MTA Számitástechnikai és Automatizálási Kutató Intézete, Közlemények 18/1977.

# ON THE SPEED OF COMPUTERS WITH PAGED AND INTERLEAVED MEMORY

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**Abstract:**

A performance measure (the speed) of computer mathematical models is defined. This measure is given as a function of hardware and program behaviour for Bélády's computer model with paged memory and Volihiman's model with interleaved memory.

KEY WORDS AND PHRASES: computer systems performance, demand paging, interleaved memory behaviour.

## 1. Intruduction

Computer performance is investigation by empirical, simulation and analytical methods [1].

Tha analytical methods is based on the analysis of mathematical by "exact" methods (e.g. queueing or Markov-chain theory, compinatorics etc.).

Due to the inacuuuracy of models the analytical method usuallly gives only a rough estimate, but the results are general and convenient for computer plannning or development.

In this lecture we recommend and analytical method, based on Bélady's[2], Coffman's [3] and Kogan's [4] methods and give some concrete formulas derived by this method.

## 2. Definition of the speed

The set $N = \{ \nu_1, \ldots, \nu_n \}$ ( $1 \leq n < \infty$) is called a program, and the sequence $\omega_T = r_1 \ldots r_T$ ($1 \leq T \leq \infty$, $r_t \in N$, $t = 1, \ldots, T$) consisiting of elements of $N$. ($T$-elements permutations with repetition) is called a program realization of lenght $T$. Denote $N^T$ the set of all possible sequences $\omega_T$. Denote $\tau[\omega_T]$ the processing time of a sequence $\omega_t$ on given computer model. The distribution of the elements of $N$ in the sequence $\omega_T$ is called program behaviour [5]. This behaviour is given by the set $D = \{ D_1, \ldots, D_T \}$ of distribution function $D_1, \ldots, D_T$ where $D_T[\omega_T]$ gives the probability of $\omega_T$ in the space of events $N^T$, that is

(2.1.) $\quad \forall \omega_T \quad 0 \leq D_T [\omega_T] \leq 1$

and

(2.2.) $\quad \forall_T \quad \sum_{\omega_T \in N^T} D_T[\omega_T] = 1.$

Further we supppose

(2.3) $$\sum_{i=1}^{n} D_{T+1}[\omega_T \nu_{.i}] = D_T[\omega_T].$$

Instead of $D_T[\omega_T]$ we use the marking $D[\omega_T]$. Denote the set of $D$'s satisfying the conditions (2.1), (2.2) and (2.3) by $D$.

In this lecture we use 6 simple behaviour model: homogeneous [6], cyclical [6], random [2], random with step [3], random with repetition [7] nad independent [5] ones, Let HOM, CYCL, RAN, $\text{STEP}_p$, $\text{REP}_p$ and $\text{IND}_{p1,\dots,p_n}$ denote then.

According to the homogeneous model the references are equivalent, that is

(2.4) $$P\{ r_1 = \nu_i \} = \frac{1}{n} \quad \text{and} \quad r_t = r_1 \quad (t = 2,3,\dots ; \quad i = 1,\dots,n).$$

This formula is equivalent to the following definition:

(2.5) $$\text{HOM}[\omega_k] = \begin{cases} \frac{1}{n}, & \text{if in } \omega_k \quad r_1 = r_2 = \dots = r_k \\ 0, & \text{otherwise.} \end{cases} \quad (k = 1,2,\dots).$$

According to the cyclical model the step $\nu_i, \nu_{i+1}$ $(\nu_{n+1} \equiv \nu_1)$ has a probability 1, that is

(2.6) $$P\{ r_1 = \nu_i \} = \frac{1}{n} \quad \text{and} \quad P\{ r_{t+1} = \nu_{i+1} \} = \begin{cases} 1, & \text{if} \quad r_t = \nu_i, \\ 0, & \text{if} \quad r_t \neq \nu_i \end{cases}$$

$$(t = 1,2,\dots ; \quad i = 1,\dots,n).$$

This formula is equivalent to the following definition:

(2.7) $$\text{CYCL}[\omega_k] = \begin{cases} \frac{1}{n}, & \text{if in } \omega_k \text{ from } r_t = \nu_i, \quad r_{t+1} = \nu_j \text{ follows } j \equiv i + 1 \ (mod\ n) \\ 0, & \text{otherwise.} \end{cases}$$

$$(t = 1,2,\dots).$$

According to the random model the references occur randomly, that is

(2.8) $P\{r_t = v_i\} = \frac{1}{n}$ $(t = 1,2, \ldots; \quad i = 1 ., \ldots ,n).$

This formula is equivalent to the following definition:

(2.9) $\mathrm{RAN}\,[\omega_k] = \frac{1}{n^k}$ $(k = 1,2, \ldots, \omega_k \epsilon N^k).$

According to the random model with repertition the repertition has a probability $p$, and other references have a probabiliy $\frac{1-p}{n-1}$:

(2.10) $$P\{r_1 = v_i\} = \frac{1}{n},\ P\{r_t = v_i\} = \begin{cases} p, & \text{if} \quad r_t = v_i, \\ \frac{1-p}{n-1} & \text{if} \quad r_t \neq v_i, \end{cases}$$

$(t = 2,3, \ldots; \quad i = 1, \ldots ,n)$

This formula is equivalent to the following definitiom:

(2.11) $\mathrm{REP}_p[\omega_k] = \frac{1}{n} \cdot p^f \left(\frac{1-p}{n-1}\right)^{k-f-1}$ $(k = 1,2, \ldots),$

where $f$ is the number of the repertitions in $\omega_k$.

Acccording to the random model with step [3] the step $v_i$, $v_{i+1} (v_{n+1} \equiv v_1$ in $\omega_k$ has a probability $p$, and other references have a probability $\frac{1-p}{n-1}$:

(2.12) $$P\{r_1 = v_i\} = \frac{1}{n};\ P\{r_t = i+1\} = \begin{cases} p, & \text{if} \quad r_{t-1} = v_i, \\ \frac{1-p}{n-1}, & \text{if} \quad r_{t-1} = v_i, \end{cases}$$

$(t = 2,3, \ldots; \quad i = 1, \ldots, n).$

This formula is equivalent to the following definition:

(2.13) $\mathrm{STEP}_p[\omega_1] = \frac{1}{n};$ $\quad \mathrm{STEP}[\omega_k] = \frac{1}{n} \cdot p^f \left(\frac{1-p}{n-1}\right)^{k-f-1}$ $(k = 1,2, \ldots),$

where $f$ is the number of the steps in $\omega_k$.

According to the independent model [5] the reference to the page $\nu_i$ has a probability $p_i$, that is

(2.14) $P\{r_t = \nu_i\} = p_i \quad (t = 1,2,\dots).$

This formula is equivalent to the following definition:

(2.15) $\mathrm{IND}_{p_1,\dots,p_n}[\omega_k] = \prod_{i=1}^{n} (p_i)^{f_i},$

where $f_i$ is the number of the references to the page $\nu_i$.

Computer performance is characterized by the number of operations in a time unit: V.V is called the speed of the computer model and is determined by the formula

(2.16) $$V \overset{\mathrm{def}}{=} \liminf_{k \to \infty} \frac{1}{\sum'_{\omega_k \in N^k} D[\omega_k] \frac{\tau[\omega_k]}{k}} .$$

If in (2.16) we have existance of the lim in addition to the lim inf, then this limit is denoted by $V'$.

Our aim is to determine the speed for various computer and program behaviour models.

## 3. The mathematical model of computers with paged memory

For the investigation of computers with paged memory we use the well-known model proposed by Bélády [2] in 1966.

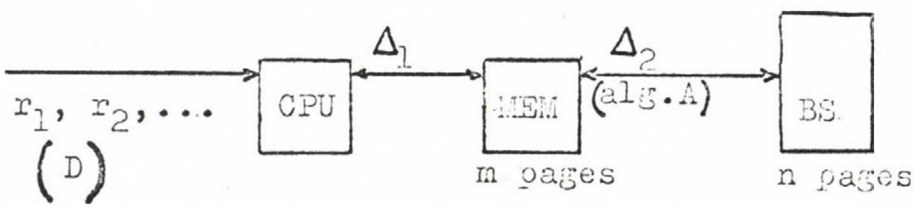


Fig. 1. The scheme of a computer with 2 level paged memory

The computer consists of a central processor unit (CPU),– an $m$-paged main memory (MEM) and an $n$-paged backing store (BS). The CPU has direct access to MEM-access time $\Delta_1$ – while an indirect access to BS-access time $\Delta_1 + \Delta_2$. The paging is controlled by a demand paging algorithm. The set of demand paging algorithms by $A$

For this model speed $V_p$ is [8]

$$(3.1)\qquad V_p = \frac{1}{\Delta_1 + \Delta_2 \cdot C},$$

where $C$ is the average cost of a reference, that is the page fault probability [5]. By definition

$$(3.2)\qquad C = C(m,n,A,D) = \limsup_{k\to\infty} \sum_{\omega_k \in N^k} D[\omega_k]\left(\frac{\sum_{i=1}^{k}\delta_i}{k}\right)$$

wherc $A \epsilon A$, $D \epsilon D$,

$$(3.3)\qquad \delta_i = \delta(i,m,n,\omega_T,A) = \begin{cases} 0, & \text{if } r_i \epsilon S_t, \\ 1, & \text{if } r_i \notin S_t, \end{cases}$$

and $S_t$ is the set of pages in MEM at time $t$. $S_t$ is the set of pages in MEM at time $t$. $S_t$ is called the memory state. If in (3.2) there exist a limit, then it is denoted by $C'$.

## 4. General assertions on the speed of computers with paged memory

**Lemma 1.** ([7]) *If* $\Delta_1 > 0$, *and* $1 \leqslant m \leqslant n < \infty$, *then*

$$(4.1)\qquad 0 = C(m,n,A,\text{HOM}) \leqslant C(m,n,A,D) \leqslant C(m,n,\text{ LRU,CYCL}) = 1,$$

*that is for the speed*

$$(4.2)\qquad \frac{1}{\Delta_1 + \Delta_2} = V_p(\Delta_1, \Delta_2, m,n,\text{LRU,CYCL}) \leqslant V_p(\Delta_1, \Delta_2, m,n,A,D) \leqslant$$

$$\leqslant V_p(\Delta_1, \Delta_2, m,n,A,\text{ HOM}) = \frac{1}{\Delta_1}$$

*holds.*

**Definition. 1.** ([9]). *The demand paging algorithms, for which*

$$(4.3)\qquad \forall T_1, \forall T_2 \sum_{i=1}^{T_1} \delta(i,m,n,\omega_{T_1},A) = \sum_{i=1}^{T_1} \delta(i,m,n,\omega_{T_1}\omega_{T_2},A)$$

*are called sequential* [6]. *The set of the sequential algorithms is denoted by* $B$.

**Lemma 2.** *If* $1 \leqslant m \leqslant n < \infty$, *then for every* $B \epsilon B$ *and for every* $D \epsilon D$

$$(4.4)\qquad C_{\inf} = \liminf \sum_{\omega_k \in N^k} D[\omega_k]\delta_k \leqslant C(m,n,B,D) \leqslant \limsup_{k\to\infty} \sum_{\omega_k \in N^k} D[\omega_k]\delta_k .$$

**Definition 2.** Let $D \epsilon D$ and $N_+^k$ $(k = 1,2, \ldots, ; N_+^k \subseteq N^k)$ be given. Denote $a_k$ the sum $\sum_{\omega_k \epsilon N_+^k} D[\omega_k]$. If

(4.5) $$\lim_{k \to \infty} a_k = 0,$$

then we shall say, that the sequence $N_+^k$ has zero limitdensity in $N^k$.

**Lemma 3.** ([7]). *If for a given* $D$ *there exist an* $m$*-tuple of pages* $\mu_1, \ldots, \mu_m$ *and* $\mu_1, \ldots, \mu_m$ *and* $\epsilon > 0$, *for whicg*

(4.6) $\forall \omega_k \quad D[\omega_k \nu_i] \geqslant \epsilon. \quad D[\omega_k]$ *holds, then the sequence* $N_+^k$ *has zero limit-density in* $N^k$, *where* $N_+^k$ *is the set of* $\omega_k \epsilon N^k$, *for which* $|S_t| = |S_t(m, \omega_k, B)| < m$.

**Definition 3.** Let $\omega_T$ be given. The sequences of lenght $(T + f)$ $(f = 0,1, \ldots)$ identical to $\omega_T$ up to the $T$-th element, are called the bundle $\Pi_f[\omega_T]$ with root $\omega_T$ and lenght $f$.

**Definition 4.** The average cost of a references $C^{\Pi}$ in a given bundle $\Pi_f[\omega_T]$ is by definition

(4.7) $$C = C^{\Pi}(m,n,A,D,\omega_T) = \lim_{k \to \infty} \sum_{\omega_k \epsilon \pi_{k-T}[\omega_T]} D[\omega_k] \delta_k,$$

where $D^{\Pi}[\omega_k]$ is the probability of sequence $\omega_k$ $(\omega_k \epsilon \Pi_{k-T}[\omega_T])$ within the bundle, that is

(4.8) $$D^{\Pi}[\omega_k] = \frac{D[\omega_k]}{\sum_{\omega_k \epsilon \pi_{k-T}[\omega_T]} D[\omega_k]}.$$

**Lemma 4.** ([7]). *Let* $N_+^k$ *denote the set of* $\omega'_k$ *not belonging to any bundle, which has a cost* $C^{\Pi}$. *If the sequence* $N_+^k$, *then* $C(m,n,B,D)$ *exists and is* $C^{\Pi}$.

**5. Theorems on the speed of computers with paged memory**

Theorem A. (*Belady*, 1966) [2]. *If* $L$ *is a nonlookahead demand paging algorithm, then*

(5.1) $$C(m,m,L,\mathrm{RAN}) = \frac{n-m}{n}.$$

Theorem B. (*Aho, Denning, Ullman* 1971/([5]). *If* $1 \leqslant m \leqslant n < \infty$.

*then*

$$\text{(5.2)} \qquad C'(m,n,\mathrm{OPT},\mathrm{IND}) = \sum_{i=m}^{n} p_i - \frac{\sum_{i=m}^{n} p_i^2}{\sum_{i=m}^{n} p_i},$$

*where* OPT *is the optimal paging algorithm, always replacing the page of* $S_t$ *with minimal* $p_i$[5].

Theorem C. (*Stoyan,* 1975/([8]). *If* $1 \leqslant m \leqslant n < \infty$, *then*

$$\text{(5.3)} \qquad C'(m,n,\mathrm{REF}_a,\mathrm{RAN}) = \frac{n-m}{n+a} \quad (a = 0,1,\ldots,m-1),$$

*where* $\mathrm{REF}_a$ *is a lookahead algorithm, which knows a references ahead, and holds required pages in the memory if possible, and chooses randomly among the others.*

Theorem 1. ([7]). *If* $1 \leqslant m \leqslant n < \infty$, *and* $0 \leqslant a \leqslant m$, *then*

$$C'(m,n\mathrm{REF}_a,\mathrm{REP}_p) =$$

$$\text{(5.4)} \qquad \frac{(n-m)(1-p)}{n-1+[\min(a,m-1)](1-p)+[\max(0,a-m+1)]\left(1-m!\,\frac{(1-p)^{m-1}}{(n-1)^{m-1}}\right)(1-p)}$$

Theorems A and B follow from theorem 1 (in cases $a = 0$, $p = \frac{1}{n}$ and $0 \leqslant a \leqslant m-1$, $p = \frac{1}{n}$).

Theorem 2. ([7]). If $1 \leqslant m \leqslant n < \infty$, then

$$\text{(5.5)} \qquad C'(m,n,PP_b,\mathrm{RAN}) = \frac{n-m}{n}\left(\frac{n-1}{n}\right)^b \qquad (b = 0,1,),$$

where $PP_b$ is a lookahead algorithm, which knows at time $t$ the next $b$ references, differing from $r_t$ and each other, and hold these pages if possible, in the memory, and chooses randomly among the others.

In case $b = 1$, $m = 2$, $n = 3$, it follows from theorem 2. the partial resolution of the problem, investigated by Bélády in 1966, namely $C'(2,3,\mathrm{MIN},\mathrm{RAN}) = \frac{2}{9}$.

Theorem 3. ([7]). If $a \geqslant 0$, then

$$C'_{(2,3,\mathrm{REF}_a,\mathrm{REP}_p)} =$$

(5.6)
$$= \frac{1-p}{2 + [\min/a, 1/]/1 - p/ + \mathrm{sign}[\max/0, a - 1/]p/1 - p^{a-1}/}$$

From this theorem it follows (in the case $a \to \infty$, when $\mathrm{REF}_a \to \mathrm{MIN}$), that

(5.7) $$C'(2,3,\mathrm{MIN},\mathrm{REP}_p) = \frac{1-p}{3}\ .$$

**6. Mathematicalmodel of computers with interleaved memory**

We investigate the following model of computers with interleaved memory due to V.E.Vulihman [10]:

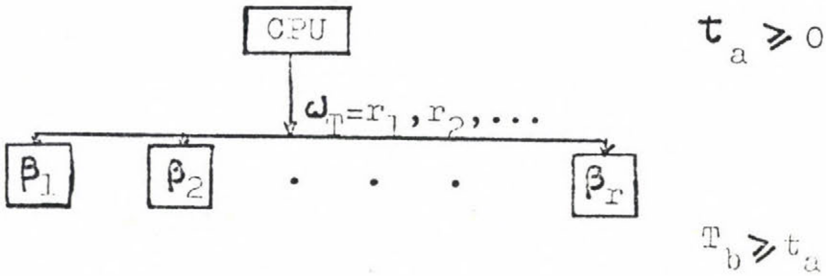


Fig. 2. Scheme of computers with interleaved memory

The computer consists of a central processor unit (CPU) and models of memory $\beta_1, \ldots, \beta_r$. The set of modules is denoted by $B$. The elements of $\omega_T$ are genoted by the CPU requiering $t_a \geqslant 0$ time per element. A request to a module reserve that moduöe for a time $T_b > t_a$ during this time other request can't be served by this module. If the module being request is occupied, then the generation of $\omega_T$ will be susepended until the modul is free.

The speed of this modul is denoted by $V_i$.

**7. General assertions on the spead of computers with interleaved memory**

Hellerman in his book [6], Bokova and Tzaturyan in their paper [11] proved the following assertions.

**Lemma 5.** ([6]). *If* $T_b > t_a \geqslant 0$, *then for every* $D \epsilon D$ *and for every* $r \geqslant 1$

(7.1) $$\frac{1}{T_b} = V_i'(t_a, T_b, r, \mathrm{HOM}) \leqslant V_i(t_a, T_b, r, D) \leqslant V_i(t_a, T_b, r, \mathrm{CYCL}) = \frac{1}{T}\min(r, \frac{T_b}{t_a}),$$

where HOM and CYCL are the homogèneous and cyclical bahaviour models.

**Definition 5.** *Let* $\rho_i(\rho_0 = 0)$ *denote the processing time of* $\omega_T$ *up to i-th element. Then the increment due to the i-th element is* $\sigma_i = \rho_i - \rho_{i-1}$.

**Example.** For every $\omega_2 \epsilon N^2$

$$(7.2) \qquad \sigma_1 = T_b + t_a, \sigma_2 = \begin{cases} T_b, & \text{if} \quad r_2 = r_1 \\ t_a, & \text{if} \quad r_2 \neq r_1 \end{cases}$$

**Lemma 6.** ([11]). *If* $T_b > t_a \geqslant 0$, *then*

$$(7.3) \qquad \operatorname*{Lim\ inf}_{k \to \infty} \frac{1}{\sum_{\omega_k \epsilon N^k} D[\omega_k]\sigma_k} \leqslant V_i(t_a, T_b, r, D)$$

$$\leqslant \limsup_{k \to \infty} \frac{1}{\sum_{\omega_k \epsilon N^k} D[\omega_k]\sigma_k}$$

**Lemma 7.** ([11]. *If* $T_b > t_a' > t_a'' \geqslant 0$, *then*

$$(7.4) \qquad V_i(t_a', T_b, r, D) \leqslant V_i(t_a'', T_b, r, D).$$

**Lemma 8.** ([11]). *If* $T_b > t_a \geqslant 0$, *and* $r' > r''$, *then*

$$(7.5) \qquad V_i(t_a, T_b, r', D) \geqslant V_i(t_a, T_b, r'', D).$$

**8. Theorems on the speed of computers with interleaved memory**

Theorem 4. ([7]). If $T_b > t_a \geqslant 0$, then for $r \geqslant 1$ $V_i'(t_a, T_b, r, \text{RAN}) \leqslant$

$$(7.6) \qquad \leqslant \frac{1}{t_a \sum_{i=1}^{r} \sum_{j=2}^{i} p_{i,j} + \left(\sum_{i=1}^{r} p_{i,1}\right)\left[\sum_{k=0}^{r-1} (\max(t_a, T_b - k \cdot t_a) \sum_{j=k+1}^{r} \frac{p_j}{j})\right]}$$

and the equality holds

a.) if $t_a = 0$, then for $r = 1$;

b.) if $\frac{T_b}{2} \leqslant t_a \leqslant T_b$, then for $r \geqslant 1$;

c.) if $0 < t_a < \frac{T_b}{2}$, then for $r = 1,2$.

In the formula (7.6)

$$p_i = \frac{r(r-1)\dots(r-i+1)}{r^{i+1}}\, i \quad (1 \leqslant i \leqslant r), \tag{7.7}$$

and

$$p_{i,j} = \frac{p_i}{\sum_{i=1}^{r} i \cdot p_i} \quad (1 \leqslant i \leqslant r, \quad 1 \leqslant j \leqslant i). \tag{7.8}$$

The following corollaries foolow from theorem 4. as special cases.

Corollary 1 (Hellerman, 1967.) ([6]). If $t_a = 0$ and $r \geqslant 1$, then

$$V_i'(0, T_b, r, \mathrm{RAN}) = \frac{\;}{T_b} \sum_{i=1}^{r} i \cdot p_i. \tag{7.9}$$

Burnett, Coffman [3] and Stone [12] proved a more general assertion.

Theorem D. (Burnett, Coffman, Stone 1974.) [3.12].

If $t_a = 0$ and $r \geqslant 1$, then

$$\begin{aligned} &V_i'(0, T_b, r, \mathrm{STEP}_p) = \\ &= \frac{2}{T_b} \sum_{k=1}^{r} \sum_{j=0}^{k-1} \binom{k-1}{j}_p j \left(\frac{1-}{n-1}\right)^{k-j-1} \cdot C_{n-j,k-j}, \end{aligned} \tag{7.10}$$

where

$$C_{n,k} = \sum_{j=0}^{k-1} \left[(-1)^j \binom{k-1}{j} (n-j-1)(n-j-2)\dots(n-k+1)\right]. \tag{7.11}$$

Corollary 2. If $\frac{T_b}{2} \leqslant t_a \leqslant T_b$ and $r \geqslant 1$, then

$$V_a'(t_a, T_b, r, \mathrm{RAN}) = \frac{1}{\frac{1}{r} T_b + (1 - \frac{1}{r}) t_a}. \tag{7.12}$$

Corollary 3. If $T_b > t_a \geq 0$, then

(7.13) $$V_b'(t_a, T_b, 2, \mathrm{RAN}) = \frac{1}{\frac{1}{3} t_a + \frac{1}{2} T_b + \frac{1}{6} \max (t_a, T_b - t_a)}.$$

On the base of the formula (7.9) it is not easy to estimate the order of $V_i'(0, T_b, r, \mathrm{RAN})$, therefore the following theorems are interesting.

Theorem E. (Hellerman, 1967)([6]). If $1 \leq r \leq 45$, then

(7.15) $$0{,}96 . r^{0,56} \leq V_i'(0, T_b, r, \mathrm{RAN}) \leq 1{,}04 \cdot r^{0,56}.$$

Theorem F. (Vulihman, 1972.)([10]). If $r \geq 1$, then

(7.16) $$V_a'(0, T_b, r, \mathrm{RAN}) \leq \sqrt{2 \Pi r}.$$

We proved the following more general theorems.

Theorem 5. ([13]). If $t_a = 0$ and $r \geq 1$, then

(7.17) $$\frac{1}{T_b} (\sqrt{\frac{\pi r}{2}} - 1) < V_i'(0, T_b, \mathrm{r}, \mathrm{RAN}) = \frac{r! \sum_{k=0}^{r-1} \frac{r^k}{k!}}{T_b \cdot r^r} \frac{1}{T_b} (\sqrt{\frac{\Pi r}{2}} + 1).$$

In our paper [7] we used a simple direct proof. Using a result due to G. Szegő [14] we can proof a formula with a smaller additive constant, which is exact.

Theorem G. (Szegő, 1928)([14]). If $q$ is a nonnegative integer number, then

(7.18) $$\frac{1}{2} e^q = 1 + \frac{q}{1!} + \frac{q^2}{2!} + \ldots + \frac{q^q}{q!} \Theta_q,$$

where $\Theta_0 = \frac{1}{2}$ and $\Theta_q$ thends monotonically to $\frac{1}{3}$ as $q \to \infty$.

Theorem 6. If $t_a = 0$ and $r \geq 1$, then

(7.19) $$V_i'(0, T_b, r, \mathrm{RAN}) = \frac{1}{T_b} (\sqrt{\frac{\Pi r}{2}} - \frac{1}{3} + \rho_r),$$

where $\rho_r$ tends monotonically to zero as $r \to \infty$ and

(7.20) $$\rho_1 = \frac{4}{3} - \sqrt{\frac{\Pi}{2}} \approx 0{,}08 \text{ and } \rho_2 = \frac{11}{6} - \sqrt{\Pi} \approx 0{,}06.$$

It seems a hard but resolvable problem to estimate the order of expression in Coffman's theorem, as a function of $p$.

## References

[1] U. Grenander, R.F. Tsao, Ouantitative methods for evaluating compuetr system performance: review and propsal (in Freiberger W., Statistical compter performance evaluation, Academic Press, New York, 1972, 3-24).

[2] L.A. Bélády, A study of replacement algorithms for a virtual storage compuetr, IBM Systems Journal, 5. No. 2. (1966), 282-288.

[3] G.J. Burnett, E,G. Jr. Coffman, A combinattorial problem related to interleaved memory systems, J. of ACM, 20, No. 1. (1973), 39-45.

[4] Коган Я.А., Марковские модели управления обменом в двухуровневой памяти ЦВМ, Автоматика и телемеханика, 28, Но. 4. /1973/ 146-154.

[5] A.V. Aho, P. J. Denning, Principles of optimal page replacement, J. of ACM, 18. No. 1. (1971), 80-93.

[6] H. Hellerman, Digital computer principles, McGraw Hill, New York, (1967).

[7] Ивани А., Исследование скорости ЭВМ со страничной и блочной организацией памяти, Кандидатская диссертация, Московский Государственный Университет, 1975.

[8] E.G. Jr. Coffman, P.J. Denning, Operating Systems Theory Englewood Cliffs, Prentice Hall, (1973).

[9] Стоян Ю.А., Оценка эффективности алгоритмов замещения, Программирование, 1, Но. 1. /1975/, 22-25.

[10] Вулихман В.Е., Исследование методов организации оперативной памяти, Кндидатская диссертация, Москва, 1972.

[11] Бокова Е.Е., Цатурян Г.К., Сопоставление известных определений скорости ЭВМ с блочной памятью, Конкурсная работа, Московский Государственный Университет, 1975, 1-14.

[12] H.S. Stone, A note on a combinatorial problem of Burnett and Coffman, Com. of ACM. 17. No. 3. (1974), 165-166.

[13] A. Iványi, I. Kátai, Lowee and upper estimates for speed of computers with blocked memory, Preprint of Eötvös L. University, Budapest, 1975. 1-14.

[14] G. Szegő, Uber einige von S. Ramanujen gestellte Aufgaben, J. of the London Mathematical Society, 3. Part 3. (1928), 225-232

Összegoglaló

Lapozott és atlapolt memóriájú számitógépek sebessége

Iványi A. – Kátai I.

Az előadásban definiáljuk a számitógépek teljesitményét jellemző mennyiséget, a sebességet. Ezt a sebességet megadjuk a hardwareő és a programviselkedési paraméterek függvényében a lapozott memóriáju számitógépek Bélády-féle és az átlapolt memóriáju számitógépek Vulihman-féle modelljére.

Kulcsszavak és kifejezések: számitógépek teljesitménye, igénye szerinti, lapozás, átlapolt memória, programviselkedés.

# Р Е З Ю М Е

## О скорости ЭВМ со страничной и блочной памятью

А. Ивани и И. Катаи

Резюме: определена мера /скорость/ производительности математических моделей ЭВМ и задана эта скорость, как функция параметров аппаратуры и поведения программ для математической модели ЭВМ со страничной памятью /предложенной Л.А. Белади/ и математической модели ЭВМ с блочной памятью /предложенной В.Е. Вулихманом/.

Ключевые слова и выражения: производительность вычислительных систем, страничная память, блочная память, поведение программ.