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Momentum-dependent superconducting order in a one-dimensional fermion system

E. Szirmai and J. Sólyom

Research Institute for Solid State Physics and Optics, H-1525 Budapest, P. O. Box 49, Hungary

E-mail: eszirmai@szfki.hu

Abstract. Recent experimental results for ultracold atomic gases as well as for high-T_c superconductors indicate that superconductivity and density-wave states may coexist. We show that the two types of order do not exclude each other in a one-dimensional fermion system if the ground state is calculated on the mean-field level. The momentum-dependent superconductivity does not need population imbalance in the inhomogeneous, charge-ordered state, the Cooper pairs condense into a state with momentum fixed by the periodicity of charge ordering.

1. Introduction

The recent experimental realizations of ultracold fermion systems provide new possibilities to analyze and understand more deeply a series of complicated phenomena like high-temperature superconductivity and the special properties of high-T_c materials in their normal phase. An important step in this direction can be made by studying the relationship between density waves and pairing, and the coexistence of these types of ordering. The phenomenon was studied, e.g., in an extended work by Micnas *et al.* [1], where the authors analyzed three-dimensional fermion lattice models with local interactions. However, fluctuations destroy true long-range order in one dimension. Indeed, the lower critical temperature of the mixed state, where zero-momentum pairs and spatial ordering were found to be present simultaneously in a narrow temperature interval, is suppressed [2]. The mean-field result can, however, be an indication of a corresponding ordering in quasi-one-dimensional systems. Recent quantum Monte Carlo calculations in one dimension [3] as well as dynamical mean-field calculations in higher dimensions [4] for ultracold fermion systems in the presence of a harmonic confinement potential also point in that direction. There is a parameter regime with dominant simultaneous instabilities to superconductivity and charge ordering. This state can be interpreted as a supersolid-like phase, and it has been found in one-dimensional fermion chains for commensurate Fermi-mixtures [5] using Luttinger-liquid description or in the strong-coupling limit for a ladder [6].

In a translation-invariant system, the Cooper pairs condense into a zero-momentum state in order to gain energy. In the presence of charge ordering that violates translational symmetry, pairs with finite momentum may have lower energy. In this paper, starting from a spatially homogeneous one-dimensional fermion system, we analyze the possibility of superconductivity occurring in the presence of a nonvanishing charge-density-wave (CDW) order parameter. Charge ordering and superconductivity are treated in a mean-field approximation and we allow for the momentum dependence of the superconducting order parameter.

2. Mean-field model

The g-ology model [7] provides us with one of the most general but mathematically well tractable model which describes a one-dimensional interacting fermion system. We consider contact interaction where all couplings between electrons with parallel spins are zero. The Hamiltonian density of the model $\mathcal{H}(x) = \mathcal{H}_0(x) + \mathcal{H}_{\rm int}(x)$ is

$$\begin{aligned}\mathcal{H}_0(x) &= \sum_\sigma \mathrm{i}v_{\rm F}(L_\sigma^\dagger \partial_x L_\sigma - R_\sigma^\dagger \partial_x R_\sigma), & (1)\\ \mathcal{H}_{\rm int}(x) &= g_1(L_\uparrow^\dagger R_\downarrow^\dagger L_\downarrow R_\uparrow + L_\downarrow^\dagger R_\uparrow^\dagger L_\uparrow R_\downarrow) + g_2(L_\uparrow^\dagger R_\downarrow^\dagger R_\downarrow L_\uparrow + L_\downarrow^\dagger R_\uparrow^\dagger R_\uparrow L_\downarrow) \\ &+ g_3(L_\uparrow^\dagger L_\downarrow^\dagger R_\downarrow R_\uparrow + R_\uparrow^\dagger R_\downarrow^\dagger L_\downarrow L_\uparrow) + g_4(L_\uparrow^\dagger L_\downarrow^\dagger L_\downarrow L_\uparrow + R_\uparrow^\dagger R_\downarrow^\dagger R_\downarrow R_\uparrow), & (2)\end{aligned}$$

where the position dependence of the fields $L(x)$ and $R(x)$ is not written out for the sake of brevity, L_σ ($L_\sigma^\dagger$) and R_σ ($R_\sigma^\dagger$) are the field operators (and their adjoints) of the left- and right-moving fermions, respectively, and $v_{\rm F}$ is the Fermi velocity.

In order to decouple the interaction terms we use a mean-field approximation taking into account different types of nonmagnetic ordering like charge-density wave and Cooper instability. Density wave is expected to appear with wave vector $\pm 2k_{\rm F}$ due to the nesting property of the Fermi surface at $\pm 2k_{\rm F}$, and the processes with large momentum transfer (g_1 and g_3) are responsible for their appearance. Whereas backward processes (g_1) and nonchiral small-momentum-transfer processes (g_2) lead to the usual zero-momentum Cooper pairs, umklapp processes (g_3) and chiral small-momentum-transfer processes (g_4) can lead to pairing with nonzero – in particular $\pm 2k_{\rm F}$ – momentum. Suppose that the inversion symmetry of the system is preserved in the various phases. The expectation values of the zero-momentum pairs is then invariant under the exchange of left- and right-moving fermions, $\langle L_\downarrow R_\uparrow \rangle = \langle R_\downarrow L_\uparrow \rangle$. For similar reasons, the expectation values of the chiral pairs with momentum $-2k_{\rm F}$ (left-moving pairs) and with momentum $2k_{\rm F}$ (right-moving pairs) are equal, $\langle L_\downarrow L_\uparrow \rangle = \langle R_\downarrow R_\uparrow \rangle$, and the averages of the particle-hole pairs with momentum $2k_{\rm F}$ and $-2k_{\rm F}$, respectively, are also equal, $\langle R_\sigma^\dagger L_\sigma \rangle = \langle L_\sigma^\dagger R_\sigma \rangle$. Therefore the phase of the charge-density-wave order parameter Δ_{DW} is $2l\pi$ with integer l, so $\Delta_{DW} = \Delta_{DW}^*$. The corresponding order parameters are defined by the expressions

$$\Delta = -\frac{1}{2}(g_1+g_2)\langle L_\uparrow^\dagger R_\downarrow^\dagger + R_\uparrow^\dagger L_\downarrow^\dagger \rangle, \qquad \Delta_c = -\frac{1}{2}(g_3+g_4)\langle L_\uparrow^\dagger L_\downarrow^\dagger + R_\uparrow^\dagger R_\downarrow^\dagger \rangle, \tag{3}$$

$$\Delta_{DW} = -\frac{1}{2}(g_1+g_3)\langle L_\sigma^\dagger R_\sigma + R_\sigma^\dagger L_\sigma \rangle. \tag{4}$$

The Hartree corrections from the g_2 and g_4 processes result in a shift of the chemical potential. This will be neglected and the energy will be measured form the Fermi energy.

The interaction term of the mean-field Hamiltonian is split into two parts: $\mathcal{H}_{MF}(x) = \mathcal{H}_0(x) + \mathcal{H}_1(x) + \mathcal{H}_2(x)$, where

$$\begin{aligned}\mathcal{H}_1(x) &= -\Delta_{DW}\sum_\sigma (L_\sigma^\dagger R_\sigma + R_\sigma^\dagger L_\sigma) - \Delta(L_\downarrow R_\uparrow + R_\downarrow L_\uparrow) - \Delta^*(R_\uparrow^\dagger L_\downarrow^\dagger + L_\uparrow^\dagger R_\downarrow^\dagger) \\ &\quad -\Delta_c(L_\downarrow L_\uparrow + R_\downarrow R_\uparrow) - \Delta_c^*(L_\uparrow^\dagger L_\downarrow^\dagger + R_\uparrow^\dagger R_\downarrow^\dagger)\,, & (5)\end{aligned}$$

and $\mathcal{H}_2$ just shifts the energy:

$$\mathcal{H}_2(x) = -\frac{\Delta_{DW}^2}{g_1+g_3} - \frac{|\Delta|^2}{g_1+g_2} - \frac{|\Delta_c|^2}{g_3+g_4}\,. \tag{6}$$

Note that the system described by the mean-field Hamiltonian $\mathcal{H}_{MF}$ is not homogeneous anymore.

3. Diagonalization procedure

At this point it is worth to switch to momentum space by introducing the Fourier transforms of the left- and right-moving fields by the definition $L_\sigma(x) = 1/\sqrt{L}\sum_k L_{k,\sigma}\mathrm{e}^{\mathrm{i}(-k_\mathrm{F}+k)x}$ and $R_\sigma(x) = 1/\sqrt{L}\sum_k R_{k,\sigma}\mathrm{e}^{\mathrm{i}(k_\mathrm{F}+k)x}$. After suitable transformations the Hamiltonian has a generalized BCS form. The diagonalization of this Hamiltonian can be achieved in the most general case by taking the linear combination of the four operators L, $L^\dagger$, R and $R^\dagger$ from the outset. In this work we are interested in the special case where first a transition to the CDW phase takes place followed by a superconducting instability of the CDW state. The Hamiltonian is then diagonalized first in the left- and right-moving fermions by the canonical transformation $\psi_{k,\uparrow} = u_k L_{k,\uparrow} + v_k R_{k,\uparrow}$, $\psi_{-k,\downarrow} = u_k^* L_{-k,\downarrow} + v_k^* R_{-k,\downarrow}$, $\chi_{k,\uparrow} = -v_k^* L_{k,\uparrow} + u_k^* R_{k,\uparrow}$ and $\chi_{-k,\downarrow} = -v_k L_{-k,\downarrow} + u_k R_{-k,\downarrow}$. The coefficients u_k and v_k can be chosen to be real, since Δ_{DW} is real due to our earlier choice, the relative phase of u_k and v_k is zero. The diagonalization conditions for u_k and v_k are very similar to those found for the usual CDW ordering with the additional relation $\Delta(v_k^2 - u_k^2) = 0$. It follows from this equation that zero-momentum pairing is incompatible with CDW ordering, $\Delta = 0$. The Hamiltonian is diagonal in the left- and right-moving fields if

$$2u_k^2 = 1 + \frac{\epsilon_k}{\sqrt{\epsilon_k^2 + \Delta_{DW}^2}}, \qquad 2v_k^2 = 1 - \frac{\epsilon_k}{\sqrt{\epsilon_k^2 + \Delta_{DW}^2}}, \qquad 2u_k v_k = \frac{\Delta_{DW}}{\sqrt{\epsilon_k^2 + \Delta_{DW}^2}}\,. \tag{7}$$

The Hamiltonian in terms of the new fermionic ψ and χ operators has the form:

$$\begin{aligned}\mathcal{H}_{MF} = \sum_k \Big[& \big[\epsilon_k(v_k^2 - u_k^2) - \Delta_{DW} 2u_k v_k\big]\psi^\dagger_{k,\uparrow}\psi_{k,\uparrow} + \big[\epsilon_k(u_k^2 - v_k^2) + \Delta_{DW} 2u_k v_k\big]\chi^\dagger_{k,\uparrow}\chi_{k,\uparrow} \\ & + \big[\epsilon_k(v_k^2 - u_k^2) - \Delta_{DW} 2u_k v_k\big]\psi^\dagger_{-k,\downarrow}\psi_{-k,\downarrow} + \big[\epsilon_k(u_k^2 - v_k^2) + \Delta_{DW} 2u_k v_k\big]\chi^\dagger_{-k,\downarrow}\chi_{-k,\downarrow} \\ & - \Delta_c\psi_{-k,\downarrow}\psi_{k,\uparrow} - \Delta_c^*\psi^\dagger_{k,\uparrow}\psi^\dagger_{-k,\downarrow} - \Delta_c\chi_{-k,\downarrow}\chi_{k,\uparrow} - \Delta_c^*\chi^\dagger_{k,\uparrow}\chi^\dagger_{-k,\downarrow}\Big] + H_2. \end{aligned} \tag{8}$$

This is a BCS Hamiltonian where the quasiparticles of the CDW state interact via a pairing potential. The result $\Delta = 0$ implies that CDW and homogeneous superconductivity cannot coexist. Due to the fact that the spectrum of the ψ and χ excitations is gapped (Δ_{DW}), the Cooper instability does not appear at arbitrary weak interactions. The binding energy of the Cooper pairs has to overcome the Peierls gap.

The BCS Hamiltonian Eq. (8) can be diagonalized by the Bogoliubov transformation: $\alpha_{k,\uparrow} = w_k\psi_{k,\uparrow} - z_k\psi^\dagger_{-k,\downarrow}$, $\alpha_{-k,\downarrow} = w_k\psi_{-k,\downarrow} + z_k\psi^\dagger_{k,\uparrow}$, $\beta_{k,\uparrow} = w'_k\chi_{k,\uparrow} - z'_k\chi^\dagger_{-k,\downarrow}$, and $\beta_{-k,\downarrow} = w'_k\chi_{-k,\downarrow} + z'_k\chi^\dagger_{k,\uparrow}$. The w_k, w'_k coefficients are chosen to be real, z_k, z'_k are complex quantities. We find

$$\begin{gathered} 2|w_k|^2 = 1 - \frac{\sqrt{\epsilon_k^2 + \Delta_{DW}^2}}{\sqrt{\epsilon_k^2 + \Delta_{DW}^2 + |\Delta_c|^2}}, \qquad 2|z_k|^2 = 1 + \frac{\sqrt{\epsilon_k^2 + \Delta_{DW}^2}}{\sqrt{\epsilon_k^2 + \Delta_{DW}^2 + |\Delta_c|^2}}, \\ 2w_k^* z_k = \frac{\Delta_c^*}{\sqrt{\epsilon_k^2 + \Delta_{DW}^2 + |\Delta_c|^2}}, \end{gathered} \tag{9}$$

and similar expressions for w'_k and z'_k. Now the Hamiltonian has the form

$$\mathcal{H} = E_{GS} + \sum_{k,\sigma}\left[E_k^\alpha \alpha^\dagger_{k,\sigma}\alpha_{k,\sigma} + E_k^\beta \beta^\dagger_{k,\sigma}\beta_{k,\sigma}\right] \tag{10}$$

with

$$E_k^{\alpha(\beta)} = \sqrt{v_\mathrm{F}^2 k^2 + \Delta_{DW}^2 + |\Delta_c|^2}, \tag{11}$$

and the ground state energy is:

$$E_{GS} = -\sum_k \sqrt{v_{\mathrm{F}}^2 k^2 + \Delta_{DW}^2 + |\Delta_c|^2} - \frac{1}{g_1 + g_3}\Delta_{DW}^2 - \frac{1}{g_3 + g_4}|\Delta_c|^2. \tag{12}$$

The above expression for the ground-state energy can be readily analyzed with respect to Δ_{DW} and Δ_c. It is found that a superconducting phase can exist in the presence of CDW ordering: E_{GS} has a minimum where both order parameters are finite for attractive interaction $g_i < 0$. The coexistence of the two types of ordering leads to the lowest ground state energy and a supersolid-like state occurs.

4. Concluding remarks

In the presented work we have analyzed the competition and coexistence of spatial inhomogeneity and superconductivity. While earlier works found a preference for zero-momentum pairing in the supersolid phase, our aim was to study the possible formation of superconducting pairs in the charge-density-wave state. That is why the diagonalization of the Hamiltonian has been carried out in two steps. A spatially inhomogeneous phase is formed in the first step and the possibility of superconductivity was studied in such a system. Thus, the unconventional Cooper pairs are bound states of the quasiparticles of the CDW phase. This can happen for sufficiently strong attraction, since the binding energy of the pairs has to be larger than the CDW gap. This requirement for the strength of the interaction can be satisfied in the regime where our weak-coupling analysis is appropriate, since Δ_{DW} is much less than the bandwidth. The evolving pairs have to follow the periodicity of the density ordering. True long range order cannot be expected in one-dimension, our results concern to the dominant instability. The method can be generalized to higher dimensions for systems with nesting Fermi surface and – based on our preliminary analysis – it leads to similar results [8]. The system can be realized with ultracold fermions in a harmonic trap strongly confined to one dimension with or without optical lattice. Note that the spatial inhomogeneity considered here is a consequence of the strong electron-electron interaction and is not due to an external periodic potential, hence the periodicity is determined by the number of particles. In this case the umklapp processes do not give contribution. If the system is on lattice, umklapp scatterings can become relevant in the half-filled case and they enhance the coexistence of charge ordering and inhomogeneous pairing.

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References

[1] Micnas R, Ranninger J, and Robaszkiewicz S 1990 *Rev. Mod. Phys.* **62** 113
[2] Levin K, Cunningham S L, and Mills D L 1974 *Phys. Rev.* B **10** 3821 and 1974 *Phys. Rev.* B **10** 3832
[3] Karim Pour F, Rigol M, Wessel S and Muramatsu A 2007 *Phys. Rev. B* **75** 161104
[4] Koga A, Higashiyama T, Inaba K, Suga S and Kawakami N 2008 *J. Phys. Soc. Jpn.* **77** 073602 and 2009 *Phys. Rev.* A **79** 013607
[5] Mathey L 2007 *Phys. Rev. B* **75** 144510
[6] Fujihara Y, Koga A and Kawakami N 2009 *Phys. Rev. A* **79** 013610
[7] Sólyom J, Adv. Phys. 1979 **28** 201
[8] Szirmai E, Sólyom J, in preparation.