Manuscript of the article: Szűcs E., Papp G., Benedek J. A study of different wavelength spectral components of the gravity field derived from various terrestrial data sets.

Appeared in: Acta Geodaetica et Geophysica, 2014, Volume 49, Issue 3, pp. 327-342.
ISSN: 2213-5812

The final publication is available at link.springer.com:

http://link.springer.com/article/10.1007/s40328-014-0061-9

A study of different wavelength spectral components of the gravity field derived from various terrestrial data sets

E. Szűcs, G. Papp. J. Benedek
Geodetic and Geophysical Institute, Research Centre for Astronomy and Earth Sciences, Hungarian Academy of Sciences. PO Box 6, H-9400, Hungary
e-mail: szucs_e@ggki.hu

Abstract

In the general scheme of gravity field modelling long-, medium- and short- wavelength constituents of the gravity field derived from e.g. geopotential model, terrestrial data and digital terrain model respectively, are routinely combined. In this study, spectral characteristics of terrestrial data sets are investigated. The estimation of spectral sensitivity of gravity related quantities such as gravity anomaly, vertical deflections and gravity gradients was accomplished through Fourier PSD and covariance analysis depending on the spatial distribution of data points. The information content of the estimated spectra were validated on global and local levels to access their further utilization. The spectra were compared to the 1D spectrum of the gravitational field derived from spherical harmonic coefficients using a high resolution global gravitational model as well as to an analytical approximation. Besides the frequency domain investigations the information content regarding the different wavelength structure comprised in terrestrial and EGM2008 model is investigated in the space domain based on covariance analysis. As a combined validation process the gravity degree variances were transformed to the necessary auto- and cross covariance functions to predict geoid height from gravity anomaly, which ensures an independent validation process of the computed spectrum. Based on the spectral characteristics of terrestrial measurement spectral weights for spectral combination were derived involving global gravity field model, gravity and gravity gradient data in gravity field modelling. To determine the geoid in the whole spectral band the specific integral kernels in the spectral domain should be modified using the suggested spectral weights.

1. Introduction

Terrestrial measurements such as geoid height, gravity anomaly, vertical deflections and gravity gradients are different functionals of the gravitational potential therefore, theoretically it is possible to recover any gravity related quantity containing the full spectral information from measurements (Schwarz 1984). Although the highest signal power of the gravitational potential can be found at low spherical harmonic degrees - i.e. 99.2% of the total power concentrated into spectral band of spherical harmonic degree 2-36 according to the analytical Tscherning-Rapp degree variance model (Tscherning and Rapp 1974) - for a comprehensive and high (cm level) precision geoid determination all available gravity related data should be combined e.g. by a least squares estimation scheme or using spectral combination techniques. The terms long, medium, short and very-short wavelength are defined in the literature, however in this study these are accommodated to the spectral content of the terrestrial data sets. The long wavelength addresses the spectral bands which can be recovered only from satellite observations, it covers long wavelength features of the gravity field up to wavelength approximately 110 km, which corresponds to spherical harmonic degree 360. The medium wavelength comprises gravity field features from wavelength approximately 110 km to 50 km. Short wavelength covers the spectral bands up to the maximal resolution of the gravity data sets, i.e. half-wavelength 2 km. Small-scale variations of the gravitational field stem from the effect of the topographic masses, the very high wavelength in this study denotes contribution of the topographic masses derived from a high pass filtered elevation data set. Possible overlapping exists between the spectra of the different observables so the above defined spectral bands can be regarded as an approximation.

Power spectra of terrestrial gravity field quantities give a deeper insight of gravity field features providing a Fourier domain representation of the data at medium and short-wavelength scale. The term spectral sensitivity was introduced by Schwarz (1984). Since then spectral characteristics of the gravity field were investigated in several studies. Forsberg (1984) analysed topography and gravity data to estimate power spectra, degree variances and covariance functions. Flury (2006) investigated the short-wavelength spectral behaviour of gravity anomalies using various data sets describing different roughness gravity field. Zhang and Featherstone (2004) used power spectra analysis besides other techniques to feature free-air, refined Bouguer and topographic-isostatic gravity anomalies at short-wavelengths to determine which data set is the smoothest and the most suitable for gravity data gridding. The spectral properties of the short- wavelength part of the gravity field were analysed by Voigt and Denker (2007) from flat to mountainous areas utilizing gravity data derived from high resolution Residual Terrain Models (RTM). Voigt and Denker (2011) compared the spectral behaviour of height anomalies and vertical deflections using analytical degree variance model. In this study it is attempted to distinguish and determine the spectral dominance of different terrestrial data types related to different functionals of the disturbing potential. The results of such an analysis could contribute the proper spectral combination of data as it is demonstrated in the following pages.

2. Spectral analysis methods

The application of spectral methods has a long history also in geophysics. It should be mentioned that the principles of these methods have been already defined in the middle of the last century in a number of textbooks and papers (e.g. Grant and West 1965, Dorman and Lewis 1970, Parker 1972) and were applied mainly for the interpretation of magnetic and gravity anomalies. It was recognized very soon at the beginning of the era of digital computers that the convolution integrals involved either in the inversion or filtering of potential field data can be efficiently solved in the Fourier domain. So the efforts to represent the gravity field of the Earth as a sum of harmonic base functions (sinusoidal, spherical, ellipsoidal depending on the coordinate system used) luckily supported by the fast computational algorithms based on Fast Fourier Transforms giving a uniform spectral approach both for analysis and synthesis.

The horizontal extension of the gravity related data up to a few hundreds kilometres allows the planar approximation in power spectral density computation, so the calculation of power spectral density distribution functions is possible by standard discrete Fourier Transform (DFT) techniques (Brigham 1974) if the data are given on equidistant grids. For continental-scale data sets tapers and methods developed for data localized on the sphere should be utilized.

A thorough description of the computational process of power spectra of gravity related quantities is given in Flury (2006) and Schwarz et al. (1990), therefore only some basic relations are repeated. Essentially there are two ways to determine power spectra of measured gravity field quantities depending on the spatial distribution of data points. One of the methods is based on gridded data sets, in this case special attention is needed to avoid data gaps during the interpolation process since these could distort the estimated spectra. The 2D power spectral density (PSD) of a gridded data set is estimated by 2D Fourier transform applying some window on the data to avoid spectral leakage (Stoica and Moses 1997). We used multitaper spectral estimation method with discrete prolate spheroidal tapers (Slepian 1983, Hanssen 1997) to reduce both leakage and variance of the estimated spectra. The advantage of multitaper spectral methods over individual estimates is that the data, which is windowed using orthogonal tapers, can be considered as a separate realization of the respecting stochastic process and variance of the estimated spectrum can be reduced using the linear combination of the individual spectra. Slepians's tapers are optimally concentrated windows in a given frequency band. The 2D Slepian concentration problem on the sphere was elaborated by Simons et al. (2006) and it is suggested to utilize those localization windows for data sets approaching a significant fraction of the surface of the Earth to avoid distortion of the spectrum. Assuming that the 2D PSD is isotropic, the estimated PSD is radially averaged to a 1D PSD. Spherical harmonic power spectrum, i.e. the well-known degree variances σ_l and planar power spectrum are closely related, the transformation is given in Forsberg (1984)

$$\sigma_l = \frac{l+0.5}{2\pi R^2} PSD(w), \tag{1}$$

where the spatial radial frequency *w* is related to the spherical harmonic degree *l* as

$$w \approx (l+0.5)/R, \tag{2}$$

and *R* is the mean radius of the Earth.

An alternative approach of computing degree variances is based on the covariance function of measured gravity related quantities, what are usually given at scattered points. An analytical model is routinely fitted to empirical covariance function to ensure that the function is positive-definite. Covariance function and degree variances form a Legendre transform pair (just like the auto-covariance function of a stochastic process relates to its power spectrum by forming a Fourier transform pair in Cartesian coordinate systems), degree variances can be computed by numerical integration of the analytical covariance function (Flury 2002) for spherical distances $\Delta\psi$

$$\sigma_l = \frac{2l+1}{2} \Delta\psi \sum_{k=0}^{K} C(\psi_k)\, P_l(cos\psi_k) sin\psi_k \tag{3}$$

where $P_l(cos\psi_k)$ denotes Legendre polynomials of degree *l*, and $\psi_k = k * \Delta\psi$ is the kth spherical distance. There are other methods of computing degree variances from covariances, for further details the reader is referred to Flury (2006).

Similarly to digital Fourier analysis the extension of the area (*D* [km]) and the grid spacing or average point distance (*d* [km]) in case of covariance function determine the recoverable spherical harmonic degree bands of the computed power spectra (Flury 2002, Voigt and Denker 2007)

$$\mathrm{l_{min}} = \frac{40000\ \mathrm{km}}{\mathrm{D}}, \quad \mathrm{l_{max}} = \frac{20000\,\mathrm{km}}{\mathrm{d}}. \tag{4}$$

A significant trend in data can bias spectral analysis, hence the low-degree part respecting the regional trend of the gravitational field described by a spherical harmonic model was removed from the available terrestrial data sets and spectral analysis was accomplished using zero mean residual gravity field quantities. Therefore, power amplitudes for longer wavelength are less accurate, anyhow these amplitudes could not be recovered due to the restricted areal extension of the data sets. The emphasis is on the analysis of medium and short-wavelength properties of the gravity field in this study.

3. Data sets

Gravity, deflection of vertical and horizontal gradient data sets of Hungary were used for the investigation of the spectral behaviour of the gravity field. Since these gravity field quantities are related to the first and second derivatives of the disturbing potential they dominate in different spectral bands of the corresponding gravity field. The following data sets were available in our computation:

- mean free-air gravity anomalies given in 1' x 1.5' grid spacing and covering the whole country (corresponds to approximately 2 km x 2 km block size according to the mean latitude of the country). Based on earlier investigations (Papp 1993) the accuracy of this interpolated/gridded data is not better than a few tenth of a mGal even if the point density in Hungary is extremely high (3 - 4 points / km2) and carefully investigated physical models (e.g. the elevation dependence of free-air gravity anomalies) are involved in the interpolation.
- point free-air gravity anomalies at 1244 points (average point distance approximately 9 km)
- 138 astrogeodetic vertical deflections both North-South and East-West components, the average point distance between stations is approximately 26 km
- at more than 26,000 torsion balance stations horizontal gravity gradients, mainly covering the flat areas (e.g. Great Hungarian Plain) of Hungary

4. Analysis of the results of power spectra computation

Validation of the estimated power spectra was performed in two ways. First, computed degree variances were compared with global gravitational model (GGM) derived and analytical degree variance models. The target quantity in gravity field modelling is geoid height, so in the second part of this section the spectra of geoid height computed from available gravity field quantities and the overlapping wavelength domains between terrestrial data and EGM2008 model based on covariance analysis were investigated. Besides the frequency and space domain investigations of frequency content of terrestrial and global gravity field model a test computation was carried out transforming gravity degree variances to auto- and cross-covariance functions of gravity data and gravity-geoid data pairs, respectively and residual geoid height predicted from gravity was compared to residual GPS/levelling data.

4.1. Comparison with degree variances from spherical harmonic model and analytical approximation

Since the EGM2008 model (Pavlis et al. 2008) is the highest resolution global gravity field model (GGM) which is available, the spectra of each gravity field quantity was compared to the corresponding gravity functional degree variances derived from the coefficients of the model. Although EGM2008 describes Earth's gravitational field with unprecedented detail, the model is completed to spherical harmonic degree and order *l,m* = 2160, which corresponds to approximately 9 km in latitude, it is not capable to feature gravity field at wavelengths corresponding to data spacing. Therefore, the analytical Tscherning-Rapp anomaly degree variance model was also utilized to characterize the gravitational signal in the recoverable spectral bands.
Degree variances of different gravity field functionals can be derived from degree variances of the disturbing potential applying the corresponding eigenvalues of the defining operators (van Gelderen and Rummel (2001)):

$$\sigma(\mathrm{G})_{\mathrm{l}} = \lambda_{\mathrm{l}}^{2} \cdot \sigma(\mathrm{T})_{\mathrm{l}}. \tag{5}$$

G denotes the gravity field functional under consideration, λ_l is the eigenvalue of the defining operator in the spectral domain and $\sigma(T)_l$ is the degree variances of the disturbing potential which can be computed from spherical harmonic coefficients of a GGM

$$\sigma(\mathrm{T})_{\mathrm{l}} = \left(\frac{\mathrm{GM}}{\mathrm{R}}\right)^{2} \sum_{\mathrm{m}=0}^{\mathrm{l}} \left(\mathrm{C_{lm}}^{2} + \mathrm{S_{lm}}^{2}\right) \tag{6}$$

or from Tscherning-Rapp variance model (Tscherning and Rapp 1974)

$$\sigma(\mathrm{T})_{\mathrm{l}} = \left(\frac{\mathrm{R}}{\mathrm{l}-1}\right)^{2} \frac{425.28\,(\mathrm{l}-1)}{(\mathrm{l}-2)\,(\mathrm{l}+24)}\, 0.999617^{\mathrm{l}+2} \cdot 10^{-10}\,, \tag{7}$$

where *GM* is the universal gravitational constant times the mass of the Earth. Relevant eigenvalues regarding this study are listed in Tabl. 1. The formal error spectrum of a GGM (ε_l) is computed by replacing the spherical harmonic coefficients in Eq. (6) with their error estimation.

Please insert Tabl. 1. near here

Spectral properties of gravity anomalies were investigated using the gridded gravity data set as well as point value anomalies. Since gravity data (5' x 5'block values) from Hungary were incorporated in the gravity data set, which was used for the determination of EGM2008 model, correlation may be exist between the model and data. Mean anomalies offer wider spectral band recovery of the gravity signal since the higher resolution of the data set compared with point anomalies. Nevertheless, point free-air anomalies can be regarded as an EGM2008 independent data set and comparing power spectra of mean and point anomalies in the common frequency bands could reveal whether there is a systematic difference between them.

In both cases, measured anomalies were reduced with ones computed from EGM2008 to *l,m* = 600. The effect of high frequency topographic masses were also removed from the local residual mean anomalies using RTM approach (Residual Terrain Modeling, Tscherning et al 1992) above the spectral content of EGM2008, since free-air anomalies are highly correlated with height and any trend in the data could distort spectral analysis. Accordingly residual anomalies were utilized for power spectra computations

$$\Delta g_{res}^{pts/avg} = \Delta g^{pts/avg} - \Delta g_{2-600}^{EGM2008} - \Delta g^{RTM} \ , \qquad (8)$$

where *pts* stands for point, *avg* for mean gravity data set.

Power spectra of gridded gravity anomalies were computed by multitaper spectral estimation. A rectangular area having an extension of 160 km x 300 km in the central part of the country where no extrapolation was necessary with a grid spacing of 2 km was selected for the computations. Degree variances of point gravity anomalies were computed using the covariance function of residual gravity anomalies. A second order Gauss-Markov model was fitted to empirical covariances (Tabl. 3) and degree variances were determined using Eq. (3). Fig. 1. shows the spectra of the gravitational signal in terms of gravity anomaly degree variances. Power spectra of measured anomalies shows similar decay as EGM2008 degree variances in spherical harmonic band approximately 1000 < *l* < 2000, the differences in amplitude are negligible. The recoverable spectral band of degree variances due to discretization is about 1000 < *l* < 2200 for point value anomalies, and 1000 < *l* < 10000 for mean anomalies respectively. The difference between degree variances computed from point and mean anomalies is in order of 10^{-3} in the common spectral bands, hence for further analysis degree variances derived from mean anomalies were used. The decay of the Tscherning- Rapp model variances by the increasing number of degree is slower, which could be attributed to the fact that this model is based on free-air gravity anomalies hence some portion of the signal amplitude is generated by topographic masses. For comparison the average topography- reduced model developed by Flury (2006) is also shown which follows a simple power law:

$$c_l^{av} = \frac{6.8 \times 10^7 \ \mathrm{mGal}^2}{(l+0.5)^{3.09}}. \qquad (9)$$

This model was derived using residual gravity anomalies from 13 data sets all around the word. EIGEN-CG01C model to *l,m* = 360 and RTM gravity anomalies were used to remove trend from data.

Fig. 1. shows that there is a good accordance between Flury's model and the degree variances computed from mean gravity anomalies. Tabl. 2. shows the spectral characteristics of mean gravity anomalies at different spectral bands. It can be seen that above the full spectral content of EGM2008 about 2.33 mGal gravity power remains.

Please insert Fig. 1. near here

Please insert Tabl. 2. near here

Due to the scarcity of data points, interpolation of vertical deflections onto a regular grid can lead to spurious signal, hence degree variances of vertical deflections were determined from the covariance functions. First EGM2008 deflections of the vertical computed to l,m = 360 was removed from measured deflections then a second-order Gauss-Markov model was fitted to the empirical covariance functions (Tabl. 3.). Analytical covariances were transformed to degree variances using Eq. (3). According to the average extension and point distance of the vertical deflection data set reasonable degree variances are in spectral band $360 < l < 1500$ approximately. As Fig. 2. shows the computed power spectra are in good accordance with EGM2008 degree variances in spectral band of about $1100 < l < 1500$. Unfortunately, the data sampling is not sufficient to study higher wavelength constituents of the gravitational signal. The global Tscherning-Rapp model based on 1° equal area free-air anomalies therefore the model gives only approximation for the decay of the gravitational field for higher frequencies. Nevertheless, this model could give a coarse assessment about the spectral behaviour of local gravity field quantities.

Please insert Fig. 2. near here

Please insert Tabl. 3. near here

Torsion balance developed by the Hungarian physicist baron Lorand Eötvös measures some components of the gravity gradient tensor. In this study the spectral properties of horizontal gravity gradients ($\frac{\partial^2 W}{\partial x \partial z}, \frac{\partial^2 W}{\partial y \partial z}$) which characterize the non- parallelism of level surfaces are investigated. Since these quantities are azimuth dependent, therefore measurements are usually transformed to other type of gravity field quantity, e.g. to vertical gravity gradient, which is an isotropic quantity and easier to handle. First, the long- wavelength component of the gravity field was removed from terrain effect-reduced horizontal gradients using EGM2008 to l,m = 360. Since torsion balance measurements are given at irregularly spaced points and further use of these quantities in geoid modelling needs the tools of Fourier transform therefore measurements were gridded onto a grid which size is 3°x 5.7° and its resolution is 1' x 1.5'. Synthetic horizontal gravity gradient data derived from EGM2008 model with spectral content of *l,m* = 360-2160 were used to supply grid nodes with no data. The transformation was solved by a convolution integral (Eötvös integral for Tzz) using the kernel function defined by Tóth et al. (2005). The Eötvös integral was evaluated by a 1D FFT algorithm implemented in an in-house software. The spectral behaviour of vertical gradient was determined via PSD estimation and is shown in Fig. 3. Computed vertical gradient has more signal power than EGM2008 model in spectral band *l,m* = 1000- 2160 and about *l* = 2500 the gradient spectrum is quickly damped and the signal amplitude remains well below 0.1 E unit at very-high frequencies. Since the accuracy of gravity gradients measured by a torsion balance is a few E unit in favourable circumstances (Völgyesi 2001) it means that statistically the integral gives low signal to noise ratio for high order terms in its numerical solution applied. Further numerical studies on the behaviour of the kernel function is needed, since it is rather singular near the computation point, e.g. kernel value of Stokes's function for 1' spherical distance is $S(\psi = 1')$ = 6,898, while the corresponding value for Eötvös kernel is $E_{zz}(\psi = 1')$ =23,639,643. Mean kernel computation strategy investigated by Hirt et al. (2011) would probably benefit the numerical evaluation of Eötvös integral.

Please insert Fig. 3. near here

4.2 Covariance analysis

Covariance function represents the statistical correlation between data points distributed in space. Autoovariance function (ACF) of geoid height determined from (1) astronomical vertical deflections and from (2) gravity anomaly was compared to covariance of EGM2008 geoid height in order to determine wavelength bands where geoid height derived from terrestrial data and the model describe mutual gravity field structures.

An astrogeodetic geoid was derived from the available astronomical deflections of the vertical data. The simple trigonometrical equation of astronomical leveling providing the undulation change ΔN in function of the deflection data ($\Theta = \sqrt{\xi^2 + \eta^2}$) and the point distance (d) was applied on the triangular network of the deflection points. This set of equations was solved by using the norm of least squares with a weighting system of $1/d^2$ and fixing one undulation value in the central part of the network. The a posteriori RMS of the unit weight measurement was 0.14 m which indicates significant signal loss resulted in by the under-sampling of the deflection field. The EGM2008 geoid (*l,m=2160*) was also determined at the astrogeodetic points and the 1st degree trend surfaces (planes) were removed from both of them. Assuming isotropy the 1D ACF-s of the residual undulations were computed and compared. Fig. 4. shows that statistically there is a very good agreement between the two undulation sets since the shape of the ACF-s are very similar. The variances, however, indicate some systematical difference because the variance of residual astro-geoid (0.51 m^2) is significantly higher then that of the residual EGM2008 geoid (0.39 m^2). This "scale" difference is also justified by the simple regression plot displayed in Fig. 5. The slope coefficient ($\Delta N_{EGM2008}/\Delta N_{astro}$) of the regression line is +0.70 +/-0.03. Both ACFs show a deterministic periodicity dominant at about 190 km wavelength. It is also clear that some low degree harmonics the wavelength of which are comparable to the extension of the area investigated influences the shape of ACFs (i.e. those do not tend to zero by the increase of distance) since the reduction of data by a fitted plane surface does not removes them completely. Because of the small number of astrogeodetic points (138) the consistency of ACF estimation was checked on a bigger number (935) of EGM2008 residual undulations. As one can see from Fig. 6. the number of points does not influence either the shape or the variance of the ACF significantly. It means that even this small number of astrogeodetic deflection of the vertical data represents reasonably the structure of the gravity field at the wavelengths below 200 km (l,m > 200) except the scale/variance difference and the very low degree constituents sufficiently modelled by a locally fitted planar surface.

Please insert Figs. 4-6. near here.

Regarding the gravity anomaly, geoid height was computed via the well-known Stokes convolution integral from block-mean gravity data of Eq. (8). Opposite to the astronomical leveling which utilizes purely terrestrial measurements, evaluation of surface integrals based on a relatively small data set inherently assume the use of synthetic data either by extension data set with synthetic data or by computation of truncation coefficient to reduce truncation effect. In this sense geoid determined from gravity data could not be considered as a fully EGM2008 independent quantity since gravity from EGM2008 was utilized outside the border of the country, however our investigations regarding the covariance analysis rely exclusively to the previously selected rectangular test area in the central part of the country. The ACF of gravimetric geoid height was computed by radially averaging the isotrop 2D planar ACF obtained by spectral method (Schwarz et al 1990). For comparison EGM2008 geoid height covariances was also derived with spectral content of l,m = 600 – 2160. ACFs of gravimetric geoid height and EGM2008 model (Fig. 7.) show similar characteristics as astrogeodetic and EGM2008 covariances (Fig. 4.). Systematic differences in variances which were manifested between astrogeodetic and EGM2008 geoid variances are also perceptible between gravimetric and EGM2008 geoid. Since terrestrial gravity data were reduced according to Eq. (8) with EGM2008 data with spectral content l,m = 2-600 the variances have smaller amplitude and the correlation length compared to the trend surface of a simple plane applied in case of astrogeodetic geoid height. Both ACFs show periodicity at wavelength about 70-80 km hence small-scale gravity field structures (wavelength below appr. 80 km) are also represented in the global model.

Please insert Fig. 7. near here.

An another way to control the computed spectra in terms of residual geoid height prediction from residual gravity anomaly, which can be regarded as local method to determine the quality of the estimated spectra. The necessary auto and cross covariance functions were determined from the mean gravity anomaly degree variances (gravity (AV) on Fig. 3.) since it is capable to recover the gravity field spectral characteristics in a wide spectral band. The geoid-gravity cross covariance function was derived by transforming gravity degree variances using the relevant eigenvalues of the defining operators. Degree variances can be transformed back to covariance function using the inverse relationship of Eq. (3)

$$\mathrm{C}(\psi) = \sum_{l=2}^{l_{max}} \sigma_l \, \mathrm{P}_l(\cos\psi). \qquad (10)$$

A local covariance function was obtained by removing certain numbers of lower degree variances (Lachapell 1975), i.e. set gravity σ_l coefficients to zero for $l < 600$. The corresponding covariance functions are depicted in Fig. 8., their parameters are given in Tabl. 3.

Please insert Fig. 8. near here

As Fig. 8. shows both functions are smooth and have small correlation distances and approach zero quite fast, hence here was no need to fit an analytical model to empirical covariances. Residual geoid heights ($\widehat{N}_{res}$) were predicted from residual gravity anomalies (Δg_{res}) according to the basic equation of LS prediction

$$\widehat{N}_{res} = \mathrm{C}^{\mathrm{N}\,\Delta \mathrm{g}} \left(\mathrm{C}^{\Delta \mathrm{g}\,\Delta \mathrm{g}} + \mathrm{D}^{\Delta \mathrm{g}\,\Delta \mathrm{g}}\right)^{-1} \Delta \mathrm{g}_{\mathrm{res}} \qquad (11)$$

where $C^{N\,\Delta g}$ denotes cross covariances between geoid and gravity, $C^{\Delta g\,\Delta g}$ is the auto covariance matrix of gravity. Geoid height was computed at 170 GPS/levelling points in the western part of the country, where high precision GPS/levelling data is available. Residual point value gravity anomalies were utilized for the numerical tractability of Eq. (11). The noise covariance matrix $D^{\Delta g\,\Delta g}$ of Δg was set to zero since point value gravity measurements have high accuracy and we simply wanted to test the capability of the method. Predicted residual geoid heights ($\widehat{N}_{res}$) were compared with the difference between measured and EGM2008 with spectral content of *l,m* = 2-600 geoid heights

$$\mathrm{N_{res}} = \mathrm{N_{GPS/lev}} - \mathrm{N_{EGM2008}^{2-600}}. \tag{12}$$

A sample cut of predicted $\widehat{N}_{res}$ and measured N_{res} geoid heights is shown in Fig.9., the statistics of each quantity are given in Tabl.4. The standard deviation of residual geoid heights is ± 6.1 cm, whereas result for predicted values is ± 4.4 cm, which could be assigned to the smoothing property of collocation. As Fig. 9. shows residual and predicted quantities are in good agreement.

Please insert Fig. 9. near here
Please insert Tabl. 4. near here

5. Combination of EGM and terrestrial measurements

In gravity field modelling spectral combination technique (Sjöberg 1980, Wenzel 1981) blends heterogeneous data sets by assigning spectral weights of their respective Laplace surface harmonics at each spectral degree *l*. It would be rather beneficial to utilize the spectral sensitivity of gravity related measurements in geoid determination, i.e. in which frequency band have their main power. Kern et al. (2003) developed a so-called quasi- deterministic weighting approach, which uses the spectra of measured gravity related quantities to determine the spectral weights. The main idea behind this method is that error degree variances of terrestrial measurements can be approximated with

$$\varepsilon_{\mathrm{l}}^{\mathrm{G_i}} = \left|\sigma_{\mathrm{l}}^{\mathrm{G_i}} - \sigma_{\mathrm{l}}^{\mathrm{model}}\right| \tag{13}$$

the absolute value of the difference between degree variances derived from the measurements and degree variances from an EGM or from an analytical model for higher spherical harmonic degrees. Combining EGM and terrestrial measurements based on $\varepsilon_l^{GGM}, \varepsilon_l^{G1}, \dots, \varepsilon_l^{Gi}$ error degree variances, where G_i denotes the *i*th terrestrial observation of gravity field functional, the larger weight is assigned to the gravity field quantity that has smaller error. For the description of calculating weights, the reader is referred to Kern et al. (2003). Kern et al. (2003) tested the proposed weighting scheme on synthetic gravity related data only, combining EGM, gravity anomaly and gravity disturbance at the long and medium wavelength frequencies. It would be worth to test the method in practice with the available data sets, therefore EGM2008 as a state-of-the-art global gravitational model was combined with gravity and vertical gravity gradient. For the determination of spectral weights degree variances of terrestrial measurements were transformed to potential degree variances. Above the spectral content of EGM2008 the Tscherning- Rapp model was used to feature the gravitational signal in high- frequency spectral domain. Since EGM2008 and Tscherning-Rapp degree variances do not describe the gravitational field seamless and due to the oscillation of signal degree variances of EGM2008 it is preferable to use a smooth, continuous curve for gravity field signal in the entire spectrum. Accordingly, a Tscherning- Rapp type model was fitted to the EGM2008 degree variances. For a possible combination of measurements (Fig. 10.), EGM2008 is used exclusively to spherical harmonic degree 1000, i.e. unit weight was assigned to EGM2008 and zero weight to the measurements for degrees l = 2-1000. Above spectral degree l = 1000 the lower the error according to Eq. (13), the larger weight is given. Weight of EGM2008 gradually decreases to zero and naturally, above degree 2160 the weights allocated to EGM2008 are set to zero. Gravity data has superior dominancy up to degree about 4000 compared to gravity gradient, while the high-frequency part of the gravity signal (above l = 4000) stems from vertical gradient in gravity field modelling.

Please insert Fig. 11. near here

It is acknowledged that there exist other methods for the determination of spectral weights, relies on the stochastic properties of measurements or using deterministic weighting approaches. The interested reader should consult Featherstone (2013) for a comprehensive and complete overview of the various weighting approaches and the resulted kernel modifications using Hotine integral.
The Wenzel-type stochastic modification has a wide acceptance, for reference see the different variants of the European Gravimetric (Quasi)geoid Model (Denker et al. 2009).
Regarding the great amount of surface gravity measurements in Hungary spectral combination seems to be a versatile method to incorporate them in gravity field modelling. The quasi-deterministic weighting scheme facilitates to combine GGM and different kind of gravity field measurements driven by their spectral characteristics.

6. Summary and outlook

In the present paper characteristics of the gravity field spectrum derived from different kind of terrestrial measurements involved in Hungarian data sets were analyzed. The computed degree variances were validated both in the frequency and the space domain. Both assessments showed that the estimated degree variances are realistic and could be utilized to determine that in which spectral band which kind of gravity field quantity should be applied for a high precision gravity field modelling. Results of the quasi-deterministic weighting approach show that the anticipated, the higher the order of the gravity functional the higher the spectral sensitivity for high frequencies, rule is valid. However, it should be emphasized that the sampling rate, i.e. discretisation of gravity field observables should be consistent with the recoverable spectral bands to avoid aliasing of the gravity signal. However, we think it is worth to determine how to combine global and local gravity data in a complementary way using the spectral properties of the data. The practical test of geoid determination using spectral combination with the proposed weight functions, i.e. integration of spectral weights in the appropriate kernel functions, is left for a future investigation.

Acknowledgements: Mean gravity and torsion balance data sets provided by the Geological and Geophysical Institute of Hungary are gratefully acknowledged.
This study was supported by the TAMOP-4.2.2.C–11/1/KONV-2012-0015 (Earth-system) project sponsored by the EU and European Social Foundation.

References

Alder, B. Fernbach, S. Rotenberg, M. Bolt, B. A. (1973) Methods in Computational Physics. Advances in Research and Applications. Academic Press, New York - San Francisco - London

Denker H, Barriot J-P, Barzaghi R, Fairhead D, Forsberg R, Ihde J, Kenyeres A, Marti U, Sarrailh M, Tziavos IN (2009): The development of the European gravimetric geoid model EGG07. In: Sideris MG (ed) Observing our changing earth, IAG symposia, vol 133. Springer-Verlag, Berlin, Heidelberg, pp 177–186

Featherstone W.E. (2013): Deterministic, stochastic, hybrid and band-limited modifications of Hotine's integral, Journal of Geodesy 87(5): 487–500.

Flury J. (2002): Schwerefeldfunktionale im Gebirge, Modellierungsgenauigkeit, Messpunktdichte und Darstellungsfehler. DGK München

Flury J. (2006): Short-wavelength spectral properties of the gravity field from a range of regional data sets. Journal of Geodesy, 79: 624-640.

Forsberg R. (1984): Local covariance functions and density distributions. Reports of the department of Geodetic Science and Surveying, Report No. 356.

Grant, F. S. and West, G. F. (1965). "Interpretation Theory in Applied Geophysics." MacGraw-Hill, New York
Hanssen A. (1997): Multidimensional multitaper spectral estimation. Signal Processing, 58: 327- 332.

Hirt C., WE. Featherstone, SJ. Claessens (2011): On the accurate numerical evaluation of geodetic convolution integrals. Journal of Geodesy 85(8): 519-538, doi: 10.1007/s00190-011-0451-5

Kern M., KP. Schwarz, N. Sneeuw (2003): A study on the combination of satellite, airborne and terrestrial gravity data. Journal of Geodesy, 77: 217- 225.

Kern M. (2003): An analysis of the combination and downward continuation of satellite. Airborne and errestrial Gravity Data, Ph.D. Thesis. UCGE Report 30172, Dept. of Geomatics Engineering, University of Calgary

Lachapelle G. (1975): Determination of the geoid using heterogeneous data. Mitteilungen der geodätischen Institute der Technischen Universität Graz, Folge 19.

Papp (1993): Trend models in the least-squares prediction of free-air gravity anomalies. Periodica Politechnica Ser. Civil Eng. Vol. 37. 109-130.

Parker, R. L. (1970). "The inverse problem of the electrical conductivity of the mantle." Geophys. J. Roy. Astron. Soc. 22, 121- 138.

Parker R.L. The rapid calculation of potential anomalies. Geophys. J. R. astr. Soc. 1972;31:447-455.

Pavlis NK., SA. Holmes, SC. Kenyon, JK. Factor (2008): An Earth Gravitational Model to Degree 2160: EGM2008, presented at the 2008 General Assembly of the European Geosciences Union, held in Vienna, Austria

Slepian D. (1983): Some comments on Fourier analysis, uncertainty and modelling. SIAM Review, 25., 3: 379-393.

Schwarz KP. (1984): Data types and their spectral properties. In: KP Schwarz (ed.), Proceedings of the Beijing International summer school on local gravity field approximation, pp. 1-66, China.

Schwarz KP., MG. Sideris, R. Forsberg (1990): The use of FFT techniques in physical geodesy. Geophys. J. Int. 100: 485 – 514.

Sjöberg LE. (1980): Least squares combination of satellite harmonics and integral formulas in physical geodesy. Gerlands Beitr. Geophys., 89, 371-377.

Tóth Gy., J. Ádám, L. Földváry, IN. Tziavos, H. Denker (2005): Calibration/ validation of GOCE data by terrestrial torsion balance observations. A Window on the Future of Geodesy, International Association of Geodesy Symposia Volume 128, pp 214-219

Tscherning CC., RH. Rapp (1974): Closed covariance expressions for gravity anomalies, geoid undulations, and deflections of the vertical implied by anomaly degree variance models. Report 208, Department of Geodesy Science Ohio State University, Columbus

van Gelderen M., R. Rummel (2001): The solution of the general geodetic boundary problem by least squares Journal of Geodesy 75: 1- 11.

Voigt C., H. Denker (2007): A study of high frequency effects in gravity field modelling. Proceed. 1st International Symposium of the International Gravity Field Service, “Gravity Field of the Earth”, Harita Dergisi, Special Issue 18, pp. 342-347, Ankara, Turkey, 2007

Voigt C., H. Denker (2011): Validation of GOCE gravity field models by astrogeodetic vertical deflections in Germany. Proceedings of the 4th International GOCE User Workshop, Munich, 31 March – 01 April, ESA Special Publication SP-696, CD-ROM, 2011

Zhang K., WE. Featherstone (2004): Investigation of the roughness of the Australian gravity field using statistical, graphical, fractal and Fourier power spectrum techniques. Survey Review, 37, 293.

Stoica P, R Moses (1997): Introduction to spectral analysis. Prentice Hall, p. 319.

Tscherning CC, Forsberg R, Knudsen P (1992) The GRAVSOFT package for geoid determination. Proc. 1. Continental Workshop on the Geoid in Europe, Prague, May 1992, pp 327–334, Research Institute of Geodesy, Topography and Cartography, Prague

Vögyesi L (2001): Geodetic applications of torsion balance measurements in Hungary. Reports on Geodesy, Warsaw University of Technology, Vol. 57, Nr. 2, pp. 203-212

Figure_1

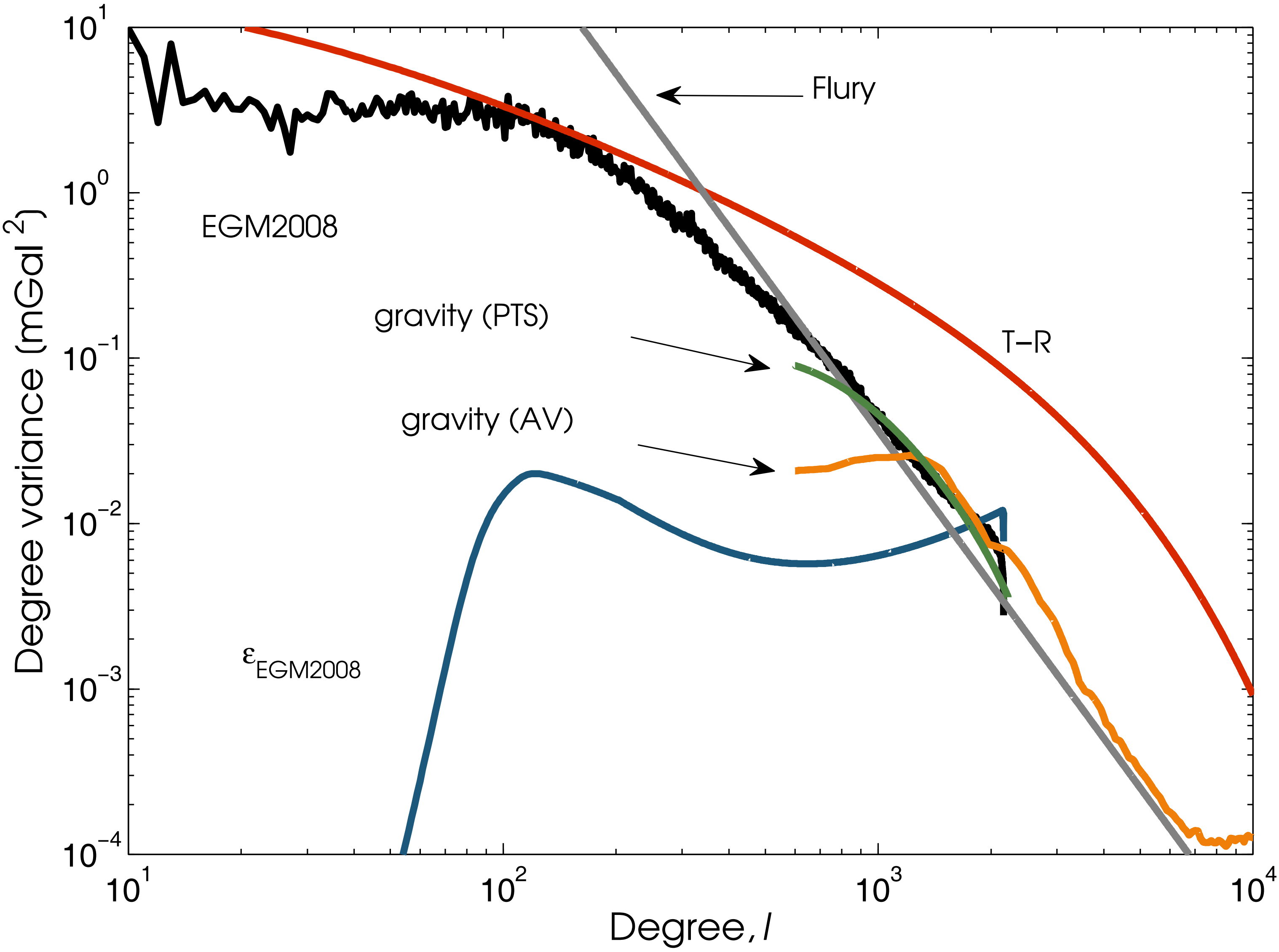

Flury
EGM2008
gravity (PTS)
T−R
gravity (AV)
$\varepsilon_{EGM2008}$
Degree variance (mGal ^{2})
Degree, l
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^1
10^2
10^3
10^4

Figure_2

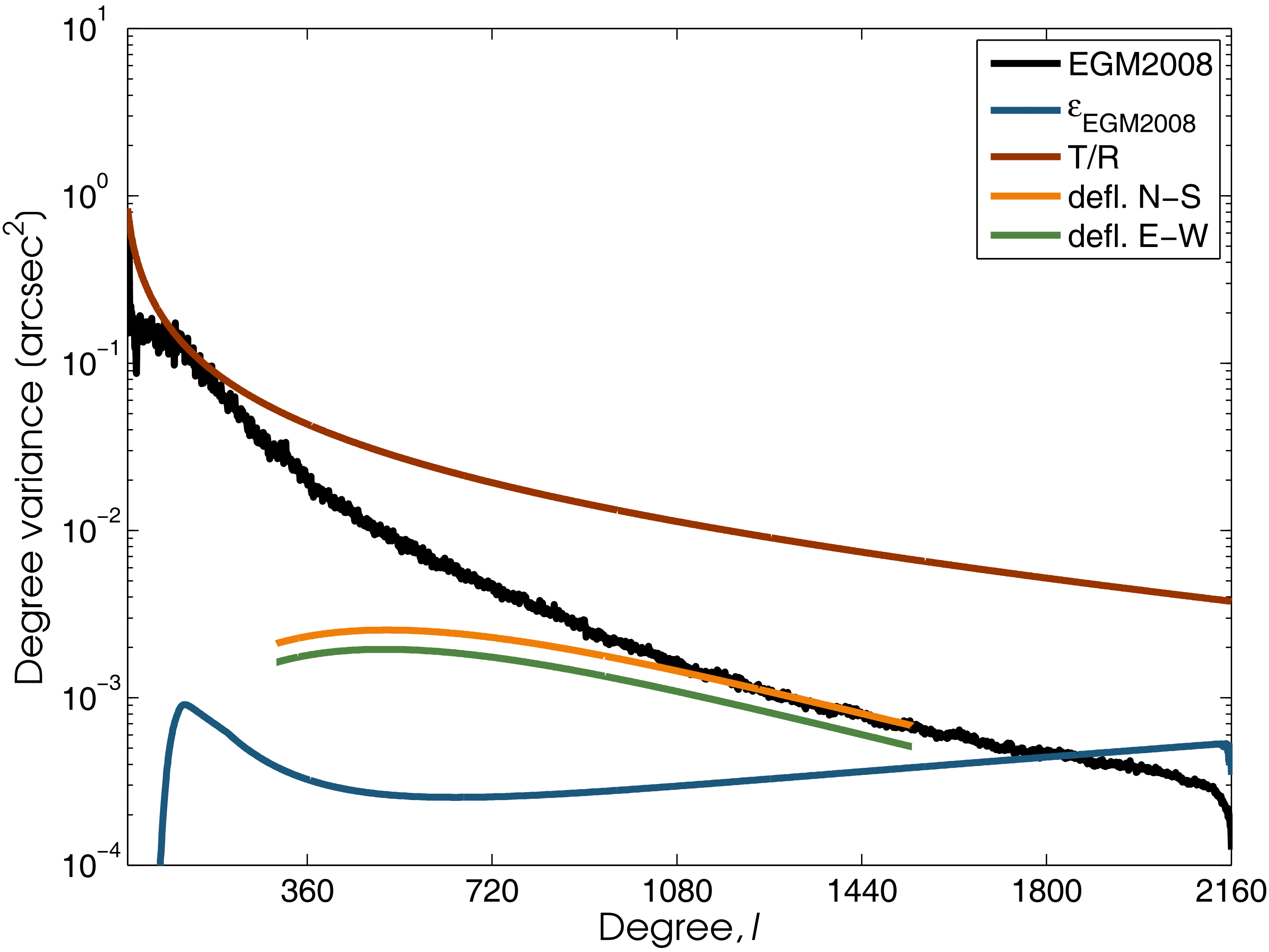
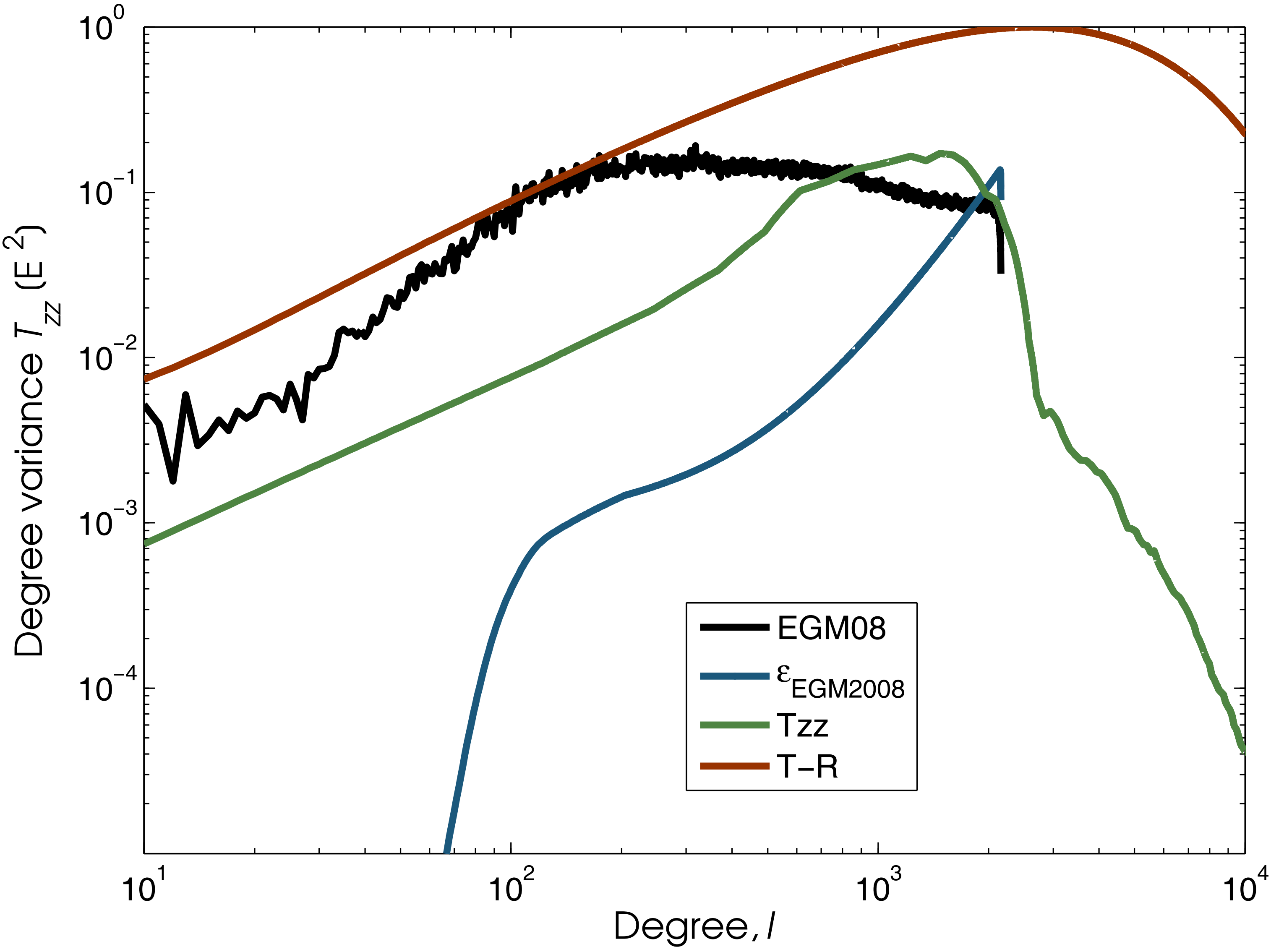

EGM2008
$\varepsilon_{EGM2008}$
T/R
defl. N−S
defl. E−W
Degree variance (arcsec^2)
Degree, l
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
360
720
1080
1440
1800
2160

Figure_3



10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
Degree variance T_{zz} (E^2)
10^1
10^2
10^3
10^4
Degree, l
EGM08
$\varepsilon_{EGM2008}$
Tzz
T−R

Figure_4

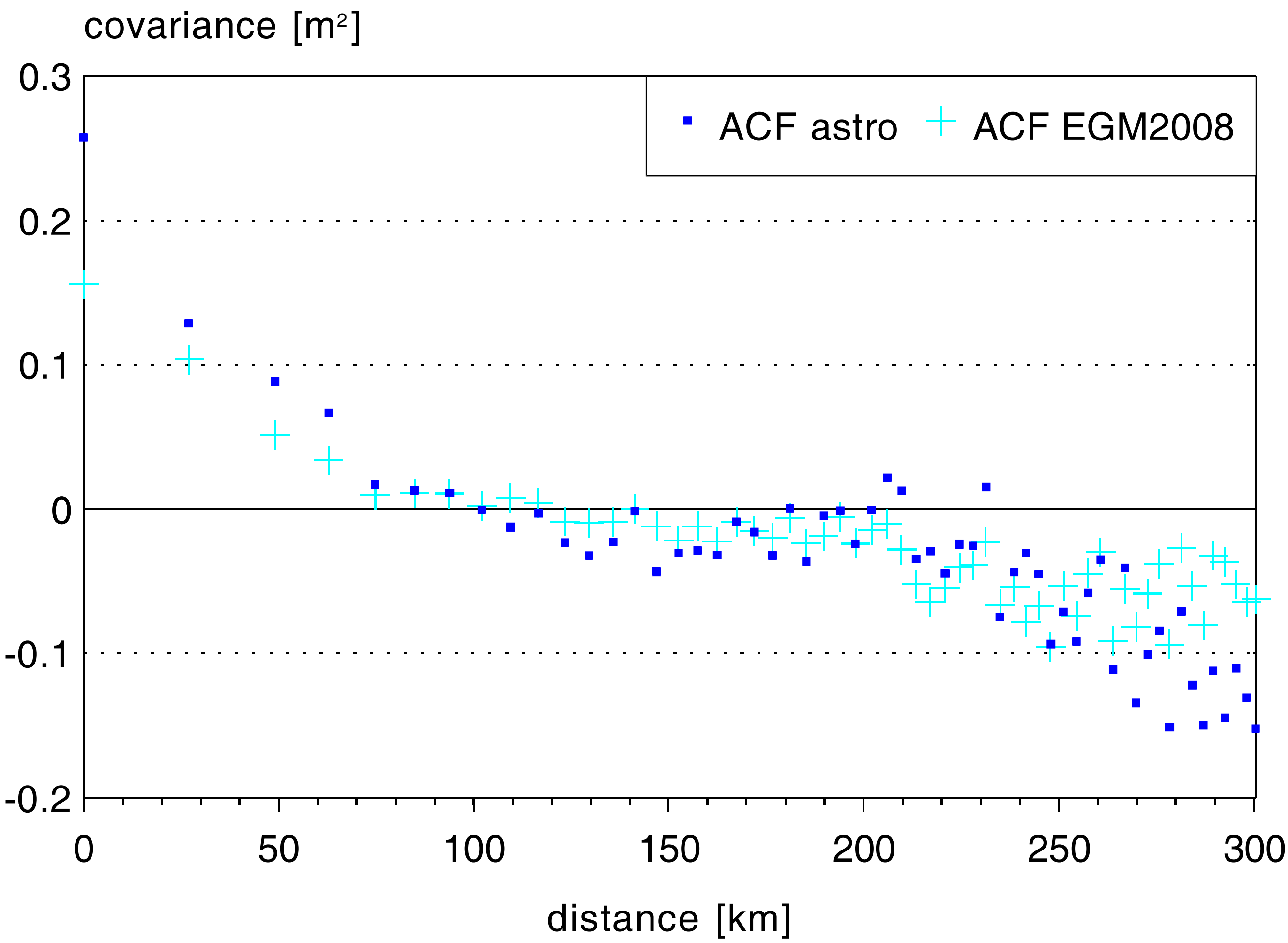

covariance [m²]
0.3
0.2
0.1
0
-0.1
-0.2
ACF astro
ACF EGM2008
0
50
100
150
200
250
300
distance [km]

Figure_5

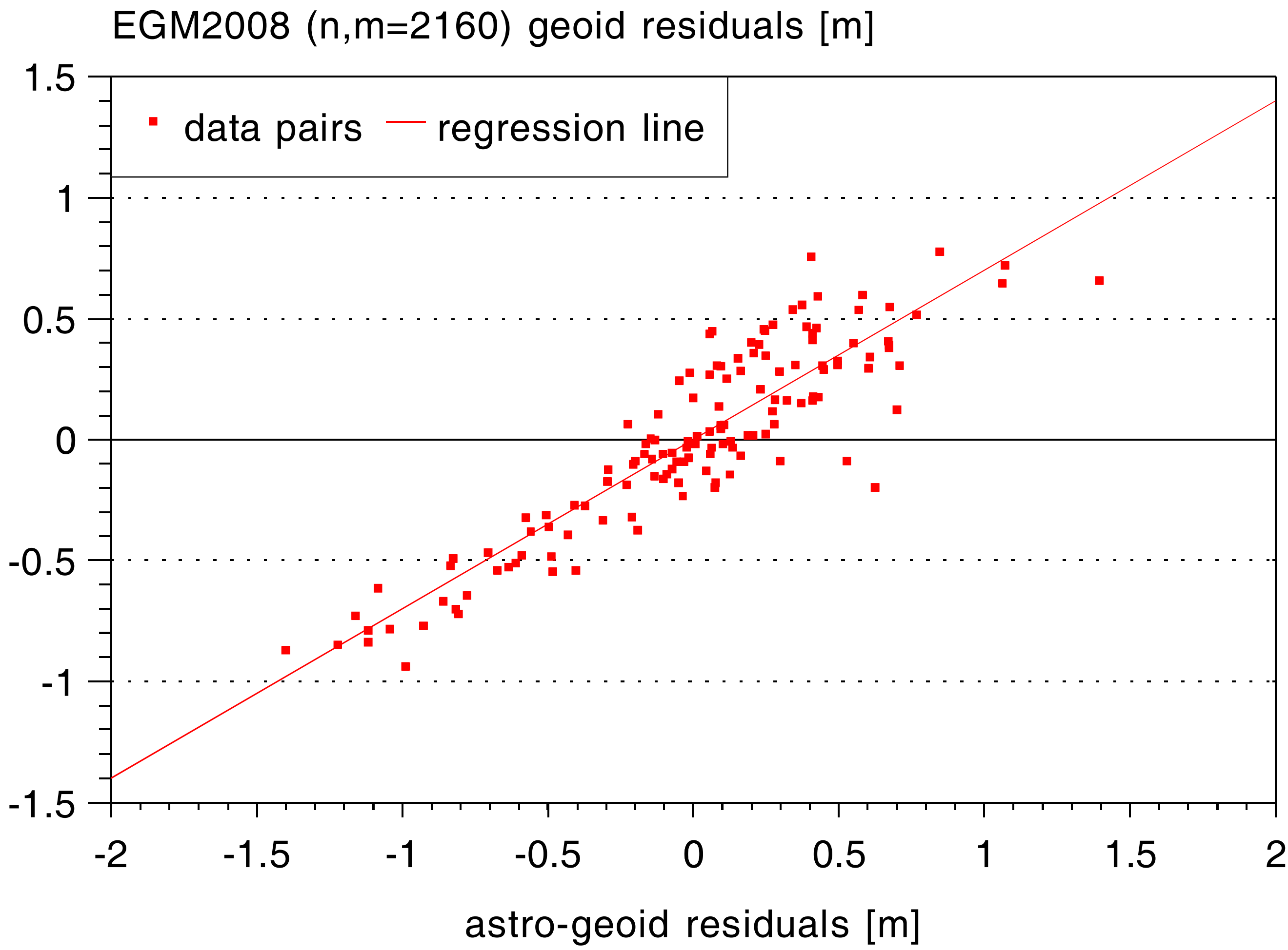

EGM2008 (n,m=2160) geoid residuals [m]
data pairs
regression line
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
astro-geoid residuals [m]

Figure_6

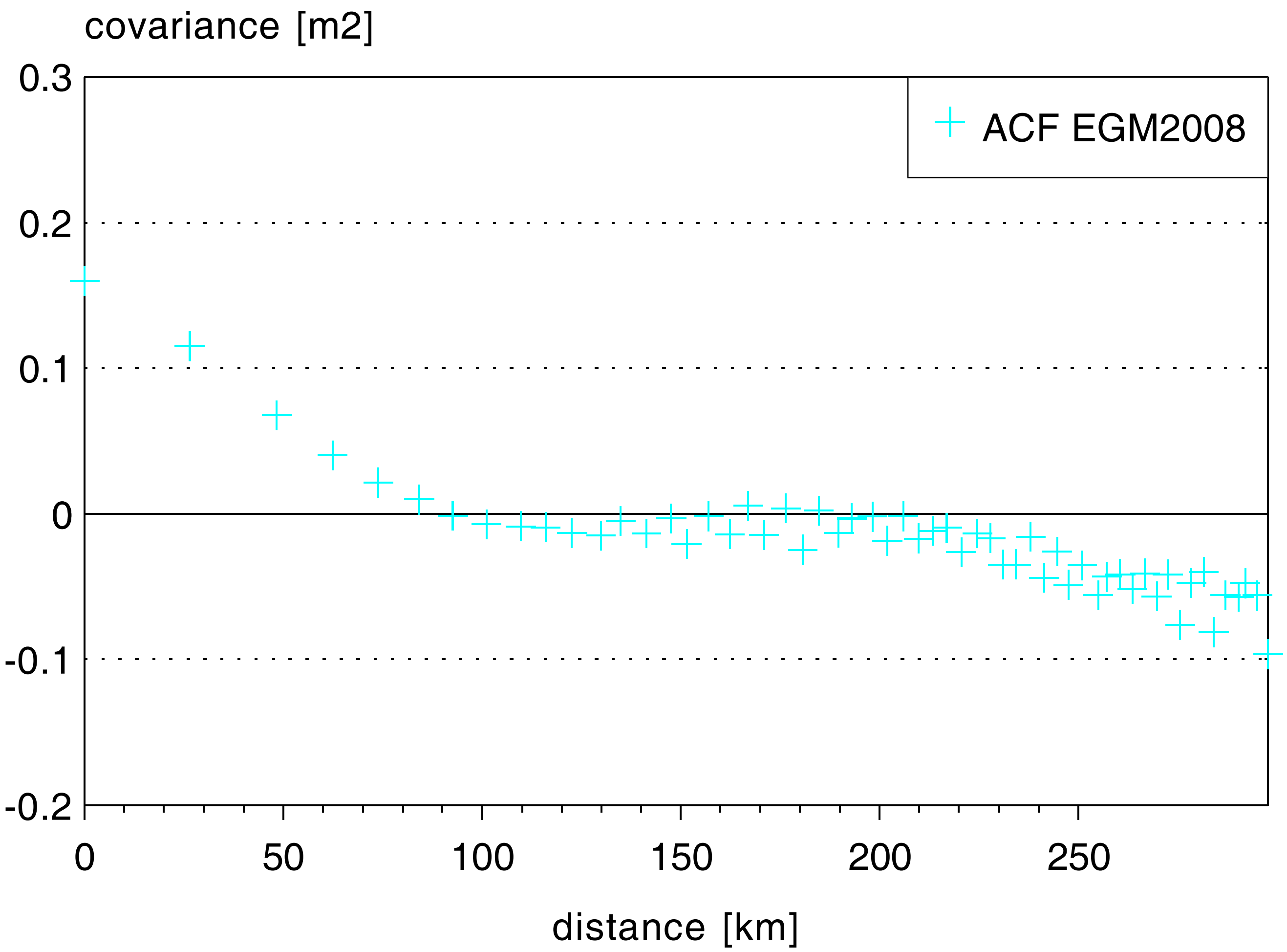

covariance [m2]
0.3
0.2
0.1
0
-0.1
-0.2
ACF EGM2008
0
50
100
150
200
250
distance [km]

Figure_7

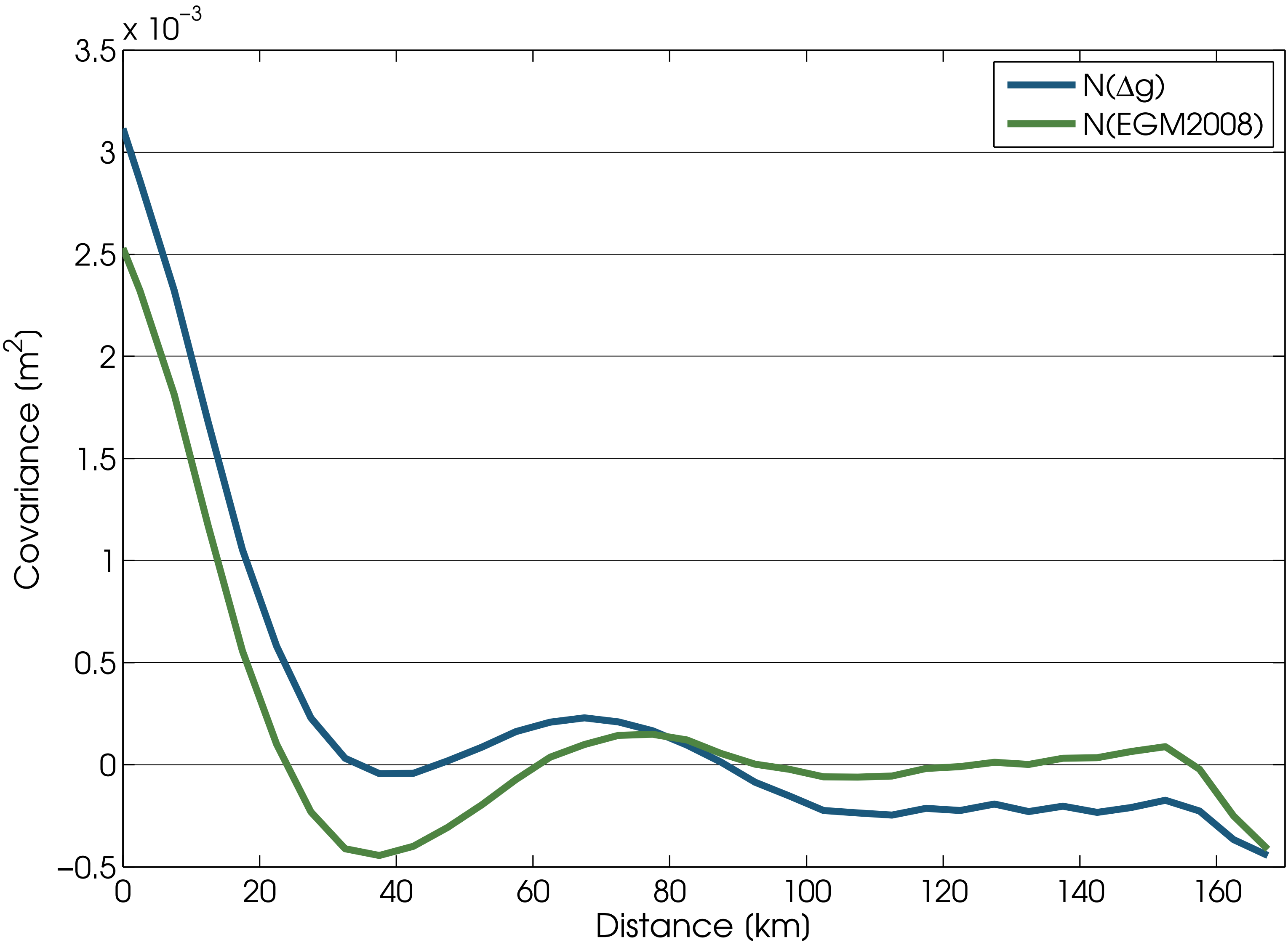
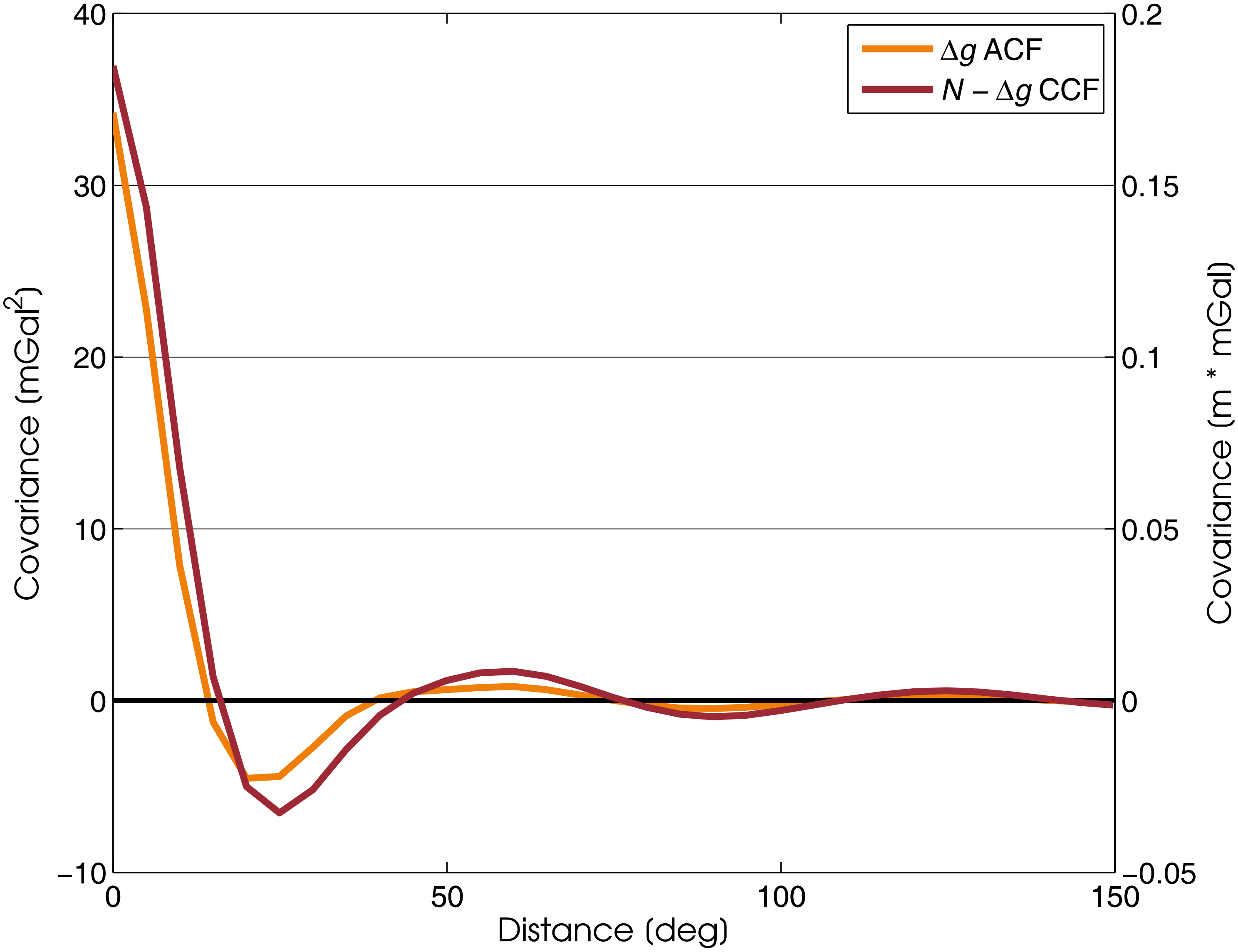
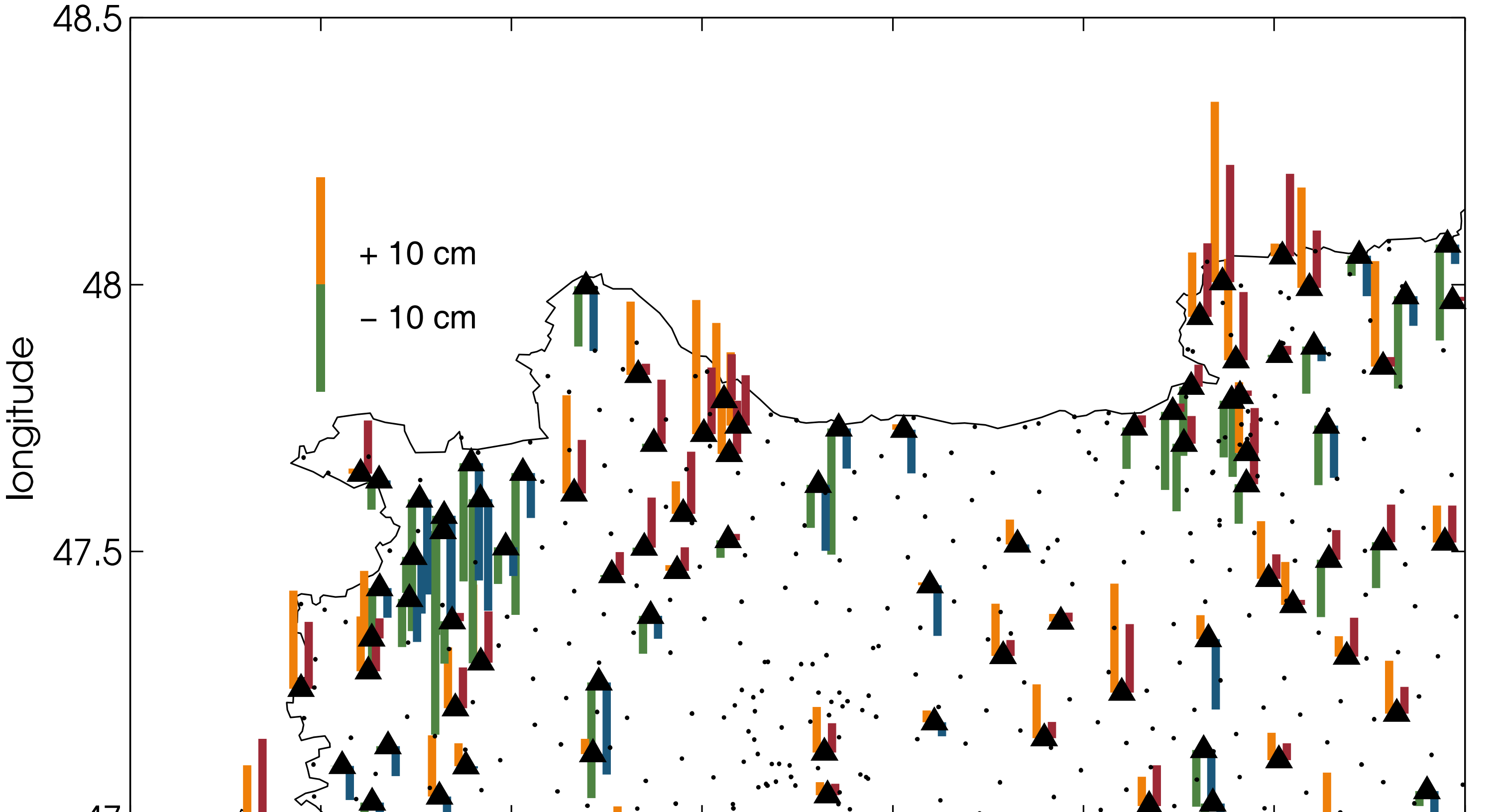

x 10^-3
3.5
3
2.5
2
1.5
1
0.5
0
−0.5
Covariance (m^2)
0
20
40
60
80
100
120
140
160
Distance (km)
N(Δg)
N(EGM2008)

Figure_8



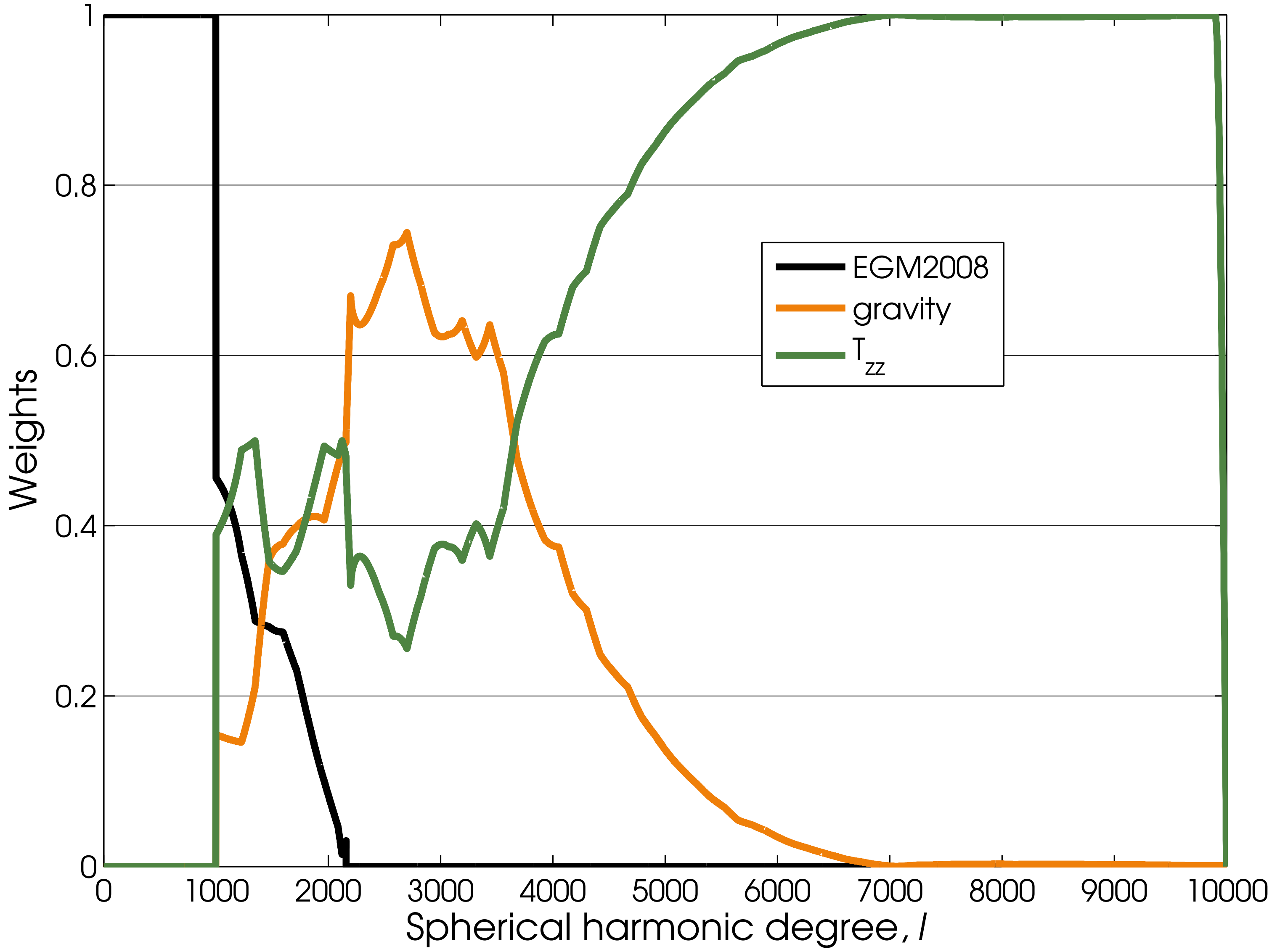
40
30
20
10
0
−10
0
50
100
150
0.2
0.15
0.1
0.05
0
−0.05
Covariance (mGal^2)
Covariance (m * mGal)
Distance (deg)
Δg ACF
$N - \Delta g$ CCF

Figure_9



48.5
48
47.5
47
16
16.5
17
17.5
18
18.5
19
19.5
longitude
latitude
+ 10 cm
− 10 cm

Figure_10



Weights
Spherical harmonic degree, l
EGM2008
gravity
T_{zz}
0
0.2
0.4
0.6
0.8
1
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000

Figure 1. Degree variances of gravity anomalies determined from mean [gravity(AV)] and point anomalies [gravity(PTS)]. For comparison EGM2008, Tscherning-Rapp [T-R] model and Flury's model gravity anomaly degree variances are also shown, with error degree variances of EGM2008

Figure 2. Degree variances for measured vertical deflection components. Analytical Tscherning-Rapp model is also depicted with EGM2008 signal and error degree variances

Figure 3. Degree variances of vertical gravity gradient with Tscherning-Rapp and EGM2008 vertical gradient signal and error degree variances

Figure 4. Autocovariance function of astrogeodetic geoid as well as EGM2008 geoid height. Astrogeodetic solution based on 138 astrogeodetic measurements. A fitted plane was removed to detrend both data

Figure 5. Scatter plot of astrogeodetic and EGM2008 geoid height

Figure 6. Geoid ACF of EGM2008 geoid heights computed at 935 points to control and verify the shape of ACF in Fig. 4.

Figure 7. ACFs of gravimetric and EGM2008 geoid heights. The long wavelength part of the gravitational field described by EGM2008 to spherical harmonic degree l = 600 was removed

Figure. 8. Local gravity (Δg ACF) and geoid- gravity (N- Δg CCF) covariance functions calculated from mean gravity anomaly [gravity(AV)] degree variances using Eq. (9)

Figure 9. Test example of predicted and residual geoid heights. Gravity anomaly points are marked by dots, triangle GPS/levelling station. Bars show positive (up) and negative (down) geoid heights respectively. Orange and green colours located on the left side of GPS/levelling marker denote N_{res} (Eq. (12)), whereas red and blue coloured columns located on right side depict predicted $\hat{N}_{res}$ (Eq. (11)) geoid height

Figure. 10. Weighting functions of EGM2008, gravity anomaly and vertical gravity gradient using quasideterministic (Kern et al. 2003) weighting approach

Table_1

Table. 1. Eigenvalues (in spectral domain) of gravity field quantities related to the disturbing potential by linear operators (van Gelderen and Rummel 2001)

functional	eigenvalue
gravity anomaly	$\frac{(l-1)}{R}$
vertical deflection	$\frac{\sqrt{l\,(l+1)}}{R}$
horizontal gradients	$\frac{(l+2)\sqrt{l\,(l+1)}}{R^2}$

Table 2. Spectral sensitivity of gravity anomaly in various spectral bands derived from EGM2008 model, measured mean gravity data and Tscherning-Rapp (T-R) degree variance model. Unit mGal

spectral band	EGM2008	gravity	T-R
721-2160	11.68	12.20	32.65
721-1080	4.74	3.15	10.93
1081-1140	3.01	2.98	8.62
1141-1800	2.20	2.10	7.10
1801-2160	1.73	1.64	6.00
2161-10000	-	2.33	11.08

Table 3. Defining parameters: variance (C_0) and correlation length of model and empirical auto and cross covariances used in this study

		C_0	correlation length[km]	unit of C_0
2^{nd} order Gauss- Markov	gravity (PTS)	99.52	8.3	$mGal^2$
	defl. N-S	3.07	7.3	$arcsec^2$
	defl. E-W	2.33	7.5	$arcsec^2$
empirical (using Eq. 10)	gravity	34.22	6.9	$mGal^2$
	gravity- geoid	0.185	8.4	mGal·m

Table_4

Table 4. Statistics of residual geoid heights [m] in sense of Eqs. (10) and (11)

	$\widehat{N}_{res}$ (Eq. (11)) (bias was removed)	N_{res} (Eq. (12))
mean	0.548	-0.006
std.	± 0.061	± 0.044
min.	-0.191	-0.116
max.	0.169	0.105