 2.5</u>:

a) If K_s^{-1} is an antikey for $S = \langle \Omega, F \rangle$ then $K_{s_1}^{-1} = K_s^{-1} - G^*$ is an antikey for $S_1 = \langle \Omega_1 , F_1 \rangle$.

b) If $K_{s_1}^{-1}$ is an antikey for $S_1 = \langle \Omega_1 , F_1 \rangle$ then $K_s^{-1} = K_{s_1}^{-1} \cup G^*$ is an antikey for $S = \langle \Omega , F \rangle$.

<u>Proof</u>:

a) Let be given K_s^{-1} an antikey for $S = \langle \Omega, F \rangle$.
We must show that $K_s^{-1} - G^*$ is an antikey for $S_1 = \langle \Omega_1, F_1 \rangle$.

i) Since $K_s^{-1} \in \mathcal{K}_s^{-1}$,for all $K \in \mathcal{K}$ then $K \nsubseteq K_s^{-1}$. It follows that $K \nsubseteq K_s^{-1} - G^*$.

ii) Let X be a subset of Ω_1 such that $K_s^{-1} - G^* \subset X$. Since $\Omega_1 = \Omega - G^* = H$ and

$K \subseteq \Omega_1$, $\forall K \in \mathcal{K}_s$, we have $K \cap G^* = \emptyset$.
It is easy to see that $K_s^{-1} \subset X \cup G^*$ and $\exists K \in \mathcal{K}_s$:
$K \subseteq X \cup G^*$. Consequently , $K \subseteq X$.

Thus, for $X \subseteq \Omega_1$ which is an extension of $K_s^{-1} - G^*$, there exists $K \in \mathcal{K}$ such that $K \subseteq X$.

Combined (i) with (ii) we concluded that $K_{s_1}^{-1} = K_s^{-1} - G^*$ is an antikey for $S_1 = \langle \Omega_1, F_1 \rangle$.

b) Suppose that $K_{s_1}^{-1}$ is an antikey for $S_1 = \langle \Omega_1, F_1 \rangle$. We prove that $K_s^{-1} = K_{s_1}^{-1} \cup G^*$ is an antikey for $S = \langle \Omega, F \rangle$.

i) Since $K_{s_1}^{-1} \in \mathcal{K}_{s_1}^{-1}$ ($\mathcal{K}_{s_1}^{-1}$ is the set of all antikeys for S_1), we have $K \nsubseteq K_{s_1}^{-1}$, $\forall K \in \mathcal{K}$.

Since $K \subseteq \bigcup\limits_{K_i \in \mathcal{K}} K_i = H$, we have $K \cap G^* = \phi$.

It follows that $K \nsubseteq K_{s_1}^{-1} \cup G^*$, $\forall K \in \mathcal{K}$.

ii) Let be given any $X \subseteq \Omega$ such that $K_{s_1}^{-1} \cup G^* \subset X$. It follows that $K_{s_1}^{-1} \subset X$. Since $K_{s_1}^{-1} \in \mathcal{K}_{s_1}^{-1}$, there exists $K \in \mathcal{K}$ such that $K \subseteq X$.

Thus for any $X \subseteq \Omega$ which is an extension of K_s^{-1}, there exists $K \in \mathcal{K}$ such that $K \subseteq X$.

The Theorem is completely proved.

In this paper we do not present the algorithm to determine the set of all M-minimal covers for any relation scheme $S = \langle \Omega, F \rangle$ also as the algorithm to recognize whether a given set $X \subseteq \Omega$ is or is not a representative set of $\mathcal{K}$.

In other words, we have not proposed an algorithm to find all the antikeys of a relation scheme.

They will be presented in a subsequent paper.

We close our paper with an example.

2.1 Example:

Let $S = \langle \Omega, F \rangle$ be a relation scheme and $\mathcal{K}_s$ be the set of all keys for S.

Where $\Omega = \{a_o, a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$

$$\mathcal{K}_s = \{K_1, K_2, K_3, K_4\}$$

$$K_1 = \{a_1, a_2\} \qquad K_2 = \{a_2, a_3, a_4\}$$

$$K_3 = \{a_2, a_4, a_5\} \qquad K_4 = \{a_4, a_6\} \ .$$

then $$\Omega_1 = H = \bigcup_{i=1}^{4} K_i = \{a_1, a_2, a_3, a_4, a_5, a_6\}$$

$$G^* = \Omega - H = \{a_o, a_7\} \ .$$

The Q_i sets of $\mathcal{K}$	The antikeys for S_1	The antikeys for S
$Q_1 = \{a_1, a_4\}$	$K^{-1} = \{a_2, a_3, a_5, a_6\}$	$K^{-1} = \{a_o, a_2, a_3, a_5, a_6, a.$
$Q_2 = \{a_2, a_6\}$	$K^{-1} = \{a_1, a_3, a_4, a_5\}$	$K^{-1} = \{a_o, a_1, a_3, a_4, a_5, a.$
$Q_3 = \{a_2, a_4\}$	$K^{-1} = \{a_1, a_3, a_5, a_6\}$	$K^{-1} = \{a_o, a_1, a_3, a_5, a_6, a.$
$Q_4 = \{a_1, a_3, a_5, a_6\}$	$K^{-1} = \{a_2, a_4\}$	$K^{-1} = \{a_o, a_2, a_4, a_7\}$

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Связь между M-минимальными покрытиями и анти-ключами в реляционных схемах

Пхам Тхе Куе

Резюме

В статье определяется понятие M-минимального покрытия реляционной схемы и доказаны его основные свойства. На основе этих свойств доказаны необходимые и достаточные условия того, чтобы $X \subset \Omega$ было множество анти-ключей /множество ключей предполагается знакомым/.

KAPCSOLAT AZ M-MINIMÁLIS LEFEDÉSEK ÉS AZ ANTI-KULCSOK KÖZÖTT A RELÁCIÓS SÉMÁBAN

Pham The Que

Összefoglaló

A szerző bevezeti a relációs séma M-minimális lefedésének fogalmát és megvizsgálja néhány tulajdonságát. Vizsgálatainak eredményeképpen szükséges és elégséges feltételt ad arra, hogy az $X \subset \Omega$ az anti-kulcsok halmaza legyen (amennyiben a kulcsok halmazát ismerjük).