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ORE EXTENSIONS OVER NEAR PSEUDO-VALUATION RINGS

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Abstract. We recall that a ring R is called near pseudo-valuation ring if every minimal prime ideal is a strongly prime ideal.

Let R be a commutative ring, σ an automorphism of R. Recall that a prime ideal P of R is σ-divided if it is comparable (under inclusion) to every σ-stable ideal I of R. A ring R is called a σ-divided ring if every prime ideal of R is σ-divided. Also a ring R is almost σ-divided ring if every minimal prime ideal of R is σ-divided.

We also recall that a prime ideal P of R is δ-divided if it is comparable (under inclusion) to every δ-invariant ideal I of R. A ring R is called a δ-divided ring if every prime ideal of R is δ-divided. A ring R is said to be almost δ-divided ring if every minimal prime ideal of R is δ-divided.

We define a Min.Spec-type endomorphism σ of a ring R ($\sigma(U) \subseteq U$ for all minimal prime ideals U of R) and a Min.Spec-type ring (if there exists a Min.Spec-type endomorphism of R). With this we prove the following. Let R be a commutative Noetherian $\mathbb{Q}$-algebra ($\mathbb{Q}$ is the field of rational numbers), δ a derivation of R. Then:

(1) R is a near pseudo valuation ring implies that $R[x;\delta]$ is a near pseudo valuation ring.
(2) R is an almost δ-divided ring if and only if $R[x;\delta]$ is an almost δ-divided ring.

We also prove a similar result for $R[x;\sigma]$, where R is a commutative Noetherian ring and σ a Min.Spec-type automorphism of R.

1. Introduction

We follow the notation as in Bhat [10], but to make the note self contained, we have the following. All rings are associative with identity. Throughout this paper R denotes a commutative ring with identity $1 \neq 0$. The nil radical of R and the prime radical of R are denoted by $N(R)$ and $P(R)$ respectively. The set of prime ideals of R is denoted by $Spec(R)$, the set of minimal prime

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ideals of R is denoted by $\text{Min.Spec}(R)$, and the set of strongly prime ideals is denoted by $S.Spec(R)$. The center of R is denoted by $Z(R)$. The field of rational numbers and the ring of integers are denoted by $\mathbb{Q}$ and $\mathbb{Z}$ respectively unless otherwise stated.

We recall that as in Hedstrom and Houston [15], an integral domain R with quotient field F, is called a pseudo-valuation domain (PVD) if each prime ideal P of R is strongly prime ($ab \in P$, $a \in F$, $b \in F$ implies that either $a \in P$ or $b \in P$). For example let $F = \mathbb{Q}(\sqrt{2})$ and $V = F + xF[[x]] = F[[x]]$. Then V is a pseudo-valuation domain. We also note that $S = \mathbb{Q} + \mathbb{Q}x + x^2V$ is not a pseudo-valuation domain (Badawi [6]). For more details on pseudo-valuation rings, the reader is refered to Badawi [6].

In Badawi, Anderson and Dobbs [7], the study of pseudo-valuation domains was generalized to arbitrary rings in the following way. A prime ideal P of R is said to be strongly prime if aP and bR are comparable (under inclusion; i.e. $aP \subseteq bR$ or $bR \subseteq aP$) for all $a, b \in R$. A ring R is said to be a pseudo-valuation ring (PVR) if each prime ideal P of R is strongly prime.

We note that a strongly prime ideal is a prime ideal, but a prime ideal need not be a strongly prime ideal. Let $R = \begin{pmatrix} \mathbb{Z} & \mathbb{Z} \\ \mathbb{Z} & \mathbb{Z} \end{pmatrix} = M_2(\mathbb{Z})$. If p is a prime number, then the ideal $P = M_2(p\mathbb{Z})$ is a prime ideal of R, but is not strongly prime, since for $a = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $b = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ we have $ab \in P$, even though $a \notin P$ and $b \notin P$.

We also note that a PVR is quasilocal by Lemma 1(b) of Badawi, Anderson and Dobbs [7]. An integral domain is a PVR if and only if it is a PVD by Proposition (3.1) of Anderson [1], Proposition (4.2) of Anderson [2] and Proposition (3) of Badawi [3].

In Badawi [5], another generalization of PVDs is given in the following way. Let R be a ring with total quotient ring Q such that $N(R)$ is a divided prime ideal of R, let $\phi\colon Q \to R_{N(R)}$ such that $\phi(a/b) = a/b$ for every $a \in R$ and every $b \in R\backslash Z(R)$. Then ϕ is a ring homomorphism from Q into $R_{N(R)}$, and ϕ restricted to R is also a ring homomorphism from R into $R_{N(R)}$ given by $\phi(r) = r/1$ for every $r \in R$. Denote $R_{N(R)}$ by T. A prime ideal P of $\phi(R)$ is called a T-strongly prime ideal if $xy \in P$, $x \in T$, $y \in T$ implies that either $x \in P$ or $y \in P$. $\phi(R)$ is said to be a T-pseudo-valuation ring (T-PVR) if each prime ideal of $\phi(R)$ is T-strongly prime. A prime ideal S of R is called ϕ-strongly prime ideal if $\phi(S)$ is a T-strongly prime ideal of $\phi(R)$. If each prime ideal of R is ϕ-strongly prime, then R is called a ϕ-pseudo-valuation ring ($\phi - PVR$).

This article concerns the study of skew polynomial rings over PVDs. Let R be a ring, σ an endomorphism of R and δ a σ-derivation of R ($\delta\colon R \to R$ is an additive map with $\delta(ab) = \delta(a)\sigma(b) + a\delta(b)$, for all $a, b \in R$). In case σ is identity, δ is just called a derivation. For example let $R = F[x]$, F a field. Then $\sigma\colon R \to R$ defined by $\sigma(f(x)) = f(0)$ is an endomorphism of R. Also let $K = \mathbb{R} \times \mathbb{R}$. Then $g\colon K \to K$ by $g(a, b) = (b, a)$ is an automorphism of K. Let σ be an automorphism of a ring R and $\delta\colon R \to R$ any map. Let $\phi\colon R \to M_2(R)$ defined by

$$\phi(r) = \begin{pmatrix} \sigma(r) & 0 \\ \delta(r) & r \end{pmatrix},$$

for all $r \in R$ be a homomorphism. Then δ is a σ-derivation of R. Also let $R = F[x]$, F a field. Then the usual differential operator $\frac{d}{dx}$ is a derivation of R.

We denote the Ore extension $R[x; \sigma, \delta]$ by $O(R)$. If I is an ideal of R such that I is σ-stable; i.e. $\sigma(I) = I$ and I is δ-invariant; i.e. $\delta(I) \subseteq I$, then we denote $I[x; \sigma, \delta]$ by $O(I)$. We would like to mention that $R[x; \sigma, \delta]$ is the usual set of polynomials with coefficients in R, i.e. $\{\sum_{i=0}^{n} x^i a_i,\ a_i \in R\}$ in which multiplication is subject to the relation $ax = x\sigma(a) + \delta(a)$ for all $a \in R$.

In case δ is the zero map, we denote the skew polynomial ring $R[x; \sigma]$ by $S(R)$ and for any ideal II of R with $\sigma(I) = I$, we denote $I[x; \sigma]$ by $S(I)$. In case σ is the identity map, we denote the differential operator ring $R[x; \delta]$ by $D(R)$ and for any ideal J of R with $\delta(J) \subseteq J$, we denote $J[x; \delta]$ by $D(J)$.

Ore-extensions (skew-polynomial rings and differential operator rings) have been of interest to many authors. For example see [10, 11, 12, 14, 16].

Recall that a ring R is called a near pseudo-valuation ring (NPVR) if each minimal prime ideal P of R is strongly prime (Bhat [12]). For example a reduced ring is NPVR.

Here the term near may not be interpreted as near ring (Bell and Mason [8]). We note that a near pseudo-valuation ring (NPVR) is a pseudo-valuation ring (PVR), but the converse is not true. For example a reduced ring is a NPVR, but need not be a PVR.

We recall that a prime ideal P of R is said to be divided if it is comparable (under inclusion) to every ideal of R. A ring R is called a divided ring if every prime ideal of R is divided (Badawi [4]). It is known (Lemma (1) of Badawi, Anderson and Dobbs [7]) that a pseudo-valuation ring is a divided ring. Recall that a ring R is called an almost divided ring if every minimal prime ideal of R is divided (Bhat [12]).

We also recall that a prime ideal P of R is σ-divided if it is comparable (under inclusion) to every σ-stable ideal I of R. A ring R is called a σ-divided ring if every prime ideal of R is σ-divided (see Bhat [10]). A ring R is said to be almost σ-divided ring if every minimal prime ideal of R is σ-divided (Bhat [12]).

A prime ideal P of R is said to be δ-divided if it is comparable (under inclusion) to every σ-stable and δ- invariant ideal I of R. A ring R is called a δ-divided ring if every prime ideal of R is δ-divided (Bhat [10]). A ring R is said to be almost δ-divided ring if every minimal prime ideal of R is δ-divided (Bhat [12]).

The author of this paper has proved in Theorems (2.6) and (2.8) of [10] the following. Let R be a ring and σ an automorphism of R. Then:

(1) If R is a commutative pseudo-valuation ring such that $x \notin P$ for any $P \in \text{Spec}(S(R))$, then $S(R)$ is also a pseudo-valuation ring.
(2) If R is a σ-divided ring such that $x \notin P$ for any $P \in \text{Spec}(S(R))$, then $S(R)$ is also a σ-divided ring.

In Theorems (2.10) and (2.11) of [10] the following results have been proved. Let R be a commutative Noetherian $\mathbb{Q}$-algebra and δ a derivation of R. Then

(1) If R is a pseudo-valuation ring, then $D(R)$ is also a pseudo-valuation ring.
(2) If R is a divided ring, then $D(R)$ is also a divided ring.

An analogue of the above results for near Pseudo-valuation rings, almost divided rings and almost δ-divided rings has been proved in (Bhat [12]), where R is a $\sigma(*)$-ring. Recall that a ring R is said to be a $\sigma(*)$-ring (σ an endomorphism of R) if $a\sigma(a) \in P(R)$ implies $a \in P(R)$ for $a \in R$ (Kwak [16]).

Theorem ([12, 2.5])**.** *Let R be a commutative Noetherian near pseudo valuation ring which is also an algebra over $\mathbb{Q}$. Let σ be an automorphism of R such that R is a $\sigma(*)$-ring and δ a σ-derivation of R. Then $O(R)$ is a Noetherian near pseudo-valuation ring.*

Theorem ([12, 2.7])**.** *If R is a commutative Noetherian almost δ-divided $\sigma(*)$-ring which is also an algebra over $\mathbb{Q}$, then $O(R)$ is a Noetherian almost δ-divided ring.*

In this paper we give a necessary and sufficient condition for $D(R)$ over a Noetherian $\mathbb{Q}$-algebra R to be a near pseudo valuation ring. We also give a necessary and sufficient condition for $D(R)$ over a Noetherian $\mathbb{Q}$-algebra R to be an almost divided ring. We prove similar results for $S(R)$ over a Noetherian ring R. These results have been proved in Theorems (2.5) and (2.7) respectively. But before that, we have the following definition:

Definition 1.1. Let R be a ring. We say that an endomorphism σ of R is Min.Spec-type if $\sigma(U) \subseteq U$ for all minimal prime ideals U of R. We say that a ring R is Min.Spec-type ring if there exists a Min.Spec-type endomorphism of R.

Example 1.2. Let $R = \begin{pmatrix} F & F \\ 0 & F \end{pmatrix}$, where F is a field. Let $\sigma\colon R \to R$ be defined by $\sigma\Big(\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}\Big) = \begin{pmatrix} a & 0 \\ 0 & c \end{pmatrix}$. Then it can be seen that σ is a Min.Spec-type endomorphism of R, and therefore, R is a Min.Spec-type ring.

Proposition 1.3. *If R is a Noetherian ring and σ is an automorphism of R such that R is a $\sigma(*)$-ring, then σ is a Min.Spec-type automorphism of R; i.e. R is a Min.Spec-type ring.*

Proof. Note that σ is an automorphism, therefore, $\sigma(U) \subseteq U$ implies that $\sigma(U) = U$. Now let R be a $\sigma(*)$-ring. We will first show that $P(R)$ is completely semiprime. Let $a \in R$ be such that $a^2 \in P(R)$. Then $a\sigma(a)\sigma(a\sigma(a)) = a\sigma(a)\sigma(a)\sigma^2(a) \in \sigma(P(R)) = P(R)$. Therefore $a\sigma(a) \in P(R)$ and hence $a \in P(R)$. So $P(R)$ is completely semiprime. Now let $U = U_1$ be a minimal prime ideal of R. Let $U_2, U_3, \ldots, U_n$ be the other minimal primes of R. Suppose that $\sigma(U) \neq U$. Then $\sigma(U)$ is also a minimal prime ideal of R. Renumber so that $\sigma(U) = U_n$. Let $a \in \cap_{i=1}^{n-1} U_i$. Then $\sigma(a) \in U_n$, and so $a\sigma(a) \in \cap_{i=1}^{n} U_i = P(R)$. Therefore $a \in P(R)$, and thus $\cap_{i=1}^{n-1} U_i \subseteq U_n$, which implies that $U_i \subseteq U_n$ for some $i \neq n$, which is impossible. Hence $\sigma(U) = U$. □

The converse of the above need not not be true. For example let $R = F[x]$, F a field. Then R is a commutative domain with $P(R) = 0$. Let $\sigma\colon R \to R$ be defined by $\sigma(f(x)) = f(0)$. Then σ is a Min.Spec-type endomorphism of R. Now let $f(x) = xa$, $0 \neq a \in F$. Then $f(x)\sigma(f(x)) \in P(R)$, but $f(x) \notin P(R)$. Therefore R is not a $\sigma(*)$-ring.

2. Ore extensions

We recall that Gabriel proved in Lemma (3.4) of [13] that if R is a Noetherian $\mathbb{Q}$-algebra and δ is a derivation of R, then $\delta(U) \subseteq U$, for all $U \in \operatorname{Min.Spec}(R)$. This result has been generalized in Theorem (2.2) of Bhat [9] for a σ-derivation δ of R and the following has been proved:

Theorem 2.1. *Let R be a Noetherian $\mathbb{Q}$-algebra. Let σ be an automorphism of R and δ a σ-derivation of R such that $\sigma(\delta(a)) = \delta(\sigma(a))$, for $a \in R$. Then $\delta(U) \subseteq U$ for all $U \in \operatorname{Min.Spec}(R)$.*

Proof. See Theorem (2.2) of Bhat [9]. □

Theorem 2.2 ([11, Theorem 3.7])**.** *Let R be a Noetherian $\mathbb{Q}$-algebra and δ be a derivation of R. Then $P \in \operatorname{Min.Spec}(D(R))$ if and only if $P = D(P \cap R)$ and $P \cap R \in \operatorname{Min.Spec}(R)$.*

Let R be a Noetherian ring. Then since $\operatorname{Min.Spec}(R)$ is finite and for any automorphism σ of R, $\sigma^j(U) \in \operatorname{Min.Spec}(R)$ for all $U \in \operatorname{Min.Spec}(R)$ and for all integers $j \geq 1$, it follows that there exists some positive integer m such that $\sigma^m(U) = U$ for all $U \in \operatorname{Min.Spec}(R)$. We denote $\cap_{j=1}^{m} \sigma^j(U)$ by U^0. With this we have the following

Theorem 2.3 ([11, Theorem 2.4])**.** *Let R be a Noetherian ring and σ an automorphism of R. Then $P \in \operatorname{Min.Spec}(S(R))$ if and only if there exists $U \in \operatorname{Min.Spec}(R)$ Such that $S(P \cap R) = P$ and $P \cap R = U^0$.*

Theorem 2.4 (Hilbert Basis Theorem)**.** *Let R be a right/left Noetherian ring. Let σ and δ be as usual. Then the ore extension $O(R) = R[x; \sigma, \delta]$ is right/left Noetherian.*

Proof. See Theorem (1.12) of Goodearl and Warfield [14]. □

Remark 1. We note if R is a ring, σ an automorphism of R and δ a σ-derivation of R such that $\sigma(\delta(a)) = \delta(\sigma(a))$ for all $a \in R$. Then σ can be extended to an automorphism of $O(R)$ by $\sigma(x) = x$; i.e. $\sigma(xa) = x\sigma(a)$ for $a \in R$. Also δ can be extended to a σ-derivation of $O(R)$ by $\delta(x) = 0$; i.e. $\delta(xa) = x\delta(a)$ for $a \in R$.

It is known (Theorem (2.10) of Bhat [10]) that if R is a commutative Noetherian $\mathbb{Q}$-algebra which is also a PVR. Then $D(R)$ is also a PVR. We generalize this result for NPVR and prove its converse also.

It is also known (Theorem (2.11) of Bhat [10]) that if R is a commutative Noetherian $\mathbb{Q}$-algebra, and is also divided, then $D(R)$ is also divided. We generalize this result for almost divided rings and prove its converse also. Towards this we prove the following:

Theorem 2.5. *Let R be a Noetherian ring, which is also an algebra over $\mathbb{Q}$. Let δ be a derivation of R. Further let any $U \in S.Spec(R)$ with $\delta(U) \subseteq U$ implies that $O(U) \in \mathrm{S.Spec}(O(R)$. Then*

1. *R is a near pseudo-valuation ring implies that $D(R)$ is a near pseudo-valuation ring.*
2. *R is an almost δ-divided ring if and only if $D(R)$ is an almost δ-divided ring.*

Proof. (1) Let R be a near pseudo-valuation ring which is also an algebra over $\mathbb{Q}$. Now $D(R)$ is Noetherian by Theorem (2.4). Let $J \in \mathrm{Min.Spec}(D(R))$. Then by Theorem (2.2) $J \cap R \in \mathrm{Min.Spec}(R)$. Now R is a near pseudo-valuation $\mathbb{Q}$-algebra, therefore $J \cap R \in S.Spec(R)$. Also $\delta(J \cap R) \subseteq J \cap R$ by Theorem (2.1). Now Theorem (2.2) implies that $D(J \cap R) = J$, and by hypothesis $D(J \cap R) \in S.Spec(D(R))$. Therefore $J \in S.Spec(D(R))$. Hence $D(R)$ is a near pseudo-valuation ring.

(2) Let R be an almost δ-divided which is also an algebra over $\mathbb{Q}$. Now $D(R)$ is Noetherian by Theorem (2.4). Let $J \in \mathrm{Min.Spec}(D(R))$ and K be an ideal of $D(R)$. Now by Theorem (2.2) $J \cap R \in \mathrm{Min.Spec}(R)$. Now R is an almost δ-divided commutative Noetherian $\mathbb{Q}$-algebra, therefore $J \cap R$ and $K \cap R$ are comparable (under inclusion), say $J \cap R \subseteq K \cap R$. Now $\delta(K \cap R) \subseteq K \cap R$ by Lemma (2.18) of Goodearl and Warfield [14]. Therefore, $D(K \cap R)$ is an ideal of $D(R)$ and so $D(J \cap R) \subseteq D(K \cap R)$. This implies that $J \subseteq K$. Hence $D(R)$ is an almost δ-divided ring.

Conversely suppose that $D(R)$ is almost δ-divided (note that δ can be extended to a derivation of $D(R)$ by Remark (1)). Let $U \in \mathrm{Min.Spec}(R)$ and V be a δ-invariant ideal of R. Now by Theorem (2.1) $\delta(U) \subseteq U$, and Theorem (2.2) implies that $D(U) \in \mathrm{Min.Spec}(D(R))$. Now $D(R)$ is an almost δ-divided ring, therefore $D(U)$ and $D(V)$ are comparable (under inclusion), say $D(U) \subseteq D(V)$. Therefore, $D(U) \cap R \subseteq D(V) \cap R$; i.e. $U \subseteq V$. Hence R is an almost δ-divided ring. □

We note that in above Theorem the hypothesis that any $U \in S.Spec(R)$ with $\delta(U) \subseteq U$ implies that $O(U) \in S.Spec(O(R)$ can not be deleted as extension of a strongly prime ideal of R need not be a strongly prime ideal of $D(R)$.

Example 2.6. $R = \mathbb{Z}_{(p)}$. This is in fact a discrete valuation domain, and therefore, its maximal ideal $P = pR$ is strongly prime. But $pR[x]$ is not strongly prime in $R[x]$ because it is not comparable with $xR[x]$ (so the condition of being strongly prime in $R[x]$ fails for $a = 1$ and $b = x$).

It is known (Theorem (2.6) of Bhat [10]) that if R is a commutative PVR such that $x \notin P$ for any $P \in \mathrm{Spec}(S(R))$. Then $S(R)$ is also a PVR. We generalize this result for NPVR and prove its converse also.

It is known (Theorem (2.8) of Bhat [10]) that if R is a σ-divided Noetherian ring such that $x \notin P$ for any $P \in \mathrm{Spec(S(R))}$. Then $S(R)$ is also a σ-divided ring. We generalize this result for NPVR and prove its converse also. Towards this we have the following:

Theorem 2.7. *Let R be a Noetherian ring. Let σ be a Min.Spec-type automorphism of R. Further let any $U \in \mathrm{S.Spec}(R)$ with $\sigma(U) = U$ implies that $O(U) \in \mathrm{S.Spec}(O(R)$. Then*

1. *R is a near pseudo-valuation ring implies that $S(R)$ is a near pseudo-valuation ring.*
2. *R is an almost σ-divided ring if and only if $S(R)$ is an almost σ-divided ring.*

Proof. (1) Let R be a near pseudo-valuation ring. Now $S(R)$ is Noetherian by Theorem (2.4). Let $J \in \mathrm{Min.Spec}(S(R))$. Then by Theorem (2.3) there exists $U \in \mathrm{Min.Spec(R)}$ Such that $S(P \cap R) = P$ and $P \cap R = U^0$. But σ being Min.Spec-type implies that $\sigma(U) = U$, and so $U^0 = U$. Now R is a near pseudo-valuation ring implies that $U \in \mathrm{S.Spec}(R)$. Now by hypothesis $S(U) \in \mathrm{S.Spec}(S(R))$. But $S(U) = P$. Therefore $P \in \mathrm{S.Spec}(S(R))$. Hence $S(R)$ is a near pseudo-valuation ring.

(2) Let R be a ring which is also almost σ-divided. Now $S(R)$ is Noetherian by Theorem (2.4). Let $J \in \mathrm{Min.Spec}(S(R))$ and K be an ideal of $S(R)$ such that $\sigma(K) = K$ (note that σ can be extended to an automorphism of $S(R)$ by Remark (1)). Now by Theorem (2.3) there exists $U \in \mathrm{Min.Spec}(R)$ Such that $S(J \cap R) = J$ and $J \cap R = U^0$. But σ being Min.Spec-type implies that $\sigma(U) = U$, and so $U^0 = U$. Now R is an almost σ-divided, therefore U and $K \cap R$ are comparable (under inclusion), say $U \subseteq K \cap R$. Therefore, $S(U) \subseteq S(K \cap R)$. This implies that $J \subseteq K$. Hence $S(R)$ is an almost σ-divided ring.

Conversely let R be a ring such that $S(R)$ is almost σ-divided. Let $U \in \mathrm{Min.Spec}(R)$ and V be a σ-stable ideal of R. Now σ being Min.Spec-type implies that $\sigma(U) = U$ and Theorem (2.3) implies that $S(U) \in \mathrm{Min.Spec}(S(R))$.

Now $S(R)$ is an almost σ-divided ring, therefore $S(U)$ and $S(V)$ are comparable (under inclusion), say $S(U) \subseteq S(V)$. Therefore, $S(U) \cap R \subseteq S(V) \cap R$; i.e. $U \subseteq V$. Hence R is an almost σ-divided ring. □

Problem. Let R be a NPVR. Let σ be an automorphism of R and δ a σ-derivation of R. Is $O(R) = R[x; \sigma, \delta]$ a NPVR?

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