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STABILITY OF PERIODIC SOLUTIONS OF ORDINARY DIFFERENTIAL EQUATIONS WITH RANDOM PARAMETERS

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The aim of this paper is to consider the stability of periodic solutions of ordinary differential equations with random parameters under stochastic perturbations of their coefficients. Generalizing some results of H. Bunke [2] and applying a theorem of A. Strauss and J.A. Yorke [5] we shall show that if the coefficients of a random system converge to the periodic stochastic processes as $t \to \infty$, then all its solutions also converge to a determined periodic solution of the limit system.

The concepts and results used in this paper can be found in [1] - [5]. The details of the proofs of the theorems formulated here can be seen in [6] and [7].

1. One has proved ([2], Theorem 3.13, p. 51) under some conditions that any solution of the perturbed system

$$\dot{y} = [A(t) + C(t)]y + z_t, \qquad (1.1)$$

where $t \in R^1$, $y \in R^n$, $A(t)$ and $C(t)$ are deterministic $n \times n$ -matrices and z_t is an n-dimensional stochastic process, converges a.e. to a determined periodic solution x_t^o of the periodic unperturbed system

$$\dot{x} = A(t)x + z_t \qquad (1.2)$$

as $t \to \infty$ if the perturbation $C(t)$ converges exponentially to zero as $t \to \infty$.

In [6] we have investigated the stability of the periodic solution x_t^o of (1.2) under action of a general nonlinear stochastic perturbation. The equivalent problem is to consider the asymptotic behaviour of solutions of system

$$\dot{y} = A(t)y + z_t + f(t,y,\omega), \tag{1.3}$$

where $f\colon R^1 \times R^n \times \Omega \to R^n$, $(\Omega,\mathfrak{U},P)$ is a probability space. In order to ensure the existence and uniqueness of solutions of (1.3) we assume that $f(t,y,\omega)$ is $\mathfrak{U}$ - measurable for all $(t,y) \in R^1 \times R^n$ and R-continuous on $R^1 \times R^n$, and there exists an R-continuous stochastic function $L(t,\omega)$ for which $\| f(t,y_1,\omega)-f(t,y_2,\omega) \| \leq L(t,\omega) \| y_1 - y_2 \|$ holds a.e. for $t \in R^1$, $y_1,y_2 \in R^n$.

Using Lyapunov's theorem on the reducibility of linear periodic systems ([3], p. 188) and a theorem of A. Strauss and J.A. Yorke ([5], Theorem 3.2), in [6] we have proved the following

Theorem 1. Assume that conditions 1,3 of Theorem 3.13 in [2] and following conditions are satisfied:

(i) Given any $\varepsilon > 0$, there exist random variables $\sigma = \sigma(\varepsilon,\omega) > 0$ and $S = S(\varepsilon,\omega) > 0$ such that

$$\| f(t,y_1,\omega) - f(t,y_2,\omega) \| \leq \varepsilon \ \|y_1 - y_2\| \qquad \text{(a.e.)}$$

provided $t \geq S$ and $\|y_1 - y_2\| < \sigma$.

(ii) $$\lim_{t\to\infty} \int_t^{t+1} \| f(s,x_s^o,\omega) \| \, ds = 0 \qquad \text{(a.e.)}.$$

Then there exist random variables $\delta = \delta(\omega) > 0$ and $T = T(\omega) \geq 0$ such that, for every $t_o \geq T$ and $y_o(\omega)$ with $\|y_o(\omega) - x^o_{t_o}(\omega)\| < \delta$ (a.e.), any solution

$y_t = y(t,\omega;\, t_o, y_o(\omega))$ of (1.3) converges a.e. to x^o_t as $t \to \infty$.

Applying Theorem 1 to linear system

$$\dot{y} = [A(t) + C_t]y + z_t + \zeta_t \,, \qquad (1.4)$$

where C_t is an R-continuous stochastic $n \times n$ - matrix and ζ_t is an R-continuous n-dimensional stochastic process, in [6] we get.

Corollary. Suppose that conditions 1,3 of Theorem 3.13 in [2] and following conditions are satisfied:

(i) $\lim_{t\to\infty} \|C_t\| = 0$ (a.e.).

(ii) There exist a number $\bar{t} \geq 0$ and a random variable $h = h(\omega)$ such that for $t \geq \bar{t}$ $\|z_t\| \leq h$ (a.e.) holds.

(iii) $\lim_{t\to\infty} \int_t^{t+1} \|\zeta_s\|\, ds = 0$ (a.e.).

Then any solution of (1.4) converges a.e. to x^o_t as $t \to \infty$.

Some sufficient conditions for the convergence in mean of solutions of (1.3) and (1.4) to x^o_t are also given in [6].

2. Generalizing a theorem of A. Ja. Dorogovcev, H. Bunke has shown ([2], Theorem 5.15, p. 120) that under some determined conditions a system of weakly nonlinear periodic random differential equations

$$\dot{x} = f(x,t,z_t) + g(x,t,z_t) \tag{2.1}$$

has a strictly periodic solution x_t^o and any solution x_t of (2.1) converges exponentially a.e. to x_t^o.

Using generalized Gronwall's lemma ([2]) and an inequality of A. Strauss and J.A. Yorke ([5], Lemma 3.5) we have proved in [7] the following

Theorem 2. Suppose that conditions 1,2,3 and 4 of Theorem 5.15 in [2] and following condition are satisfied: $h \in C[R^n \times R^1 \times R^m \to R^n]$ and there is a function $\Psi : R^1 \times R^m \to R^1$ such that $\int_t^{t+1} \Psi(s,z_s)ds$ is R-continuous, $\| h(x,t,z_t) \| \leq \Psi(t,z_t)$ (a.e.) for all $(x,t) \in R^n \times R^1$ and

$$\lim_{t\to\infty} e^{\delta t} \int_t^{t+1} \Psi(s,z_s)ds = 0 \qquad \text{(a.e.)} \tag{2.2}$$

with some $\delta > 0$ hold. Then any solution x_t of

$$\dot{x} = f(x,t,z_t) + g(x,t,z_t) + h(x,t,z_t) \tag{2.3}$$

converges exponentially a.e. to the solution x_t^o of (2.1).

If instead of (2.2) we only suppose that

$\lim_{t\to\infty} \int_t^{t+1} \Psi(s,z_s)ds = 0$ (a.e.), then any solution x_t of

(2.3) converges a.e. to x_t^o, but in general not exponentially.

The results of this paper were applied to the vibration equation with random parameters ([8]).

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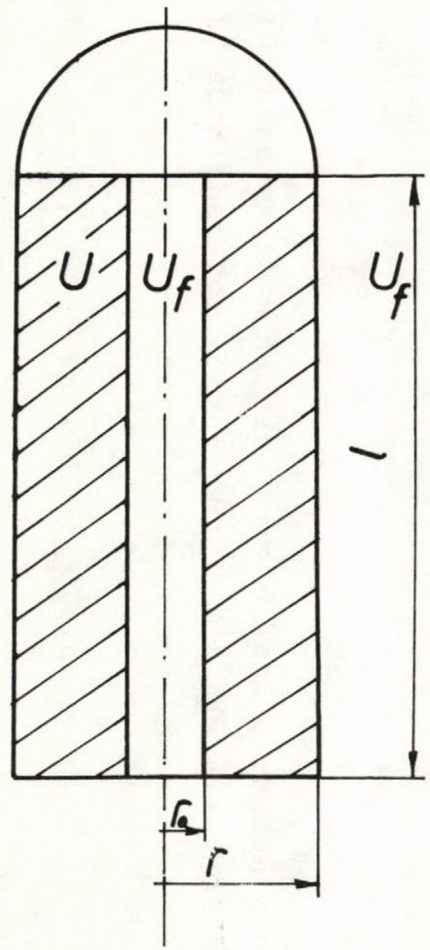
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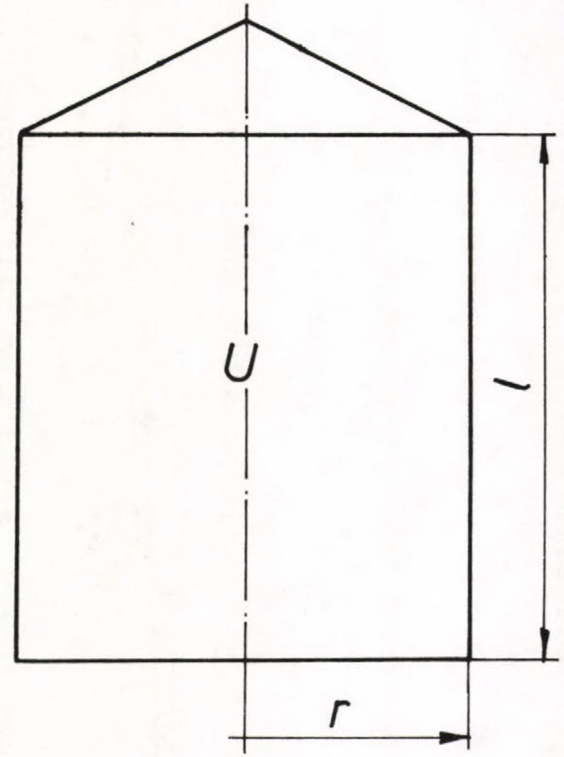
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