

Nucleon microscopy in proton-nucleus scattering via analysis of bremsstrahlung emission

Sergei P. Maydanyuk*

*Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou, 730000, China and
Institute for Nuclear Research, National Academy of Sciences of Ukraine, Kiev, 03680, Ukraine*

Peng-Ming Zhang[†] and Li-Ping Zou[‡]

Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou, 730000, China

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We investigate an idea, how to use analysis of the bremsstrahlung photons to study the internal structure of proton under nuclear reaction with nucleus. A new model is constructed to describe bremsstrahlung emission of photons which accompanies the scattering of protons off nuclei. Our bremsstrahlung formalism uses many-nucleon basis that allows to analyze coherent and incoherent bremsstrahlung emissions. As scattered proton can be under the influence of strong forces and produces the largest bremsstrahlung contribution to full spectrum, we focus on accurate determination of its quantum evolution concerning nucleus basing on quantum mechanics. For such a motivation, we at first time generalize Pauli equation with interacting potential describing evolution of fermion inside strong field, with including the electromagnetic form-factors of nucleon basing on DIS theory. Anomalous magnetic momenta of nucleons reinforce our motivation to develop such a formalism. The full bremsstrahlung spectrum in our model (after renormalization) is dependent on form-factors of the scattered proton.

In our calculations we choose the scattering of $p + {}^{197}\text{Au}$ at proton beam energy of 190 MeV, where experimental bremsstrahlung data were obtained with high accuracy. We show that the full bremsstrahlung spectrum is sensitive to the form-factors of the scattered proton. In the limit without such form-factors, we reconstruct our previous result (where internal structure of the scattered proton was not studied).

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I. INTRODUCTION

Understanding the internal structure of nucleons has attracted researchers for a long time. The first experimental investigations of structure of protons were performed in inelastic electron-proton scattering at high energies in SLAC in 1968 (see Ref. [1], p. 96). Now high energy lepton-nucleon scattering (deep inelastic scattering, DIS) plays a key role in determining internal (partonic) structure of protons. Relevant information is summarized in reports [see Review PDG [2], also reviews [3–7]]. Investigations of nucleon-nucleon collisions at TEVATRON (Fermilab), RHIC (Brookhaven), LHC (CERN) show us new important information [2].

Moreover, we know that full information of nuclear forces (strong interactions) cannot be obtained on the basis of analysis of any type of reactions between two nucleons (clear example is fusion in nuclei, see Ref. [8], also reviews [9–24]). So, analysis of interactions between two nucleons is not enough (for example, Nijmegen data set [25]), and we have to include to analysis systems with more nucleons (i.e. nuclei). From microscopic point of view, nuclear interactions should be completely formed with internal structure of nucleons. This situation leads naturally to new investigations of interactions (reactions) between nucleons and nuclei, in order to obtain more complete information about internal structure of nucleons.

The bremsstrahlung emission of photons accompanying nuclear reactions is a traditional sector in nuclear physics, which has been causing much interest for a long time (see reviews [26, 27]). This is because of such photons provide rich independent information about the studied nuclear process. Dynamics of the nuclear process, interactions between nucleons, types of nuclear forces, structure of nuclei, quantum effects, anisotropy (deformations) can be included in the

*Electronic address: maidan@kinr.kiev.ua

[†]Electronic address: zhpm@impcas.ac.cn

[‡]Electronic address: zoulp@impcas.ac.cn

model describing the bremsstrahlung emission. At the same time, measurements of such photons and their analysis provide information about all these aspects, and verify suitability of the models. So, bremsstrahlung photons is the tool to obtain experimental information for all above questions.

Anomalous magnetic momenta for neutrons (and protons) is another strong motivation, why there is sense to include internal structure of nucleons to the bremsstrahlung formalism in nuclear reactions (even for low energies of beams). As we shown in Ref. [28], magnetic momenta of nucleons play an important role in formation of the magnetic emission in proton-nucleus scattering. According to our estimations [28], the electric and magnetic bremsstrahlung emissions have similar magnitudes in such a reaction. It was shown in Ref. [29], that incoherent emission in experimental bremsstrahlung data [30] for scattering $p + {}^{208}\text{Pb}$ at 145 MeV of the proton beam energy is not small at low energies (at 20–120 MeV of photons emitted, see Fig. 5 for details in that paper). Such a type of emission is formed due to individual interactions between the scattered proton and nucleons of nucleus. However, we did not take into account anomalous magnetic momenta of nucleons in our model, calculations and analysis of bremsstrahlung experimental information. As anomalous magnetic moment for neutron is essentially different from its Dirac magnetic moment, one can suppose that after inclusion of anomalous momenta to the model changes of results [29] can be not small (for example, this can give essentially different estimation of role of incoherent emission on background of the full bremsstrahlung spectrum). So, inclusion of internal structure of nucleons to the bremsstrahlung model for nuclear reactions for energies from low to high is motivated task.

In this paper, we focus on solution how to realize ideas described above. We construct a new bremsstrahlung model for proton nucleus scattering, where we include internal structure of the scattered proton. As starting basis for such developments, we use our previous formalism [28, 29] applied for proton nucleus scattering. We use the first approximation of generalisation of Dirac equation to describe system of nucleons with interacting potential. Of course, such an equation should describe spinor properties of fermions, and should have fully quantum description of system of nucleons, in full correspondence with quantum mechanics. As we found, this is many-nucleon generalisation of Pauli equation with interacting potential investigated in Refs. [28, 29, 31–33]. This way allows us to construct formalism with some connection with many-nucleon bremsstrahlung developments of other researchers [34–46] (here, evolution of nucleons is investigated as many body problem of quantum mechanics that allows to save quantum properties maximally completely). On the other side, our formalism is started from approximation of Dirac equation, so it allows to include next relativistic corrections in quantum way. DIS theory provides accurate description of internal structure of nucleons via form-factors, so we implement this formalism to our bremsstrahlung theory. We estimate new bremsstrahlung contributions of emitted photons, caused by such new addition to the model. The obtained bremsstrahlung probabilities already dependent on internal structure of the scattered proton. We analyze and estimate such a dependence.

The paper is organized by the following way. In Sec. II we present our new model of the bremsstrahlung photons emitted during proton nucleus scattering. In Sec. III we give the results of study for the scattering of $p + {}^{197}\text{Au}$ at proton beam energy of 190 MeV. We summarize conclusions in Sect. IV. We present main part of calculations in Appendixes (see Sect. A–E).

II. MODEL

A. Generalized equation of Pauli for spinor particle with mass m in field $V(\mathbf{r})$ with electromagnetic form factors

Let us consider Dirac equation for nucleon with mass m inside field $V(\mathbf{r})$ (see [47], p. 21 (1.2.3), p. 32):

$$i\hbar \frac{\partial \psi}{\partial t} = \left\{ c \alpha \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right) + \beta mc^2 + ze A_0 + V(\mathbf{r}) \right\} \psi. \quad (1)$$

This equation is written in coordinates of Euclidian space (in frameworks of formalism in [47], which includes potentials in Dirac equation). In particular, we have the following relations between coordinates and corresponding momenta of Euclidian and pseudo-Euclidian spaces:

$$x_{1,2,3}^{(\text{ev})} = \mathbf{r}, \quad x_4^{(\text{ev})} = it, \quad p_{1,2,3}^{(\text{ev})} = -i\hbar \frac{d}{dx_{1,2,3}}, \quad p_4^{(\text{ev})} = i p_0^{(\text{ps})}. \quad (2)$$

We change the wave function as

$$\Psi = \psi e^{i mc^2 t / \hbar}, \quad \frac{\partial \Psi}{\partial t} = e^{i mc^2 t / \hbar} \left(\frac{\partial \psi}{\partial t} + \frac{i mc^2}{\hbar} \psi \right), \quad \frac{\partial \psi}{\partial t} = e^{-i mc^2 t / \hbar} \frac{\partial \Psi}{\partial t} - \frac{i mc^2}{\hbar} \psi \quad (3)$$

and equation is transformed to the following:

$$-\beta\hbar\frac{\partial\Psi}{\partial t} = \left\{ i c \beta \alpha \left(\mathbf{p} - \frac{ze}{c}\mathbf{A} \right) + i m c^2 + i \beta [ze A_0 + V(\mathbf{r}) - m c^2] \right\} \Psi. \quad (4)$$

We rewrite this equation via matrixes γ_μ (we define them, according to Ref. [47]):

$$\gamma_4 = \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma_k = -i \beta \alpha_k = \begin{pmatrix} 0 & -i \sigma_k \\ i \sigma_k & 0 \end{pmatrix} \quad (5)$$

and obtain:

$$-\hbar\gamma_4\frac{\partial\Psi}{\partial t} = \left\{ -c\gamma_k \left(\mathbf{p} - \frac{ze}{c}\mathbf{A} \right) + i m c^2 + i \gamma_4 [ze A_0 + V(\mathbf{r}) - m c^2] \right\} \Psi. \quad (6)$$

In order to describe internal structure of the scattered proton, we introduce matrixes Γ_μ of DIS theory [we define them according to formalism in Ref. [1], see (3.6) p. 78; we apply approximation where we neglect structure of nucleons of nucleus] instead of Dirac's matrixes γ_μ :

$$\gamma_\mu \rightarrow \Gamma_\mu = A \gamma_\mu + B p'_\mu + C p_\mu + i D p'^\nu \sigma_{\mu\nu} + i E p^\nu \sigma_{\mu\nu}. \quad (7)$$

A, B, C, D, E are functions depended on the transferred momentum q^2 between the scattered proton and nucleon of nucleus. They characterize internal structure of the scattered proton. p and p' are momenta of the scattered proton before its interaction with virtual photon (emitted by nucleon of the nucleus) and after it. One of motivations to use transition (7) in the formalism is the following. At $q^2 \rightarrow 0$ we should obtain $A(q^2 = 0) = 1$, and components B, C, D, E should describe (anomalous) magnetic moment of the scattered proton.

Now equation (6) is rewritten as

$$-\hbar\Gamma_4\frac{\partial\Psi}{\partial t} = \left\{ -c\Gamma_k \left(\mathbf{p} - \frac{ze}{c}\mathbf{A} \right) + i m c^2 + i \Gamma_4 [ze A_0 + V(\mathbf{r}) - m c^2] \right\} \Psi. \quad (8)$$

Substituting explicit form (7) of matrixes Γ_μ , this equation is transformed as

$$\begin{aligned} & -\hbar (A \gamma_4 + B p'_4 + C p_4 + i D p'^\nu \sigma_{4\nu} + i E p^\nu \sigma_{4\nu}) \frac{\partial\Psi}{\partial t} = \\ & = \left\{ -c (A \gamma_k + B p'_k + C p_k + i D p'^\nu \sigma_{k\nu} + i E p^\nu \sigma_{k\nu}) \left(\mathbf{p} - \frac{ze}{c}\mathbf{A} \right) + i m c^2 + \right. \\ & \left. + i (A \gamma_4 + B p'_4 + C p_4 + i D p'^\nu \sigma_{4\nu} + i E p^\nu \sigma_{4\nu}) [ze A_0 + V(\mathbf{r}) - m c^2] \right\} \Psi \end{aligned} \quad (9)$$

Using Gordon transformation and conditions $B = C, E = -D$ from DIS theory (see [1], p. 79), equation (9) is transformed to

$$\begin{aligned} -\hbar (A \gamma_4 + i B q^\nu \sigma_{4\nu}) \frac{\partial\Psi}{\partial t} &= \left\{ -c (A \gamma_k + i B q^\nu \sigma_{k\nu}) \left(\mathbf{p} - \frac{ze}{c}\mathbf{A} \right) + i m c^2 + \right. \\ & \left. + i (A \gamma_4 + i B q^\nu \sigma_{4\nu}) [ze A_0 + V(\mathbf{r}) - m c^2] \right\} \Psi, \end{aligned} \quad (10)$$

where $q^\mu = p'^\mu - p^\mu$, $q^2 = \mathbf{q}^2 + q_4^2$. In DIS theory $A = F_1$ and $B = F_2$ are Dirac and Pauli form-factors of nucleon. According to Ref. [48], F_1 and F_2 represent electric charge and (anomalous) magnetic moment of nucleon, they are

$$F_1^p(Q^2 = -q^2 = 0) = 1, \quad F_1^n(0) = 0, \quad F_2^p(0) = 1.793, \quad F_2^n(0) = -1.913. \quad (11)$$

We rewrite bi-spinor wave function via spinor components as

$$\Psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}. \quad (12)$$

Taking explicit form of matrixes γ_μ (5) into account, we rewrite equation (10) by components as

$$\begin{aligned} -\hbar \left\{ F_1 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + i F_2 q^\nu \sigma_{4\nu} \right\} \begin{pmatrix} \frac{\partial \varphi}{\partial t} \\ \frac{\partial \chi}{\partial t} \end{pmatrix} = \left\{ -c \left[F_1 \begin{pmatrix} 0 & -i\sigma_k \\ i\sigma_k & 0 \end{pmatrix} + i F_2 q^\nu \sigma_{k\nu} \right] \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right) + i mc^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \right. \\ \left. + i \left[F_1 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + i F_2 q^\nu \sigma_{4\nu} \right] [ze A_0 + V(\mathbf{r}) - mc^2] \right\} \begin{pmatrix} \varphi \\ \chi \end{pmatrix}. \end{aligned} \quad (13)$$

According to definition of $\sigma_{\mu\nu}$ (see [47], p. 23) we have:

$$\sigma_{\mu\nu} = \frac{1}{2i}(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu) \quad (14)$$

and

$$\sigma_{44} = 0, \quad \sigma_{k4} = \sigma_k \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_{km} = \frac{\sigma_k \sigma_m - \sigma_m \sigma_k}{2i} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \varepsilon_{kmj} \sigma_j \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (15)$$

where we use property of commutator of Pauli matrixes (see Ref. [49], p. 32, $i, j, k = 1, 2, 3$):

$$\sigma_i \sigma_j = \delta_{ij} I + i \varepsilon_{ijk} \sigma_k, \quad (16)$$

and ε_{ijk} is unit antisymmetric tensor, $\varepsilon_{123} = 1$. Substituting these components to (13), we obtain:

$$\begin{aligned} i \hbar F_1 \frac{\partial \varphi}{\partial t} - \hbar F_2 q^k \sigma_k \frac{\partial \chi}{\partial t} &= \left\{ -c \varepsilon_{kmj} F_2 q^k \sigma_j \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right) + mc^2 + F_1 [ze A_0 + V(\mathbf{r}) - mc^2] \right\} \varphi + \\ &+ \left\{ c [F_1 \sigma_m - F_2 q^4 \sigma_m] \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right) - i F_2 q^k \sigma_k [ze A_0 + V(\mathbf{r}) - mc^2] \right\} \chi, \\ -i \hbar F_1 \frac{\partial \chi}{\partial t} - \hbar F_2 q^k \sigma_k \frac{\partial \varphi}{\partial t} &= \left\{ -c [F_1 \sigma_m + F_2 q^4 \sigma_m] \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right) - i F_2 q^k \sigma_k [ze A_0 + V(\mathbf{r}) - mc^2] \right\} \varphi + \\ &+ \left\{ -c \varepsilon_{kmj} F_2 q^k \sigma_j \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right) + mc^2 - F_1 [ze A_0 + V(\mathbf{r}) - mc^2] \right\} \chi. \end{aligned} \quad (17)$$

We shall solve system of equations (17). We shall find equations in dependence on $\frac{\partial \varphi}{\partial t}$ and $\frac{\partial \chi}{\partial t}$. The first equation is obtained, after summarizing the second equation with the first one (17) with multiplication on corresponding factors:

$$\begin{aligned} &i \hbar \left\{ F_1^2 - F_2^2 (q^k \sigma_k)^2 \right\} \frac{\partial \varphi}{\partial t} - \\ &- \left\{ c F_2 \left[-\varepsilon_{kmj} F_1 q^k \sigma_j - i (F_1 + F_2 q^4) q^k \sigma_k \sigma_m \right] \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right) + \right. \\ &+ F_1 mc^2 + [F_1^2 + F_2^2 (q^k \sigma_k)^2] [ze A_0 + V(\mathbf{r}) - mc^2] \left. \right\} \varphi = \\ &= \left\{ c [F_1 (F_1 - F_2 q^4) \sigma_m - i F_2^2 q^k \sigma_k \varepsilon_{nmj} q^n \sigma_j] \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right) + i mc^2 F_2 q^k \sigma_k - 2i F_1 F_2 q^k \sigma_k [ze A_0 + V(\mathbf{r}) - mc^2] \right\} \chi. \end{aligned} \quad (18)$$

The second new equation is obtained, when we remove the second equation of system (17) from the first one with multiplication on corresponding coefficients:

$$\begin{aligned} &i \hbar \left\{ F_1^2 - F_2^2 (q^k \sigma_k)^2 \right\} \frac{\partial \chi}{\partial t} = \\ &= \left\{ c \left[-i F_2^2 q^k \sigma_k \varepsilon_{nmj} q^n \sigma_j + F_1 (F_1 + F_2 q^4) \sigma_m \right] \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right) + i F_2 q^k \sigma_k mc^2 + \right. \\ &+ 2i F_1 F_2 q^k \sigma_k [ze A_0 + V(\mathbf{r}) - mc^2] \left. \right\} \varphi + \\ &+ \left\{ c \left[i F_2 q^k \sigma_k (F_1 - F_2 q^4) \sigma_m + F_1 \varepsilon_{kmj} F_2 q^k \sigma_j \right] \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right) - \right. \\ &- F_1 mc^2 + [F_1^2 + F_2^2 (q^k \sigma_k)^2] [ze A_0 + V(\mathbf{r}) - mc^2] \left. \right\} \chi. \end{aligned} \quad (19)$$

One can see that at limit of

$$F_1 = 1 \quad F_2 = 0, \quad (20)$$

the obtained equations (18) and (19) are transformed to known system of equations (1.3.3) in [47], p. 32] (for one particle with addition of the potential V):

$$\begin{aligned} i\hbar \frac{\partial \varphi}{\partial t} &= c \boldsymbol{\sigma} \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right) \chi + ze A_0 \varphi + V(\mathbf{r}) \varphi, \\ i\hbar \frac{\partial \chi}{\partial t} &= c \boldsymbol{\sigma} \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right) \varphi - 2mc^2 \chi + ze A_0 \chi + V(\mathbf{r}) \chi. \end{aligned} \quad (21)$$

Now we shall apply expansion over powers of $1/c$. Here, we follow idea given in [47] (see p. 32–33 in that book). Let us assume that χ has similar magnitude as φ/c . In obtaining new equation in the first approximation, one can omit all terms with χ (with exception of $2m_i c^2 \chi$ which includes c^2 ; but we keep terms with $\frac{ze}{c} \mathbf{A}$) in the second equation (19). We obtain:

$$\begin{aligned} &[F_1 + F_1^2 + F_2^2 (q^k \sigma_k)^2] \chi = \\ &= \left\{ \frac{1}{mc} \left[-iF_2^2 q^k \sigma_k \varepsilon_{nmj} q^n \sigma_j + F_1 (F_1 + F_2 q^4) \sigma_m \right] \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right) + i(1 - 2F_1) F_2 q^k \sigma_k \right\} \varphi. \end{aligned} \quad (22)$$

We take into account properties of Pauli matrixes:

$$(\sigma_1)^2 = (\sigma_2)^2 = (\sigma_3)^2 = 1, \quad \sigma_i \sigma_j = \delta_{ij} I + i \varepsilon_{ijk} \sigma_k, \quad (23)$$

where ε_{ijk} is unit antisymmetric tensor, $\varepsilon_{123} = 1$. We find:

$$(q^k \sigma_k)^2 = I \mathbf{q}^2, \quad (24)$$

where I is unit matrix. From (22) we obtain:

$$f(|\mathbf{q}|) \chi = \left\{ \frac{1}{mc} \left[-iF_2^2 q^k \sigma_k \varepsilon_{nmj} q^n \sigma_j + F_1 (F_1 + F_2 q^4) \sigma_m \right] \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right) + i(1 - 2F_1) F_2 q^k \sigma_k \right\} \varphi, \quad (25)$$

where we introduced a new notation:

$$f(|\mathbf{q}|) = F_1 + F_1^2 + F_2^2 (q^k \sigma_k)^2 = F_1 + F_1^2 + F_2^2 \mathbf{q}^2. \quad (26)$$

As next step, we have to substitute this equation to equation (18). Taking (26) into account, we obtain:

$$\begin{aligned} &i\hbar \left\{ F_1^2 - F_2^2 \mathbf{q}^2 \right\} f(|\mathbf{q}|) \frac{\partial \varphi}{\partial t} - \left\{ c F_2 \left[-\varepsilon_{kmj} F_1 q^k \sigma_j - i(F_1 + F_2 q^4) q^k \sigma_k \sigma_m \right] \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right) + \right. \\ &+ F_1 mc^2 + (F_1^2 + F_2^2 \mathbf{q}^2) (ze A_0 + V(\mathbf{r}) - mc^2) \left. \right\} \cdot f(|\mathbf{q}|) \varphi = \\ &= \left\{ c \left[F_1 (F_1 - F_2 q^4) \sigma_m - iF_2^2 q^k \sigma_k \varepsilon_{nmj} q^n \sigma_j \right] \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right) + i mc^2 F_2 q^k \sigma_k - 2i F_1 F_2 q^k \sigma_k [ze A_0 + V(\mathbf{r}) - mc^2] \right\} \times \\ &\times \left\{ \frac{1}{mc} \left[-iF_2^2 q^k \sigma_k \varepsilon_{nmj} q^n \sigma_j + F_1 (F_1 + F_2 q^4) \sigma_m \right] \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right) + i(1 - 2F_1) F_2 q^k \sigma_k \right\} \varphi. \end{aligned} \quad (27)$$

So, we obtain a new equation which depends on one spinor function φ only. This equation is generalization of Pauli equation [see Eqs. (1.3.5)–(1.3.7) in Ref. [47], p. 33], but with included electromagnetic form-factors of nucleon and the interacting potential $V(\mathbf{r})$ [along to formalism in Ref. [47], see p. 48–60]. It is convenient to rewrite this equation in compact form:

$$i\hbar \left\{ F_1^2 - F_2^2 \mathbf{q}^2 \right\} f(|\mathbf{q}|) \frac{\partial \varphi}{\partial t} = A \cdot f(|\mathbf{q}|) \cdot \varphi + B \cdot \varphi, \quad (28)$$

where

$$\begin{aligned} A &= c F_2 \left[-\varepsilon_{kmj} F_1 q^k \sigma_j - i(F_1 + F_2 q^4) q^k \sigma_k \sigma_m \right] \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right) + \\ &+ F_1 mc^2 + (F_1^2 + F_2^2 \mathbf{q}^2) (ze A_0 + V(\mathbf{r}) - mc^2), \end{aligned} \quad (29)$$

$$B = \left\{ c \left[F_1 (F_1 - F_2 q^4) \sigma_m - i F_2^2 q^k \sigma_k \varepsilon_{nmj} q^n \sigma_j \right] \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right) + i m c^2 F_2 q^k \sigma_k - 2i F_1 F_2 q^k \sigma_k [ze A_0 + V(\mathbf{r}) - mc^2] \right\} \times \\ \times \left\{ \frac{1}{mc} \left[-i F_2^2 q^k \sigma_k \varepsilon_{nmj} q^n \sigma_j + F_1 (F_1 + F_2 q^4) \sigma_m \right] \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right) + i (1 - 2 F_1) F_2 q^k \sigma_k \right\}. \quad (30)$$

After calculations, we obtain the following solutions for functions A and B (see Appendix A):

$$B - B_1 + A \cdot f(|\mathbf{q}|) = i c F_2 \left\{ b_1 q^m + b_2 \varepsilon_{mjl} q^j \sigma_l \right\} \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right)_m + m c^2 b_3, \quad (31)$$

where

$$m B_1 = a_1 \left[\left(\mathbf{p}_i - \frac{z_i e}{c} \mathbf{A}_i \right)^2 - \frac{z_i e}{c} \boldsymbol{\sigma} \mathbf{H} \right] + (F_2^4 \mathbf{q}^2 + a_2 + a_3) \left(\mathbf{q} \mathbf{p} - \frac{ze}{c} \mathbf{q} \mathbf{A} \right)^2 + m \bar{B}_{10}, \quad (32)$$

$$m \bar{B}_{10} = i \varepsilon_{lm'k} \sigma_k q^l q^m \left\{ (a_2 - a_3) (p_m p_{m'} - \frac{ze}{c} (A_{m'} p_m + A_m p_{m'})) + \frac{z^2 e^2}{c^2} A_m A_{m'} \right\} + \frac{i \hbar z e}{c} \left[a_2 \frac{dA_{m'}}{dx_m} - a_3 \frac{dA_m}{dx_{m'}} \right], \quad (33)$$

and

$$b_1 = F_1^2 (1 - F_1) + F_1 F_2 (3 F_1 - 1) q^4 - F_2^2 (F_1 + F_2 q^4) \mathbf{q}^2 - \frac{2 F_1^2}{m c^2} (F_1 + F_2 q^4) [ze A_0 + V(\mathbf{r})], \\ b_2 = i \left[2 F_1^2 + F_1 F_2 (3 F_1 - 1) q^4 - F_2^2 (2 + F_2 q^4) \mathbf{q}^2 - \frac{2 F_1}{m c^2} (F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2) (ze A_0 + V(\mathbf{r})) \right], \\ b_3 = \left\{ F_1^2 (1 - F_1^2) - (1 - 2 F_1^2) F_2^2 \mathbf{q}^2 - F_2^4 \mathbf{q}^4 \right\} + \frac{1}{m c^2} \left\{ F_1^3 (1 + F_1) + F_1 F_2^2 (3 - 2 F_1) \mathbf{q}^2 + F_2^4 \mathbf{q}^4 \right\} [ze A_0 + V(\mathbf{r})]. \quad (34)$$

Coefficients a_1 , a_2 and a_3 are defined in Appendix A 3 [see Eqs. (A30) and (A35)].

B. Operator of emission of bremsstrahlung photons

In this paper we use approximation $A_0 = 0$. On such a basis, from Eqs. (26), (31)–(33) we obtain (see Appendix B, for details)

$$i \hbar \left\{ F_1^3 (1 + F_1) - F_1 F_2^2 \mathbf{q}^2 - F_2^4 \mathbf{q}^4 \right\} \frac{\partial \varphi}{\partial t} = (h_0 + h_{\gamma 0} + h_{\gamma 1}) \cdot \varphi, \quad (35)$$

where

$$h_0 = \frac{a_1 \mathbf{p}_i^2}{m} + \left(F_1^3 (1 + F_1) + F_1 F_2^2 (3 - 2 F_1) \mathbf{q}^2 + F_2^4 \mathbf{q}^4 \right) V(\mathbf{r}) + \\ + m c^2 \left[F_1^2 (1 - F_1^2) - (1 - 2 F_1^2) F_2^2 \mathbf{q}^2 - F_2^4 \mathbf{q}^4 \right] - \\ - i \frac{2 F_1 F_2}{m c} \left\{ F_1 (F_1 + F_2 q^4) q^m + i (F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2) \varepsilon_{mjl} q^j \sigma_l \right\} V(\mathbf{r}) \mathbf{p}_m + \\ + \frac{1}{m} \left\{ (F_2^4 \mathbf{q}^2 + a_2 + a_3) (\mathbf{q} \mathbf{p})^2 + i \varepsilon_{lm'k} \sigma_k q^l q^m (a_2 - a_3) p_m p_{m'} \right\} + \\ + i c F_2 \left\{ \left[F_1^2 (1 - F_1) + F_1 F_2 (3 F_1 - 1) q^4 - F_2^2 (F_1 + F_2 q^4) \mathbf{q}^2 \right] q^m + \right. \\ \left. + i \left[2 F_1^2 + F_1 F_2 (3 F_1 - 1) q^4 - F_2^2 (2 + F_2 q^4) \mathbf{q}^2 \right] \varepsilon_{mjl} q^j \sigma_l \right\} \mathbf{p}_m, \quad (36)$$

$$h_{\gamma 0} = \frac{a_1}{m} \left[-\frac{z_i e}{c} (-i \hbar \operatorname{div} \mathbf{A} + 2 \mathbf{A} \mathbf{p}) + \frac{z_i^2 e^2}{c^2} \mathbf{A}^2 - \frac{z_i e}{c} \boldsymbol{\sigma} \mathbf{H} \right], \\ h_{\gamma 1} = -i c F_2 \left\{ b_1 q^m + b_2 \varepsilon_{mjl} q^j \sigma_l \right\} \frac{ze}{c} \mathbf{A}_m + \frac{1}{m} \left\{ (F_2^4 \mathbf{q}^2 + a_2 + a_3) \left[-2 (\mathbf{q} \mathbf{p}) \frac{ze}{c} (\mathbf{q} \mathbf{A}) + \frac{z^2 e^2}{c^2} (\mathbf{q} \mathbf{A})^2 \right] + \right. \\ \left. + i \varepsilon_{lm'k} \sigma_k q^l q^m \left[(a_2 - a_3) \left(-\frac{ze}{c} (A_{m'} p_m + A_m p_{m'}) \right) + \frac{z^2 e^2}{c^2} A_m A_{m'} \right] + \frac{i \hbar z e}{c} \left(a_2 \frac{dA_{m'}}{dx_m} - a_3 \frac{dA_m}{dx_{m'}} \right) \right\}. \quad (37)$$

Here, the first term $h_{\gamma 0}$ is operator, describing electric and magnetic emissions of the bremsstrahlung photons without characteristics of the internal structure of the scattered proton. Peculiarities of such types of the (coherent and incoherent) bremsstrahlung emission in the proton-nucleus scattering were studied in details in Refs. [28, 29]. The second term $h_{\gamma 1}$ is operator of emission, describing contribution to the full bremsstrahlung spectrum caused taking the internal structure of the scattered proton into account.

C. Elastic scattering of virtual photon on proton

For the first estimations of emission of the bremsstrahlung photons on the basis of the developed formalism, we shall analyze elastic scattering of virtual photon on proton (scattered off the nucleus). So, energy of this proton in the scattering is conserved and we have:

$$q_4 = 0, \quad \mathbf{q}^2 = -q^2 = Q^2. \quad (38)$$

From Eqs. (A30) and (A35) we calculate coefficients:

$$a_1 = (F_1^2 - F_2^2 Q^2)^2 = \frac{a_2^2}{F_2^4}, \quad a_2 = a_3 = F_2^2 [F_1^2 - F_2^2 Q^2]. \quad (39)$$

$$\begin{aligned} b_1 &= F_1^2(1 - F_1) - F_1 F_2^2 Q^2 - \frac{2 F_1^3}{m c^2} V(\mathbf{r}), \\ b_2 &= i \left[2 F_1^2 - 2 F_2^2 Q^2 - \frac{2 F_1}{m c^2} (F_1^2 - F_2^2 Q^2) V(\mathbf{r}) \right], \\ b_3 &= \left\{ F_1^2 (1 - F_1^2) - (1 - 2 F_1^2) F_2^2 Q^2 - F_2^4 Q^4 \right\} + \frac{1}{m c^2} \left\{ F_1^3 (1 + F_1) + F_1 F_2^2 (3 - 2 F_1) Q^2 + F_2^4 Q^4 \right\} V(\mathbf{r}). \end{aligned} \quad (40)$$

DIS theory provides kinematic relation for virtual photon and proton (see Eq. (3.7) in Ref. [1], p. 79; we do not take the internal structure of nucleon of the nucleus into account)¹. We obtain:

$$\mathbf{q}\mathbf{p} = -\frac{1}{2} Q^2. \quad (41)$$

We shall use QED representation for the vector potential of the bremsstrahlung emission:

$$\mathbf{A} = \sum_{\alpha=1,2} \sqrt{\frac{2\pi\hbar c^2}{w_{\text{ph}}}} \mathbf{e}^{(\alpha),*} e^{-i\mathbf{k}_{\text{ph}}\mathbf{r}}. \quad (42)$$

Here, $\mathbf{e}^{(\alpha)}$ are unit vectors of polarization of the photon emitted ($\mathbf{e}^{(\alpha),*} = \mathbf{e}^{(\alpha)}$), \mathbf{k}_{ph} is wave vector of the photon and $w_{\text{ph}} = k_{\text{ph}}c = |\mathbf{k}_{\text{ph}}|c$. Vectors $\mathbf{e}^{(\alpha)}$ are perpendicular to \mathbf{k}_{ph} in Coulomb calibration. We have two independent polarizations $\mathbf{e}^{(1)}$ and $\mathbf{e}^{(2)}$ for the photon with impulse \mathbf{k}_{ph} ($\alpha = 1, 2$). We have properties:

$$\left[\mathbf{k}_{\text{ph}} \times \mathbf{e}^{(1)} \right] = k_{\text{ph}} \mathbf{e}^{(2)}, \quad \left[\mathbf{k}_{\text{ph}} \times \mathbf{e}^{(2)} \right] = -k_{\text{ph}} \mathbf{e}^{(1)}, \quad \left[\mathbf{k}_{\text{ph}} \times \mathbf{e}^{(3)} \right] = 0, \quad \sum_{\alpha=1,2,3} \left[\mathbf{k}_{\text{ph}} \times \mathbf{e}^{(\alpha)} \right] = k_{\text{ph}} (\mathbf{e}^{(2)} - \mathbf{e}^{(1)}). \quad (43)$$

We also need in determination of the scalar multiplication of the vectors \mathbf{q} and \mathbf{A} . As the first approximation, we shall introduce a new angle φ_{ph} between these vectors:

$$\mathbf{q}\mathbf{A} = qA \cdot \sin \varphi_{ph}. \quad (44)$$

After calculations we obtain the following terms for the hamiltonian (see Appendix C, for details):

$$\begin{aligned} h_0 &= \frac{a_1 \mathbf{p}^2}{m} + \left(F_1^3 (1 + F_1) + F_1 F_2^2 (3 - 2 F_1) Q^2 + F_2^4 Q^4 + i \frac{F_1^3 F_2 Q^2}{m c} \right) [ze A_0 + V(\mathbf{r})] + \\ &+ mc^2 \left[F_1^2 (1 - F_1^2) - (1 - 2 F_1^2) F_2^2 Q^2 - F_2^4 Q^4 \right] + \frac{Q^4}{4m} (F_2^4 Q^2 + 2a_2) - \frac{i c F_2 Q^2}{2} \left[F_1^2 (1 - F_1) - F_2^2 F_1 Q^2 \right] + \\ &+ \left\{ \frac{2 F_1 F_2}{m c} [ze A_0 + V(\mathbf{r})] - 2c F_2 \right\} (F_1^2 - F_2^2 Q^2) \varepsilon_{mjl} q^j \sigma_l \mathbf{p}_m. \end{aligned} \quad (45)$$

$$\begin{aligned} h_{\gamma 1} &= ze F_2 \sqrt{\frac{\pi \hbar c^2}{w_{\text{ph}}}} e^{-i\mathbf{k}_{\text{ph}}\mathbf{r}} \left\{ 2Q \sin \varphi_{ph} \left[-i b_1 + \frac{F_2 Q^2}{m c} (2F_1^2 - F_2^2 Q^2) \right] - \right. \\ &\left. - i \sqrt{2} b_2 \varepsilon_{mjl} q^j \sigma_l \sum_{\alpha=1,2} \mathbf{e}_m^{(\alpha),*} + \frac{4ze}{m c} \sqrt{\frac{\pi \hbar}{w_{\text{ph}}}} F_2 Q^2 (2F_1^2 - F_2^2 Q^2) e^{-i\mathbf{k}_{\text{ph}}\mathbf{r}} \sin^2 \varphi_{ph} \right\}. \end{aligned} \quad (46)$$

$h_{\gamma 0}$ is not changed.

¹ In our formalism developed in the space representation, application of DIS kinematic relations for virtual photons is approximation.

D. Matrix element of emission of bremsstrahlung photons

Now we calculate the full matrix element of the bremsstrahlung emission. We define it, using as basis our previous formalism [28, 29, 31–33] (see also Refs. [50–59]) as

$$F_{fi} \equiv F_{fi,0} + F_{fi,1} = \langle k_f | h_{\gamma 0} + h_{\gamma 1} | k_i \rangle = \int \psi_f^*(\mathbf{r}) (h_{\gamma 0} + h_{\gamma 1}) \psi_i(\mathbf{r}) d\mathbf{r}, \quad (47)$$

where $\psi_i(\mathbf{r}) = |k_i\rangle$ and $\psi_f(\mathbf{r}) = |k_f\rangle$ are the stationary wave functions of the proton-nucleus system in the initial i -state (i.e. state before emission of the bremsstrahlung photon) and final f -state (i.e. state after emission of this photon) which do not contain number of photons emitted.

In further development of our formalism, we shall assume that it is impossible to fix polarization of virtual photon concerning polarization of the bremsstrahlung photon. So, we have to average the matrix elements of emission over angle φ_{ph} and obtain (see Appendix D for details):

$$\begin{aligned} F_{fi,0} &= \langle k_f | h_{\gamma 0} | k_i \rangle = Z_{\text{eff}} \frac{e}{mc} \sqrt{\frac{2\pi\hbar c^2}{w}} \{p_{\text{el}} + p_{\text{mag},1} + p_{\text{mag},2}\}, \\ F_{fi,1} &= \langle k_f | h_{\gamma 1} | k_i \rangle = Z_{\text{eff}} e F_2 \sqrt{\frac{\pi\hbar c^2}{w_{\text{ph}}}} \{p_{\text{q},1} + p_{\text{q},2} + p_{\text{q},3}\}, \end{aligned} \quad (48)$$

where

$$\begin{aligned} p_{\text{el}} &= i \sum_{\alpha=1,2} \mathbf{e}^{(\alpha)} \langle k_f | e^{-i\mathbf{k}\mathbf{r}} \nabla | k_i \rangle, \\ p_{\text{mag},1} &= \frac{1}{2} \sum_{\alpha=1,2} \langle k_f | e^{-i\mathbf{k}\mathbf{r}} \boldsymbol{\sigma} \cdot [\mathbf{e}^{(\alpha)} \times \nabla] | k_i \rangle, \\ p_{\text{mag},2} &= -i \frac{1}{2} \sum_{\alpha=1,2} [\mathbf{k} \times \mathbf{e}^{(\alpha)}] \langle k_f | e^{-i\mathbf{k}\mathbf{r}} \boldsymbol{\sigma} | k_i \rangle, \end{aligned} \quad (49)$$

$$\begin{aligned} \tilde{p}_{\text{q},1} &= i A_1(Q, F_1, F_2) \langle k_f | e^{-i\mathbf{k}_{\text{ph}}\mathbf{r}} | k_i \rangle + i B_1(Q, F_1, F_2) \langle k_f | e^{-i\mathbf{k}_{\text{ph}}\mathbf{r}} V(\mathbf{r}) | k_i \rangle, \\ \tilde{p}_{\text{q},2} &= i A_2(Q, F_1, F_2) \langle k_f | e^{-i\mathbf{k}_{\text{ph}}\mathbf{r}} | k_i \rangle + i B_2(Q, F_1, F_2) \langle k_f | e^{-i\mathbf{k}_{\text{ph}}\mathbf{r}} V(\mathbf{r}) | k_i \rangle, \\ \tilde{p}_{\text{q},3} &= i A_3(Q, F_1, F_2) \langle k_f | e^{-i\mathbf{k}_{\text{ph}}\mathbf{r}} | k_i \rangle \end{aligned} \quad (50)$$

and

$$\begin{aligned} A_1(Q, F_1, F_2) &= -\frac{4Q}{\pi} \left\{ [F_1^2(1 - F_1) - F_1 F_2^2 Q^2] + i \frac{F_2 Q^2}{\pi mc} (2F_1^2 - F_2^2 Q^2) \right\}, \\ B_1(Q, F_1, F_2) &= 8Q \frac{F_1^3}{\pi mc^2}, \\ A_2(Q, F_1, F_2) &= -i 2 (F_1^2 - F_2^2 Q^2) \sqrt{2} \varepsilon_{mj l} q^j \sigma_l \sum_{\alpha=1,2} \mathbf{e}_m^{(\alpha),*}, \\ B_2(Q, F_1, F_2) &= i 2 (F_1^2 - F_2^2 Q^2) \frac{\sqrt{2} F_1}{mc^2} \varepsilon_{mj l} q^j \sigma_l \sum_{\alpha=1,2} \mathbf{e}_m^{(\alpha),*}, \\ A_3(Q, F_1, F_2) &= -i \frac{2ze}{mc} \sqrt{\frac{\pi\hbar}{w_{\text{ph}}}} F_2 Q^2 (2F_1^2 - F_2^2 Q^2). \end{aligned} \quad (51)$$

One can see that the magnetic moment of the scattered proton gives own correction to the full magnetic bremsstrahlung emission via components $p_{\text{q},1}$, $p_{\text{q},2}$ and $p_{\text{q},3}$ (i.e. the magnetic field of the full nuclear system is changed). Now a new physical question has been appeared about magnitude of such a magnetic emission. Also a new type of space distribution of the emitted photons is appeared via term $\langle k_f | e^{-i\mathbf{k}_{\text{ph}}\mathbf{r}} V(\mathbf{r}) | k_i \rangle$.

E. Wave function of nuclear system and summation over spinor states

We define the wave function of the proton in field of the nucleus, according to formalism in Ref. [28] (see Sect. C, Eqs. (12)–(13) in that paper). We construct it in form of bilinear combination of eigenfunctions of orbital and spinor subsystems (as Eq. (1.4.2) in [47], p. 42). However, we shall assume that it is not possible to fix experimentally states for selected M (eigenvalue of momentum operator \hat{J}_z). So, we shall be interesting in superposition over all states with different M and define the wave function so²:

$$\varphi_{jl}(\mathbf{r}, s) = R_l(r) \sum_{m=-l}^l \sum_{\mu=\pm 1/2} C_{lm1/2\mu}^{j,M=m+\mu} Y_{lm}(\mathbf{n}_r) v_\mu(s), \quad (52)$$

where $R(r)$ is radial scalar function (not dependent on m at the same l), $\mathbf{n}_r = \mathbf{r}/r$ is unit vector directed along \mathbf{r} , $Y_{lm}(\mathbf{n}_r)$ are spherical functions (we use definition (28,7)–(28,8), p. 119 in [60]), $C_{lm1/2\mu}^{jM}$ are Clebsh-Gordon coefficients, s is variable of spin, $M = m + \mu$ and $l = j \pm 1/2$. For convenience of calculations we shall use spacial wave function as

$$\varphi_{lm}(\mathbf{r}) = R_l(r) Y_{lm}(\mathbf{n}_r). \quad (53)$$

Using representation (52) for the wave functions, we calculate the matrix elements (49) and (50), according to formalism in Ref. [28] [see Sect. C, Eqs. (14)–(23) in that paper]:

$$\begin{aligned} p_{\text{el}} &= i \sum_{m_f, m_i} \sum_{\mu_i, \mu_f=\pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} \cdot (\mathbf{e}^{(1)} + \mathbf{e}^{(2)}) \left\langle k_f \left| e^{-i \mathbf{k} \mathbf{r}} \nabla \right| k_i \right\rangle_{\mathbf{r}}, \\ p_{\text{mag}, 1} &= \frac{1}{2} \sum_{m_f, m_i} \sum_{\mu_i, \mu_f=\pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} \cdot \left[\mathbf{e}_x + \mathbf{e}_y i \left\{ \delta_{\mu_i, +1/2} - \delta_{\mu_i, -1/2} \right\} + \mathbf{e}_z \right] \times \\ &\quad \times \left[\sum_{\alpha=1,2} e^{(\alpha)} \times \left\langle k_f \left| e^{-i \mathbf{k} \mathbf{r}} \nabla \right| k_i \right\rangle_{\mathbf{r}} \right], \\ p_{\text{mag}, 2} &= \frac{-i k}{2} \sum_{m_f, m_i} \sum_{\mu_i, \mu_f=\pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} \cdot \left[-1 + i \left\{ \delta_{\mu_i, +1/2} - \delta_{\mu_i, -1/2} \right\} \right] \left\langle k_f \left| e^{-i \mathbf{k} \mathbf{r}} \right| k_i \right\rangle_{\mathbf{r}} \end{aligned} \quad (54)$$

and we obtain formulas for new matrix elements:

$$\begin{aligned} \tilde{p}_{q,1} &= i \sum_{m_f, m_i} \sum_{\mu_i, \mu_f=\pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} \times \\ &\quad \times \left\{ A_1(Q, F_1, F_2) \left\langle k_f \left| e^{-i \mathbf{k}_{\text{ph}} \mathbf{r}} \right| k_i \right\rangle_{\mathbf{r}} + B_1(Q, F_1, F_2) \left\langle k_f \left| e^{-i \mathbf{k}_{\text{ph}} \mathbf{r}} V(\mathbf{r}) \right| k_i \right\rangle_{\mathbf{r}} \right\}, \\ \tilde{p}_{q,3} &= i \sum_{m_f, m_i} \sum_{\mu_i, \mu_f=\pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} \cdot A_3(Q, F_1, F_2) \left\langle k_f \left| e^{-2i \mathbf{k}_{\text{ph}} \mathbf{r}} \right| k_i \right\rangle_{\mathbf{r}}. \end{aligned} \quad (55)$$

Here, $\langle k_f | \dots | k_i \rangle_{\mathbf{r}}$ is one-component matrix element

$$\left\langle k_f \left| \hat{f} \right| k_i \right\rangle_{\mathbf{r}} \equiv \int R_f^*(r) Y_{l_f m_f}(\mathbf{n}_r)^* \hat{f} R_i(r) Y_{l_i m_i}(\mathbf{n}_r) d\mathbf{r}, \quad (56)$$

where integration should be performed over space coordinates only. Here, we orient frame vectors \mathbf{e}_x , \mathbf{e}_y and \mathbf{e}_z so, that \mathbf{e}_z be directed along to \mathbf{k} . Then, vectors \mathbf{e}_x and \mathbf{e}_y can be directed along $\mathbf{e}^{(1)}$ and $\mathbf{e}^{(2)}$, correspondingly. In Coulomb gauge we have:

$$\mathbf{e}_x = \mathbf{e}^{(1)}, \quad \mathbf{e}_y = \mathbf{e}^{(2)}, \quad |\mathbf{e}_x| = |\mathbf{e}_y| = |\mathbf{e}_z| = 1, \quad |\mathbf{e}^{(3)}| = 0. \quad (57)$$

² Here, the function (52) is spinor (i.e. two component) solution of equation (35) (which is generalization of the Pauli equation). At the same time, wave function (12) is bi-spinor (i.e. four component) solution of the Dirac equation. Along to QED formalism (see Ref. [47], p. 42–44), the wave function (35) is fully characterized by quantum number l (while two components of unite solution (12) of the Dirac equation have different values of l , and so different radial components in the spherically symmetric consideration). So, representation (52) in determination of solution of equation (35) is correct.

F. Calculations of matrix elements of emission in multipolar expansion

As nest step, we apply multipolar expansion in order to calculate the matrix elements of bremsstrahlung emission. Such calculations are straightforward and they are presented in Appendix E. Calculation of radial integrals in the obtained solutions (E13), (E14), (E18), (E22) and (E26) for the matrix elements in that Appendix is the most difficult numeric part in this research. But, they do not depend on μ , and also m_i, m_f . So, one can rewrite the matrix elements, performing summation over such quantum numbers.

1. Representation for matrix element p_{el}

For p_{el} from (E13) and (E19) we obtain:

$$\begin{aligned}
 p_{\text{el}}^M &= \sqrt{\frac{\pi}{2}} \sum_{l_{\text{ph}}=1} (-i)^{l_{\text{ph}}} \sqrt{2l_{\text{ph}}+1} \cdot \left\{ \sqrt{\frac{l_i}{2l_i+1}} K_{\text{el}}^M(l_i, l_f, l_{\text{ph}}, l_i-1) \cdot [J_1(l_i, l_f, l_{\text{ph}}) + (l_i+1) \cdot J_2(l_i, l_f, l_{\text{ph}})] - \right. \\
 &\quad \left. - \sqrt{\frac{l_i+1}{2l_i+1}} K_{\text{el}}^M(l_i, l_f, l_{\text{ph}}, l_i+1) \cdot [J_1(l_i, l_f, l_{\text{ph}}) - l_i \cdot J_2(l_i, l_f, l_{\text{ph}})] \right\}, \\
 p_{\text{el}}^E &= \sqrt{\frac{\pi}{2}} \sum_{l_{\text{ph}}=1} (-i)^{l_{\text{ph}}} \sqrt{2l_{\text{ph}}+1} \times \\
 &\quad \times \left\{ \sqrt{\frac{l_i(l_{\text{ph}}+1)}{(2l_i+1)(2l_{\text{ph}}+1)}} \cdot K_E(l_i, l_f, l_{\text{ph}}, l_i-1, l_{\text{ph}}-1) \cdot [J_1(l_i, l_f, l_{\text{ph}}-1) + (l_i+1) \cdot J_2(l_i, l_f, l_{\text{ph}}-1)] - \right. \\
 &\quad - \sqrt{\frac{l_i l_{\text{ph}}}{(2l_i+1)(2l_{\text{ph}}+1)}} \cdot K_E(l_i, l_f, l_{\text{ph}}, l_i-1, l_{\text{ph}}+1) \cdot [J_1(l_i, l_f, l_{\text{ph}}+1) + (l_i+1) \cdot J_2(l_i, l_f, l_{\text{ph}}+1)] + \\
 &\quad + \sqrt{\frac{(l_i+1)(l_{\text{ph}}+1)}{(2l_i+1)(2l_{\text{ph}}+1)}} \cdot K_E(l_i, l_f, l_{\text{ph}}, l_i+1, l_{\text{ph}}-1) \cdot [J_1(l_i, l_f, l_{\text{ph}}-1) - l_i \cdot J_2(l_i, l_f, l_{\text{ph}}-1)] - \\
 &\quad \left. - \sqrt{\frac{(l_i+1) l_{\text{ph}}}{(2l_i+1)(2l_{\text{ph}}+1)}} \cdot K_E(l_i, l_f, l_{\text{ph}}, l_i+1, l_{\text{ph}}+1) \cdot [J_1(l_i, l_f, l_{\text{ph}}+1) - l_i \cdot J_2(l_i, l_f, l_{\text{ph}}+1)] \right\},
 \end{aligned} \tag{58}$$

where

$$\begin{aligned}
 K_{\text{el}}^M(l_i, l_f, l_{\text{ph}}, l_i-1) &= \sum_{\mu=\pm 1} h_{\mu} \cdot i \mu \sum_{m_i, m_f} \sum_{\mu_i, \mu_f=\pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} I_M(l_i, l_f, l_{\text{ph}}, l_i-1, \mu), \\
 K_{\text{el}}^M(l_i, l_f, l_{\text{ph}}, l_i+1) &= \sum_{\mu=\pm 1} h_{\mu} \cdot i \mu \sum_{m_i, m_f} \sum_{\mu_i, \mu_f=\pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} I_M(l_i, l_f, l_{\text{ph}}, l_i+1, \mu).
 \end{aligned} \tag{59}$$

$$\begin{aligned}
 K_E(l_i, l_f, l_{\text{ph}}, l_i-1, l_{\text{ph}}-1) &= \sum_{\mu=\pm 1} h_{\mu} \sum_{m_i, m_f} \sum_{\mu_i, \mu_f=\pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} I_E(l_i, l_f, l_{\text{ph}}, l_i-1, l_{\text{ph}}-1, \mu), \\
 K_E(l_i, l_f, l_{\text{ph}}, l_i-1, l_{\text{ph}}+1) &= \sum_{\mu=\pm 1} h_{\mu} \sum_{m_i, m_f} \sum_{\mu_i, \mu_f=\pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} I_E(l_i, l_f, l_{\text{ph}}, l_i-1, l_{\text{ph}}+1, \mu), \\
 K_E(l_i, l_f, l_{\text{ph}}, l_i+1, l_{\text{ph}}-1) &= \sum_{\mu=\pm 1} h_{\mu} \sum_{m_i, m_f} \sum_{\mu_i, \mu_f=\pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} I_E(l_i, l_f, l_{\text{ph}}, l_i+1, l_{\text{ph}}-1, \mu), \\
 K_E(l_i, l_f, l_{\text{ph}}, l_i+1, l_{\text{ph}}+1) &= \sum_{\mu=\pm 1} h_{\mu} \sum_{m_i, m_f} \sum_{\mu_i, \mu_f=\pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} I_E(l_i, l_f, l_{\text{ph}}, l_i+1, l_{\text{ph}}+1, \mu).
 \end{aligned} \tag{60}$$

2. Representation for matrix element $p_{\text{mag},1}$

For electric and magnetic components of $p_{\text{mag},1}$ from (E13) and (E22) we have:

$$\begin{aligned}
p_{\text{mag},1}^M &= \frac{1}{2} \sqrt{\frac{\pi}{2}} \sum_{l_{\text{ph}}=1} (-i)^{l_{\text{ph}}} \sqrt{2l_{\text{ph}}+1} \cdot \left\{ \sqrt{\frac{l_i}{2l_i+1}} K_{\text{mag},1}^M(l_i, l_f, l_{\text{ph}}, l_i-1) \cdot [J_1(l_i, l_f, l_{\text{ph}}) + (l_i+1) \cdot J_2(l_i, l_f, l_{\text{ph}})] - \right. \\
&\quad \left. - \sqrt{\frac{l_i+1}{2l_i+1}} K_{\text{mag},1}^M(l_i, l_f, l_{\text{ph}}, l_i+1) \cdot [J_1(l_i, l_f, l_{\text{ph}}) - l_i \cdot J_2(l_i, l_f, l_{\text{ph}})] \right\}, \\
p_{\text{mag},1}^E &= \frac{1}{2} \sqrt{\frac{\pi}{2}} \sum_{l_{\text{ph}}=1} (-i)^{l_{\text{ph}}} \sqrt{2l_{\text{ph}}+1} \times \\
&\quad \times \left\{ \sqrt{\frac{l_i(l_{\text{ph}}+1)}{(2l_i+1)(2l_{\text{ph}}+1)}} \cdot K_E(l_i, l_f, l_{\text{ph}}, l_i-1, l_{\text{ph}}-1) \cdot [J_1(l_i, l_f, l_{\text{ph}}-1) + (l_i+1) \cdot J_2(l_i, l_f, l_{\text{ph}}-1)] - \right. \\
&\quad - \sqrt{\frac{l_i l_{\text{ph}}}{(2l_i+1)(2l_{\text{ph}}+1)}} \cdot K_E(l_i, l_f, l_{\text{ph}}, l_i-1, l_{\text{ph}}+1) \cdot [J_1(l_i, l_f, l_{\text{ph}}+1) + (l_i+1) \cdot J_2(l_i, l_f, l_{\text{ph}}+1)] + \\
&\quad + \sqrt{\frac{(l_i+1)(l_{\text{ph}}+1)}{(2l_i+1)(2l_{\text{ph}}+1)}} \cdot K_E(l_i, l_f, l_{\text{ph}}, l_i+1, l_{\text{ph}}-1) \cdot [J_1(l_i, l_f, l_{\text{ph}}-1) - l_i \cdot J_2(l_i, l_f, l_{\text{ph}}-1)] - \\
&\quad \left. - \sqrt{\frac{(l_i+1)l_{\text{ph}}}{(2l_i+1)(2l_{\text{ph}}+1)}} \cdot K_E(l_i, l_f, l_{\text{ph}}, l_i+1, l_{\text{ph}}+1) \cdot [J_1(l_i, l_f, l_{\text{ph}}+1) - l_i \cdot J_2(l_i, l_f, l_{\text{ph}}+1)] \right\},
\end{aligned} \tag{61}$$

where

$$\begin{aligned}
K_{\text{mag},1}^M(l_i, l_f, l_{\text{ph}}, l_i-1) &= \sum_{\mu=\pm 1} h_\mu \cdot i \sum_{m_i, m_f} \sum_{\mu_i, \mu_f=\pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} I_M(l_i, l_f, l_{\text{ph}}, l_i-1, \mu), \\
K_{\text{mag},1}^M(l_i, l_f, l_{\text{ph}}, l_i+1) &= \sum_{\mu=\pm 1} h_\mu \cdot i \sum_{m_i, m_f} \sum_{\mu_i, \mu_f=\pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} I_M(l_i, l_f, l_{\text{ph}}, l_i+1, \mu). \\
K_{\text{mag},1}^E(l_i, l_f, l_{\text{ph}}, l_i-1, l_{\text{ph}}-1) &= \sum_{\mu=\pm 1} h_\mu \mu \sum_{m_i, m_f} \sum_{\mu_i, \mu_f=\pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} I_E(l_i, l_f, l_{\text{ph}}, l_i-1, l_{\text{ph}}-1, \mu), \\
K_{\text{mag},1}^E(l_i, l_f, l_{\text{ph}}, l_i-1, l_{\text{ph}}+1) &= \sum_{\mu=\pm 1} h_\mu \mu \sum_{m_i, m_f} \sum_{\mu_i, \mu_f=\pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} I_E(l_i, l_f, l_{\text{ph}}, l_i-1, l_{\text{ph}}+1, \mu), \\
K_{\text{mag},1}^E(l_i, l_f, l_{\text{ph}}, l_i+1, l_{\text{ph}}-1) &= \sum_{\mu=\pm 1} h_\mu \mu \sum_{m_i, m_f} \sum_{\mu_i, \mu_f=\pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} I_E(l_i, l_f, l_{\text{ph}}, l_i+1, l_{\text{ph}}-1, \mu), \\
K_{\text{mag},1}^E(l_i, l_f, l_{\text{ph}}, l_i+1, l_{\text{ph}}+1) &= \sum_{\mu=\pm 1} h_\mu \mu \sum_{m_i, m_f} \sum_{\mu_i, \mu_f=\pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} I_E(l_i, l_f, l_{\text{ph}}, l_i+1, l_{\text{ph}}+1, \mu).
\end{aligned} \tag{62}$$

3. Representation for matrix element $p_{\text{mag},2}$

For electric and magnetic components of $p_{\text{mag},2}$ from (E13) and (E22) we have:

$$\begin{aligned}
p_{\text{mag},2}^M &= \frac{1}{2} \sqrt{\frac{\pi}{2}} k \sum_{l_{\text{ph}}=1} (-i)^{l_{\text{ph}}} \sqrt{2l_{\text{ph}}+1} \cdot K_{\text{mag},2}^M(l_i, l_f, l_{\text{ph}}, l_{\text{ph}}) \cdot \tilde{J}(l_i, l_f, l_{\text{ph}}), \\
p_{\text{mag},2}^E &= \frac{1}{2} \sqrt{\frac{\pi}{2}} k \sum_{l_{\text{ph}}=1} (-i)^{l_{\text{ph}}} \sqrt{2l_{\text{ph}}+1} \cdot \left\{ \sqrt{\frac{l_{\text{ph}}+1}{2l_{\text{ph}}+1}} \cdot K_{\text{mag},2}^E(l_i, l_f, l_{\text{ph}}, l_{\text{ph}}-1) \cdot \tilde{J}(l_i, l_f, l_{\text{ph}}-1) - \right. \\
&\quad \left. - \sqrt{\frac{l_{\text{ph}}}{2l_{\text{ph}}+1}} \cdot K_{\text{mag},2}^E(l_i, l_f, l_{\text{ph}}, l_{\text{ph}}+1) \cdot \tilde{J}(l_i, l_f, l_{\text{ph}}+1) \right\},
\end{aligned} \tag{64}$$

where

$$\begin{aligned}
K_{\text{mag},2}^M(l_i, l_f, l_{\text{ph}}, l_{\text{ph}}) &= \sum_{\mu=\pm 1} i\mu \sum_{m_i, m_f} \sum_{\mu_i, \mu_f=\pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} \times \\
&\times \left[-1 + i \left\{ \delta_{\mu_i, +1/2} - \delta_{\mu_i, -1/2} \right\} \right] \tilde{I}(l_i, l_f, l_{\text{ph}}, l_{\text{ph}}, \mu), \\
K_{\text{mag},2}^E(l_i, l_f, l_{\text{ph}}, l_{\text{ph}} - 1) &= \sum_{\mu=\pm 1} \sum_{m_i, m_f} \sum_{\mu_i, \mu_f=\pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} \left[-1 + i \left\{ \delta_{\mu_i, +1/2} - \delta_{\mu_i, -1/2} \right\} \right] \times \\
&\times \tilde{I}(l_i, l_f, l_{\text{ph}}, l_{\text{ph}} - 1, \mu), \\
K_{\text{mag},2}^E(l_i, l_f, l_{\text{ph}}, l_{\text{ph}} + 1) &= \sum_{\mu=\pm 1} \sum_{m_i, m_f} \sum_{\mu_i, \mu_f=\pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} \left[-1 + i \left\{ \delta_{\mu_i, +1/2} - \delta_{\mu_i, -1/2} \right\} \right] \times \\
&\times \tilde{I}(l_i, l_f, l_{\text{ph}}, l_{\text{ph}} + 1, \mu).
\end{aligned} \tag{65}$$

4. Representation for matrix element $\tilde{p}_{q,1}$

For electric and magnetic components of $\tilde{p}_{q,1}$ from (E14), (E22) and (E26) we have:

$$\begin{aligned}
\tilde{p}_{q,1}^M &= \sqrt{\frac{\pi}{2}} \sum_{l_{\text{ph}}=1} (-i)^{l_{\text{ph}}} \sqrt{2l_{\text{ph}}+1} \cdot K_{q,1}^M(l_i, l_f, l_{\text{ph}}, l_{\text{ph}}) \left\{ A_1(Q, F_1, F_2) \cdot \tilde{J}(l_i, l_f, l_{\text{ph}}) + B_1(Q, F_1, F_2) \cdot \check{J}(l_i, l_f, l_{\text{ph}}) \right\}, \\
\tilde{p}_{q,1}^E &= \sqrt{\frac{\pi}{2}} \sum_{l_{\text{ph}}=1} (-i)^{l_{\text{ph}}} \sqrt{2l_{\text{ph}}+1} \times \\
&\times \left\{ K_{q,1}^E(l_i, l_f, l_{\text{ph}}, l_{\text{ph}} - 1) \cdot \sqrt{\frac{l_{\text{ph}}+1}{2l_{\text{ph}}+1}} \cdot \left[A_1(Q, F_1, F_2) \cdot \tilde{J}(l_i, l_f, l_{\text{ph}} - 1) + B_1(Q, F_1, F_2) \cdot \check{J}(l_i, l_f, l_{\text{ph}} - 1) \right] - \right. \\
&- K_{q,1}^E(l_i, l_f, l_{\text{ph}}, l_{\text{ph}} + 1) \cdot \sqrt{\frac{l_{\text{ph}}}{2l_{\text{ph}}+1}} \cdot \left[A_1(Q, F_1, F_2) \cdot \tilde{J}(l_i, l_f, l_{\text{ph}} + 1) + B_1(Q, F_1, F_2) \cdot \check{J}(l_i, l_f, l_{\text{ph}} + 1) \right] \left. \right\},
\end{aligned} \tag{66}$$

where

$$\begin{aligned}
K_{q,1}^M(l_i, l_f, l_{\text{ph}}, l_{\text{ph}}) &= \sum_{\mu=\pm 1} i\mu \sum_{m_i, m_f} \sum_{\mu_i, \mu_f=\pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} \cdot \tilde{I}(l_i, l_f, l_{\text{ph}}, l_{\text{ph}}, \mu), \\
K_{q,1}^E(l_i, l_f, l_{\text{ph}}, l_{\text{ph}}) &= \sum_{\mu=\pm 1} \sum_{m_i, m_f} \sum_{\mu_i, \mu_f=\pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} \cdot \tilde{I}(l_i, l_f, l_{\text{ph}}, l_{\text{ph}}, \mu).
\end{aligned} \tag{67}$$

G. Probability of emission of the bremsstrahlung photon

We define the probability of the emitted bremsstrahlung photons on the basis of the full matrix element p_{fi} in frameworks of formalism given in [28, 29, 31] and we do not repeat it in this paper. In result, we obtain the bremsstrahlung probability as

$$\frac{dP}{dw_{\text{ph}}} = \frac{e^2}{2\pi c^5} \frac{w_{\text{ph}} E_i}{m_p^2 k_i} |p_{fi}|^2. \tag{68}$$

In further analysis we will calculate the different contributions of the emitted photons to the full bremsstrahlung spectrum. For estimation of the interesting contribution, we just use the corresponding term p_{el} , $p_{\text{mag},1}$, $p_{\text{mag},2}$ or $p_{q,1}$.

III. ANALYSIS, DISCUSSIONS

Let us estimate the bremsstrahlung probability accompanying the scattering of protons off nuclei, using formalism above. For calculations and analysis we choose the reaction of $p + {}^{197}\text{Au}$ at proton beam energy of 190 MeV,

where experimental bremsstrahlung data [30] were obtained with high accuracy. Wave function of relative motion between proton and center-of-mass of nucleus is determined concerning to the proton-nucleus potential in form of $V(r) = v_c(r) + v_N(r) + v_{so}(r) + v_l(r)$, where $v_c(r)$, $v_N(r)$, $v_{so}(r)$, and $v_l(r)$ are Coulomb, nuclear, spin-orbital, and centrifugal components defined with parameters in Eqs. (46)–(47) in Ref. [29].

We analyzed these data in our previous paper [29] in details (without consideration of the internal structure of nucleons). In particular, we constructed formalism describing the coherent and incoherent emissions of the bremsstrahlung photons. We found that inclusion of the incoherent emission to the model allows to improve essentially agreement between calculations and experimental data [30]. So, in the current research we focus on estimation of role of the internal structure of the scattered proton in forming the bremsstrahlung spectrum. In order to preform such an analysis clearly and obtain the first estimations, we shall neglect by incoherent emission (which will make analysis and formalism to be essentially more complicated) at current step. But, it turns out that this is enough to obtain the first conclusions about our approach from such an analysis.

At first, we analyze contributions of the electric and magnetic emissions, given by terms p_{el} , $p_{mag,1}$ and $p_{mag,2}$, to the full bremsstrahlung spectrum. Results of such calculations are presented in Fig. 1 (a). We see that the contributions

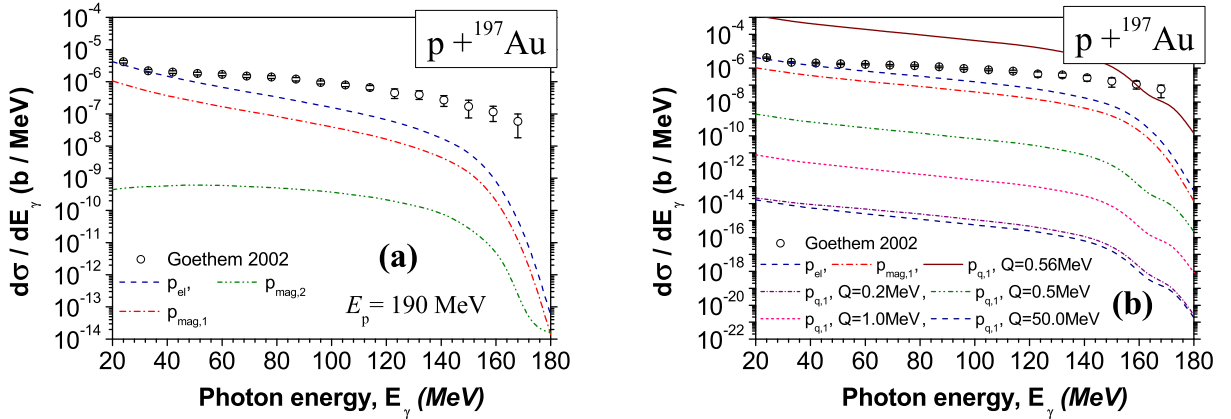


FIG. 1: (Color online) The calculated electric and magnetic bremsstrahlung emissions in the scattering of protons off the ^{197}Au nuclei at energy of proton beam of $E_p = 190$ MeV in comparison with experimental data [30] [components are defined in Eqs. (53), (58)–(65)]: (a) electric emission defined by p_{el} (blue dashed line), magnetic emission defined by $p_{mag,1}$ (red dash-dotted line) and magnetic emission defined by $p_{mag,2}$ (green dash-double dotted line). (b) Contributions of the bremsstrahlung emission given by term $p_{q,1}$ with different Q in comparison with electric and magnetic emissions.

from electrical and first magnetic terms p_{el} and $p_{mag,1}$ are similar. But, such two contributions are essentially larger than contribution for the second magnetic term $p_{mag,2}$. This result is in complete agreement with analysis given in Ref. [28]. After reconstruction of the logic given in Ref. [28], now we include the internal structure of the scattered proton to calculations. Such calculations for contribution of the emitted photons caused by internal structure of the scattered proton are presented in next Fig. 1 (b), where we provide the bremsstrahlung contributions from $p_{q,1}$ in dependence on different values for Q . From this figure one can see, that these spectra are essentially different. Moreover, at some values of Q this contribution of the emitted photons is even larger than the electric and magnetic contributions presented in Fig. 1 (a).

Comparing Eqs. (49) and (50), one can find that matrix element $p_{q,1}$ has different dependence on energy of the emitted photons in comparison with p_{el} , $p_{mag,1}$ and $p_{mag,2}$. Moreover, dependence of $p_{q,1}$ on energy of photons is changed in variations of Q . So, the summarized bremsstrahlung spectrum with inclusion of term $p_{q,1}$ will be changed even after renormalization of calculations at different Q on the same experimental data. We use such an idea in order to find value for Q , which gives the most close agreement between calculations and experimental data. In order to realize such an idea, we use our functions of errors previously introduced to the nuclear bremsstrahlung theory (see Eqs. (23)–(24) in Ref. [31], Eqs. (20) in Ref. [63], also Ref. [8]), which we reformulate as

$$\varepsilon(Q) = \frac{1}{N} \sum_{k=1}^N \frac{|\sigma^{(\text{theor})}(E_k, Q) - \sigma^{(\text{exp})}(E_k)|}{\sigma^{(\text{exp})}(E_k)}. \quad (69)$$

Here, $\sigma^{(\text{theor})}(E_k)$ and $\sigma^{(\text{exp})}(E_k)$ are theoretical and experimental bremsstrahlung cross-sections at energy E_k of the emitted photon, the summation is performed over experimental data ($N = 17$ for data in Ref. [30]). Such calculations are presented in Fig. 2. Here, we find $Q = 50$ MeV corresponding to the minimal function of errors (69).

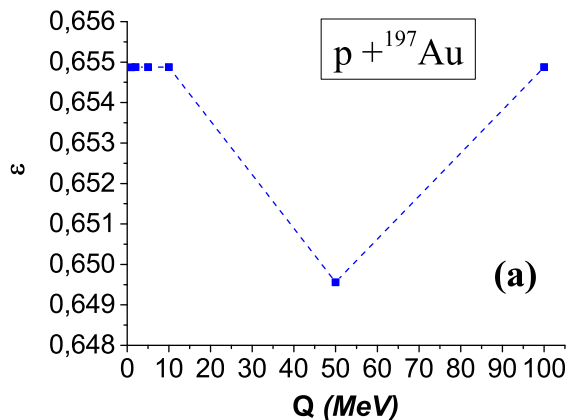


FIG. 2: (Color online) Function of errors defined in Eq. (69) in dependence on Q for bremsstrahlung emissions in the scattering of protons off the ^{197}Au nuclei at energy of proton beam of $E_p = 190$ MeV [we calculate the full matrix element as summation of terms p_{el} , $p_{\text{mag},1}$, $p_{\text{mag},2}$ and $p_{\text{q},1}(Q)$; experimental data are used from Ref. [30]].

Note that at limit of neglecting of the internal structure of the scattering proton via limit of $F_1(Q) \rightarrow 1$, $F_2(Q) \rightarrow 0$, we reconstruct our previous results in Ref. [28] completely.

IV. CONCLUSIONS

In this paper we investigate an idea, how to use analysis of the bremsstrahlung photons to study the internal structure of proton, which is under nuclear reaction with nucleus (which can be light, middle or superheavy). We construct a new model describing bremsstrahlung emission of photons which accompanies the scattering of protons off nuclei.

From physical grounds, emission of photons is formed as result of relative motions (accelerations) of nucleons of the nucleus-target and the scattering proton. By such a reason, we construct the bremsstrahlung formalism from many-nucleon basis [29, 31]. This allows to analyze different contributions of coherent and incoherent bremsstrahlung emissions.

In the model, we focus on the new description of internal structure of the scattered proton³. We are interested if such a structure can be visible in the full bremsstrahlung spectrum. To realize this aim, we implement electromagnetic form-factors for nucleons on the basis of DIS theory to our model. As a result, in our formalism the full bremsstrahlung spectrum is dependent on such form-factors of the scattered proton. In the limit without such form-factors, we reconstruct our previous results in Ref. [28] completely. As the scattered proton can be under influence of strong forces and gives the largest bremsstrahlung contribution to full spectrum, we focus on maximally accurate description of its evolution concerning nucleons of nucleus. Quantum effects and evolution of such a complicated nuclear system are well described by the scattering theory that has been deeply studied and tested experimentally well. From such motivations, (for the first time) we generalize Pauli equation with interacting potential (describing quantum evolution of fermion inside strong field), with including the formalism of electromagnetic form-factors of nucleon. Note that the idea of generalizations of Pauli equation has been successfully applied in studying coherent bremsstrahlung for the proton-nucleus scattering [28], and allows to add incoherent processes from individual nucleon-nucleon interactions [29].

In order to analyze and test our approach, for calculations we choose the reaction of $p + ^{197}\text{Au}$ at proton beam energy of 190 MeV, where experimental bremsstrahlung data [30] were obtained with high accuracy. Anomalous magnetic momenta of nucleons (important in estimations of bremsstrahlung) reinforce our motivation to develop formalism at such energies of protons. Conclusions from analysis of this model are the following:

1. At first, we analyze contributions of the electric and magnetic emissions, defined by terms p_{el} , $p_{\text{mag},1}$ and $p_{\text{mag},2}$, to the full bremsstrahlung spectrum (see Fig. 1 (a)). We find that the electrical and first magnetic contributions from terms p_{el} and $p_{\text{mag},1}$ are similar. But, such two contributions are essentially larger than contribution

³ At current formalism, we neglect structure of nucleons of nucleus-target.

from the second magnetic term $p_{\text{mag},2}$. So, we reconstruct completely our old result [28] (where the electric and magnetic coherent emissions were studied in details in such a reaction) without inclusion of the internal structure of the scattered proton into analysis and calculations.

2. In next step, we analyze and estimate the contribution of bremsstrahlung emission after we include the internal structure of scattered proton to the model and calculations. This is a new type of bremsstrahlung emission defined by term $p_{q,1}$, which we introduce to the bremsstrahlung theory in nuclear physics (see Fig. 1 (b)). This emission depends on the form-factors of scattered proton. We find that at some value of Q such an emission can be larger in comparison with the electrical and magnetic emissions (presented in Fig. 1 (a) and studied in Ref. [28]).
3. Important advance of our approach is in that such a dependence of the bremsstrahlung spectra on form-factors exists also after renormalization of calculations on experimental data. Using idea of function of errors (69) (see also Eqs. (23)–(24) in Ref. [31], Eqs. (20) in Ref. [63], Ref. [8]), we extract proper value for Q (we obtain $Q = 50$ MeV), which corresponds to the most close agreement between calculated full spectrum and experimental data [30] (see Fig. 2).
4. In the limit of neglecting of the internal structure of the scattering proton (at $F_1(Q) \rightarrow 1$, $F_2(Q) \rightarrow 0$), we reconstruct results in Ref. [28] completely.

Our results confirm that the full bremsstrahlung spectrum is sensitive to the form-factors of scattered proton (characterizing its internal structure). This is the first indication that this is possible to construct a new type of microscopy (with higher resolution in comparison with existed one), to study internal structure of nucleons in experimental way by means of bremsstrahlung analysis.

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Appendix A: Solution of equation (28)

1. Calculations of parameters for Eq. (28)

In this Appendix we solve equation (28). At first, we shall transform function A in Eq. (29). Taking the following property into account

$$\sigma_i \sigma_j = \delta_{ij} I + i \varepsilon_{ijk} \sigma_k, \quad \varepsilon_{ijk} \varepsilon_{ilm} = \delta_{jl} \delta_{km} - \delta_{jk} \delta_{lm}, \quad (\text{A1})$$

we obtain:

$$-\varepsilon_{kmj} F_1 q^k \sigma_j - i(F_1 + F_2 q^4) q^k \sigma_k \sigma_m = -i(F_1 + F_2 q^4) q^m + F_2 q^4 q^k \varepsilon_{kmj} \sigma_j. \quad (\text{A2})$$

Taking this expression into account, now we simplify A in Eq. (29):

$$\begin{aligned} A = & c F_2 \left[-i(F_1 + F_2 q^4) q^m + F_2 q^4 q^k \varepsilon_{kmj} \sigma_j \right] \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right)_m + \\ & + F_1 mc^2 + (F_1^2 + F_2^2 \mathbf{q}^2) (ze A_0 + V(\mathbf{r}) - mc^2). \end{aligned} \quad (\text{A3})$$

We transform function B in Eq. (30) as

$$\begin{aligned}
B = & B_1 + i c F_2 \left\{ \left[F_1 (F_1 - F_2 q^4) \sigma_m - i F_2^2 q^k \sigma_k \varepsilon_{nmj} q^n \sigma_j \right] (1 - 2 F_1) q^k \sigma_k + \right. \\
& + q^k \sigma_k \left[-i F_2^2 q^k \sigma_k \varepsilon_{nmj} q^n \sigma_j + F_1 (F_1 + F_2 q^4) \sigma_m \right] - \\
& - 2 F_1 q^k \sigma_k [ze A_0 + V(\mathbf{r}) - mc^2] \frac{1}{mc^2} \left[-i F_2^2 q^k \sigma_k \varepsilon_{nmj} q^n \sigma_j + F_1 (F_1 + F_2 q^4) \sigma_m \right] \left. \right\} \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right) - \\
& - mc^2 (1 - 2 F_1) F_2^2 \mathbf{q}^2 + 2 (1 - 2 F_1) F_1 F_2^2 [ze A_0 + V(\mathbf{r}) - mc^2] \mathbf{q}^2,
\end{aligned} \tag{A4}$$

where

$$\begin{aligned}
B_1 = & \frac{1}{m} \left[[F_1^2 - F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \sigma_m + F_2^2 q^m q^l \sigma_l \right] \left(p_m - \frac{ze}{c} A_m \right) \times \\
& \times \left[[F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \sigma_{m'} + F_2^2 q^{m'} q^l \sigma_l \right] \left(p_{m'} - \frac{ze}{c} A_{m'} \right).
\end{aligned} \tag{A5}$$

Here, we taken property (A8) into account. It is obtained so. Taking properties (A1) into account, we simplify term $(i, j, k = 1, 2, 3)$

$$\begin{aligned}
i F_2^2 q^k \sigma_k \varepsilon_{nmj} q^n \sigma_j &= i F_2^2 q^k q^n \varepsilon_{nmj} (\sigma_k \sigma_j) = i F_2^2 q^k q^n \varepsilon_{nmj} (\delta_{kj} I + i \varepsilon_{kjl} \sigma_l) = i F_2^2 q^k q^n \varepsilon_{nmj} \delta_{kj} + i F_2^2 q^k q^n \varepsilon_{nmj} i \varepsilon_{kjl} \sigma_l = \\
&= i F_2^2 q^k q^n \varepsilon_{nmk} + F_2^2 q^k q^k \sigma_m - F_2^2 q^m q^l \sigma_l = i F_2^2 q^k q^n \varepsilon_{nmk} + F_2^2 \mathbf{q}^2 \sigma_m - F_2^2 q^m q^l \sigma_l.
\end{aligned} \tag{A6}$$

Here, the first term equas to zero, as summation is performed over two indexes of antisymmetric ε_{nmk} :

$$q^k q^n \varepsilon_{nmk} = -q^k q^n \varepsilon_{nkm} = -\frac{1}{2} (q^k q^n \varepsilon_{nkm} + q^n q^k \varepsilon_{knm}) = -\frac{1}{2} q^k q^n (\varepsilon_{nkm} - \varepsilon_{nkm}) = 0. \tag{A7}$$

So, we write:

$$i F_2^2 q^k \sigma_k \varepsilon_{nmj} q^n \sigma_j = F_2^2 \mathbf{q}^2 \sigma_m - F_2^2 q^m q^l \sigma_l. \tag{A8}$$

Now we simplify the first term in final expression in Eq. (A4):

$$\begin{aligned}
F_1 (F_1 - F_2 q^4) \sigma_m - i F_2^2 q^k \sigma_k \varepsilon_{nmj} q^n \sigma_j &= F_1 (F_1 - F_2 q^4) \sigma_m - F_2^2 \mathbf{q}^2 \sigma_m + F_2^2 q^m q^l \sigma_l = \\
&= [F_1^2 - F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \sigma_m + F_2^2 q^m q^l \sigma_l.
\end{aligned} \tag{A9}$$

Basing on Eqs. (A9) and (24), we find:

$$\left[F_1 (F_1 - F_2 q^4) \sigma_m - i F_2^2 q^k \sigma_k \varepsilon_{nmj} q^n \sigma_j \right] q^k \sigma_k = [F_1^2 - F_1 F_2 q^4] q^m + i [F_1^2 - F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \varepsilon_{mjl} q^j \sigma_l. \tag{A10}$$

For the second term in Eq. (A4) we find:

$$-i F_2^2 q^k \sigma_k \varepsilon_{nm'j} q^n \sigma_j + F_1 (F_1 + F_2 q^4) \sigma_{m'} = [F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \sigma_{m'} + F_2^2 q^{m'} q^l \sigma_l. \tag{A11}$$

and, using logic of transformations (A10), we obtain:

$$\begin{aligned}
\left[-i F_2^2 q^k \sigma_k \varepsilon_{nmj} q^n \sigma_j + F_1 (F_1 + F_2 q^4) \sigma_m \right] q^k \sigma_k &= [F_1^2 + F_1 F_2 q^4] q^m + i [F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \varepsilon_{mjl} q^j \sigma_l.
\end{aligned} \tag{A12}$$

For the third term in Eq. (A4) [taking formula (A12) into account] we have

$$\begin{aligned}
& -2 F_1 q^k \sigma_k [ze A_0 + V(\mathbf{r}) - mc^2] \frac{1}{mc^2} \left[-i F_2^2 q^k \sigma_k \varepsilon_{nmj} q^n \sigma_j + F_1 (F_1 + F_2 q^4) \sigma_m \right] = \\
& = -\frac{2 F_1}{mc^2} [ze A_0 + V(\mathbf{r})] \cdot \left\{ [F_1^2 + F_1 F_2 q^4] q^m + i [F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \varepsilon_{mjl} q^j \sigma_l \right\} + \\
& + 2 F_1 \cdot \left\{ [F_1^2 + F_1 F_2 q^4] q^m + i [F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \varepsilon_{mjl} q^j \sigma_l \right\}.
\end{aligned} \tag{A13}$$

Summation of this expression and term (A12) equals to

$$\begin{aligned}
& \left[-iF_2^2 q^k \sigma_k \varepsilon_{nmj} q^n \sigma_j + F_1 (F_1 + F_2 q^4) \sigma_m \right] q^k \sigma_k - \\
& - 2 F_1 q^k \sigma_k [ze A_0 + V(\mathbf{r}) - mc^2] \frac{1}{mc^2} \left[-iF_2^2 q^k \sigma_k \varepsilon_{nmj} q^n \sigma_j + F_1 (F_1 + F_2 q^4) \sigma_m \right] = \\
& = -\frac{2F_1}{mc^2} [ze A_0 + V(\mathbf{r})] \cdot \left\{ [F_1^2 + F_1 F_2 q^4] q^m + i [F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \varepsilon_{mjl} q^j \sigma_l \right\} + \\
& + (2F_1 + 1) \cdot \left\{ [F_1^2 + F_1 F_2 q^4] q^m + i [F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \varepsilon_{mjl} q^j \sigma_l \right\}.
\end{aligned} \tag{A14}$$

Now we simplify B in Eq. (A4) further:

$$\begin{aligned}
B &= B_1 + icF_2 \left\{ (1 - 2F_1) \left[(F_1^2 - F_1 F_2 q^4) q^m + i (F_1^2 - F_1 F_2 q^4 - F_2^2 \mathbf{q}^2) \varepsilon_{mjl} q^j \sigma_l \right] - \right. \\
& - \frac{2F_1}{mc^2} [ze A_0 + V(\mathbf{r})] \cdot \left\{ [F_1^2 + F_1 F_2 q^4] q^m + i [F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \varepsilon_{mjl} q^j \sigma_l \right\} + \\
& + (2F_1 + 1) \cdot \left\{ [F_1^2 + F_1 F_2 q^4] q^m + i [F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \varepsilon_{mjl} q^j \sigma_l \right\} \left. \right\} \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right)_m - \\
& - mc^2 (1 - 2F_1) F_2^2 \mathbf{q}^2 + 2(1 - 2F_1) F_1 F_2^2 [ze A_0 + V(\mathbf{r}) - mc^2] \mathbf{q}^2.
\end{aligned} \tag{A15}$$

Let us simplify term at $(\mathbf{p} - \frac{ze}{c} \mathbf{A})$. We have

$$\begin{aligned}
& (1 - 2F_1) \left[(F_1^2 - F_1 F_2 q^4) q^m + i (F_1^2 - F_1 F_2 q^4 - F_2^2 \mathbf{q}^2) \varepsilon_{mjl} q^j \sigma_l \right] - \\
& - \frac{2F_1}{mc^2} [ze A_0 + V(\mathbf{r})] \cdot \left\{ [F_1^2 + F_1 F_2 q^4] q^m + i [F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \varepsilon_{mjl} q^j \sigma_l \right\} + \\
& + (2F_1 + 1) \cdot \left\{ [F_1^2 + F_1 F_2 q^4] q^m + i [F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \varepsilon_{mjl} q^j \sigma_l \right\} = \\
& = \left[(1 - 2F_1) (F_1^2 - F_1 F_2 q^4) + (2F_1 + 1) (F_1^2 + F_1 F_2 q^4) - \frac{2F_1}{mc^2} (ze A_0 + V(\mathbf{r})) (F_1^2 + F_1 F_2 q^4) \right] q^m + \\
& + i \left[(1 - 2F_1) (F_1^2 - F_1 F_2 q^4 - F_2^2 \mathbf{q}^2) + (2F_1 + 1) (F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2) - \right. \\
& - \left. \frac{2F_1}{mc^2} (ze A_0 + V(\mathbf{r})) (F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2) \right] \varepsilon_{mjl} q^j \sigma_l.
\end{aligned} \tag{A16}$$

We consider summation of two terms:

$$(1 - 2F_1) (F_1^2 - F_1 F_2 q^4) + (2F_1 + 1) (F_1^2 + F_1 F_2 q^4) = 2F_1^2 + 4F_1 \cdot F_1 F_2 q^4 = 2F_1^2 (1 + 2F_2 q^4). \tag{A17}$$

Then expression (A16) can be rewritten as

$$\begin{aligned}
& \left[2F_1^2 (1 + 2F_2 q^4) - \frac{2F_1}{mc^2} (ze A_0 + V(\mathbf{r})) (F_1^2 + F_1 F_2 q^4) \right] q^m + \\
& + i \left[2F_1^2 (1 + 2F_2 q^4) - (1 - 2F_1) F_2^2 \mathbf{q}^2 - (2F_1 + 1) F_2^2 \mathbf{q}^2 - \frac{2F_1}{mc^2} (ze A_0 + V(\mathbf{r})) (F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2) \right] \varepsilon_{mjl} q^j \sigma_l = \\
& = \left[2F_1^2 (1 + 2F_2 q^4) - \frac{2F_1}{mc^2} (ze A_0 + V(\mathbf{r})) (F_1^2 + F_1 F_2 q^4) \right] q^m + \\
& + i \left[2F_1^2 (1 + 2F_2 q^4) - 2F_2^2 \mathbf{q}^2 - \frac{2F_1}{mc^2} (ze A_0 + V(\mathbf{r})) (F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2) \right] \varepsilon_{mjl} q^j \sigma_l.
\end{aligned} \tag{A18}$$

Taking this expression into account, we rewrite solution for B in Eq. (A15):

$$B = B_1 + B_2 \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right)_m - mc^2 (1 - 2F_1) F_2^2 \mathbf{q}^2 + 2(1 - 2F_1) F_1 F_2^2 [ze A_0 + V(\mathbf{r}) - mc^2] \mathbf{q}^2, \tag{A19}$$

where

$$\begin{aligned}
B_2 &= icF_2 \left\{ \left[2F_1^2 (1 + 2F_2 q^4) - \frac{2F_1}{mc^2} (ze A_0 + V(\mathbf{r})) (F_1^2 + F_1 F_2 q^4) \right] q^m + \right. \\
& + i \left[2F_1^2 (1 + 2F_2 q^4) - 2F_2^2 \mathbf{q}^2 - \frac{2F_1}{mc^2} (ze A_0 + V(\mathbf{r})) (F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2) \right] \varepsilon_{mjl} q^j \sigma_l \left. \right\}.
\end{aligned} \tag{A20}$$

As next step, we calculate such a term:

$$\begin{aligned}
& B_2 + c F_2 \left[-i (F_1 + F_2 q^4) q^m + F_2 q^4 q^k \varepsilon_{kmj} \sigma_j \right] \cdot f(|\mathbf{q}|) = \\
& = ic F_2 \left[2 F_1^2 (1 + 2 F_2 q^4) - \frac{2 F_1}{mc^2} (ze A_0 + V(\mathbf{r})) (F_1^2 + F_1 F_2 q^4) - (F_1 + F_2 q^4) f(|\mathbf{q}|) \right] q^m - \\
& - c F_2 \left[2 F_1^2 (1 + 2 F_2 q^4) - 2 F_2^2 \mathbf{q}^2 - \frac{2 F_1}{mc^2} (ze A_0 + V(\mathbf{r})) (F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2) - F_2 q^4 f(|\mathbf{q}|) \right] \varepsilon_{mjl} q^j \sigma_l.
\end{aligned} \tag{A21}$$

We obtain:

$$B - B_1 + A \cdot f(|\mathbf{q}|) = ic F_2 \left\{ b_1 q^m + b_2 \varepsilon_{mjl} q^j \sigma_l \right\} \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right)_m + mc^2 b_3, \tag{A22}$$

where

$$\begin{aligned}
b_1 &= 2 F_1^2 (1 + 2 F_2 q^4) - \frac{2 F_1}{mc^2} (ze A_0 + V(\mathbf{r})) (F_1^2 + F_1 F_2 q^4) - (F_1 + F_2 q^4) f(|\mathbf{q}|), \\
b_2 &= i \left[2 F_1^2 (1 + 2 F_2 q^4) - 2 F_2^2 \mathbf{q}^2 - \frac{2 F_1}{mc^2} (ze A_0 + V(\mathbf{r})) (F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2) - F_2 q^4 f(|\mathbf{q}|) \right], \\
b_3 &= \frac{1}{mc^2} \left\{ -mc^2 (1 - 2 F_1) F_2^2 \mathbf{q}^2 + 2 (1 - 2 F_1) F_1 F_2^2 [ze A_0 + V(\mathbf{r}) - mc^2] \mathbf{q}^2 + \right. \\
& \quad \left. + [F_1 mc^2 + (F_1^2 + F_2^2 \mathbf{q}^2) (ze A_0 + V(\mathbf{r}) - mc^2)] f(|\mathbf{q}|) \right\}.
\end{aligned} \tag{A23}$$

2. Calculations of b_1, b_2, b_3

Taking into account Eq. (26)

$$f(|\mathbf{q}|) = F_1 + F_1^2 + F_2^2 \mathbf{q}^2,$$

we have

$$\begin{aligned}
b_1 &= 2 F_1^2 (1 + 2 F_2 q^4) - \frac{2 F_1}{mc^2} (ze A_0 + V(\mathbf{r})) (F_1^2 + F_1 F_2 q^4) - (F_1 + F_2 q^4) [F_1 + F_1^2 + F_2^2 \mathbf{q}^2] = \\
&= F_1^2 (1 - F_1) + F_1 F_2 (3 F_1 - 1) q^4 - F_2^2 (F_1 + F_2 q^4) \mathbf{q}^2 - \frac{2 F_1^2}{mc^2} (F_1 + F_2 q^4) (ze A_0 + V(\mathbf{r})).
\end{aligned} \tag{A24}$$

Now we simplify solution for b_2 . Taking Eq. (26) into account, we obtain:

$$\begin{aligned}
b_2 &= i \left[2 F_1^2 (1 + 2 F_2 q^4) - 2 F_2^2 \mathbf{q}^2 - \frac{2 F_1}{mc^2} (ze A_0 + V(\mathbf{r})) (F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2) - F_2 q^4 (F_1 + F_1^2 + F_2^2 \mathbf{q}^2) \right] = \\
&= i \left[2 F_1^2 + F_2 (3 F_1^2 - F_1) q^4 - F_2^2 (2 + F_2 q^4) \mathbf{q}^2 - \frac{2 F_1}{mc^2} (F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2) (ze A_0 + V(\mathbf{r})) \right].
\end{aligned} \tag{A25}$$

We simplify solution for b_3 , taking (26) into account:

$$mc^2 b_3 = mc^2 \left\{ F_1^2 (1 - F_1^2) - (1 - 2 F_1^2) F_2^2 \mathbf{q}^2 - F_2^4 \mathbf{q}^4 \right\} + \left\{ F_1^3 (1 + F_1) + F_1 F_2^2 (3 - 2 F_1) \mathbf{q}^2 + F_2^4 \mathbf{q}^4 \right\} [ze A_0 + V(\mathbf{r})]. \tag{A26}$$

We summarize the found solution:

$$\begin{aligned}
b_1 &= F_1^2 (1 - F_1) + F_1 F_2 (3 F_1 - 1) q^4 - F_2^2 (F_1 + F_2 q^4) \mathbf{q}^2 - \frac{2 F_1^2}{mc^2} (F_1 + F_2 q^4) [ze A_0 + V(\mathbf{r})], \\
b_2 &= i \left[2 F_1^2 + F_1 F_2 (3 F_1 - 1) q^4 - F_2^2 (2 + F_2 q^4) \mathbf{q}^2 - \frac{2 F_1}{mc^2} (F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2) (ze A_0 + V(\mathbf{r})) \right], \\
b_3 &= \left\{ F_1^2 (1 - F_1^2) - (1 - 2 F_1^2) F_2^2 \mathbf{q}^2 - F_2^4 \mathbf{q}^4 \right\} + \frac{1}{mc^2} \left\{ F_1^3 (1 + F_1) + F_1 F_2^2 (3 - 2 F_1) \mathbf{q}^2 + F_2^4 \mathbf{q}^4 \right\} [ze A_0 + V(\mathbf{r})].
\end{aligned} \tag{A27}$$

3. Calculation of B_1

Let us calculate B_1 in Eq. (A5):

$$\begin{aligned}
mB_1 = & \left[[F_1^2 - F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \sigma_m \right] \left(p_m - \frac{ze}{c} A_m \right) \cdot \left[[F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \sigma_{m'} \right] \left(p_{m'} - \frac{ze}{c} A_{m'} \right) + \\
& + F_2^2 q^m q^l \sigma_l \left(p_m - \frac{ze}{c} A_m \right) \cdot \left[[F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \sigma_{m'} \right] \left(p_{m'} - \frac{ze}{c} A_{m'} \right) + \\
& + \left[[F_1^2 - F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \sigma_m \right] \left(p_m - \frac{ze}{c} A_m \right) \cdot F_2^2 q^{m'} q^l \sigma_l \left(p_{m'} - \frac{ze}{c} A_{m'} \right) + \\
& + F_2^2 q^m q^l \sigma_l \left(p_m - \frac{ze}{c} A_m \right) \cdot F_2^2 q^{m'} q^l \sigma_l \left(p_{m'} - \frac{ze}{c} A_{m'} \right).
\end{aligned} \tag{A28}$$

We simplify the first term in the obtained expression:

$$\begin{aligned}
& \left[[F_1^2 - F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \sigma_m \right] \left(p_m - \frac{ze}{c} A_m \right) \cdot \left[[F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \sigma_{m'} \right] \left(p_{m'} - \frac{ze}{c} A_{m'} \right) = \\
& = [(F_1^2 - F_2^2 \mathbf{q}^2)^2 - F_1^2 F_2^2 (q^4)^2] \cdot \left[\sigma \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right) \right]^2 = a_1 \cdot \left[\sigma \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right) \right]^2,
\end{aligned} \tag{A29}$$

where

$$a_1 = (F_1^2 - F_2^2 \mathbf{q}^2)^2 - F_1^2 F_2^2 (q^4)^2. \tag{A30}$$

Using properties of Dirac's matrices, we have

$$\left[\sigma \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right) \right]^2 = \left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right)^2 - \frac{ze}{c} \sigma \mathbf{H}, \tag{A31}$$

where $\mathbf{H} = \mathbf{rot} \mathbf{A}$ is magnetic field. Substituting this equation to Eq. (A29), we obtain:

$$\begin{aligned}
& \left[[F_1^2 - F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \sigma_m \right] \left(p_m - \frac{ze}{c} A_m \right) \cdot \left[[F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \sigma_{m'} \right] \left(p_{m'} - \frac{ze}{c} A_{m'} \right) = \\
& = a_1 \left[\left(\mathbf{p} - \frac{ze}{c} \mathbf{A} \right)^2 - \frac{ze}{c} \sigma \mathbf{H} \right].
\end{aligned} \tag{A32}$$

We simplify fourth term in the obtained Eq. (A28) [$m, m' = 1, 2, 3$]:

$$F_2^2 q^m q^l \sigma_l \left(p_m - \frac{ze}{c} A_m \right) \cdot F_2^2 q^{m'} q^l \sigma_l \left(p_{m'} - \frac{ze}{c} A_{m'} \right) = F_2^4 \mathbf{q}^2 \left(q^m p_m - \frac{ze}{c} q^m A_m \right)^2 = F_2^4 \mathbf{q}^2 \left(\mathbf{q} \mathbf{p} - \frac{ze}{c} \mathbf{q} \mathbf{A} \right)^2. \tag{A33}$$

Now we calculate summation of the second and third terms in Eq. (A28):

$$\begin{aligned}
& F_2^2 q^m q^l \sigma_l \left(p_m - \frac{ze}{c} A_m \right) \cdot \left[[F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \sigma_{m'} \right] \left(p_{m'} - \frac{ze}{c} A_{m'} \right) + \\
& + \left[[F_1^2 - F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \sigma_m \right] \left(p_m - \frac{ze}{c} A_m \right) \cdot F_2^2 q^{m'} q^l \sigma_l \left(p_{m'} - \frac{ze}{c} A_{m'} \right) = \\
& = F_2^2 [F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \mathbf{q} \sigma q^m \left(p_m - \frac{ze}{c} A_m \right) \cdot \sigma_{m'} \left(p_{m'} - \frac{ze}{c} A_{m'} \right) + \\
& + F_2^2 [F_1^2 - F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \sigma_m \mathbf{q} \sigma \left(p_m - \frac{ze}{c} A_m \right) \cdot q^{m'} \left(p_{m'} - \frac{ze}{c} A_{m'} \right).
\end{aligned} \tag{A34}$$

Introducing new functions:

$$a_2 = F_2^2 [F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2], \quad a_3 = F_2^2 [F_1^2 - F_1 F_2 q^4 - F_2^2 \mathbf{q}^2], \tag{A35}$$

we rewrite this summation as

$$\begin{aligned}
& F_2^2 q^m q^l \sigma_l \left(p_m - \frac{ze}{c} A_m \right) \cdot \left[[F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \sigma_{m'} \right] \left(p_{m'} - \frac{ze}{c} A_{m'} \right) + \\
& + \left[[F_1^2 - F_1 F_2 q^4 - F_2^2 \mathbf{q}^2] \sigma_m \right] \left(p_m - \frac{ze}{c} A_m \right) \cdot F_2^2 q^{m'} q^l \sigma_l \left(p_{m'} - \frac{ze}{c} A_{m'} \right) = \\
& = a_2 \mathbf{q} \sigma q^m \left(p_m - \frac{ze}{c} A_m \right) \cdot \sigma_{m'} \left(p_{m'} - \frac{ze}{c} A_{m'} \right) + a_3 \sigma_m \mathbf{q} \sigma \left(p_m - \frac{ze}{c} A_m \right) \cdot q^{m'} \left(p_{m'} - \frac{ze}{c} A_{m'} \right).
\end{aligned} \tag{A36}$$

So, we obtain the following expression for B_1 :

$$\begin{aligned} mB_1 &= a_1 \left[\left(\mathbf{p}_i - \frac{z_i e}{c} \mathbf{A}_i \right)^2 - \frac{z_i e}{c} \boldsymbol{\sigma} \mathbf{H} \right] + F_2^4 \mathbf{q}^2 \left(\mathbf{q} \mathbf{p} - \frac{z e}{c} \mathbf{q} \mathbf{A} \right)^2 + \\ &+ a_2 \mathbf{q} \boldsymbol{\sigma} q^m \left(p_m - \frac{z e}{c} A_m \right) \cdot \sigma_{m'} \left(p_{m'} - \frac{z e}{c} A_{m'} \right) + a_3 \sigma_m \mathbf{q} \boldsymbol{\sigma} \left(p_m - \frac{z e}{c} A_m \right) \cdot q^{m'} \left(p_{m'} - \frac{z e}{c} A_{m'} \right) = \\ &= a_1 \left[\left(\mathbf{p}_i - \frac{z_i e}{c} \mathbf{A}_i \right)^2 - \frac{z_i e}{c} \boldsymbol{\sigma} \mathbf{H} \right] + F_2^4 \mathbf{q}^2 \left(\mathbf{q} \mathbf{p} - \frac{z e}{c} \mathbf{q} \mathbf{A} \right)^2 + m B_{10}, \end{aligned} \quad (\text{A37})$$

where

$$mB_{10} = a_2 \mathbf{q} \boldsymbol{\sigma} q^m \left(p_m - \frac{z e}{c} A_m \right) \cdot \sigma_{m'} \left(p_{m'} - \frac{z e}{c} A_{m'} \right) + a_3 \sigma_m \mathbf{q} \boldsymbol{\sigma} \left(p_m - \frac{z e}{c} A_m \right) \cdot q^{m'} \left(p_{m'} - \frac{z e}{c} A_{m'} \right). \quad (\text{A38})$$

Taking properties (A1) into account, we simplify the first term in Eq. (A38):

$$\begin{aligned} a_2 \mathbf{q} \boldsymbol{\sigma} q^m \left(p_m - \frac{z e}{c} A_m \right) \cdot \sigma_{m'} \left(p_{m'} - \frac{z e}{c} A_{m'} \right) &= a_2 q^l \sigma_l \sigma_{m'} q^m \left(p_m - \frac{z e}{c} A_m \right) \cdot \left(p_{m'} - \frac{z e}{c} A_{m'} \right) = \\ &= a_2 q^{m'} q^m \left(p_m - \frac{z e}{c} A_m \right) \left(p_{m'} - \frac{z e}{c} A_{m'} \right) + a_2 i \varepsilon_{lm'k} q^l q^m \sigma_k \left(p_m - \frac{z e}{c} A_m \right) \left(p_{m'} - \frac{z e}{c} A_{m'} \right) \end{aligned} \quad (\text{A39})$$

and the second term in Eq. (A38):

$$\begin{aligned} a_3 \sigma_m \mathbf{q} \boldsymbol{\sigma} \left(p_m - \frac{z e}{c} A_m \right) \cdot q^{m'} \left(p_{m'} - \frac{z e}{c} A_{m'} \right) &= a_3 q^l q^{m'} \sigma_m \sigma_l \left(p_m - \frac{z e}{c} A_m \right) \cdot \left(p_{m'} - \frac{z e}{c} A_{m'} \right) = \\ &= a_3 q^m q^{m'} \left(p_m - \frac{z e}{c} A_m \right) \left(p_{m'} - \frac{z e}{c} A_{m'} \right) + a_3 i \varepsilon_{mlk} q^l q^{m'} \sigma_k \left(p_m - \frac{z e}{c} A_m \right) \left(p_{m'} - \frac{z e}{c} A_{m'} \right). \end{aligned} \quad (\text{A40})$$

We find summation of these two terms:

$$mB_{10} = (a_2 + a_3) \left(\mathbf{q} \mathbf{p} - \frac{z e}{c} \mathbf{q} \mathbf{A} \right)^2 + m \bar{B}_{10}, \quad (\text{A41})$$

where

$$m \bar{B}_{10} = a_2 i \varepsilon_{lm'k} q^l q^m \sigma_k \left(p_m - \frac{z e}{c} A_m \right) \left(p_{m'} - \frac{z e}{c} A_{m'} \right) + a_3 i \varepsilon_{mlk} q^l q^{m'} \sigma_k \left(p_m - \frac{z e}{c} A_m \right) \left(p_{m'} - \frac{z e}{c} A_{m'} \right). \quad (\text{A42})$$

Now we rewrite the found solution (A37) as

$$mB_1 = a_1 \left[\left(\mathbf{p}_i - \frac{z_i e}{c} \mathbf{A}_i \right)^2 - \frac{z_i e}{c} \boldsymbol{\sigma} \mathbf{H} \right] + (F_2^4 \mathbf{q}^2 + a_2 + a_3) \left(\mathbf{q} \mathbf{p} - \frac{z e}{c} \mathbf{q} \mathbf{A} \right)^2 + m \bar{B}_{10}. \quad (\text{A43})$$

4. Calculation of \bar{B}_{10}

Let us rewrite Eq. (A42):

$$m \bar{B}_{10} = a_2 i \varepsilon_{lm'k} q^l q^m \sigma_k \left(p_m - \frac{z e}{c} A_m \right) \left(p_{m'} - \frac{z e}{c} A_{m'} \right) + a_3 i \varepsilon_{mlk} q^l q^{m'} \sigma_k \left(p_m - \frac{z e}{c} A_m \right) \left(p_{m'} - \frac{z e}{c} A_{m'} \right).$$

We find summations:

$$\begin{aligned} i a_2 \varepsilon_{lm'k} q^l q^m \sigma_k p_m p_{m'} + i a_3 \varepsilon_{mlk} q^l q^{m'} \sigma_k p_m p_{m'} &= i (a_2 - a_3) \varepsilon_{lm'k} \sigma_k q^l q^m p_m p_{m'}, \\ i a_2 \varepsilon_{lm'k} q^l q^m \sigma_k \frac{z e}{c} A_m \frac{z e}{c} A_{m'} + i a_3 \varepsilon_{mlk} q^l q^{m'} \sigma_k \frac{z e}{c} A_m \frac{z e}{c} A_{m'} &= i (a_2 - a_3) \frac{z^2 e^2}{c^2} \varepsilon_{lm'k} \sigma_k q^l q^m A_m A_{m'}. \end{aligned} \quad (\text{A44})$$

Now let us consider term:

$$i a_2 \varepsilon_{lm'k} q^l q^m \sigma_k \left[p_m \frac{z e}{c} A_{m'} + \frac{z e}{c} A_m p_{m'} \right] = i \frac{z e}{c} a_2 \varepsilon_{lm'k} q^l q^m \sigma_k \left[-i \hbar \frac{dA_{m'}}{dx_m} + A_{m'} p_m + A_m p_{m'} \right] \quad (\text{A45})$$

and we obtain:

$$\begin{aligned} i a_2 \varepsilon_{lm'k} q^l q^m \sigma_k \left[p_m \frac{z e}{c} A_{m'} + \frac{z e}{c} A_m p_{m'} \right] + i a_3 \varepsilon_{mlk} q^l q^{m'} \sigma_k \left[p_m \frac{z e}{c} A_{m'} + \frac{z e}{c} A_m p_{m'} \right] &= \\ = i \frac{z e}{c} \varepsilon_{lm'k} q^l q^m \sigma_k \left[-i \hbar \left(a_2 \frac{dA_{m'}}{dx_m} - a_3 \frac{dA_m}{dx_{m'}} \right) + (a_2 - a_3) (A_{m'} p_m + A_m p_{m'}) \right]. \end{aligned} \quad (\text{A46})$$

Now we find the final solution for \bar{B}_{10} , performing summation in (A44)–(A46):

$$m\bar{B}_{10} = i\varepsilon_{lm'k}\sigma_k q^l q^m \left\{ (a_2 - a_3) \left(p_m p_{m'} - \frac{ze}{c} (A_{m'} p_m + A_m p_{m'}) + \frac{z^2 e^2}{c^2} A_m A_{m'} \right) + \frac{i\hbar ze}{c} \left[a_2 \frac{dA_{m'}}{dx_m} - a_3 \frac{dA_m}{dx_{m'}} \right] \right\}. \quad (\text{A47})$$

We write the found solutions for the coefficients a_1 , a_2 and a_3 :

$$a_1 = (F_1^2 - F_2^2 \mathbf{q}^2)^2 - F_1^2 F_2^2 (q^4)^2, \quad a_2 = F_2^2 [F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2], \quad a_3 = F_2^2 [F_1^2 - F_1 F_2 q^4 - F_2^2 \mathbf{q}^2]. \quad (\text{A48})$$

Appendix B: Operator of emission of bremsstrahlung photons

In approximation of $A_0 = 0$, and using (26)

$$f(|\mathbf{q}|) = F_1 + F_1^2 + F_2^2 \mathbf{q}^2,$$

we rewrite equation (28) as

$$i\hbar \left\{ F_1^3 (1 + F_1) - F_1 F_2^2 \mathbf{q}^2 - F_2^4 \mathbf{q}^4 \right\} \frac{\partial \varphi}{\partial t} = h_0 \cdot \varphi + h_\gamma \varphi, \quad (\text{B1})$$

where

$$\begin{aligned} h_0 &= i c F_2 [b_1 q^m + b_2 \varepsilon_{mjl} q^j \sigma_l] \mathbf{p}_m + mc^2 b_3 + \frac{1}{m} \left\{ a_1 \mathbf{p}_i^2 + (F_2^4 \mathbf{q}^2 + a_2 + a_3) (\mathbf{qp})^2 + i \varepsilon_{lm'k} \sigma_k q^l q^m (a_2 - a_3) p_m p_{m'} \right\}, \\ h_\gamma &= -i c F_2 \left\{ b_1 q^m + b_2 \varepsilon_{mjl} q^j \sigma_l \right\} \frac{ze}{c} \mathbf{A}_m + \\ &+ \frac{1}{m} \left\{ a_1 \left[\left(-\frac{ze}{c} (\mathbf{pA} + \mathbf{Ap}) \right) + \frac{z_i^2 e^2}{c^2} \mathbf{A}^2 - \frac{z_i e}{c} \boldsymbol{\sigma} \mathbf{H} \right] + (F_2^4 \mathbf{q}^2 + a_2 + a_3) \left[-2(\mathbf{qp}) \frac{ze}{c} (\mathbf{qA}) + \frac{z^2 e^2}{c^2} (\mathbf{qA})^2 \right] + \right. \\ &\left. + i \varepsilon_{lm'k} \sigma_k q^l q^m \left[(a_2 - a_3) \left(-\frac{ze}{c} (A_{m'} p_m + A_m p_{m'}) + \frac{z^2 e^2}{c^2} A_m A_{m'} \right) + \frac{i\hbar ze}{c} \left(a_2 \frac{dA_{m'}}{dx_m} - a_3 \frac{dA_m}{dx_{m'}} \right) \right] \right\}. \end{aligned} \quad (\text{B2})$$

One can separate explicitly terms with interacting potential in hamiltonian h_0 . We calculate:

$$\begin{aligned} &i c F_2 [b_1 q^m + b_2 \varepsilon_{mjl} q^j \sigma_l] \mathbf{p}_m + mc^2 b_3 = \\ &= i c F_2 \left\{ \left[F_1^2 (1 - F_1) + F_1 F_2 (3F_1 - 1) q^4 - F_2^2 (F_1 + F_2 q^4) \mathbf{q}^2 \right] q^m + \right. \\ &+ i \left[2 F_1^2 + F_1 F_2 (3F_1 - 1) q^4 - F_2^2 (2 + F_2 q^4) \mathbf{q}^2 \right] \varepsilon_{mjl} q^j \sigma_l \left. \right\} \mathbf{p}_m + mc^2 \left[F_1^2 (1 - F_1^2) - (1 - 2 F_1^2) F_2^2 \mathbf{q}^2 - F_2^4 \mathbf{q}^4 \right] + \\ &- i \frac{2F_1 F_2}{mc} \left\{ F_1 (F_1 + F_2 q^4) q^m + i (F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2) \varepsilon_{mjl} q^j \sigma_l \right\} [ze A_0 + V(\mathbf{r})] \mathbf{p}_m + \\ &+ \left(F_1^3 (1 + F_1) + F_1 F_2^2 (3 - 2F_1) \mathbf{q}^2 + F_2^4 \mathbf{q}^4 \right) [ze A_0 + V(\mathbf{r})] \end{aligned} \quad (\text{B3})$$

and obtain:

$$\begin{aligned} h_0 &= \frac{a_1 \mathbf{p}_i^2}{m} + \left(F_1^3 (1 + F_1) + F_1 F_2^2 (3 - 2F_1) \mathbf{q}^2 + F_2^4 \mathbf{q}^4 \right) [ze A_0 + V(\mathbf{r})] + \\ &+ mc^2 \left[F_1^2 (1 - F_1^2) - (1 - 2 F_1^2) F_2^2 \mathbf{q}^2 - F_2^4 \mathbf{q}^4 \right] - \\ &- i \frac{2F_1 F_2}{mc} \left\{ F_1 (F_1 + F_2 q^4) q^m + i (F_1^2 + F_1 F_2 q^4 - F_2^2 \mathbf{q}^2) \varepsilon_{mjl} q^j \sigma_l \right\} [ze A_0 + V(\mathbf{r})] \mathbf{p}_m + \\ &+ \frac{1}{m} \left\{ (F_2^4 \mathbf{q}^2 + a_2 + a_3) (\mathbf{qp})^2 + i \varepsilon_{lm'k} \sigma_k q^l q^m (a_2 - a_3) p_m p_{m'} \right\} + \\ &+ i c F_2 \left\{ \left[F_1^2 (1 - F_1) + F_1 F_2 (3F_1 - 1) q^4 - F_2^2 (F_1 + F_2 q^4) \mathbf{q}^2 \right] q^m + \right. \\ &+ i \left[2 F_1^2 + F_1 F_2 (3F_1 - 1) q^4 - F_2^2 (2 + F_2 q^4) \mathbf{q}^2 \right] \varepsilon_{mjl} q^j \sigma_l \left. \right\} \mathbf{p}_m. \end{aligned} \quad (\text{B4})$$

In operator of emission we separate term, corresponding to old our formalism in Ref. [28], where virtual photons were not included into analysis. From Eq. (B2) we obtain:

$$h_\gamma = h_{\gamma 0} + h_{\gamma 1}, \quad (\text{B5})$$

where

$$\begin{aligned}
h_{\gamma 0} &= \frac{a_1}{m} \left[-\frac{z_i e}{c} (-i\hbar \mathbf{div} \mathbf{A} + 2 \mathbf{A} \mathbf{p}) + \frac{z_i^2 e^2}{c^2} \mathbf{A}^2 - \frac{z_i e}{c} \boldsymbol{\sigma} \mathbf{H} \right], \\
h_{\gamma 1} &= -i c F_2 \left\{ b_1 q^m + b_2 \varepsilon_{mjl} q^j \sigma_l \right\} \frac{ze}{c} \mathbf{A}_m + \frac{1}{m} \left\{ (F_2^4 q^2 + a_2 + a_3) \left[-2(\mathbf{qp}) \frac{ze}{c} (\mathbf{qA}) + \frac{z^2 e^2}{c^2} (\mathbf{qA})^2 \right] + \right. \\
&\quad \left. + i \varepsilon_{lm'k} \sigma_k q^l q^m \left[(a_2 - a_3) \left(-\frac{ze}{c} (A_{m'} p_m + A_m p_{m'}) + \frac{z^2 e^2}{c^2} A_m A_{m'} \right) + \frac{i\hbar ze}{c} \left(a_2 \frac{dA_{m'}}{dx_m} - a_3 \frac{dA_m}{dx_{m'}} \right) \right] \right\}. \tag{B6}
\end{aligned}$$

The first term $h_{\gamma 0}$ is operator of emission in old formalism in Ref. [28], without inclusion of the virtual photons and possibility to consider internal structure of the scattered proton. The second term $h_{\gamma 1}$ is correction of old operator of emission $h_{\gamma 0}$, which is appeared after inclusion of virtual photons to formalism.

Appendix C: Elastic scattering of virtual photon on proton (scattered off nucleus)

Let us calculate hamiltonians h_0 and h_γ for the elastic scattering of the virtual photon off proton. From Eq. (36) we obtain:

$$\begin{aligned}
h_0 &= \frac{a_1 \mathbf{p}^2}{m} + \left(F_1^3 (1 + F_1) + F_1 F_2^2 (3 - 2F_1) Q^2 + F_2^4 Q^4 \right) V(\mathbf{r}) + \\
&\quad + mc^2 \left[F_1^2 (1 - F_1^2) - (1 - 2F_1^2) F_2^2 Q^2 - F_2^4 Q^4 \right] - \\
&\quad - i \frac{2F_1 F_2}{mc} \left\{ F_1^2 q^m + i (F_1^2 - F_2^2 Q^2) \varepsilon_{mjl} q^j \sigma_l \right\} V(\mathbf{r}) \mathbf{p}_m + \\
&\quad + \frac{1}{m} (F_2^4 Q^2 + 2a_2) (\mathbf{qp})^2 + i c F_2 \left\{ \left[F_1^2 (1 - F_1) - F_2^2 F_1 Q^2 \right] q^m + i \left[2F_1^2 - 2F_2^2 Q^2 \right] \varepsilon_{mjl} q^j \sigma_l \right\} \mathbf{p}_m. \tag{C1}
\end{aligned}$$

$h_{\gamma 0}$ is not changed. From Eq. (37) for $h_{\gamma 1}$ we obtain:

$$\begin{aligned}
h_{\gamma 1} &= -i ze F_2 \left\{ b_1 \mathbf{qA} + b_2 \varepsilon_{mjl} q^j \sigma_l \mathbf{A}_m \right\} + \\
&\quad + \frac{1}{m} \left\{ (F_2^4 Q^2 + 2a_2) \left[-2(\mathbf{qp}) \frac{ze}{c} (\mathbf{qA}) + \frac{z^2 e^2}{c^2} (\mathbf{qA})^2 \right] + i \varepsilon_{lm'k} \sigma_k q^l q^m \frac{i\hbar ze}{c} a_2 \left(\frac{dA_{m'}}{dx_m} - \frac{dA_m}{dx_{m'}} \right) \right\} = \\
&= -i ze F_2 \left\{ b_1 \mathbf{qA} + b_2 \varepsilon_{mjl} q^j \sigma_l \mathbf{A}_m \right\} + \\
&\quad + \frac{ze}{mc} \left\{ F_2^2 (2F_1^2 - F_2^2 Q^2) \left[-2(\mathbf{qp}) (\mathbf{qA}) + \frac{ze}{c} (\mathbf{qA})^2 \right] - \hbar a_2 \varepsilon_{lm'k} \sigma_k q^l q^m \left(\frac{dA_{m'}}{dx_m} - \frac{dA_m}{dx_{m'}} \right) \right\}. \tag{C2}
\end{aligned}$$

For the elastic scattering we use kinematic relation (41), and unperturbed hamiltonian h_0 from Eq. (C1) is simplified as

$$\begin{aligned}
h_0 &= \frac{a_1 \mathbf{p}^2}{m} + \left(F_1^3 (1 + F_1) + F_1 F_2^2 (3 - 2F_1) Q^2 + F_2^4 Q^4 + i \frac{F_1^3 F_2 Q^2}{mc} \right) [ze A_0 + V(\mathbf{r})] + \\
&\quad + mc^2 \left[F_1^2 (1 - F_1^2) - (1 - 2F_1^2) F_2^2 Q^2 - F_2^4 Q^4 \right] + \frac{Q^4}{4m} (F_2^4 Q^2 + 2a_2) - \frac{i c F_2 Q^2}{2} \left[F_1^2 (1 - F_1) - F_2^2 F_1 Q^2 \right] + \\
&\quad + \left\{ \frac{2F_1 F_2}{mc} [ze A_0 + V(\mathbf{r})] - 2c F_2 \right\} (F_1^2 - F_2^2 Q^2) \varepsilon_{mjl} q^j \sigma_l \mathbf{p}_m. \tag{C3}
\end{aligned}$$

Operator of emission $h_{\gamma 1}$ from Eq. (C2) is transformed as

$$\begin{aligned}
h_{\gamma 1} &= -i ze F_2 \left\{ b_1 \mathbf{qA} + b_2 \varepsilon_{mjl} q^j \sigma_l \mathbf{A}_m \right\} + \\
&\quad + \frac{ze}{mc} \left\{ F_2^2 (2F_1^2 - F_2^2 Q^2) \left[Q^2 (\mathbf{qA}) + \frac{ze}{c} (\mathbf{qA})^2 \right] - \hbar a_2 \varepsilon_{lm'k} \sigma_k q^l q^m \left(\frac{dA_{m'}}{dx_m} - \frac{dA_m}{dx_{m'}} \right) \right\}. \tag{C4}
\end{aligned}$$

We assume that last term, having Plank constant, is smaller essentially in comparison with other terms. In such a case, we neglect by such a term and obtain:

$$h_{\gamma 1} = -i ze F_2 \left\{ b_1 \mathbf{qA} + b_2 \varepsilon_{mjl} q^j \sigma_l \mathbf{A}_m \right\} + \frac{ze}{mc} F_2^2 (2F_1^2 - F_2^2 Q^2) \left[Q^2 (\mathbf{qA}) + \frac{ze}{c} (\mathbf{qA})^2 \right]. \tag{C5}$$

For the QED representation (42) for the vector potential of the electromagnetic field, we introduce new angle φ_{ph} between vectors \mathbf{q} and \mathbf{A} for determination of the scalar multiplication of them:

$$\mathbf{q}\mathbf{A} = qA \cdot \sin \varphi_{ph}. \quad (\text{C6})$$

We find properties:

$$\mathbf{q}\mathbf{A} = 2Q \sqrt{\frac{\pi\hbar c^2}{w_{ph}}} e^{-i\mathbf{k}_{ph}\mathbf{r}} \sin \varphi_{ph}, \quad (\mathbf{q}\mathbf{A})^2 = 4Q^2 \frac{\pi\hbar c^2}{w_{ph}} e^{-2i\mathbf{k}_{ph}\mathbf{r}} \sin^2 \varphi_{ph}, \quad (\text{C7})$$

and we calculate operator of bremsstrahlung emission, related with the virtual photons:

$$\begin{aligned} h_{\gamma 1} = & ze F_2 \sqrt{\frac{\pi\hbar c^2}{w_{ph}}} e^{-i\mathbf{k}_{ph}\mathbf{r}} \left\{ 2Q \sin \varphi_{ph} \left[-i b_1 + \frac{F_2 Q^2}{mc} (2F_1^2 - F_2^2 Q^2) \right] - \right. \\ & \left. - i \sqrt{2} b_2 \varepsilon_{mjl} q^j \sigma_l \sum_{\alpha=1,2} \mathbf{e}_m^{(\alpha),*} + \frac{4ze}{mc} \sqrt{\frac{\pi\hbar}{w_{ph}}} F_2 Q^2 (2F_1^2 - F_2^2 Q^2) e^{-i\mathbf{k}_{ph}\mathbf{r}} \sin^2 \varphi_{ph} \right\}. \end{aligned} \quad (\text{C8})$$

Appendix D: Matrix element of emission of bremsstrahlung photons

1. Calculations of p_{eq} , $p_{mag,2}$ and $p_{mag,2}$

Taking into account Eqs. (45) and (46) for the operator of emission, we obtain:

$$\begin{aligned} F_{fi,0} &= \langle k_f | h_{\gamma 0} | k_i \rangle = \\ &= \left\langle k_f \left| Z_{\text{eff}} \frac{e}{mc} \sqrt{\frac{2\pi\hbar c^2}{w}} \sum_{\alpha=1,2} e^{-i\mathbf{k}\mathbf{r}} \left(i \mathbf{e}^{(\alpha)} \nabla - \frac{1}{2} \boldsymbol{\sigma} \cdot [\nabla \times \mathbf{e}^{(\alpha)}] + i \frac{1}{2} \boldsymbol{\sigma} \cdot [\mathbf{k} \times \mathbf{e}^{(\alpha)}] \right) \right| k_i \right\rangle = \\ &= Z_{\text{eff}} \frac{e}{mc} \sqrt{\frac{2\pi\hbar c^2}{w}} \sum_{\alpha=1,2} \left\langle k_f \left| e^{-i\mathbf{k}\mathbf{r}} \left(i \mathbf{e}^{(\alpha)} \nabla - \frac{1}{2} \boldsymbol{\sigma} \cdot [\mathbf{e}^{(\alpha)} \times \nabla] + i \frac{1}{2} \boldsymbol{\sigma} \cdot [\mathbf{k} \times \mathbf{e}^{(\alpha)}] \right) \right| k_i \right\rangle = \\ &= Z_{\text{eff}} \frac{e}{mc} \sqrt{\frac{2\pi\hbar c^2}{w}} \left\{ i \sum_{\alpha=1,2} \mathbf{e}^{(\alpha)} \langle k_f | e^{-i\mathbf{k}\mathbf{r}} \nabla | k_i \rangle - \frac{1}{2} \sum_{\alpha=1,2} \langle k_f | e^{-i\mathbf{k}\mathbf{r}} \boldsymbol{\sigma} \cdot [\mathbf{e}^{(\alpha)} \times \nabla] | k_i \rangle + \right. \\ &\quad \left. + i \frac{1}{2} \sum_{\alpha=1,2} [\mathbf{k} \times \mathbf{e}^{(\alpha)}] \langle k_f | e^{-i\mathbf{k}\mathbf{r}} \boldsymbol{\sigma} | k_i \rangle \right\}, \end{aligned}$$

$$\begin{aligned} F_{fi,1} &= \langle k_f | h_{\gamma 1} | k_i \rangle = \\ &= \left\langle k_f \left| ze F_2 \sqrt{\frac{\pi\hbar c^2}{w_{ph}}} e^{-i\mathbf{k}_{ph}\mathbf{r}} \left\{ 2Q \sin \varphi_{ph} \left[-i b_1 + \frac{F_2 Q^2}{mc} (2F_1^2 - F_2^2 Q^2) \right] - \right. \right. \right. \\ &\quad \left. \left. - i \sqrt{2} b_2 \varepsilon_{mjl} q^j \sigma_l \sum_{\alpha=1,2} \mathbf{e}_m^{(\alpha),*} + \frac{4ze}{mc} \sqrt{\frac{\pi\hbar}{w_{ph}}} F_2 Q^2 (2F_1^2 - F_2^2 Q^2) e^{-i\mathbf{k}_{ph}\mathbf{r}} \sin^2 \varphi_{ph} \right\} \right| k_i \right\rangle = \\ &= ze F_2 \sqrt{\frac{\pi\hbar c^2}{w_{ph}}} \left\{ 2Q \sin \varphi_{ph} \langle k_f | e^{-i\mathbf{k}_{ph}\mathbf{r}} \left[-i b_1 + \frac{F_2 Q^2}{mc} (2F_1^2 - F_2^2 Q^2) \right] | k_i \rangle - \right. \\ &\quad \left. - i \sqrt{2} \varepsilon_{mjl} q^j \sigma_l \sum_{\alpha=1,2} \mathbf{e}_m^{(\alpha),*} \langle k_f | e^{-i\mathbf{k}_{ph}\mathbf{r}} b_2 | k_i \rangle + \frac{4ze}{mc} \sqrt{\frac{\pi\hbar}{w_{ph}}} F_2 Q^2 (2F_1^2 - F_2^2 Q^2) \sin^2 \varphi_{ph} \langle k_f | e^{-2i\mathbf{k}_{ph}\mathbf{r}} | k_i \rangle \right\} \end{aligned}$$

or

$$\begin{aligned} F_{fi,0} &= \langle k_f | h_{\gamma 0} | k_i \rangle = Z_{\text{eff}} \frac{e}{mc} \sqrt{\frac{2\pi\hbar c^2}{w}} \{ p_{el} + p_{mag,1} + p_{mag,2} \}, \\ F_{fi,1} &= \langle k_f | h_{\gamma 1} | k_i \rangle = Z_{\text{eff}} e F_2 \sqrt{\frac{\pi\hbar c^2}{w_{ph}}} \{ p_{q,1} + p_{q,2} + p_{q,3} \}, \end{aligned} \quad (\text{D1})$$

where

$$\begin{aligned}
p_{\text{el}} &= i \sum_{\alpha=1,2} \mathbf{e}^{(\alpha)} \left\langle k_f \left| e^{-i \mathbf{k} \mathbf{r}} \nabla \right| k_i \right\rangle, \\
p_{\text{mag},1} &= \frac{1}{2} \sum_{\alpha=1,2} \left\langle k_f \left| e^{-i \mathbf{k} \mathbf{r}} \boldsymbol{\sigma} \cdot [\mathbf{e}^{(\alpha)} \times \nabla] \right| k_i \right\rangle, \\
p_{\text{mag},2} &= -i \frac{1}{2} \sum_{\alpha=1,2} [\mathbf{k} \times \mathbf{e}^{(\alpha)}] \left\langle k_f \left| e^{-i \mathbf{k} \mathbf{r}} \boldsymbol{\sigma} \right| k_i \right\rangle,
\end{aligned} \tag{D2}$$

$$\begin{aligned}
p_{\text{q},1} &= 2Q \sin \varphi_{ph} \left\langle k_f \left| e^{-i \mathbf{k}_{ph} \mathbf{r}} \left[-i b_1 + \frac{F_2 Q^2}{mc} (2F_1^2 - F_2^2 Q^2) \right] \right| k_i \right\rangle, \\
p_{\text{q},2} &= -i \sqrt{2} \varepsilon_{mjl} q^j \sigma_l \sum_{\alpha=1,2} \mathbf{e}_m^{(\alpha),*} \left\langle k_f \left| e^{-i \mathbf{k}_{ph} \mathbf{r}} b_2 \right| k_i \right\rangle, \\
p_{\text{q},3} &= \frac{4ze}{mc} \sqrt{\frac{\pi \hbar}{w_{ph}}} F_2 Q^2 (2F_1^2 - F_2^2 Q^2) \sin^2 \varphi_{ph} \left\langle k_f \left| e^{-2i \mathbf{k}_{ph} \mathbf{r}} \right| k_i \right\rangle.
\end{aligned} \tag{D3}$$

2. Calculations of $p_{\text{q},1}$, $p_{\text{q},2}$ and $p_{\text{q},3}$, and averaging over polarizations of virtual photons

At $A_0 = 0$

$$\begin{aligned}
b_1 &= F_1^2(1 - F_1) - F_1 F_2^2 Q^2 - \frac{2F_1^3}{mc^2} V(\mathbf{r}), \\
b_2 &= i \left[2F_1^2 - 2F_2^2 Q^2 - \frac{2F_1}{mc^2} (F_1^2 - F_2^2 Q^2) V(\mathbf{r}) \right].
\end{aligned} \tag{D4}$$

Substitute such solutions to Eqs. (D3) and obtain:

$$\begin{aligned}
p_{\text{q},1} &= 2Q \sin \varphi_{ph} \left\{ -i \left[F_1^2(1 - F_1) - F_1 F_2^2 Q^2 \right] + \frac{F_2 Q^2}{mc} (2F_1^2 - F_2^2 Q^2) \right\} \left\langle k_f \left| e^{-i \mathbf{k}_{ph} \mathbf{r}} \right| k_i \right\rangle + \\
&\quad + 2Q \sin \varphi_{ph} i \frac{2F_1^3}{mc^2} \left\langle k_f \left| e^{-i \mathbf{k}_{ph} \mathbf{r}} V(\mathbf{r}) \right| k_i \right\rangle, \\
p_{\text{q},2} &= 2(F_1^2 - F_2^2 Q^2) \sqrt{2} \varepsilon_{mjl} q^j \sigma_l \sum_{\alpha=1,2} \mathbf{e}_m^{(\alpha),*} \left\langle k_f \left| e^{-i \mathbf{k}_{ph} \mathbf{r}} \right| k_i \right\rangle - \\
&\quad - 2(F_1^2 - F_2^2 Q^2) \frac{\sqrt{2} F_1}{mc^2} \varepsilon_{mjl} q^j \sigma_l \sum_{\alpha=1,2} \mathbf{e}_m^{(\alpha),*} \left\langle k_f \left| e^{-i \mathbf{k}_{ph} \mathbf{r}} V(\mathbf{r}) \right| k_i \right\rangle.
\end{aligned} \tag{D5}$$

We assume that there is no way to fix direction of polarization of the virtual photons (concerning to vectors of polarization of the bremsstrahlung photons) experimentally. So, we have to integrate the matrix elements $p_{\text{q},i}$ over all such a possible directions (i.e. we integrate over angle φ_{ph} , $i = 1, 2, 3$):

$$\tilde{p}_{\text{q},i} = N \cdot \int_0^\pi p_{\text{q},i} d\varphi_{ph}, \quad N = \frac{1}{\pi}. \tag{D6}$$

Taking into account that

$$\int_0^\pi \sin \varphi_{ph} d\varphi_{ph} = 2, \quad \int_0^\pi d\varphi_{ph} = \pi, \quad \int_0^\pi \sin^2 \varphi_{ph} d\varphi_{ph} = \frac{\pi}{2}, \tag{D7}$$

from (D5) and (D3) we obtain:

$$\begin{aligned}
\tilde{p}_{\text{q},1} &= i A_1(Q, F_1, F_2) \left\langle k_f \left| e^{-i \mathbf{k}_{ph} \mathbf{r}} \right| k_i \right\rangle + i B_1(Q, F_1, F_2) \left\langle k_f \left| e^{-i \mathbf{k}_{ph} \mathbf{r}} V(\mathbf{r}) \right| k_i \right\rangle, \\
\tilde{p}_{\text{q},2} &= i A_2(Q, F_1, F_2) \left\langle k_f \left| e^{-i \mathbf{k}_{ph} \mathbf{r}} \right| k_i \right\rangle + i B_2(Q, F_1, F_2) \left\langle k_f \left| e^{-i \mathbf{k}_{ph} \mathbf{r}} V(\mathbf{r}) \right| k_i \right\rangle, \\
\tilde{p}_{\text{q},3} &= i A_3(Q, F_1, F_2) \left\langle k_f \left| e^{-i 2\mathbf{k}_{ph} \mathbf{r}} \right| k_i \right\rangle,
\end{aligned} \tag{D8}$$

where

$$\begin{aligned}
A_1(Q, F_1, F_2) &= -\frac{4Q}{\pi} \left\{ \left[F_1^2 (1 - F_1) - F_1 F_2^2 Q^2 \right] + i \frac{F_2 Q^2}{\pi m c} (2F_1^2 - F_2^2 Q^2) \right\}, \\
B_1(Q, F_1, F_2) &= 8Q \frac{F_1^3}{\pi m c^2}, \\
A_2(Q, F_1, F_2) &= -i 2 (F_1^2 - F_2^2 Q^2) \sqrt{2} \varepsilon_{mjl} q^j \sigma_l \sum_{\alpha=1,2} \mathbf{e}_m^{(\alpha),*}, \\
B_2(Q, F_1, F_2) &= i 2 (F_1^2 - F_2^2 Q^2) \frac{\sqrt{2} F_1}{m c^2} \varepsilon_{mjl} q^j \sigma_l \sum_{\alpha=1,2} \mathbf{e}_m^{(\alpha),*}, \\
A_3(Q, F_1, F_2) &= -i \frac{2ze}{mc} \sqrt{\frac{\pi \hbar}{w_{\text{ph}}}} F_2 Q^2 (2F_1^2 - F_2^2 Q^2).
\end{aligned} \tag{D9}$$

Appendix E: Calculations of matrix elements of emission in multipolar expansion

We shall calculate the following matrix elements:

$$\left\langle k_f \left| e^{-i\mathbf{k}\mathbf{r}} \right| k_i \right\rangle_{\mathbf{r}} = \int \varphi_f^*(\mathbf{r}) e^{-i\mathbf{k}\mathbf{r}} \varphi_i(\mathbf{r}) d\mathbf{r}, \quad \left\langle k_f \left| e^{-i\mathbf{k}\mathbf{r}} \frac{\partial}{\partial \mathbf{r}} \right| k_i \right\rangle_{\mathbf{r}} = \int \varphi_f^*(\mathbf{r}) e^{-i\mathbf{k}\mathbf{r}} \frac{\partial}{\partial \mathbf{r}} \varphi_i(\mathbf{r}) d\mathbf{r}. \tag{E1}$$

1. Expansion of the vector potential \mathbf{A} by multipoles

Let us expand the vectorial potential \mathbf{A} of electromagnetic field by multipolar terms. According to Ref. [61] [see (2.106), p. 58], in the spherical symmetric approximation we have:

$$\xi_\mu e^{i\mathbf{k}\mathbf{r}} = \mu \sqrt{2\pi} \sum_{l=1} (2l+1)^{1/2} i^l \cdot \left[\mathbf{A}_{l\mu}(\mathbf{r}, M) + i\mu \mathbf{A}_{l\mu}(\mathbf{r}, E) \right], \tag{E2}$$

where (see [61], (2.73) in p. 49, (2.80) in p. 51)

$$\begin{aligned}
\mathbf{A}_{l\mu}(\mathbf{r}, M) &= j_l(kr) \mathbf{T}_{l,\mu}(\mathbf{n}_r), \\
\mathbf{A}_{l\mu}(\mathbf{r}, E) &= \sqrt{\frac{l+1}{2l+1}} j_{l-1}(kr) \mathbf{T}_{l-1,\mu}(\mathbf{n}_r) - \sqrt{\frac{l}{2l+1}} j_{l+1}(kr) \mathbf{T}_{l+1,\mu}(\mathbf{n}_r).
\end{aligned} \tag{E3}$$

Here, $\mathbf{A}_{l\mu}(\mathbf{r}, M)$ and $\mathbf{A}_{l\mu}(\mathbf{r}, E)$ are *magnetic* and *electric multipoles*, $j_l(kr)$ is *spherical Bessel function of order l* , $\mathbf{T}_{l\mu,\mu}(\mathbf{n}_r)$ are *vector spherical harmonics*. We orient the frame so that axis z be directed along the vector \mathbf{k} (see [61], (2.105) in p. 57). According to [61] (see p. 45), the functions $\mathbf{T}_{l\mu,\mu}(\mathbf{n}_r)$ have the following form ($\xi_0 = 0$):

$$\mathbf{T}_{jl,m}(\mathbf{n}_r) = \sum_{\mu=\pm 1} (l, 1, j | m - \mu, \mu, m) Y_{l,m-\mu}(\mathbf{n}_r) \xi_\mu, \tag{E4}$$

where $(l, 1, j | m - \mu, \mu, m)$ are *Clebsh-Gordon coefficients*, $Y_{lm}(\theta, \varphi)$ are *spherical functions* defined, according to [60] (see p. 119, (28,7)–(28,8)). From eq.app. (E2) one can obtain such a formula (at $\mathbf{e}^{(3)} = 0$):

$$e^{-i\mathbf{k}\mathbf{r}} = \frac{1}{2} \sum_{\mu=\pm 1} \xi_\mu \mu \sqrt{2\pi} \sum_{l=1} (2l+1)^{1/2} (-i)^l \cdot \left[\mathbf{A}_{l\mu}^*(\mathbf{r}, M) - i\mu \mathbf{A}_{l\mu}^*(\mathbf{r}, E) \right]. \tag{E5}$$

2. Spherically symmetric nucleus

Using (E5), for (E2) we find:

$$\begin{aligned}
\left\langle k_f \left| e^{-i\mathbf{k}\mathbf{r}} \right| k_i \right\rangle_{\mathbf{r}} &= \sqrt{\frac{\pi}{2}} \sum_{l_{\text{ph}}=1} (-i)^{l_{\text{ph}}} \sqrt{2l_{\text{ph}}+1} \sum_{\mu=\pm 1} \left[\mu \tilde{p}_{l_{\text{ph}}\mu}^M - i \tilde{p}_{l_{\text{ph}}\mu}^E \right], \\
\left\langle k_f \left| e^{-i\mathbf{k}\mathbf{r}} \frac{\partial}{\partial \mathbf{r}} \right| k_i \right\rangle_{\mathbf{r}} &= \sqrt{\frac{\pi}{2}} \sum_{l_{\text{ph}}=1} (-i)^{l_{\text{ph}}} \sqrt{2l_{\text{ph}}+1} \sum_{\mu=\pm 1} \xi_\mu \mu \times \left[p_{l_{\text{ph}}\mu}^M - i\mu p_{l_{\text{ph}}\mu}^E \right],
\end{aligned} \tag{E6}$$

where

$$p_{l_{\text{ph}}\mu}^M = \int \varphi_f^*(\mathbf{r}) \left(\frac{\partial}{\partial \mathbf{r}} \varphi_i(\mathbf{r}) \right) \mathbf{A}_{l_{\text{ph}}\mu}^*(\mathbf{r}, M) d\mathbf{r}, \quad p_{l_{\text{ph}}\mu}^E = \int \varphi_f^*(\mathbf{r}) \left(\frac{\partial}{\partial \mathbf{r}} \varphi_i(\mathbf{r}) \right) \mathbf{A}_{l_{\text{ph}}\mu}^*(\mathbf{r}, E) d\mathbf{r}, \quad (\text{E7})$$

and

$$\tilde{p}_{l_{\text{ph}}\mu}^M = \xi_\mu \int \varphi_f^*(\mathbf{r}) \varphi_i(\mathbf{r}) \mathbf{A}_{l_{\text{ph}}\mu}^*(\mathbf{r}, M) d\mathbf{r}, \quad \tilde{p}_{l_{\text{ph}}\mu}^E = \xi_\mu \int \varphi_f^*(\mathbf{r}) \varphi_i(\mathbf{r}) \mathbf{A}_{l_{\text{ph}}\mu}^*(\mathbf{r}, E) d\mathbf{r}. \quad (\text{E8})$$

Now we shall calculate components in Eqs. (54). For the first and third items we obtain:

$$\begin{aligned} p_{\text{el}} &= i \sqrt{\frac{\pi}{2}} \sum_{m_i, m_f} \sum_{\mu_i, \mu_f = \pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} \cdot \sum_{l_{\text{ph}}=1} (-i)^{l_{\text{ph}}} \sqrt{2l_{\text{ph}}+1} \cdot \left[p_{l_{\text{ph}}}^M - i p_{l_{\text{ph}}}^E \right], \\ p_{\text{mag}, 2} &= \frac{-i k}{2} \sqrt{\frac{\pi}{2}} \sum_{m_i, m_f} \sum_{\mu_i, \mu_f = \pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} \times \\ &\times \left[-1 + i \left\{ \delta_{\mu_i, +1/2} - \delta_{\mu_i, -1/2} \right\} \right] \cdot \sum_{l_{\text{ph}}=1} (-i)^{l_{\text{ph}}} \sqrt{2l_{\text{ph}}+1} \cdot \left[\tilde{p}_{l_{\text{ph}}}^M - i \tilde{p}_{l_{\text{ph}}}^E \right], \end{aligned} \quad (\text{E9})$$

where

$$p_{l_{\text{ph}}}^M = \sum_{\mu=\pm 1} h_\mu \mu p_{l_{\text{ph}}\mu}^M, \quad p_{l_{\text{ph}}}^E = \sum_{\mu=\pm 1} h_\mu p_{l_{\text{ph}}\mu}^E, \quad \tilde{p}_{l_{\text{ph}}}^M = \sum_{\mu=\pm 1} \mu \tilde{p}_{l_{\text{ph}}\mu}^M, \quad \tilde{p}_{l_{\text{ph}}}^E = \sum_{\mu=\pm 1} \tilde{p}_{l_{\text{ph}}\mu}^E. \quad (\text{E10})$$

Now we shall analyze the second item in Eqs. (54) and find:

$$\begin{aligned} p_{\text{mag}, 1} &= \frac{1}{2} \sum_{m_i, m_f} \sum_{\mu_i, \mu_f = \pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} \cdot \left[\frac{1}{\sqrt{2}} (\xi_{-1} - \xi_{+1}) + \frac{i}{\sqrt{2}} (\xi_{-1} + \xi_{+1}) i \left\{ \delta_{\mu_i, +1/2} - \right. \right. \\ &\left. \left. - \delta_{\mu_i, -1/2} \right\} + \mathbf{e}_z \right] \times \left[\sum_{\mu=\pm 1} h_\mu \xi_\mu^* \times \sqrt{\frac{\pi}{2}} \sum_l (-i)^l \sqrt{2l+1} \sum_{\mu'=\pm 1} \xi_{\mu'} \mu' \times \left[p_{l_{\text{ph}}}^M - i p_{l_{\text{ph}}}^E \right] \right]. \end{aligned} \quad (\text{E11})$$

Taking properties (43) into account, we calculate Eq. (E11):

$$p_{\text{mag}, 1} = -\frac{1}{2} \sqrt{\frac{\pi}{2}} \sum_{m_i, m_f} \sum_{\mu_i, \mu_f = \pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} \sum_{l_{\text{ph}}} (-i)^{l_{\text{ph}}} \sqrt{2l_{\text{ph}}+1} \cdot \sum_{\mu=\pm 1} i h_\mu \mu \left[\mu p_{l_{\text{ph}}\mu}^M - i p_{l_{\text{ph}}\mu}^E \right]. \quad (\text{E12})$$

So, we have found all components in Eqs. (54):

$$\begin{aligned} p_{\text{el}} &= \sqrt{\frac{\pi}{2}} \sum_{l_{\text{ph}}=1} (-i)^{l_{\text{ph}}} \sqrt{2l_{\text{ph}}+1} \cdot \sum_{\mu=\pm 1} h_\mu \cdot \sum_{m_i, m_f} \sum_{\mu_i, \mu_f = \pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} \left[i \mu p_{l_{\text{ph}}\mu}^{M m_i m_f} + p_{l_{\text{ph}}\mu}^{E m_i m_f} \right], \\ p_{\text{mag}, 1} &= \frac{1}{2} \sqrt{\frac{\pi}{2}} \sum_{l_{\text{ph}}=1} (-i)^{l_{\text{ph}}} \sqrt{2l_{\text{ph}}+1} \cdot \sum_{\mu=\pm 1} h_\mu \mu \sum_{m_i, m_f} \sum_{\mu_i, \mu_f = \pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} \left[i \mu p_{l_{\text{ph}}\mu}^{M m_i m_f} + p_{l_{\text{ph}}\mu}^{E m_i m_f} \right], \\ p_{\text{mag}, 2} &= \sqrt{\frac{\pi}{8}} k \sum_{l_{\text{ph}}=1} (-i)^{l_{\text{ph}}} \sqrt{2l_{\text{ph}}+1} \cdot \sum_{\mu=\pm 1} \sum_{m_i, m_f} \sum_{\mu_i, \mu_f = \pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} \times \\ &\times \left[-1 + i \left\{ \delta_{\mu_i, +1/2} - \delta_{\mu_i, -1/2} \right\} \right] \cdot \left[i \mu \tilde{p}_{l_{\text{ph}}\mu}^{M m_i m_f} + \tilde{p}_{l_{\text{ph}}\mu}^{E m_i m_f} \right]. \end{aligned} \quad (\text{E13})$$

3. Matrix element $\tilde{p}_{q,1}$ at spherically symmetric description of nucleus

On the basis of the obtained formalism for the matrix elements above, we write matrix element $\tilde{p}_{q,1}$:

$$\begin{aligned} \tilde{p}_{q,1} &= \sqrt{\frac{\pi}{2}} \sum_{l_{\text{ph}}=1} (-i)^{l_{\text{ph}}} \sqrt{2l_{\text{ph}}+1} \cdot \sum_{\mu=\pm 1} \sum_{m_f, m_i} \sum_{\mu_i, \mu_f = \pm 1/2} C_{l_f m_f 1/2 \mu_f}^{j_f M_f, *} C_{l_i m_i 1/2 \mu_i}^{j_i M_i} \times \\ &\times \left\{ A_1(Q, F_1, F_2) \left[i \mu \tilde{p}_{l_{\text{ph}}\mu}^{M m_i m_f} + \tilde{p}_{l_{\text{ph}}\mu}^{E m_i m_f} \right] + B_1(Q, F_1, F_2) \left[i \mu \tilde{p}_{l_{\text{ph}}\mu}^{M m_i m_f} + \tilde{p}_{l_{\text{ph}}\mu}^{E m_i m_f} \right] \right\}, \end{aligned} \quad (\text{E14})$$

where $\tilde{p}_{l_{\text{ph}}\mu}^M$ and $\tilde{p}_{l_{\text{ph}}\mu}^E$ are defined in Eqs. (E10) and

$$\tilde{p}_{l_{\text{ph}}\mu}^{Mm_i m_f} = \xi_\mu \int \varphi_f^*(\mathbf{r}) \varphi_i(\mathbf{r}) V(\mathbf{r}) \mathbf{A}_{l_{\text{ph}}\mu}^*(\mathbf{r}, M) d\mathbf{r}, \quad \tilde{p}_{l_{\text{ph}}\mu}^{Em_i m_f} = \xi_\mu \int \varphi_f^*(\mathbf{r}) \varphi_i(\mathbf{r}) V(\mathbf{r}) \mathbf{A}_{l_{\text{ph}}\mu}^*(\mathbf{r}, E) d\mathbf{r}. \quad (\text{E15})$$

4. Calculations of components $p_{l_{\text{ph}}\mu}^M$, $p_{l_{\text{ph}}\mu}^E$ and $\tilde{p}_{l_{\text{ph}}\mu}^M$, $\tilde{p}_{l_{\text{ph}}\mu}^E$: case of $l_i \neq 0$

Let us consider a case, when the full nuclear system has $l_i \neq 0$ in the initial state. Using gradient formula, we obtain:

$$\begin{aligned} \frac{\partial}{\partial \mathbf{r}} \varphi_i(\mathbf{r}) &= \frac{\partial}{\partial \mathbf{r}} \left\{ R_i(r) Y_{l_i m_i}(\mathbf{n}_r^i) \right\} = \\ &= \sqrt{\frac{l_i}{2l_i+1}} \left(\frac{dR_i(r)}{dr} + \frac{l_i+1}{r} R_i(r) \right) \mathbf{T}_{l_i l_i-1, m_i}(\mathbf{n}_r^i) - \sqrt{\frac{l_i+1}{2l_i+1}} \left(\frac{dR_i(r)}{dr} - \frac{l_i}{r} R_i(r) \right) \mathbf{T}_{l_i l_i+1, m_i}(\mathbf{n}_r^i). \end{aligned} \quad (\text{E16})$$

Using such a formula, for the magnetic component $p_{l_{\text{ph}}\mu}^M$ $l_i \neq 0$ we have:

$$\begin{aligned} p_{l_{\text{ph}}\mu}^M &= \int_0^{+\infty} dr \int d\Omega r^2 \varphi_f^*(\mathbf{r}) \left(\frac{\partial}{\partial \mathbf{r}} \varphi_i(\mathbf{r}) \right) \mathbf{A}_{l_{\text{ph}}\mu}^*(\mathbf{r}, M) = \\ &= \int_0^{+\infty} dr \int d\Omega r^2 R_f^*(r) Y_{l_f m_f}^*(\mathbf{n}_r^f) \left\{ \sqrt{\frac{l_i}{2l_i+1}} \left(\frac{dR_i(r)}{dr} + \frac{l_i+1}{r} R_i(r) \right) \mathbf{T}_{l_i l_i-1, m_i}(\mathbf{n}_r^i) - \right. \\ &\quad \left. - \sqrt{\frac{l_i+1}{2l_i+1}} \left(\frac{dR_i(r)}{dr} - \frac{l_i}{r} R_i(r) \right) \mathbf{T}_{l_i l_i+1, m_i}(\mathbf{n}_r^i) \right\} j_{l_{\text{ph}}}(kr) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}, \mu}^*(\mathbf{n}_{\text{ph}}) = \\ &= \sqrt{\frac{l_i}{2l_i+1}} \int_0^{+\infty} R_f^*(r) \left(\frac{dR_i(r)}{dr} + \frac{l_i+1}{r} R_i(r) \right) j_{l_{\text{ph}}}(kr) r^2 dr \cdot \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{l_i l_i-1, m_i}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega - \\ &\quad - \sqrt{\frac{l_i+1}{2l_i+1}} \int_0^{+\infty} R_f^*(r) \left(\frac{dR_i(r)}{dr} - \frac{l_i}{r} R_i(r) \right) j_{l_{\text{ph}}}(kr) r^2 dr \cdot \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{l_i l_i+1, m_i}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega. \end{aligned}$$

For electric component $p_{l_{\text{ph}}\mu}^E$ we have:

$$\begin{aligned} p_{l_{\text{ph}}\mu}^E &= \int_0^{+\infty} dr \int d\Omega r^2 \varphi_f^*(\mathbf{r}) \left(\frac{\partial}{\partial \mathbf{r}} \varphi_i(\mathbf{r}) \right) \mathbf{A}_{l_{\text{ph}}\mu}^*(\mathbf{r}, E) = \\ &= \int_0^{+\infty} dr \int d\Omega r^2 R_f^*(r) Y_{l_f m_f}^*(\mathbf{n}_r^f) \cdot \left\{ \sqrt{\frac{l_i}{2l_i+1}} \left(\frac{dR_i(r)}{dr} + \frac{l_i+1}{r} R_i(r) \right) \mathbf{T}_{l_i l_i-1, m_i}(\mathbf{n}_r^i) - \right. \\ &\quad \left. - \sqrt{\frac{l_i+1}{2l_i+1}} \left(\frac{dR_i(r)}{dr} - \frac{l_i}{r} R_i(r) \right) \mathbf{T}_{l_i l_i+1, m_i}(\mathbf{n}_r^i) \right\} \times \\ &\quad \times \left\{ \sqrt{\frac{l_{\text{ph}}+1}{2l_{\text{ph}}+1}} j_{l_{\text{ph}}-1}(kr) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}-1, \mu}^*(\mathbf{n}_{\text{ph}}) - \sqrt{\frac{l_{\text{ph}}}{2l_{\text{ph}}+1}} j_{l_{\text{ph}}+1}(kr) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}+1, \mu}^*(\mathbf{n}_{\text{ph}}) \right\} = \\ &= \sqrt{\frac{l_i}{2l_i+1}} \sqrt{\frac{l_{\text{ph}}+1}{2l_{\text{ph}}+1}} \int_0^{+\infty} R_f^*(r) \left(\frac{dR_i(r)}{dr} + \frac{l_i+1}{r} R_i(r) \right) j_{l_{\text{ph}}-1}(kr) r^2 dr \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{l_i l_i-1, m_i}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}-1, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega - \\ &\quad - \sqrt{\frac{l_i}{2l_i+1}} \sqrt{\frac{l_{\text{ph}}}{2l_{\text{ph}}+1}} \int_0^{+\infty} R_f^*(r) \left(\frac{dR_i(r)}{dr} + \frac{l_i+1}{r} R_i(r) \right) j_{l_{\text{ph}}+1}(kr) r^2 dr \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{l_i l_i-1, m_i}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}+1, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega + \\ &\quad + \sqrt{\frac{l_i+1}{2l_i+1}} \sqrt{\frac{l_{\text{ph}}+1}{2l_{\text{ph}}+1}} \int_0^{+\infty} R_f^*(r) \left(\frac{dR_i(r)}{dr} - \frac{l_i}{r} R_i(r) \right) j_{l_{\text{ph}}-1}(kr) r^2 dr \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{l_i l_i+1, m_i}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}-1, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega - \\ &\quad - \sqrt{\frac{l_i+1}{2l_i+1}} \sqrt{\frac{l_{\text{ph}}}{2l_{\text{ph}}+1}} \int_0^{+\infty} R_f^*(r) \left(\frac{dR_i(r)}{dr} - \frac{l_i}{r} R_i(r) \right) j_{l_{\text{ph}}+1}(kr) r^2 dr \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{l_i l_i+1, m_i}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}+1, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega. \end{aligned}$$

So, components $p_{l_{\text{ph}}\mu}^M$ and $p_{l_{\text{ph}}\mu}^E$ have form:

$$\begin{aligned}
p_{l_{\text{ph}}\mu}^M &= \sqrt{\frac{l_i}{2l_i+1}} \int_0^{+\infty} R_f^*(r) \left(\frac{dR_i(r)}{dr} + \frac{l_i+1}{r} R_i(r) \right) j_{l_{\text{ph}}}(kr) r^2 dr \cdot \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{l_i l_i-1, m_i}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega - \\
&- \sqrt{\frac{l_i+1}{2l_i+1}} \int_0^{+\infty} R_f^*(r) \left(\frac{dR_i(r)}{dr} - \frac{l_i}{r} R_i(r) \right) j_{l_{\text{ph}}}(kr) r^2 dr \cdot \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{l_i l_i+1, m_i}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega, \\
p_{l_{\text{ph}}\mu}^E &= \sqrt{\frac{l_i}{2l_i+1}} \sqrt{\frac{l_{\text{ph}}+1}{2l_{\text{ph}}+1}} \int_0^{+\infty} R_f^*(r) \left(\frac{dR_i(r)}{dr} + \frac{l_i+1}{r} R_i(r) \right) j_{l_{\text{ph}}-1}(kr) r^2 dr \times \\
&\times \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{l_i l_i-1, m_i}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}-1, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega - \\
&- \sqrt{\frac{l_i}{2l_i+1}} \sqrt{\frac{l_{\text{ph}}}{2l_{\text{ph}}+1}} \int_0^{+\infty} R_f^*(r) \left(\frac{dR_i(r)}{dr} + \frac{l_i+1}{r} R_i(r) \right) j_{l_{\text{ph}}+1}(kr) r^2 dr \times \\
&\times \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{l_i l_i-1, m_i}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}+1, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega + \\
&+ \sqrt{\frac{l_i+1}{2l_i+1}} \sqrt{\frac{l_{\text{ph}}+1}{2l_{\text{ph}}+1}} \int_0^{+\infty} R_f^*(r) \left(\frac{dR_i(r)}{dr} - \frac{l_i}{r} R_i(r) \right) j_{l_{\text{ph}}-1}(kr) r^2 dr \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{l_i l_i+1, m_i}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}-1, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega - \\
&- \sqrt{\frac{l_i+1}{2l_i+1}} \sqrt{\frac{l_{\text{ph}}}{2l_{\text{ph}}+1}} \int_0^{+\infty} R_f^*(r) \left(\frac{dR_i(r)}{dr} - \frac{l_i}{r} R_i(r) \right) j_{l_{\text{ph}}+1}(kr) r^2 dr \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{l_i l_i+1, m_i}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}+1, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega.
\end{aligned} \tag{E17}$$

Let us introduce the following notions:

$$\begin{aligned}
J_1(l_i, l_f, n) &= \int_0^{+\infty} \frac{dR_i(r, l_i)}{dr} R_f^*(l_f, r) j_n(kr) r^2 dr, \\
J_2(l_i, l_f, n) &= \int_0^{+\infty} R_i(r, l_i) R_f^*(l_f, r) j_n(kr) r dr, \\
I_M(l_i, l_f, l_{\text{ph}}, l_1, \mu) &= \int_0^{+\infty} Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{l_i l_1, m_i}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega, \\
I_E(l_i, l_f, l_{\text{ph}}, l_1, l_2, \mu) &= \int_0^{+\infty} Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{l_i l_1, m_i}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}} l_2, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega.
\end{aligned} \tag{E18}$$

Then, one can rewrite Eqs. (E17) as

$$\begin{aligned}
p_{l_{\text{ph}}\mu}^M &= \sqrt{\frac{l_i}{2l_i+1}} I_M(l_i, l_f, l_{\text{ph}}, l_i-1, \mu) \cdot \left\{ J_1(l_i, l_f, l_{\text{ph}}) + (l_i+1) \cdot J_2(l_i, l_f, l_{\text{ph}}) \right\} - \\
&- \sqrt{\frac{l_i+1}{2l_i+1}} I_M(l_i, l_f, l_{\text{ph}}, l_i+1, \mu) \cdot \left\{ J_1(l_i, l_f, l_{\text{ph}}) - l_i \cdot J_2(l_i, l_f, l_{\text{ph}}) \right\}, \\
p_{l_{\text{ph}}\mu}^E &= \sqrt{\frac{l_i(l_{\text{ph}}+1)}{(2l_i+1)(2l_{\text{ph}}+1)}} \cdot I_E(l_i, l_f, l_{\text{ph}}, l_i-1, l_{\text{ph}}-1, \mu) \cdot \left\{ J_1(l_i, l_f, l_{\text{ph}}-1) + (l_i+1) \cdot J_2(l_i, l_f, l_{\text{ph}}-1) \right\} - \\
&- \sqrt{\frac{l_i l_{\text{ph}}}{(2l_i+1)(2l_{\text{ph}}+1)}} \cdot I_E(l_i, l_f, l_{\text{ph}}, l_i-1, l_{\text{ph}}+1, \mu) \cdot \left\{ J_1(l_i, l_f, l_{\text{ph}}+1) + (l_i+1) \cdot J_2(l_i, l_f, l_{\text{ph}}+1) \right\} + \\
&+ \sqrt{\frac{(l_i+1)(l_{\text{ph}}+1)}{(2l_i+1)(2l_{\text{ph}}+1)}} \cdot I_E(l_i, l_f, l_{\text{ph}}, l_i+1, l_{\text{ph}}-1, \mu) \cdot \left\{ J_1(l_i, l_f, l_{\text{ph}}-1) - l_i \cdot J_2(l_i, l_f, l_{\text{ph}}-1) \right\} - \\
&- \sqrt{\frac{(l_i+1) l_{\text{ph}}}{(2l_i+1)(2l_{\text{ph}}+1)}} \cdot I_E(l_i, l_f, l_{\text{ph}}, l_i+1, l_{\text{ph}}+1, \mu) \cdot \left\{ J_1(l_i, l_f, l_{\text{ph}}+1) - l_i \cdot J_2(l_i, l_f, l_{\text{ph}}+1) \right\}.
\end{aligned} \tag{E19}$$

By the same way, we find components $\tilde{p}_{l_{\text{ph}}\mu}^M$ and $\tilde{p}_{l_{\text{ph}}\mu}^E$:

$$\begin{aligned}\tilde{p}_{l_{\text{ph}}\mu}^M &= \int_0^{+\infty} R_f^*(r) R_i(r) j_{l_{\text{ph}}}(kr) r^2 dr \cdot \boldsymbol{\xi}_\mu \int Y_{l_f m_f}^*(\mathbf{n}_r^f) Y_{l_i m_i}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega, \\ \tilde{p}_{l_{\text{ph}}\mu}^E &= \sqrt{\frac{l_{\text{ph}}+1}{2l_{\text{ph}}+1}} \int_0^{+\infty} R_f^*(r) R_i(r) j_{l_{\text{ph}}-1}(kr) r^2 dr \cdot \boldsymbol{\xi}_\mu \int Y_{l_f m_f}^*(\mathbf{n}_r^f) Y_{l_i m_i}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}-1, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega - \\ &\quad - \sqrt{\frac{l_{\text{ph}}}{2l_{\text{ph}}+1}} \int_0^{+\infty} R_f^*(r) R_i(r) j_{l_{\text{ph}}+1}(kr) r^2 dr \cdot \boldsymbol{\xi}_\mu \int Y_{l_f m_f}^*(\mathbf{n}_r^f) Y_{l_i m_i}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}+1, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega.\end{aligned}\quad (\text{E20})$$

Introducing new integrals:

$$\begin{aligned}\tilde{J}(l_i, l_f, n) &= \int_0^{+\infty} R_i(r) R_f^*(l, r) j_n(kr) r^2 dr, \\ \tilde{I}(l_i, l_f, l_{\text{ph}}, n, \mu) &= \boldsymbol{\xi}_\mu \int Y_{l_i m_i}(\mathbf{n}_r^i) Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{l_{\text{ph}} n, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega,\end{aligned}\quad (\text{E21})$$

we rewrite (E20) as

$$\begin{aligned}\tilde{p}_{l_{\text{ph}}\mu}^M &= \tilde{I}(l_i, l_f, l_{\text{ph}}, l_{\text{ph}}, \mu) \cdot \tilde{J}(l_i, l_f, l_{\text{ph}}), \\ \tilde{p}_{l_{\text{ph}}\mu}^E &= \sqrt{\frac{l_{\text{ph}}+1}{2l_{\text{ph}}+1}} \tilde{I}(l_i, l_f, l_{\text{ph}}, l_{\text{ph}}-1, \mu) \cdot \tilde{J}(l_i, l_f, l_{\text{ph}}-1) - \sqrt{\frac{l_{\text{ph}}}{2l_{\text{ph}}+1}} \tilde{I}(l_i, l_f, l_{\text{ph}}, l_{\text{ph}}+1, \mu) \cdot \tilde{J}(l_i, l_f, l_{\text{ph}}+1).\end{aligned}\quad (\text{E22})$$

5. Calculation of terms $\check{p}_{l_{\text{ph}}\mu}^M$ and $\check{p}_{l_{\text{ph}}\mu}^E$: case of $l_i \neq 0$

In this Section we calculate components $\check{p}_{l_{\text{ph}}\mu}^M$ and $\check{p}_{l_{\text{ph}}\mu}^E$ at $l_i \neq 0$. From (E15) we have:

$$\check{p}_{l_{\text{ph}}\mu}^M = \boldsymbol{\xi}_\mu \int \varphi_f^*(\mathbf{r}) \varphi_i(\mathbf{r}) V(\mathbf{r}) \mathbf{A}_{l_{\text{ph}}\mu}^*(\mathbf{r}, M) d\mathbf{r}, \quad \check{p}_{l_{\text{ph}}\mu}^E = \boldsymbol{\xi}_\mu \int \varphi_f^*(\mathbf{r}) \varphi_i(\mathbf{r}) V(\mathbf{r}) \mathbf{A}_{l_{\text{ph}}\mu}^*(\mathbf{r}, E) d\mathbf{r}, \quad (\text{E23})$$

$$\varphi_i(\mathbf{r}) = R_i(r) Y_{l_i m_i}(\mathbf{n}_r^i), \quad \varphi_f(\mathbf{r}) = R_f(r) Y_{l_f m_f}(\mathbf{n}_r^f). \quad (\text{E24})$$

For magnetic component $\check{p}_{l_{\text{ph}}\mu}^M$ $l_i \neq 0$ we obtain:

$$\begin{aligned}\check{p}_{l_{\text{ph}}\mu}^M &= \boldsymbol{\xi}_\mu \int_0^{+\infty} dr \int d\Omega r^2 \varphi_f^*(\mathbf{r}) \varphi_i(\mathbf{r}) V(\mathbf{r}) \mathbf{A}_{l_{\text{ph}}\mu}^*(\mathbf{r}, M) = \\ &= \boldsymbol{\xi}_\mu \int_0^{+\infty} dr \int d\Omega r^2 R_f^*(r) Y_{l_f m_f}^*(\mathbf{n}_r^f) R_i(r) Y_{l_i m_i}(\mathbf{n}_r^i) V(\mathbf{r}) j_{l_{\text{ph}}}(kr) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}, \mu}^*(\mathbf{n}_{\text{ph}}) = \\ &= \int_0^{+\infty} R_f^*(r) R_i(r) V(\mathbf{r}) j_{l_{\text{ph}}}(kr) r^2 dr \cdot \boldsymbol{\xi}_\mu \int Y_{l_f m_f}^*(\mathbf{n}_r^f) Y_{l_i m_i}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega.\end{aligned}$$

For electric component $\check{p}_{l_{\text{ph}}\mu}^E$ we have:

$$\begin{aligned}
\check{p}_{l_{\text{ph}}\mu}^E &= \xi_\mu \int_0^{+\infty} dr \int d\Omega r^2 \varphi_f^*(\mathbf{r}) \varphi_i(\mathbf{r}) V(\mathbf{r}) \mathbf{A}_{l_{\text{ph}}\mu}^*(\mathbf{r}, E) = \\
&= \xi_\mu \int_0^{+\infty} dr \int d\Omega r^2 R_f^*(r) Y_{l_f m_f}^*(\mathbf{n}_r^f) R_i(r) Y_{l_i m_i}(\mathbf{n}_r^i) V(\mathbf{r}) \times \\
&\times \left\{ \sqrt{\frac{l_{\text{ph}}+1}{2l_{\text{ph}}+1}} j_{l_{\text{ph}}-1}(kr) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}-1, \mu}^*(\mathbf{n}_{\text{ph}}) - \sqrt{\frac{l_{\text{ph}}}{2l_{\text{ph}}+1}} j_{l_{\text{ph}}+1}(kr) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}+1, \mu}^*(\mathbf{n}_{\text{ph}}) \right\} = \\
&= \sqrt{\frac{l_{\text{ph}}+1}{2l_{\text{ph}}+1}} \int_0^{+\infty} R_f^*(r) R_i(r) V(\mathbf{r}) j_{l_{\text{ph}}-1}(kr) r^2 dr \cdot \xi_\mu \int Y_{l_f m_f}^*(\mathbf{n}_r^f) Y_{l_i m_i}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}-1, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega - \\
&- \sqrt{\frac{l_{\text{ph}}}{2l_{\text{ph}}+1}} \int_0^{+\infty} R_f^*(r) R_i(r) V(\mathbf{r}) j_{l_{\text{ph}}+1}(kr) r^2 dr \cdot \xi_\mu \int Y_{l_f m_f}^*(\mathbf{n}_r^f) Y_{l_i m_i}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}+1, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega.
\end{aligned}$$

So, components $\check{p}_{l_{\text{ph}}\mu}^M$ and $\check{p}_{l_{\text{ph}}\mu}^E$ have form:

$$\begin{aligned}
\check{p}_{l_{\text{ph}}\mu}^M &= \int_0^{+\infty} R_f^*(r) R_i(r) V(\mathbf{r}) j_{l_{\text{ph}}}(kr) r^2 dr \cdot \xi_\mu \int Y_{l_f m_f}^*(\mathbf{n}_r^f) Y_{l_i m_i}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega, \\
\check{p}_{l_{\text{ph}}\mu}^E &= \sqrt{\frac{l_{\text{ph}}+1}{2l_{\text{ph}}+1}} \int_0^{+\infty} R_f^*(r) R_i(r) V(\mathbf{r}) j_{l_{\text{ph}}-1}(kr) r^2 dr \cdot \xi_\mu \int Y_{l_f m_f}^*(\mathbf{n}_r^f) Y_{l_i m_i}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}-1, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega - \\
&- \sqrt{\frac{l_{\text{ph}}}{2l_{\text{ph}}+1}} \int_0^{+\infty} R_f^*(r) R_i(r) V(\mathbf{r}) j_{l_{\text{ph}}+1}(kr) r^2 dr \cdot \xi_\mu \int Y_{l_f m_f}^*(\mathbf{n}_r^f) Y_{l_i m_i}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}+1, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega.
\end{aligned} \tag{E25}$$

Using notation (E21) for the angular integral $\tilde{I}(l_i, l_f, l_{\text{ph}}, n, \mu)$, we rewrite Eqs. (E25) as

$$\begin{aligned}
\check{p}_{l_{\text{ph}}\mu}^M &= \tilde{I}(l_i, l_f, l_{\text{ph}}, l_{\text{ph}}, \mu) \cdot \check{J}(l_i, l_f, l_{\text{ph}}), \\
\check{p}_{l_{\text{ph}}\mu}^E &= \sqrt{\frac{l_{\text{ph}}+1}{2l_{\text{ph}}+1}} \tilde{I}(l_i, l_f, l_{\text{ph}}, l_{\text{ph}}-1, \mu) \cdot \check{J}(l_i, l_f, l_{\text{ph}}-1) - \sqrt{\frac{l_{\text{ph}}}{2l_{\text{ph}}+1}} \tilde{I}(l_i, l_f, l_{\text{ph}}, l_{\text{ph}}+1, \mu) \cdot \check{J}(l_i, l_f, l_{\text{ph}}+1),
\end{aligned} \tag{E26}$$

where we introduce the following new denotation:

$$\check{J}(l_i, l_f, n) = \int_0^{+\infty} R_i(r) R_{l_f}^*(r) V(\mathbf{r}) j_n(kr) r^2 dr. \tag{E27}$$

6. Calculation of components $p_{l_{\text{ph}}\mu}^M$ and $p_{l_{\text{ph}}\mu}^E$ and $\check{p}_{l_{\text{ph}}\mu}^M, \check{p}_{l_{\text{ph}}\mu}^E$: case of $l_i = 0$

In this section let us analyze case of $l_i = 0$. We have:

$$\varphi_i(\mathbf{r}) = R_i(r) Y_{00}(\mathbf{n}_r^i). \tag{E28}$$

Using gradient formula (see [61], (2.56), p. 46):

$$\frac{\partial}{\partial \mathbf{r}} f(r) Y_{lm}(\mathbf{n}_r) = \sqrt{\frac{l}{2l+1}} \left(\frac{df}{dr} + \frac{l+1}{r} f \right) \mathbf{T}_{l-1, m}(\mathbf{n}_r) - \sqrt{\frac{l+1}{2l+1}} \left(\frac{df}{dr} - \frac{l}{r} f \right) \mathbf{T}_{l+1, m}(\mathbf{n}_r) \tag{E29}$$

and taking into account Eq. (E28), we obtain:

$$\frac{\partial}{\partial \mathbf{r}} \varphi_i(\mathbf{r}) = -\frac{dR_i(r)}{dr} \mathbf{T}_{01, 0}(\mathbf{n}_r^i). \tag{E30}$$

Using this relation and Eq. (E3), we transform expressions (E15). For the magnetic component $p_{l_\mu}^M$ we obtain:

$$\begin{aligned}
p_{l_{\text{ph}}\mu}^{M0m_f} &= \int_0^{+\infty} dr \int d\Omega r^2 \varphi_f^*(\mathbf{r}) \left(\frac{\partial}{\partial \mathbf{r}} \varphi_i(\mathbf{r}) \right) \mathbf{A}_{l_{\text{ph}}\mu}^*(\mathbf{r}, M) = \\
&= \int_0^{+\infty} dr \int d\Omega r^2 R_f^*(r) Y_{l_f m_f}^*(\mathbf{n}_r^f) \left(-\frac{dR_i(r)}{dr} \mathbf{T}_{01,0}(\mathbf{n}_r^i) \right) j_{l_{\text{ph}}}(kr) \mathbf{T}_{l_{\text{ph}}l_{\text{ph}},\mu}^*(\mathbf{n}_{\text{ph}}) = \\
&= - \int_0^{+\infty} R_f^*(r) \frac{dR_i(r)}{dr} j_{l_{\text{ph}}}(kr) r^2 dr \cdot \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{01,0}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}}l_{\text{ph}},\mu}^*(\mathbf{n}_{\text{ph}}) d\Omega.
\end{aligned}$$

For the electric component $p_{l_{\text{ph}}\mu}^E$ we obtain:

$$\begin{aligned}
p_{l_{\text{ph}}\mu}^{E0m_f} &= \int_0^{+\infty} dr \int d\Omega r^2 \varphi_f^*(\mathbf{r}) \left(\frac{\partial}{\partial \mathbf{r}} \varphi_i(\mathbf{r}) \right) \mathbf{A}_{l_{\text{ph}}\mu}^*(\mathbf{r}, E) = \\
&= \int_0^{+\infty} dr \int d\Omega r^2 R_f^*(r) Y_{l_f m_f}^*(\mathbf{n}_r^f) \left(-\frac{dR_i(r)}{dr} \mathbf{T}_{01,0}(\mathbf{n}_r^i) \right) \times \\
&\quad \times \left\{ \sqrt{\frac{l_{\text{ph}}+1}{2l_{\text{ph}}+1}} j_{l_{\text{ph}}-1}(kr) \mathbf{T}_{l_{\text{ph}}l_{\text{ph}}-1,\mu}^*(\mathbf{n}_{\text{ph}}) - \sqrt{\frac{l_{\text{ph}}}{2l_{\text{ph}}+1}} j_{l_{\text{ph}}+1}(kr) \mathbf{T}_{l_{\text{ph}}l_{\text{ph}}+1,\mu}^*(\mathbf{n}_{\text{ph}}) \right\} = \\
&= -\sqrt{\frac{l_{\text{ph}}+1}{2l_{\text{ph}}+1}} \int_0^{+\infty} R_f^*(r) \frac{dR_i(r)}{dr} j_{l_{\text{ph}}-1}(kr) r^2 dr \cdot \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{01,0}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}}l_{\text{ph}}-1,\mu}^*(\mathbf{n}_{\text{ph}}) d\Omega - \\
&\quad + \sqrt{\frac{l_{\text{ph}}}{2l_{\text{ph}}+1}} \int_0^{+\infty} R_f^*(r) \frac{dR_i(r)}{dr} j_{l_{\text{ph}}+1}(kr) r^2 dr \cdot \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{01,0}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}}l_{\text{ph}}+1,\mu}^*(\mathbf{n}_{\text{ph}}) d\Omega.
\end{aligned}$$

So, the components $p_{l_{\text{ph}}\mu}^M$ and $p_{l_{\text{ph}}\mu}^E$ have form:

$$\begin{aligned}
p_{l_{\text{ph}}\mu}^{M0m_f} &= - \int_0^{+\infty} R_f^*(r) \frac{dR_i(r)}{dr} j_{l_{\text{ph}}}(kr) r^2 dr \cdot \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{01,0}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}}l_{\text{ph}},\mu}^*(\mathbf{n}_{\text{ph}}) d\Omega, \\
p_{l_{\text{ph}}\mu}^{E0m_f} &= -\sqrt{\frac{l_{\text{ph}}+1}{2l_{\text{ph}}+1}} \int_0^{+\infty} R_f^*(r) \frac{dR_i(r)}{dr} j_{l_{\text{ph}}-1}(kr) r^2 dr \cdot \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{01,0}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}}l_{\text{ph}}-1,\mu}^*(\mathbf{n}_{\text{ph}}) d\Omega - \\
&\quad + \sqrt{\frac{l_{\text{ph}}}{2l_{\text{ph}}+1}} \int_0^{+\infty} R_f^*(r) \frac{dR_i(r)}{dr} j_{l_{\text{ph}}+1}(kr) r^2 dr \cdot \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{01,0}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}}l_{\text{ph}}+1,\mu}^*(\mathbf{n}_{\text{ph}}) d\Omega.
\end{aligned} \tag{E31}$$

We introduce the following definitions:

$$\begin{aligned}
J(l_f, n) &= \int_0^{+\infty} \frac{dR_i(r)}{dr} R_f^*(l, r) j_n(kr) r^2 dr, \\
I(l_f, l_{\text{ph}}, n, \mu) &= \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{01,0}(\mathbf{n}_r^i) \mathbf{T}_{l_{\text{ph}}n,\mu}^*(\mathbf{n}_{\text{ph}}) d\Omega.
\end{aligned} \tag{E32}$$

Then one can write Eqs. (E31) as

$$\begin{aligned}
p_{l_{\text{ph}}\mu}^{M0m_f} &= -I(l_f, l_{\text{ph}}, l_{\text{ph}}, \mu) \cdot J(l_f, l_{\text{ph}}), \\
p_{l_{\text{ph}}\mu}^{E0m_f} &= -\sqrt{\frac{l_{\text{ph}}+1}{2l_{\text{ph}}+1}} I(l_f, l_{\text{ph}}, l_{\text{ph}}-1, \mu) \cdot J(l_f, l_{\text{ph}}-1) + \sqrt{\frac{l_{\text{ph}}}{2l_{\text{ph}}+1}} I(l_f, l_{\text{ph}}, l_{\text{ph}}+1, \mu) \cdot J(l_f, l_{\text{ph}}+1).
\end{aligned} \tag{E33}$$

By the same way, we find the components $\tilde{p}_{l_{\text{ph}}\mu}^{M0m_f}$ and $\tilde{p}_{l_{\text{ph}}\mu}^{E0m_f}$:

$$\begin{aligned}\tilde{p}_{l_{\text{ph}}\mu}^{M0m_f} &= \int_0^{+\infty} R_f^*(r) R_i(r) j_{l_{\text{ph}}}(kr) r^2 dr \cdot \boldsymbol{\xi}_\mu \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega, \\ \tilde{p}_{l_{\text{ph}}\mu}^{E0m_f} &= \sqrt{\frac{l_{\text{ph}}+1}{2l_{\text{ph}}+1}} \int_0^{+\infty} R_f^*(r) R_i(r) j_{l_{\text{ph}}-1}(kr) r^2 dr \cdot \boldsymbol{\xi}_\mu \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}-1, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega - \\ &\quad - \sqrt{\frac{l_{\text{ph}}}{2l_{\text{ph}}+1}} \int_0^{+\infty} R_f^*(r) R_i(r) j_{l_{\text{ph}}+1}(kr) r^2 dr \cdot \boldsymbol{\xi}_\mu \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}+1, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega.\end{aligned}\quad (\text{E34})$$

Introducing new integrals:

$$\begin{aligned}\tilde{J}(l_f, n) &= \int_0^{+\infty} R_i(r) R_f^*(l, r) j_n(kr) r^2 dr, \\ \tilde{I}(l_f, l_{\text{ph}}, n, \mu) &= \boldsymbol{\xi}_\mu \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{l_{\text{ph}} n, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega,\end{aligned}\quad (\text{E35})$$

we rewrite Eq. (E34) as

$$\begin{aligned}\tilde{p}_{l_{\text{ph}}\mu}^{M0m_f} &= \tilde{I}(l_f, l_{\text{ph}}, l_{\text{ph}}, \mu) \cdot \tilde{J}(l_f, l_{\text{ph}}), \\ \tilde{p}_{l_{\text{ph}}\mu}^{E0m_f} &= \sqrt{\frac{l_{\text{ph}}+1}{2l_{\text{ph}}+1}} \tilde{I}(l_f, l_{\text{ph}}, l_{\text{ph}}-1, \mu) \cdot \tilde{J}(l_f, l_{\text{ph}}-1) - \sqrt{\frac{l_{\text{ph}}}{2l_{\text{ph}}+1}} \tilde{I}(l_f, l_{\text{ph}}, l_{\text{ph}}+1, \mu) \cdot \tilde{J}(l_f, l_{\text{ph}}+1).\end{aligned}\quad (\text{E36})$$

7. Calculation of components $\check{p}_{l_{\text{ph}}\mu}^M$ and $\check{p}_{l_{\text{ph}}\mu}^E$: case of $l_i = 0$

Now we calculate the components $\check{p}_{l_{\text{ph}}\mu}^M$ and $\check{p}_{l_{\text{ph}}\mu}^E$ at $l_i = 0$. From Eqs. (E15) and (E28) we have:

$$\check{p}_{l_{\text{ph}}\mu}^{M0m_f} = \boldsymbol{\xi}_\mu \int \varphi_f^*(\mathbf{r}) \varphi_i(\mathbf{r}) V(\mathbf{r}) \mathbf{A}_{l_{\text{ph}}\mu}^*(\mathbf{r}, M) d\mathbf{r}, \quad \check{p}_{l_{\text{ph}}\mu}^{E0m_f} = \boldsymbol{\xi}_\mu \int \varphi_f^*(\mathbf{r}) \varphi_i(\mathbf{r}) V(\mathbf{r}) \mathbf{A}_{l_{\text{ph}}\mu}^*(\mathbf{r}, E) d\mathbf{r}, \quad (\text{E37})$$

$$\varphi_i(\mathbf{r}) = R_i(r) Y_{00}(\mathbf{n}_r^i), \quad \varphi_f(\mathbf{r}) = R_f(r) Y_{l_f m_f}(\mathbf{n}_r^f). \quad (\text{E38})$$

For the magnetic component $\check{p}_{l_{\text{ph}}\mu}^{M0m_f}$ we obtain:

$$\begin{aligned}\check{p}_{l_{\text{ph}}\mu}^{M0m_f} &= \boldsymbol{\xi}_\mu \int_0^{+\infty} dr \int d\Omega r^2 \varphi_f^*(\mathbf{r}) \varphi_i(\mathbf{r}) V(\mathbf{r}) \mathbf{A}_{l_{\text{ph}}\mu}^*(\mathbf{r}, M) = \\ &= \boldsymbol{\xi}_\mu \int_0^{+\infty} dr \int d\Omega r^2 R_f^*(r) Y_{l_f m_f}^*(\mathbf{n}_r^f) R_i(r) V(\mathbf{r}) j_{l_{\text{ph}}}(kr) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}, \mu}^*(\mathbf{n}_{\text{ph}}) = \\ &= \boldsymbol{\xi}_\mu \int_0^{+\infty} R_f^*(r) R_i(r) V(\mathbf{r}) j_{l_{\text{ph}}}(kr) r^2 dr \cdot \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega.\end{aligned}$$

For the electric component $p_{l_{\text{ph}}\mu}^{E0m_f}$ we obtain:

$$\begin{aligned}
\check{p}_{l_{\text{ph}}\mu}^{E0m_f} &= \xi_\mu \int_0^{+\infty} dr \int d\Omega r^2 \varphi_f^*(\mathbf{r}) \varphi_i(\mathbf{r}) V(\mathbf{r}) \mathbf{A}_{l_{\text{ph}}\mu}^*(\mathbf{r}, E) = \\
&= \xi_\mu \int_0^{+\infty} dr \int d\Omega r^2 R_f^*(r) Y_{l_f m_f}^*(\mathbf{n}_r^f) R_i(r) V(\mathbf{r}) \times \\
&\quad \times \left\{ \sqrt{\frac{l_{\text{ph}}+1}{2l_{\text{ph}}+1}} j_{l_{\text{ph}}-1}(kr) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}-1, \mu}^*(\mathbf{n}_{\text{ph}}) - \sqrt{\frac{l_{\text{ph}}}{2l_{\text{ph}}+1}} j_{l_{\text{ph}}+1}(kr) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}+1, \mu}^*(\mathbf{n}_{\text{ph}}) \right\} = \\
&= \xi_\mu \sqrt{\frac{l_{\text{ph}}+1}{2l_{\text{ph}}+1}} \int_0^{+\infty} R_f^*(r) R_i(r) V(\mathbf{r}) j_{l_{\text{ph}}-1}(kr) r^2 dr \cdot \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}-1, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega - \\
&\quad - \xi_\mu \sqrt{\frac{l_{\text{ph}}}{2l_{\text{ph}}+1}} \int_0^{+\infty} R_f^*(r) R_i(r) V(\mathbf{r}) j_{l_{\text{ph}}+1}(kr) r^2 dr \cdot \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}+1, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega.
\end{aligned}$$

So, the components $p_{l_{\text{ph}}\mu}^{M0m_f}$ and $p_{l_{\text{ph}}\mu}^{E0m_f}$ have form:

$$\begin{aligned}
\check{p}_{l_{\text{ph}}\mu}^{M0m_f} &= \xi_\mu \int_0^{+\infty} R_f^*(r) R_i(r) V(\mathbf{r}) j_{l_{\text{ph}}}(kr) r^2 dr \cdot \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega, \\
\check{p}_{l_{\text{ph}}\mu}^{E0m_f} &= \xi_\mu \sqrt{\frac{l_{\text{ph}}+1}{2l_{\text{ph}}+1}} \int_0^{+\infty} R_f^*(r) R_i(r) V(\mathbf{r}) j_{l_{\text{ph}}-1}(kr) r^2 dr \cdot \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}-1, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega - \\
&\quad - \xi_\mu \sqrt{\frac{l_{\text{ph}}}{2l_{\text{ph}}+1}} \int_0^{+\infty} R_f^*(r) R_i(r) V(\mathbf{r}) j_{l_{\text{ph}}+1}(kr) r^2 dr \cdot \int Y_{l_f m_f}^*(\mathbf{n}_r^f) \mathbf{T}_{l_{\text{ph}} l_{\text{ph}}+1, \mu}^*(\mathbf{n}_{\text{ph}}) d\Omega.
\end{aligned} \tag{E39}$$

Using Eq. (E35) for the angular integral, we rewrite (E39) as

$$\begin{aligned}
\check{p}_{l_{\text{ph}}\mu}^{M0m_f} &= \tilde{I}(l_f, l_{\text{ph}}, l_{\text{ph}}, \mu) \cdot \check{J}(0, l_f, l_{\text{ph}}), \\
\check{p}_{l_{\text{ph}}\mu}^{E0m_f} &= \sqrt{\frac{l_{\text{ph}}+1}{2l_{\text{ph}}+1}} \tilde{I}(l_f, l_{\text{ph}}, l_{\text{ph}}-1, \mu) \cdot \check{J}(0, l_f, l_{\text{ph}}-1) - \sqrt{\frac{l_{\text{ph}}}{2l_{\text{ph}}+1}} \tilde{I}(l_f, l_{\text{ph}}, l_{\text{ph}}+1, \mu) \cdot \check{J}(0, l_f, l_{\text{ph}}+1).
\end{aligned} \tag{E40}$$

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