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A NOTE ON UNITS IN $\mathbb{F}_{p^m}D_{2p^n}$

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Abstract. Let p be a prime and $\mathbb{F}_{p^m}D_{2p^n}$ be the group algebra of the dihedral group D_{2p^n} of order $2p^n$ over $\mathbb{F}_{p^m} = GF(p^m)$. In this note, the structure of the unitary subgroup of the group of units of $\mathbb{F}_{p^m}D_{2p^n}$ with respect to canonical involution $*$ is established when $p > 2$. The unit group of the group algebra $\mathbb{F}_{p^m}D_{2p^n}$ is discussed. It is shown that any unit in $\mathbb{F}_{2^m}D_{2^n}$ is expressible as a product of a unitary unit and a symmetric unit. Additionally the structure of the center of the maximal p-subgroup of the unit group $\mathcal{U}(\mathbb{F}_{p^m}D_{2p^n})$ is given when $p > 2$.

1. Introduction

Let FG be the group algebra of the finite group G over the field F and $\mathcal{U}(FG)$ be its unit group. Study of units and their properties is one of the main research problems in group ring theory. Results obtained in this direction are also useful for the investigation of Lie properties of group rings, isomorphism problem and other open questions in this area(see, for example, [1]).

The map $g \mapsto g^{-1}$ of G can be extended linearly to an anti-automorphism $a \mapsto a^*$ of FG, called the *classical involution* of FG. This extension leaves $\mathcal{U}(FG)$ setwise invariant. An element $u \in \mathcal{U}(FG)$ is called a symmetric unit if $u^* = u$ and a unitary unit if $u^* = u^{-1}$. Let $\mathcal{U}_*(FG)$ be the subgroup of $\mathcal{U}(FG)$ formed by the unitary units in FG and $S_*(FG)$ be the set of symmetric units of FG. Classical involution and symmetric units were studied by V. Bovdi et al. in [3, 4, 5]. In [11], K. Kaur and M. Khan described the structure of $\mathcal{U}(\mathbb{F}_2D_{2p})$ and $\mathcal{U}_*(\mathbb{F}_2D_{2p})$ for an odd prime p. The structure of $\mathcal{U}_*(\mathbb{F}_{2^m}D_8)$ and in general, that of $\mathcal{U}_*(\mathbb{F}_{2^m}D_{2^n})$ was determined in [8] and [13] respectively. In this note, we study the units in $\mathbb{F}_{p^m}D_{2p^n}$. The structure of the unitary subgroup $\mathcal{U}_*(\mathbb{F}_{p^m}D_{2p^n})$ is established when p is an odd prime. It is shown that any unit in $\mathbb{F}_{2^m}D_{2^n}$ is expressible as a product of a unitary unit and a symmetric unit. The structure of the center of the maximal p-subgroup of $\mathcal{U}(\mathbb{F}_{p^m}D_{2p^n})$ is also given.

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The following presentation of D_{2p^n} shall be used:

$$\langle\ a,\ b \mid a^{p^n},\ b^2,\ b^{-1}ab = a^{-1}\rangle$$

2. Preliminaries

For a normal subgroup N of G, the natural homomorphism

$$G \to G/N,\ g \mapsto gN$$

can be extended to an F-algebra homomorphism

$$\varepsilon_N\colon FG \to F(G/N)$$

defined by $\sum\limits_{g\in G} a_g g \mapsto \sum\limits_{g\in G} a_g gN,\ a_g \in F$. The kernel of ε_N, denoted by $\Delta(G,N)$, is the ideal of FG generated by $\{x-1 \mid x \in N\}$ in FG and $FG/\Delta(G,N) \cong F(G/N)$. It can be seen that $\Delta(G,N) = \Delta(N)\ FG = FG\ \Delta(N)$, where $\Delta(N) = \Delta(N,N)$.

Let $J(FG)$ denote the Jacobson radical of the group algebra FG. From [12, Chap. 8, Lem. 1.17], it follows that if G is a locally finite p-group and F is a field of characteristic $p>0$, then $J(FG) = \Delta(G)$. Hence

$$\mathcal{U}(FG) = (1 + J(FG)) \times F^*.$$

That is, $\mathcal{U}(FG) = \{\ x \in FG \mid \varepsilon_G(x) \neq 0\ \}$.

The following is a more general result.

Lemma 2.1. *Let k be a perfect field and G be a finite group. Then*

$$\mathcal{U}(kG) \cong (1 + J(kG)) \rtimes \mathcal{U}\left(\frac{kG}{J(kG)}\right).$$

Proof. Observe that

$$1 \longrightarrow 1 + J(kG) \xrightarrow{inc} \mathcal{U}(kG) \xrightarrow{\psi} \mathcal{U}\left(\frac{kG}{J(kG)}\right) \longrightarrow 1$$

is a short exact sequence of groups, where $\psi(x) = x + J(kG)\ \forall\ x \in \mathcal{U}(kG)$.

By *Wedderburn-Malcev* theorem [6, Thm. 6.2.1], it follows that there exists a semisimple subalgebra B of kG such that

$$kG = B \oplus J(kG)$$

and thus for each $x + J(kG) \in \dfrac{kG}{J(kG)}$, there exists a unique $x_B \in B$ such that

$$x + J(kG) = x_B + J(kG).$$

Define $\phi\colon \mathcal{U}\left(\dfrac{kG}{J(kG)}\right) \to \mathcal{U}(kG)$ as

$$\phi\left(x + J(kG)\right) = x_B, \quad x + J(kG) \in \mathcal{U}\left(\frac{kG}{J(kG)}\right).$$

Then ϕ is a group homomorphism such that $\psi\ o\ \phi = id \mid_{\mathcal{U}(kG/J(kG))}$ and hence

$$\mathcal{U}(kG) \cong (1 + J(kG)) \rtimes \mathcal{U}\left(\frac{kG}{J(kG)}\right). \qquad \square$$

3. The unitary subgroup $\mathcal{U}_*(\mathbb{F}_{p^m}D_{2p^n})$

The unitary subgroup $\mathcal{U}_*(\mathbb{F}_{2^m}D_{2^n})$ was discussed in [13]. As a consequence of the following theorem, we shall obtain the structure of $\mathcal{U}_*(\mathbb{F}_{p^m}D_{2p^n})$ when $p > 2$.

Theorem 3.1. *Let F be a field of characteristic $p > 2$, G be a finite group having an abelian p-subgroup A of index 2 and an element b that inverts every element of A. Then*

$$\mathcal{U}_*(FG) = \mathcal{U}_*(FA) \rtimes \langle b\rangle.$$

Proof. Observe that $z = b^2 \in A$. Thus $z^b = z^{-1}$ which implies $z = z^{-1}$ and hence $b^4 = 1$. But since $|G| = 2|A|$, we find that $o(b) = 2$.

If $X \in \mathcal{U}_*(FG)$, then $X = Y + Zb$ for some $Y,\ Z \in FA$ such that $\varepsilon_G(Y) \neq 0$ or $\varepsilon_G(Z) \neq 0$.

If $\varepsilon_G(Y) \neq 0$, then $Y \in \mathcal{U}(FA)$ and hence it is possible to write

$$X = Y(1 + Wb)$$

where $W = Y^{-1}Z = \sum_{a\in A} \alpha_a a \in FA$.

Now $XX^* = 1$. This implies

$$\begin{aligned}
&Y(1 + Wb)\,(Y(1 + Wb))^* = 1,\\
\Rightarrow\ &Y(1 + Wb)(1 + Wb)^*Y^* = 1,\\
\Rightarrow\ &Y^*Y\left(1 + 2Wb + (Wb)^2\right) = 1,\\
\Rightarrow\ &Y^*Y = 1 \text{ and } W = 0,\\
\Rightarrow\ &X = Y \in \mathcal{U}_*(FA).
\end{aligned}$$

However if $\varepsilon_G(Y) = 0$, then $\varepsilon_G(Z) \neq 0$. Since $Xb = Z + Yb \in \mathcal{U}_*(FG)$, therefore via similar arguments as above, we find $Xb = Z \in \mathcal{U}_*(FA)$.

Also $b^{-1}Cb = C^* = C^{-1} \in \mathcal{U}_*(FA)$ for all $C \in \mathcal{U}_*(FA)$. Thus in either case, $X \in \mathcal{U}_*(FA) \rtimes \langle b\rangle$ and $\mathcal{U}_*(FG) = \mathcal{U}_*(FA) \rtimes \langle b\rangle$. $\square$

Remark 1. The basis of $V_*(\mathbb{F}_{p^m}A) = \{u \in \mathcal{U}_*(\mathbb{F}_{p^m}A) \mid \varepsilon_A(u) = 1\}$ is known from [2, Theorem 3] which simplifies the structure of $\mathcal{U}_*(\mathbb{F}_{p^m}G)$ as

$$\mathcal{U}_*(\mathbb{F}_{p^m}A) = V_*(\mathbb{F}_{p^m}A) \times \langle -1\rangle.$$

Corollary 3.2. *Let F be a field of characteristic $p > 2$. Then*

$$\mathcal{U}_*(FD_{2p^n}) = \mathcal{U}_*(F\langle a\rangle) \rtimes \langle b\rangle.$$

Moreover, if $F = \mathbb{F}_{p^m}$, then

$$\mathcal{U}_* \left(FD_{2p^n}\right) \cong \left(\prod_{i=1}^{n} C_{p^i}^{m_i} \times C_2 \right) \rtimes C_2,$$

where $m_n = \dfrac{m(p-1)}{2}$ *and* $m_i = \dfrac{mp^{n-i-1}(p-1)^2}{2}$ $\forall\ i,\ 1 \leq i \leq n-1.$

Proof. Using [2, Theorem 1], we find

$$V_*(\mathbb{F}_{p^m} C_{p^n}) = \prod_{i=1}^{n} C_{p^i}^{m_i}. \qquad \square$$

4. Units in $\mathbb{F}_{p^m} D_{2p^n}$

Lemma 4.1. *Let H be the subset of $\mathbb{F}_{2^m} D_{2^{n+1}}$ consisting of the elements of the form*

$$1 + \sum_{i=1}^{c} \alpha_i \left(a^i + a^{-i}\right) + \sum_{i=0}^{c} \beta_i b \left(a^i + a^{-i-1}\right)$$

where $\alpha_i,\ \beta_i \in \mathbb{F}_{2^m}$ and $c = 2^{n-1} - 1$. Then H is an abelian subgroup of $1 + J(\mathbb{F}_{2^m} D_{2^{n+1}})$ and $H \subseteq S_*(\mathbb{F}_{2^m} D_{2^{n+1}})$.

Proof. It is apparent that $H \subseteq 1 + J(\mathbb{F}_{2^m} D_{2^{n+1}})$.

Let

$$u_1 = 1 + \sum_{i=1}^{c} \alpha_i \left(a^i + a^{-i}\right) + \sum_{i=0}^{c} \beta_i b \left(a^i + a^{-i-1}\right),$$

$$u_2 = 1 + \sum_{i=1}^{c} \alpha_i' \left(a^i + a^{-i}\right) + \sum_{i=0}^{c} \beta_i' b \left(a^i + a^{-i-1}\right)$$

be any two elements of H.

Then

$$\begin{aligned} u_1 u_2 = 1 &+ \sum_{i=1}^{c} \left(\alpha_i + \alpha_i'\right) \left(a^i + a^{-i}\right) + \sum_{i=0}^{c} \left(\beta_i + \beta_i'\right) b \left(a^i + a^{-i-1}\right) \\ &+ \sum_{i=1}^{c} \sum_{j=1}^{c} \alpha_i \alpha_j' \left(a^{i+j} + a^{-i-j} + a^{i-j} + a^{-i+j}\right) \\ &+ \sum_{i=1}^{c} \sum_{j=0}^{c} \left(\alpha_i \beta_j' + \alpha_i' \beta_j\right) b \left(a^{i+j} + a^{-i-j-1} + a^{-i+j} + a^{i-j-1}\right) \\ &+ \sum_{i=0}^{c} \sum_{j=0}^{c} \beta_i \beta_j' \left(a^{i-j} + a^{-i+j} + a^{i+j+1} + a^{-i-j-1}\right) \in H \end{aligned}$$

and $u_1 u_2 = u_2 u_1$.

Since $[b(a^i+a^{-i-1})]^2=(a^{2i+1}+a^{-2i-1})$, it is apparent that $u^{2^n}=1$ and $u^*=u$, $u\in H$. ☐

Theorem 4.2. $1+J(\mathbb{F}_{2^m}D_{2^{n+1}})=\mathcal{U}_*(\mathbb{F}_{2^m}D_{2^{n+1}})H$, *where* H *is the group defined in Lemma 4.1.*

Proof. Observe that $\mathcal{U}_*(\mathbb{F}_{2^m}D_{2^{n+1}})\subseteq 1+J(\mathbb{F}_{2^m}D_{2^{n+1}})$.

Let

$$u=1+\sum_{i=1}^{2^{n-1}-1}\alpha_i\left(a^i+a^{-i}\right)+\sum_{i=0}^{2^{n-1}-1}\beta_i b\left(a^i+a^{-i-1}\right)\in\mathcal{U}_*(\mathbb{F}_{2^m}D_{2^{n+1}})\cap H.$$

As a consequence $u^2=1$ and

$$\sum_{i=1}^{2^{n-1}-1}\alpha_i^2\left(a^{2i}+a^{-2i}\right)+\sum_{i=0}^{2^{n-1}-1}\beta_i^2\left(a^{2i+1}+a^{-2i-1}\right)=0,$$

moreover

$$\sum_{i=1}^{2^{n-2}-1}\left(\alpha_i+\alpha_{2^{n-1}-i}\right)^2\left(a^{2i}+a^{-2i}\right)+\sum_{i=0}^{2^{n-2}-1}\left(\beta_i+\beta_{2^{n-1}-i-1}\right)^2\left(a^{2i+1}+a^{-2i-1}\right)=0,$$

and

$$u=\left(\sum_{i=1}^{2^{n-2}-1}\alpha_i\left(a^i+a^{-i}\right)+\sum_{i=0}^{2^{n-2}-1}\beta_i\left(a^i+a^{-i-1}\right)\right)\left(1+a^{2^{n-1}}\right)+\alpha_{2^{n-2}}(a^{2^{n-2}}+a^{-2^{n-2}}).$$

Thus $\mid\mathcal{U}_*(\mathbb{F}_{2^m}D_{2^{n+1}})\cap H\mid=2^{2^{n-1}m}$. Also from [5], it is known that

$$\mid\mathcal{U}_*(\mathbb{F}_{2^m}D_{2^{n+1}})\mid=2^{3.2^{n-1}m},$$

showing that

$$\mid\mathcal{U}_*(\mathbb{F}_{2^m}D_{2^{n+1}})H\mid=2^{m(3.2^{n-1}+(2^n-1)-2^{n-1})}=\mid 1+J(\mathbb{F}_{2^m}D_{2^{n+1}})\mid$$

and hence

$$1+J(\mathbb{F}_{2^m}D_{2^{n+1}})=\mathcal{U}_*(\mathbb{F}_{2^m}D_{2^{n+1}})H.$$ ☐

Corollary 4.3. *Every unit in* $\mathbb{F}_{2^m}D_{2^n}$ *is expressible as a product of a unitary unit and a symmetric unit.*

Proof. Since $\mathcal{U}(\mathbb{F}_{2^m}D_{2^n})=(1+J(\mathbb{F}_{2^m}D_{2^n}))\times\mathbb{F}_{2^m}^*$, the proof follows. ☐

It follows from Lemma 2.1 that in order to study the structure of $\mathcal{U}(FG)$, it is important to study its subgroup $1+J(FG)$. In [14], M. Khan et al. showed that $\mathcal{Z}(1+J(\mathbb{F}_{3^m}D_6))$ is an elementary abelian 3-group. The result was improved by J. Gildea in [7] and for any odd prime p, the center of maximal

p-subgroup of $\mathcal{U}(\mathbb{F}_{p^n} D_{2p})$ was described as an elementary abelian p-group using an established isomorphism between $\mathbb{F}_{p^n} D_{2p}$ and $M_{2p}(\mathbb{F}_{p^n})$ [9]. In this context, we now prove a generalized result.

Theorem 4.4. *Let p be any odd prime. Then*
(a) $\mathcal{U}(\mathbb{F}_{p^m} D_{2p^n}) \cong V_{m,n} \rtimes \left(\mathbb{F}^*_{p^m} \times \mathbb{F}^*_{p^m} \right)$, *where* $V_{m,n} = 1 + J(\mathbb{F}_{p^m} D_{2p^n})$, *the maximal p-subgroup of* $\mathcal{U}(\mathbb{F}_{p^m} D_{2p^n})$.
(b) If $n \geq 2$, then $\mathcal{Z}(V_{m,n})$ is a group of exponent p^n and

$$\mathcal{Z}(V_{m,n}) \cong \prod_{i=1}^{n} C_{p^i}^{\,k_i}$$

where

$$k_1 = m\left(\frac{p^{n-2}(p-1)^2}{2} + 1\right), \quad k_n = m\left(\frac{p-1}{2}\right),$$

and

$$k_t = m\left(\frac{p^{n-t-1}(p-1)^2}{2}\right) \qquad \text{for all } t,\ 1 < t < n.$$

Proof. Let $G = D_{2p^n}$ and $H = \langle a | a^{p^n} \rangle$. Since $H \trianglelefteq G$, we have

$$\Delta(G,H) = \Delta(H)\ \mathbb{F}_{p^m} G = J(\mathbb{F}_{p^m} H)\ \mathbb{F}_{p^m} G = J(\mathbb{F}_{p^m} G)$$

by [12, Ch. 7, Theorem 2.7], as $p \nmid [G:H]$. Thus

$$\frac{\mathbb{F}_{p^m} G}{J\left(\mathbb{F}_{p^m} G\right)} \cong \mathbb{F}_{p^m}(G/H) \cong \mathbb{F}_{p^m} \oplus \mathbb{F}_{p^m}$$

and

$$\mathcal{U}(\mathbb{F}_{p^m} G) \cong (1 + J(\mathbb{F}_{p^m} G)) \rtimes \left(\mathbb{F}^*_{p^m} \times \mathbb{F}^*_{p^m}\right).$$

Using [10, Proposition 1.9, p. 110], $J(\mathbb{F}_{p^m} G)^{p^n} = (0)$ and $1 + J(\mathbb{F}_{p^m} G)$ is a p-group. This proves part (a) of the Theorem.

Now $\mathcal{B} = \{ b^i(a^j - 1) \mid 0 \leq i \leq 1,\ 1 \leq j \leq p^n - 1\}$ is a basis of $J\left(\mathbb{F}_{p^m} G\right) = \Delta(G,H)$. Consequently,

$$X = \sum_{i=1}^{p^n-1} \alpha_i (a^i - 1) + \sum_{i=1}^{p^n-1} \beta_i b(a^i - 1) \in \mathcal{Z}(J(\mathbb{F}_{p^m} G)) \iff XY = YX$$

for all $Y \in \mathcal{B}$. In particular, $X \in \mathcal{Z}(J(\mathbb{F}_{p^n}[G]))$, and $X(a-1) = (a-1)X$, which implies

$$\beta_1 = \beta_{p^n-1},\ \beta_2 = -\left(\sum_{i=1}^{p^n-1} \beta_i\right) = \beta_{p^n-2}$$

and $\beta_{t-1} = \beta_{t+1}$, for all $1 < t < p^n - 1$. It means

$$\beta_2 = -\left(\frac{p^n-1}{2}\right)(\beta_1 + \beta_2)$$

and $\beta_2=\beta_1$ consequently

$$\beta_t=\beta \text{ (say)}, \quad 1\leq t\leq p^n-1.$$

Therefore

$$X=\sum_{i=1}^{p^n-1}\alpha_i(a^i-1)+\beta b\left(\sum_{i=1}^{p^n-1}(a^i-1)\right)=\sum_{i=1}^{p^n-1}\alpha_i(a^i-1)+\beta b\hat{a},$$

where $\hat{a}=1+a+\cdots+a^{p^n-1}$.

Also $X\in\mathcal{Z}(J(\mathbb{F}_{p^n}[G]))$ which implies

$$\begin{aligned}&Xb(a-1)=b(a-1)X\\ &\Rightarrow \alpha_i=\alpha_{p^n-i}\ \forall\ i,\ 1\leq i\leq\frac{p^n-1}{2}\\ &\Rightarrow X=\sum_{i=1}^{\frac{p^n-1}{2}}\alpha_i(a^i+a^{-i}-2)+\beta b\hat{a}.\end{aligned}$$

Since

$$\left(\sum_{i=1}^{\frac{p^n-1}{2}}\alpha_i(a^i+a^{-i}-2)+\beta b\hat{a}\right)Y=Y\left(\sum_{i=1}^{\frac{p^n-1}{2}}\alpha_i(a^i+a^{-i}-2)+\beta b\hat{a}\right)$$

for all $Y\in\mathcal{B},\ \alpha_i,\beta\in\mathbb{F}_{p^n}$, we conclude that

$$\left\{\ a^i+a^{-i}-2\ \middle|\ 1\leq i\leq\frac{p^n-1}{2}\right\}\cup\{b\hat{a}\}$$

forms a basis of $\mathcal{Z}(J(\mathbb{F}_{p^m}G))$.

Observe that $\mathcal{Z}(V_{m,n})=1+\mathcal{Z}(J(\mathbb{F}_{p^m}G))$ and $o(1+(a+a^{-1}-2))=p^n$. Thus $\mathcal{Z}(V_{m,n})$ is a group of exponent p^n and by fundamental theorem of abelian groups, we have

$$\mathcal{Z}(V_{m,n})\cong\prod_{i=1}^{n}C_{p^i}^{\ k_i}$$

where $k_i\geq 0$.

Let $X\in\mathcal{Z}\left(J(\mathbb{F}_{p^m}G)\right)$. Then

$$X=\beta b\hat{a}+\sum_{i=1}^{\frac{p-1}{2}}\sum_{j=0}^{n-1}\sum_{r=0}^{p^{n-j-1}-1}\beta_{i,j,r}(a^{p^j(rp+i)}+a^{-p^j(rp+i)}-2).$$

For any $t,\ 1\leq t\leq n-1$, $(1+X)^{p^t}=1$

$$\begin{aligned}&\Rightarrow X^{p^t}=0\\ &\Rightarrow\sum_{i=1}^{\frac{p-1}{2}}\sum_{j=0}^{n-t-1}\sum_{r=0}^{p^{n-j-t-1}-1}B_{i,j,r,t}^{\ p^t}(a^{(rp+i)p^{t+j}}+a^{-(rp+i)p^{t+j}}-2)=0\end{aligned}$$

where $B_{i,j,r,t} = \sum_{s=0}^{p^t-1} \beta_{i,j,r+sp^{n-j-t-1}}$.

From above we conclude that the number of elements of order $\leq p^t$ in $\mathcal{Z}(V_{m,n})$ is $p^{mN_{n,t}}$, where

$$\begin{aligned} N_{n,t} &= \left(\frac{p-1}{2}\right)\left(p^t-1\right)\sum_{j=0}^{n-t-1} p^{n-j-t-1} + \left(\frac{p-1}{2}\right)\sum_{j=n-t}^{n-1} p^{n-j-1} + 1 \\ &= \frac{\left(p^t-1\right)\left(p^{n-t}-1\right)}{2} + \frac{\left(p^t-1\right)}{2} + 1 \\ &= \frac{p^{n-t}\left(p^t-1\right)}{2} + 1 \end{aligned}$$

Thus

$$\sum_{i=1}^{t} ik_i + t\sum_{i=t+1}^{n} k_i = mN_{n,t} \forall t,\ 1 \leq t \leq n-1$$

and

$$\sum_{i=1}^{n} ik_i = m\left(\frac{p^n+1}{2}\right).$$

The rest follows by solving the above system of equations over $\mathbb{F}_{p^m}$. ☐

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