 Közlemények 31/1984. 33-43

LANGUAGES OF SYNTHESIZED CONCURRENT SYSTEMS

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1. Introduction

This paper deals with the analysis of behaviours of concurrent systems synthesized from particular parts. Such systems have attracted a great deal of attention. Important contributions are [1,3,4,5].

We propose in this paper to describe a set of computation sequences of a concurrent system by mean of the formal languages. This, to our mind, has a little sense in understanding concurrent systems more essentially.

In the second section, we shall defined synthesized languages and in the third one, its application will be discussed.

2. Synthesized language

The concept defined in this section closely concernes the concept of the synchronization behaviours introduced by Nivat [5].

Let L_1, L_2 be languages on alphabets Σ_1, Σ_2, respectively, $\Sigma = \Sigma_1 \cap \Sigma_2$. Homomorphisms h_1 and h_2 are defined on Σ_1 and Σ_2, take values in Σ.

$$h_1(a) = if\ a \in \Sigma \quad then\ a\ else\ e,$$
$$h_2(a) = if\ a \in \Sigma \quad then\ a\ else\ e,$$

where e is the emty word.

Suppose $L = h_1(L_1) \cap h_2(L_2)$. Each element of L is considered as a sequence of synchronization elements of L_1 and L_2.

(The reason will be seen later.)

Definition: The language

$$\bigcup_{w \in L} h_1^{-1}(w) \circledast h_2^{-1}(w)$$

is called a synthesized language (SL in short form) of L_1 and L_2.

In our definition, $\circledast$ is the symbol of the projetive product defined by E.Knuth, Gy.György and L.Ronyai [1].

To make the definition more clear, we should note that:

$$u \in h_1^{-1}(w) \circledast h_2^{-1}(w) \quad iff$$

$$u = a_1 a_2, \ldots, a_n,$$

$$w_1 = a_{i_1} a_{i_2}, \ldots, a_{i_l} \in h_1^{-1}(w), \quad i_1 < i_2 <, \ldots, < i_n,$$

$$w_2 = a_{j_1} a_{j_2}, \ldots, a_{j_s} \in h_2^{-1}(w), \quad j_1 < j_2 <, \ldots, < j_s,$$

$$\{i_1, i_2, \ldots, i_l\} \cup \{j_1, j_2, \ldots, j_s\} = \{1, 2, \ldots, n\}.$$

and w is the longest common word of w_1 and w_2. Considering L_i as sets of behaviours of processes $Q_i, i=1,2$, we can identify L with the sets of sequences of synchronization promitives of these processes.

A synthesized language of two regular languages is regular. This fact can be proped by constructing an automaton from ones recognizing the component languages.

Assume that M_1, M_2 are automatons recognizing L_1 and L_2:

$$M_1 = (Q_1, \Sigma_1, \delta_1, q_1^o, F_1)$$

$$M_2 = (Q_2, \Sigma_2, \delta_2, q_2^o, F_2) .$$

We construct automaton M as follows:

$$M = (Q, \Sigma_1 \cup \Sigma_2, \delta, q^o, F)$$

$$Q = Q_1 x Q_2, q^o = (q_1^o, q_2^o),\ F = F_1 x F_2\ ,$$

$$\delta((q_1,q_2),a) = \begin{cases} (\delta_1(q_1,a),q_2) & \text{if a } \in \Sigma_1 \setminus \Sigma, \\ (q_1,\delta_2(q_2,a)) & \text{if a } \in \Sigma_2 \setminus \Sigma, \\ (\delta_1(q_1,a),\delta_2(q_2,a)) & \text{if a } \in \Sigma. \end{cases}$$

It is easy to prove that the language recognized by M is SL of L_1 and L_2 . The proof is omitted.

By induction we also defined the SL of the n given languages $L_1, L_2, \ldots, L_n$. That is, the SL of $L_1, L_2, \ldots, L_n$ is the SL of $L_1, L_2, \ldots, L_{n-1}$ and L_n. It can be seen that this definition does not depend on the order of $L_1, L_2, \ldots, L_n$.

The concept introduced above can be used to study state-machine decomposable systems, such as a sets of behaviours of vector of processes introduced by Nivat [5]. In this paper we shall apply this concept to present the set of computation sequences of the computation system introduced by Janicki [2,3].

3. Languages generated by S-nets

S-nets (simple nets) introduced by Janicki [2] are a particular case of Petri nets.

Definition (2)

A simple net (S-net) $N=(T,P)$ consist of:

i) a set T of transitions,

ii) $P \subseteq 2^T x 2^T$ - a relation over 2^T, a set of places,

where T and P satisfy the following conditions:

$$(\forall a \in T)(\exists p,q \in P):(a \in left(p) \cap right(q));\ P=\emptyset \ \ iff \ \ T=\emptyset.$$

$$((x,y) \in X x X,\ right(x,y)=y,\ \ left(x,y)=x).$$

Denote by SNETS the set of S-nets. For $N_1,N_2 \in SNETS, N_1 \subseteq N_2$ iff $P_1 \subseteq P_2$. Relations is a partial order over SNETS.

We shall adopt the following notations:

$$N_1 \cup N_2 = sup\{N_1,N_2\},\quad N_1 \cap N_2 = inf\{N_1,N_2\},$$

$$\bigcup_{N \in S} N = sup\{N | N \in S\},\qquad \bigcap_{N \in S} N = inf\{N | N \in S\}.$$

<u>Theorem 1 [2]:</u> Algebra (SNETS,$\cup$,$\cap$) is a complete lattice with the smallest element $(\emptyset,\emptyset)$

A marked S-net $M=(N,B,E)$ consists of:

a) S-net N,
b) $B \in 2^P$ - initial marking, and
c) $E \in 2^P$ - final marking.

1-step reaching relation $R_1^N : T \to 2^P x 2^P$ is defined by:

$$(\forall M_1,M_2 \in 2^P),(M_1,M_2) \in R_1^N(a) \quad iff$$

$$M_1 - {}^\bullet a = M_2 - a^\bullet \, and \, {}^\bullet a \subseteq M_1,\ a^\bullet \subseteq M_2.$$

(where $a^\bullet = \{p \in P | a \in right(p)\}$, ${}^\bullet a = \{p \in P | a \in left(p)\}$.
A word $w=a_1 a_2,\ldots,a_n$ is called a firing sequence from marking M' to marking M'' if there exists a sequence of markings $M' = M_o, M_1,\ldots,M_n = M''$ such that:

$$(M_{i-1},M_i) \in R_1^N(a_i),\quad i=1,2,\ldots,n.$$

The language $L(M)$ of M is the set of all firing sequences from the initial marking to the final marking.

The main result can be stated as follows:

<u>Theorem 2:</u> Let $M^1=(N_1,B_1,E_1)$ and $M^2=(N_2,B_2,E_2)$ be marked S-nets. Suppose $M^o=(N_1 \cup N_2, B_2 \cup B_2, E_1 \cup E_2)$.

Then the language generated by M is the SL of $L(M^1)$ and $L(M^2)$.

<u>Proof:</u> Denote

$$L_1=L(M_1^1),\ L_2=L(M_2^2),\ L=L(M^o),$$

$$\bar{T}=T_1\cap T_2,\ T=T_1\cup T_2,\ P=P_1\cup P_2,\ B=B_1\cup B_2,\ E=E_1\cup E_2.$$

Our task is to prove that:

$$L=SL \ \text{of}\ L_1 \ \text{and}\ L_2.$$

Taking $w=a_1a_2,\ldots,a_m\in L$, we show that w belongs to SL of L_1 and L_2. By definition of $L(M)$, there exist $M_1,M_2,\ldots,M_m\in 2^P$ such that:

$$(M_{i-1},M_i)\in R_1^N(a_i),\ i=1,2,\ldots,m,\ M_o=E,M_m=E. \qquad (2)$$

Assume that $a_{i_1},a_{i_2},\ldots,a_{i_l}$ is the subsequence of w containing occurences of transitions in T, $a_{j_1},a_{j_2},\ldots,a_{j_s}$ is the subsequence of w containing occurences of transitions in T_2, and $c_1c_2,\ldots,c_q$ is the longest common subsequence of $a_{i_1}a_{i_2}\ldots a_{i_l}$ and $a_{j_1}a_{j_2}\ldots a_{j_s}$ containing occurences of transitions in $\bar{T}$. That is,

$$\{j_1,j_2,\ldots,j_s\}\cup\{i_1,i_2,\ldots,i_1\}=\{1,2,\ldots,m\},$$

$$j_1<j_2<\ldots<j_s,\quad i_1<i_2<\ldots<i_l\ .$$

We shall show that:

$$(M_{i_{h-1}}\cap P_1,\ M_{i_h}\cap P_1)\in R_1^{N_1}(a_{i_h})\ ,\quad h=1,2,\ldots,l,$$

and

$$(M_{j_{k-1}}\cap P_2,\ M_{j_k}\cap P_2)\in R_1^{N_2}(a_{i_k}),\quad k=1,2,\ldots,s.$$

This follows from the fact that N_1 and N_2 are S-nets. Since N_1 and N_2 are S-nets,

$$\forall t \notin T_1 \Rightarrow \dot{\dot{t}} \cup t^{\bullet\bullet} \subseteq P_2 \backslash P_1,$$
$$\forall t \notin T_2 \Rightarrow \dot{\dot{t}} \cup t^{\bullet\bullet} \subseteq P_1 \backslash P_2.$$

Consequently,

$$\forall h=1,2,\ldots,l, \quad if \quad i_{h-1}<i_h-1 \quad then$$
$$M_{i_{n-1}} \cap P_1 = M_{i_{n-1}+1} \cap P_1 = \ldots = M_{i_n-1} \cap P_1,$$
$$\forall k=1,2,\ldots,s, \quad if \quad j_{k-1}<j_k-1 \quad then$$
$$M_{j_{k-1}} \cap P_2 = M_{j_{k-1}+1} \cap P_2 = \ldots = M_{j_k-1} \cap P_2,$$
$$if \quad j_s<m \quad then \quad M_{j_s+1} \cap P_2 = \ldots = M_m \cap P_2,$$
$$if \quad i_l<m \quad then \quad M_{i_l+1} \cap P_1 = \ldots = M_m \cap P_1.$$

Since $M_o \cap P_1 = B_1$, $M_o \cap P_2 = B_2$, $M_m \cap P_1 = E_1$, $M_m \cap P_2 = E_2$ and (2) $a_{i_1} a_{i_2} \ldots a_{i_l} \in L_1$, $a_{j_1} a_{j_2} \ldots a_{j_s} \in L_2$. By definition of SL, w in SL of L_1, L_2.

Reversely, take $w=a_1 a_2 \ldots a_m$ in SL of L_1 and L_2. By definition of SL, we have:

$$w_1 = a_{i_1} a_{i_2} \ldots a_{i_l} \in L_1 \quad (if \quad w=e, \ i_l=0),$$
$$w_2 = a_{j_1} a_{j_2} \ldots a_{j_s} \in L \quad (if \quad w=e, \ j_s=0),$$

$w=a_{r_1} a_{r_2} \ldots a_{r_q}$ is the longest common subsequence of w_1 and w_2 (if $w=e$, $r=o$),

$$\{i_1, i_2, \ldots, i_l\} \cup \{j_1, j_2, \ldots, j_s\} = \{1, 2, \ldots, m\},$$
$$\{i_1, i_2, \ldots, i_l\} \cap \{j_1, j_2, \ldots, j_s\} = \{r_1, r_2, \ldots, r_{r_q}\},$$
$$i_1 < i_2 < \ldots < i_{i_l}, \quad j_1 < j_2 < \ldots < j_s, \quad r_1 < r_2 < \ldots < r_{r_q},$$

($i_o=j_o=r_o=0$ by convention).

Therefore, there exist $M^1_{i_o}, M^1_{i_1}, \ldots, M^1_{i_l}$ and $M^2_{j_c}, M^2_{j_1}, \ldots, M^2_{j_s}$ such that:

$$\forall h=1,2,\ldots,l,\quad M^1_{i_h} \in 2^{P_1}, (M^1_{i_{h-1}}, M^1_{i_h}) \in R_1^{N_1}(a_{i_h}),$$

$$\forall k=1,2,\ldots,s,\quad M^2_{j_k} \in 2^{P_2}, (M^2_{j_{k-1}}, M^2_{j_k}) \in R_1^{N_2}(a_{j_k}),$$

$$M^1_{i_o}=B_1,\ M^1_{i_l}=E_1,\ M^2_{j_o}=E_2,\ M^2_{j_s}=E_2.$$

We construct a sequence $M_o, M_1, \ldots, M_m$ of markings of N by:

$$M_o = B,$$
$$M_i=M^1_i \cup M^2_i \quad if \quad i\in\{r_1,r_2,\ldots,r_q\}.$$

In remain cases, if $i=i_h$ with $h\in\{1,2,\ldots,1\}$ then $M_i=M^1_{i_h} \cup M^2_{j_k}$ with k in $\{1,2,\ldots,s\}$ so that j_k is the bigest number, which is less than i, if $i=j_k$ with k in $\{1,2,\ldots,s\}$ then $M_i=M^1_{i_h} \cup M^2_{j_k}$ with h in $\{1,2,\ldots,l\}$ so that i_h is the bigest number which is less than j_k. (3)

We claim that

$$\forall i=1,2,\ldots,m,\ (M_{i-1},M_i)\in R_1^N(a_i).$$

There are the following cases:

a) $i=i_h$, $h\in\{1,2,\ldots,l\}$, $i_h\notin\{r_1,r_2,\ldots,r_q\}$. In this case, $M_i=M^1_{i_h} \cup M^2_{j_k}$ with k defined in the definition of M_i and $i-1=i_{h-1}$ (i_{h-1} can be in $\{r_1,r_2,\ldots,r_q\}$ or $i-1=j_k$ (j_k does not belong to $\{r_1,r_2,\ldots,r_q\}$). By definition, $M_{i-1} = M^1_{i_{k-1}} \cup M^2_{j_k}$. Noting in this case $\dot{a}_{i_h} \cup \dot{a}_{i_h} \in P_1$ we have

$(M_{i-1}, M_i) \in R_1^N(a_i)$.

b) $i=j_k$, $k\in\{1,2,\ldots,l\}$ and $j_k\in\{r_1,r_2,\ldots,r_q\}$. In exactly the same way with interchanging the role of N_1 and N_2, we have $(M_{i-1}, M_i) \in R_1^N(a_i)$.

c) $i\in\{r_1,r_2,\ldots,r_q\}$. In this case, there exist h and k such that $j_k=i_h=i$. So, $M_i=M^1_{i_h}=M^l_{j_k}$.

If $i-1=i_{h-1}=j_{k-1}$, then $M_{i-1}=M^1_{i_{h-1}} \cup M^2_{j_{k-1}}$.

If $i-1=i_{h-1}\neq j_{k-1}$, then j_{k-1} is the begest number in $\{j_1,\ldots,j_s\}$ which is less then i_{h-1}; if $i-1=j_{k-1}\neq i_{h-1}$ then i_{h-1} is the begest number in $\{i_1,\ldots,i_l\}$ which is less then j_{k-1}. So, in any case of $i-1$, $M_{i-1}=M^1_{i_{h-1}} \cup M^2_{j_{k-1}}$, and since that:

$$(M_{i-1}, M_i) \in R_1^N(a_i).$$

The proved property of sequence $M_o, M_1, \ldots, M_m$ shows that $w\in L(M^o)$.

This completes the proof of our theorem.

Corollary: The language generated by proper net $M=(N,B,E)$ is the sythesized language of languages generated by sequential components creating it.

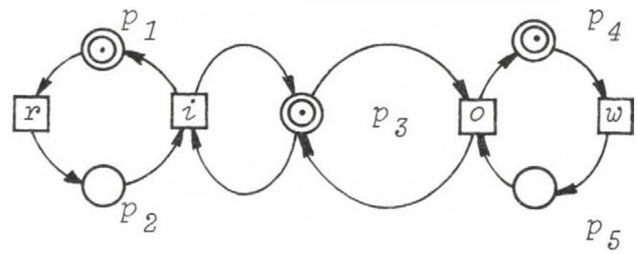
Proof: Since M is proper net, there are elementary nets $N_1, N_2, \ldots, N_k$ such that:

$$N = N_1 \cup N_2 \cup \ldots \cup N_k.$$

For every $i, M^i=(N_i, B\cap P_i, E\cap E_i)$ is a marked S-net $(i=1,\ldots,k)$. The corollary is an immediate consequence of theorem 2.

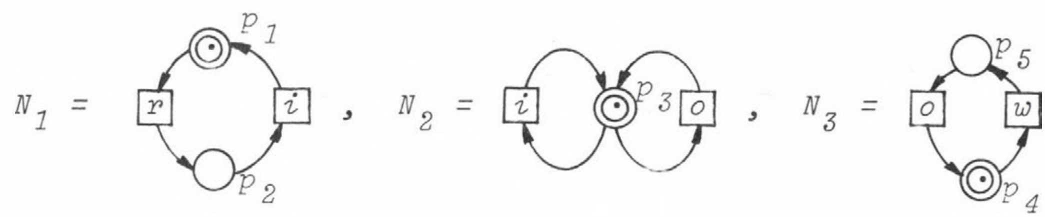
Janicki has used proper nets to model parallel computation systems. The corollary given above can be used in proving the correctness of synthesizing concurrent systems from sequential parts.

<u>Example:</u> Consider the following proper net:

$N =$ 

where: r : read,
i : input,
o : output,
w : write, ⨀ : initial place, ◎ final place.

This net is the union of three sequential components:



$L_1=(ri)^*$, $L_2=(i^*o^*)^*$, $L_3=(wo)^*$

By the corollary, we have:

$$L_N = \{[\{r^n w^m o^m i^n \mid n \geq 0, m \geq 0\}]\} ,$$

here $[\{r^n w^m o^m i^n \mid n \geq 0, m \geq 0\}]$ is the trace language on $\{r,w,i,o\}$ under the relation $I=\{(r,w),(r,o),(o,i),(i,w)\}$.

L_N is the behaviours of read-write systems.

<u>Conclusion:</u>

In this paper we have only considered representation of behaviours of systems synthesized from their components. The combination of concept given above and trace languages used in searching parallel system will be considered in the future.

The author would like ot thank prof.E.Knuth for helpful advices.

REFERENCE

[1] E.Knuth,Gy.Györy, L.Ronyai: A study of the projection operation. Apllication and theory of Petri nets, 52, Springer-Verlag, Berlin Heidelberg-New York 1982.

[2] Ryszard Janicki: An algebraic structure of Petri nets. Lecture Notes in computer science, Vol.83,1980.

[3] Ryszard Janicki: A construction of concurrency relation. Lecture Notes in computer science, IV/1981.

[4] Peter Hendersen, Yechezkel Zalestein: Sychronization problem solvable by generalized PV-systems. JACM, Janury, 1980, Vol.27, n-1

[5] Nivat,M.: Synchronization of concurrent processes. In pre. N-80-3, Janvier 1980. Paris.

[6] E.Knuth: Petri nets and regular trace languages. 25-th. April, 1978, the University of Newcastle Upon Time, computing labor

Ö S S Z E F O G L A L Á S

SZINTETIZÁLT KONKURENS RENDSZEREK NYELVEI

Dang Vah Hung

A dolgozatban a szerző bevezeti a "szinkron-nyelv" fogalmát, amely Knuth Elŏd [6] által már korábban definiált fogalomnak továbbfejlesztése. A szerző a fogalom segitségével tanulmányozza a Janicki [3] által bevezetett rendszereket. A dolgozat fő célja jobb betekintést nyerni a parallel rendszerek strukturájába.

ЯЗЫКИ СИНТЕЗИРОВАННЫХ КОНКРУРЕНТНЫХ СИСТЕМ

Данг Ван Хунг

В работе вводится понятие синхронного языка, которое является развитием базисного понятия введенного Кнутхом для изучения систем введенных Яницким. Цель работы состоит в том, чтобы лучше понять поведение параллельных систем.