# Particle model for description of operation of screw conveyors

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**KEY WORDS:** materials handling, screw conveyor, critical angular velocity, maximum angular velocity.

**ABSTRACT**
The study is a contribution to the theory of screw conveyors. In contrast with previous investigations the author starts from the differential equation of the screws with general position and derives some valuable conclusions from its solution.

## INTRODUCTION

The object of this study is the screw conveyor which is one of the oldest conveyor, that is used for mainly horizontal, vertical and inclined conveying of dry, non sticky bulky and dusty materials. It can be mentioned, that the operational principle of screws is suitable for not only conveying but mixing and pressing. First of all, this study deals with theoretical questions of conveying, but it is possible that the results can be utilized in a wider range of other areas.

For the theoretical investigation we will use one of the simplest mechanical models, the so-called particle that is known as a rough approximation of reality therefore does not reflect exactly the real processes. In spite of this, in general, from the differential equation coming from the equation of motion can be drawn valuable conclusions, or can be gained approximate solutions that satisfy the demands of practice.

## METHODS

*Differential equation of particle moving on helical*

Granular material in the tube can be considered as an individual particle ($P$). Let us analyse the motion of the particle with respect to a set of moving coordinate axes $P,\mathbf{t},\mathbf{n},\mathbf{b},$ which are in turn moving in some known way with respect to an inertial (fixed) reference system $O,x,y,z$. For the purpose of general description let axe $z$ enclose $\delta$ angle with the direction of horizontal (*Fig. 1*). Moreover, let us assume that the particle moves on a helical being on the edge of screw blade and bordered by the tube and the coefficients of friction are constant.

As it is known, equation of motion to a rotating reference system can be obtained so that the transport and *Coriolis* inertial forces are added to the forces of interaction with other bodies acting on the particle. After this all equations and theorems of mechanics for relative motion of the particle can be written exactly like the equations of absolute motion. In our case the equation of motion:

(1) $$m\ddot{\mathbf{r}}=\mathbf{G}+\mathbf{S}_1+\mathbf{S}_2+\mathbf{B}+\mathbf{N}+\mathbf{F}_s+\mathbf{F}_c\,,$$

where: $m$ mass of the particle,
$\ddot{\mathbf{r}}$ relative acceleration of the particle,
$\mathbf{S}_1$ friction force on the screw blade,
$\mathbf{S}_2$ friction force on the tube,
$\mathbf{B}$ constrained force on the screw blade,
$\mathbf{N}$ constrained force on the tube,
$\overline{\omega}_0$ angular velocity of the shaft of the screw,
$\dot{s}$ relative velocity of the particle,

$\mathbf{G}=m\mathbf{g}$ force of gravitation,
$\mathbf{F}_s=-m[\overline{\omega}_0\times(\overline{\omega}_0\times\mathbf{r})]$ transport force,
$\mathbf{F}_c=-m(2\overline{\omega}_0\times\dot{s}\mathbf{t})$ a *Coriolis* force.

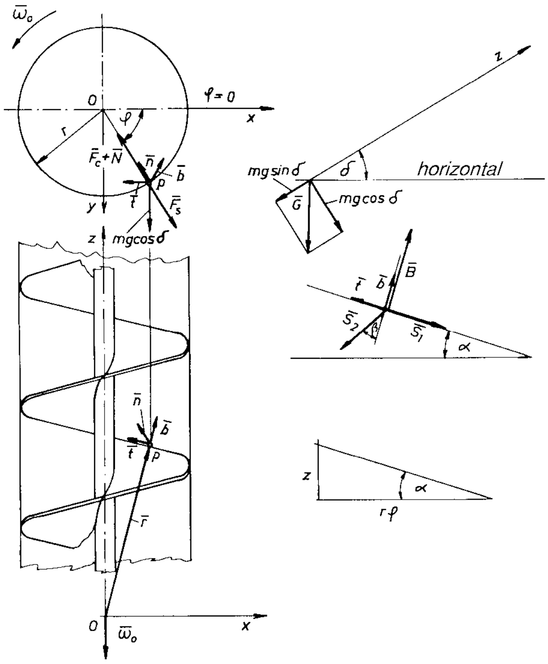


**Fig. 1** Coordinate systems and forces acting on the particle

Unit vectors $\mathbf{t},\mathbf{n},\mathbf{b}$ in the reference system *O,x,y,z*

(2) $$\mathbf{t}=\cos\alpha\begin{bmatrix}-\sin\varphi\\ \cos\varphi\\ \mathrm{tg}\alpha\end{bmatrix},\quad \frac{\mathbf{n}}{R}=\frac{\cos^2\alpha}{r}\begin{bmatrix}-\cos\varphi\\ -\sin\varphi\\ 0\end{bmatrix},\quad \mathbf{b}=\begin{bmatrix}\sin\alpha\sin\varphi\\ -\sin\alpha\cos\varphi\\ \cos\alpha\end{bmatrix},$$

where: $\alpha$ helix angle at radius *r*,
$\varphi$ angle made by *r* with the *x* coordinate axe,
$r$ projection of the radius vector **r** on the plane *x,y*,
$R=r/\cos^2\alpha$ the radius of the curvature.

Forces acting on the particle transforming into the moving coordinate system and substituting into formula (1), we obtain the differential equation of particle moving on helical [1]:

(3) $$m\ddot{s}=mg\cos\alpha\cos\varphi\cos\delta-mg\sin\alpha\sin\delta-\mu_1|\mathbf{B}|+\mu_2|\mathbf{N}|\sin\beta$$

$$|\mathbf{N}|=mg\sin\varphi\cos\delta+mr\omega_0^2-2m\omega_0\dot{s}\cos\alpha+m\frac{\dot{s}^2}{R},$$

$$|\mathbf{B}|=mg\sin\alpha\cos\varphi\cos\delta+mg\cos\alpha\sin\delta+\mu_2|\mathbf{N}|\cos\beta,$$

where $\beta$ is the direction of travel of the particle with respect to binormal vector.

The above non-linear second order differential equation particularly consists of the rule of motion for horizontal and vertical screw conveyors. Substitution $\delta=0$ and $\delta=\pi/2$ yields the equations for horizontal and vertical screw conveyors.

## RESULT, DISCUSSION

*Solution of the differential equation*

For the solution of differential equation (3) it is necessary to introduce notation relative angle of motion $\varphi_r$. According to *Fig. 2*:

$$ds = \frac{r}{\cos\alpha} d\varphi_r .$$

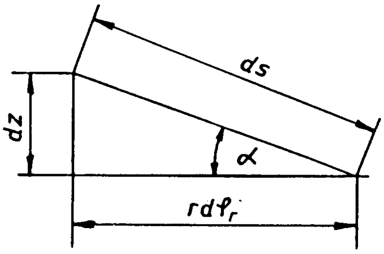


**Fig. 2**

Division by $dt$ yields the relative velocity:

$$\dot{s} = \frac{ds}{dt} = \frac{r}{\cos\alpha}\dot{\varphi}_r ,$$

and the relative acceleration

$$\ddot{s} = \frac{r}{\cos\alpha}\ddot{\varphi}_r ,$$

where $\dot{\varphi}_r$ is the relative angular velocity and $\ddot{\varphi}_r$ is the relative angular acceleration.

Substituting these expressions into equations (3) and reducing them to the appropriate form we obtain:

(4/*a*) $$\ddot{\varphi}_r = \frac{\cos\alpha}{r}\left( g\cos\alpha\cos\varphi\cos\delta - g\sin\alpha\sin\delta - \mu_1\frac{|\mathbf{B}|}{m} + \mu_2\frac{|\mathbf{N}|}{m}\sin\beta \right)$$

where:

(4/*b*) $$\frac{|\mathbf{N}|}{m} = g\sin\varphi\cos\delta + r\omega_0^2 - 2r\omega_0\dot{\varphi}_r + r\dot{\varphi}_r^2 ,$$

(4/*c*) $$\frac{|\mathbf{B}|}{m} = g\sin\alpha\cos\varphi\cos\delta + g\cos\alpha\sin\delta + \mu_2\frac{|\mathbf{N}|}{m}\cos\beta,$$

(4/*d*) $$\sin\beta = \frac{\omega_0\cos^2\alpha - \dot{\varphi}_r}{\sqrt{\omega_0^2\cos^2\alpha - 2\omega_0\dot{\varphi}_r\cos^2\alpha + \dot{\varphi}_r^2}}$$

(4/*e*) $$\cos\beta = \frac{\omega_0\cos\alpha\sin\alpha}{\sqrt{\omega_0^2\cos^2\alpha - 2\omega_0\dot{\varphi}_r\cos^2\alpha + \dot{\varphi}_r^2}}$$

From equations (4) we have to eliminate angle $\varphi$ which characterizes absolute position of the particle, more precisely we have to write it as a function of the relative angle of motion.

Let $\varphi_0$ be the angle at the time $t$=0. Because rotation of the screw is negative and relative rotation of the particle is positive, the

(4/*f*) $$\varphi = \varphi_0 - (\omega_0 t - \varphi_r).$$

Initial conditions for numerical solution of initial value problem can be given easily. At the time $t$=0 the particle locates at angle $\varphi=\varphi_0$ and the relative angle of motion is $\varphi_r(0)=0$, the relative angular velocity is also 0 i.e. $\dot{\varphi}_r(0) = 0$.

As it is known differential equations of higher order can be converted into system of differential equations of first order and after conversion we can use the one of the well-known methods for solution. In general, the differential equation of second order (4) is:

$$\ddot{\varphi}_r = f(\dot{\varphi}_r, \varphi_r, t) .$$

Let be $\dot{\varphi}_r = z$, after substitution the system of differential equations is:

$$\dot{z} = f(z, \varphi_r, t) ,$$

$$z = g(z, \varphi_r, t).$$

The initial conditions are:

$$\varphi_r(0) = 0 \text{ és } z(0) = 0.$$

The graphs of particular solutions obtained by the *Runge-Kutta* method of 4th order for a horizontal and an inclined screw conveyor characterized by parameters $\mu_1$=0.36, $\mu_2$=0.6, $\alpha$ =14.3°, $\delta$=0° and 40°, $r$=0.125 m, $\omega_0$=10 1/s and initial conditions $\varphi_0$=161.93°, 142.37° can be followed in *Fig. 3*.

The absolute position of the particles ($\varphi$) as the function of time are shown in *Fig. 3/a.* At first the curves decrease strictly and monotonously, later, after location of minimum the curves tend to a constant value by damped oscillation.

The functions of the relative angular velocity $\dot{\varphi}_r$ can be seen in *Fig. 3/c*. The functions of the relative acceleration cut the axe of time at the location of maximum of the functions of the relative angular velocity (*Fig. 3/d*). After the maximum location of the functions of the relative angular velocity decrease monotonously and oscillates around value $\dot{\varphi}_r$=10 1/s with decreasing amplitude and increasing period, i.e. the relative angular velocity tends to the angular velocity of the screw. It means, the particle no longer accelerates and its velocity becomes constant. All these can be followed also in *Fig. 3/d*, where after the minimum location the value of relative acceleration increases monotonously and tends to zero.

Speciality of the solution is that in the section of the damped motion the direction of the absolute velocity tends to ~$\beta$=–14.3°, which is equal to $-\alpha$, i.e. it is parallel to the axe $z$ (*Fig. 3/b* ).

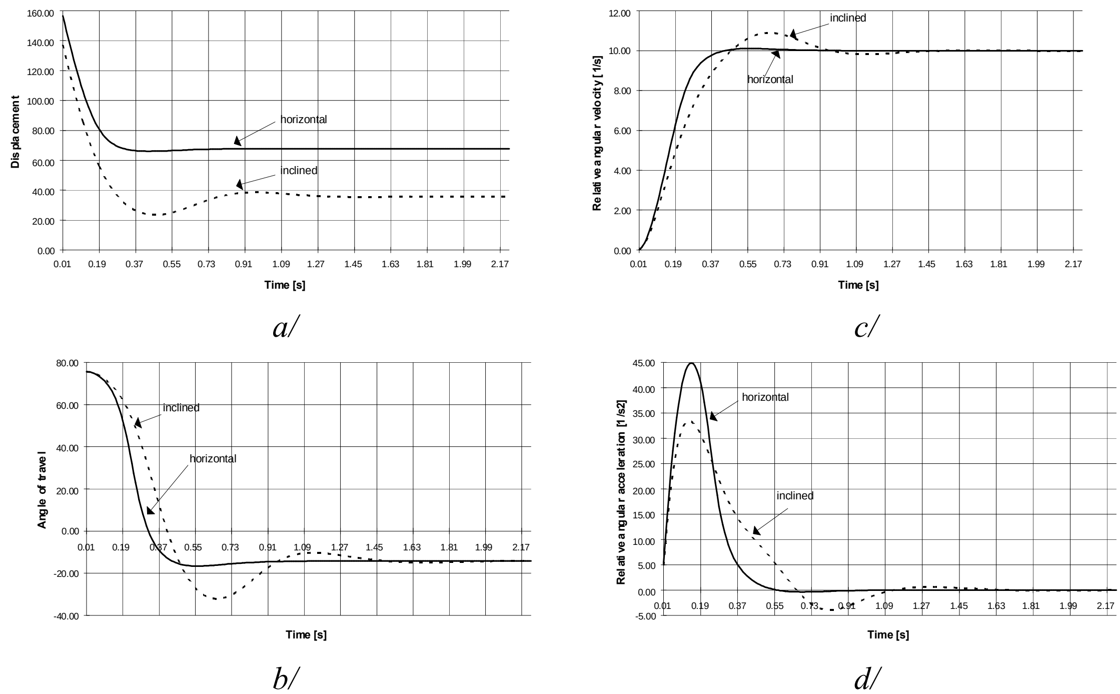


**Fig. 3:** Particular solutions of the differential equation
($\varphi_0$=$\pi$/2 $\mu_1$=0.36, $\mu_2$=0.6, $\alpha$=14.3°, $r$=0.125 m, $\omega_0$=10 1/s)

The damping of the curves are hardly perceptible because the amplitudes of the oscillation are very small. The time of the oscillation depends on the accuracy of the solution. When the space of the iteration is chosen very small e.g. $h$=0.00001, then the time of damping for $\varphi$, $\dot{\varphi}_r$, $\ddot{\varphi}_r$ and $\beta$ values are very long. Fortunately, the amplitudes of the oscillation are became rapidly negligible. In the practice, in the case of screws with ordinarily revolution, this time is less than 1 s, therefore the screw conveyors reach the steady-state during less than 1/2 revolution, henceforth values of $\varphi$, $\dot{\varphi}_r$, $\ddot{\varphi}_r$ and $\beta$ can be considered constant.

In the possession of the particular solutions and graphs let us try to describe the motion of the particle. At the time $t$=0 the particle travels with the screw blade and its relative velocity is zero, therefore the absolute velocity (**v**) is equal to the circumferential velocity of the screw blade ($\mathbf{v_k}$), and its direction is $\beta=\pi/2-\alpha$. At the initial position ($\varphi_0$) the particle slips on the screw blade (assuming that its conditions are given) and gradually accelerates. During this time the magnitude and direction ($\beta$) of the absolute velocity vector change. In the first section of the motion the vector **v** inclines right to the binormal vector **b** and the second section inclines left. Thus the path of the motion will be an irregular spiral elevating in the direction of the rotation. In the interval $\varphi$=0-$\pi$/2, at displacement ($\varphi=\varphi_a$) determining by the parameters $\mu_1$, $\mu_2$, $\alpha$, $\delta$ the motion of the particle reaches steady-state. The relative acceleration becomes zero i.e. $\ddot{\varphi}_r = 0$, and components of the forces acting on the particle in direction **t** will be equilibrium. The relative velocity will be:

$$\dot{\varphi}_r = \omega_0 ,$$

and the direction of the absolute velocity will be:

$$\beta=-\alpha.$$

The differential equation and its solution does not say too much for technical designers who are rather interested in gaining new information about steady-state. However, the key to this information is also the same system of equations (4). Therefore in the next sections we will investigate the conditions for relative motion and analyse the steady-state.

*Conditions of the relative motion*

It can be a question, how to choose the angle belonging to $t$=0. Having determine the motion of the particle we know the angle $\varphi_0$ has to be in such an interval where the conditions of the relative motion are given. Therefore the question is, how to determine the low border of this interval. Beginning of the relative motion at the time $t$=0 $\varphi_r(0)$=0, $\dot{\varphi}_r(0) = 0$ and in borderline case the $\ddot{\varphi}_r(0) = 0$. Moreover, we know that at this time $\mathbf{v}=\mathbf{v_k}$, namely the direction of vector **v**, angle $\beta$ is equal to $\pi/2-\alpha$, therefore

$$\sin\beta = \sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha ,$$

and

$$\cos\beta = \cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha .$$

Substituting expressions above and initial conditions at the time $t$=0 into equations (4) we obtain the following algebraical system of equations:

(5/*a*) $$g\cos\alpha\cos\varphi\cos\delta - g\sin\alpha\sin\delta - \mu_1\frac{|\mathbf{B}|}{m} + \mu_2\frac{|\mathbf{N}|}{m}\cos\alpha = 0,$$

where

(5/*b*) $$\frac{|\mathbf{N}|}{m} = g\sin\varphi\cos\delta + r\,\omega_0^2,$$

(5/*c*) $$\frac{|\mathbf{B}|}{m} = g\sin\alpha\cos\varphi\cos\delta + g\cos\alpha\sin\delta + \mu_2\frac{|\mathbf{N}|}{m}\sin\alpha.$$

Writing equations (5/*b* and *c*) into (5/*a*), whence

$$\cos\varphi + \mu_2\sin\varphi - \left(\mathrm{tg}\delta\frac{\sin\alpha + \mu_1\cos\alpha}{\cos\alpha - \mu_1\sin\alpha} - \frac{\mu_2 r\omega_0^2}{g\cos\delta}\right) = 0.$$

Let us introduce notation

$$C = \mathrm{tg}\delta\frac{\sin\alpha + \mu_1\cos\alpha}{\cos\alpha - \mu_1\sin\alpha} - \frac{\mu_2 r\omega_0^2}{g\cos\delta},$$

in this way

$$\cos\varphi + \mu_2\sin\varphi - C = 0.$$

After using the trigonometric identity $\cos\varphi = \sqrt{1-\sin^2\varphi}$, we obtain

(6) $$\sin^2\varphi - \frac{2C\mu_2}{\mu_2^2+1}\sin\varphi + \frac{C^2-1}{\mu_2^2+1} = 0.$$

The positive root ($\varphi_1$) of equation (6) determines a placement where the relative motion can start. Therefore, when we give the initial conditions for $\varphi_0$ it has to be:

$$\varphi_0 \le \pi - \varphi_1.$$

The condition for right operation of screw conveyors is that the real solution of (25) exists, namely the discriminator of the equation must be positive,

$$\frac{4C^2\mu_2^2}{(\mu_2^2+1)^2} \ge 4\frac{C^2-1}{\mu_2^2+1}.$$

Inserting value of $C$ and using notation $\mu_1$=tg$\rho$, we get

$$\frac{\mu_2^2}{\mu_2^2+1} \ge 1 - \frac{1}{\left(\mathrm{tg}\delta\,\mathrm{tg}(\alpha+\rho) - \frac{\mu_2 r\omega_0^2}{g\cos\delta}\right)^2}.$$

Let us solve the inequality above for $\omega_0$. After some elementary steps, we have

$$\omega_0^4 - \frac{2g\sin\delta\,\mathrm{tg}(\alpha+\rho)}{\mu_2 r}\omega_0^2 + \frac{g^2}{\mu_2^2 r^2}\left[\sin^2\delta\,\mathrm{tg}^2(\alpha+\rho) - \cos^2\delta(\mu_2^2+1)\right] \le 0.$$

Let us introduce notation

$$p = \frac{2g\sin\delta\,\mathrm{tg}(\alpha+\rho)}{\mu_2 r},$$

$$q = \frac{g^2}{\mu_2^2 r^2}\left[\sin^2\delta\,\mathrm{tg}^2(\alpha+\rho) - \cos^2\delta(\mu_2^2+1)\right],$$

and using them, we get

$$\omega_0^4 - p\omega_0^2 + q \le 0.$$

Transforming the right side of the inequality into full square, whence

$$\left(\omega_0^2 - \frac{p}{2}\right)^2 \le \left(\frac{p}{2}\right)^2 - q\,.$$

Solutions of the inequality are:

$$-\sqrt{\left(\frac{p}{2}\right)^2 - q} \le \left(\omega_0^2 - \frac{p}{2}\right) \le \sqrt{\left(\frac{p}{2}\right)^2 - q}\,.$$

Since $\omega_0$ is on the second power, the feasible solution is:

$$\omega_0^2 \le \sqrt{\left(\frac{p}{2}\right)^2 - q} + \frac{p}{2}.$$

Putting back value of $p$ and $q$, we obtain the **maximum angular velocity:**

(7) $$\omega_{0\,\max} \le \sqrt{\frac{g}{\mu_2 r}\left[\cos\delta\sqrt{\mu_2^2+1} + \sin\delta\ \mathrm{tg}(\alpha+\rho)\right]}.$$

Above this upper value the condition of the relative motion are not given. In the case of **horizontal screw conveyors** $\delta$=0, therefore

(8) $$\omega_{0\,\max} \le \sqrt{\frac{g}{\mu_2 r}\sqrt{\mu_2^2+1}}\,.$$

Consequently, the inclined and horizontal screw conveyors with given $\mu_1$, $\mu_2$, $\alpha$, $\delta$ parameters have a maximum angular velocity ($\omega_{0\max}$). Equation (6) has not got real solution above this value, namely greater angular velocity than $\omega_{0\,\max}$ the condition of the relative motion are not given. Therefore, there is no reason for increasing the revolution of inclined and horizontal screw conveyors above the determined limit. This theoretical result is supported by empirical formulas and its explanation that can be found in the literature.

In the case of **vertical screw conveyors** beginning of the relative motion is independent of angle $\varphi$, i.e. inserting $\delta$=$\pi$/2 into equation (5) the members consisting $\varphi$ are eliminated, thus

$$-g\sin\alpha - \mu_1\frac{|\mathbf{B}|}{m} + \mu_2\frac{|\mathbf{N}|}{m}\cos\alpha \ge 0,$$

$$\frac{|\mathbf{N}|}{m} = r\,\omega_0^2,$$

$$\frac{|\mathbf{B}|}{m} = g\cos\alpha + \mu_2\frac{|\mathbf{N}|}{m}\sin\alpha,$$

whence

$$-\frac{\sin\alpha + \mu_1\cos\alpha}{\cos\alpha - \mu_1\sin\alpha} + \frac{\mu_2 r\omega_0^2}{g} \ge 0,$$

and substituting $\mu_1$=tg$\rho$, we have the result:

(9) $$\omega_{0crit} \ge \sqrt{\frac{g}{\mu_2 r}\mathrm{tg}(\alpha+\rho)}\,,$$

which is identical the well known **critical angular velocity** given in the cited references [2], [4].

The formulas (7-9) have great importance in the practice since they determine the maximum or minimum angular velocity. Above the maximum or under minimum angular velocity the conditions for relative motion are missed, practically, screw conveyors do not work.

*Examination of steady-state motion*

In context of the solution and steady-state motion there are two important practical questions: (1) what is the direction of absolute velocity in steady-state and where is the placement of steady-state, (2) whether steady-state exist everywhere or not in the interval $\delta$=0-$\pi$/2.

In the particular solutions examined before, at first the angular velocity of relative motion of particle increased, later after the maximum place it decreased monotonously and tended to $\omega_0$, namely in steady-state $\dot{\varphi}_r = \omega_0$ and $\ddot{\varphi}_r = 0$. The numerical examination of the process leaded to the next conclusion. In the case of unchanged $\mu_1$, $\mu_2$, $\alpha$, $\delta$ parameters the particle reach the steady-state at the same place $\varphi=\varphi_a$ independently choosing of $\omega_0$ and $\varphi_0$, but naturally assumed that $\omega_0 \leq \omega_{0max}$, and at placement $\varphi_0$ the condition of relative motion are given. Correctness of this result can be easily realized if we examine the equations (4/*b*) and (4/*d*) thoroughly. Substituting the result of steady-state $\dot{\varphi}_r = \omega_0$ into (4/*b*) and (4/*d*) then $\omega_0$ is eliminated and $\varphi_a$ will be only the function of $\mu_1$, $\mu_2$, $\alpha$, $\delta$ parameters.

After a short digression we return to the original questions. At first approach let us suppose that in steady-state $\dot{\varphi}_r = \omega_0$ therefore let us substitute $\dot{\varphi}_r = \omega_0$ into equation (4/*d*), whence

$$\sin\beta = \frac{\omega_0 \cos^2\alpha - \omega_0}{\sqrt{\omega_0^2 \cos^2\alpha - 2\omega_0^2 \cos^2\alpha + \omega_0^2}} = \frac{\cos^2\alpha - 1}{\sqrt{\cos^2\alpha - 2\cos^2\alpha + 1}},$$

$$\sin\beta = \frac{\cos^2\alpha - 1}{\sin\alpha} = -\frac{\sin^2\alpha}{\sin\alpha} = -\sin\alpha,$$

from which $\beta=-\alpha$, namely in steady-state direction of absolute velocity is parallel with the shaft of the screw. (Let us remember the shaft of the screw was fitted to axle *z*.)

For the second part of the first question (where is the placement of steady-state) we can gain answer such a way, that we substitute $\dot{\varphi}_r = \omega_0$, $\ddot{\varphi}_r = 0$ and $\beta=-\alpha$ into system of equations (4). In this way the right side of (4/*a*) will be 0 and the members of (4/*b*) which consist $\omega_0$ will eliminate, namely

(10/*a*) $$g\cos\alpha\cos\varphi\cos\delta - g\sin\alpha\sin\delta - \mu_1\frac{|\mathbf{B}|}{m} - \mu_2\frac{|\mathbf{N}|}{m}\sin\alpha = 0,$$

where

(10/*b*) $$\frac{|\mathbf{N}|}{m} = g\sin\varphi\cos\delta,$$

(10/*c*) $$\frac{|\mathbf{B}|}{m} = g\sin\alpha\cos\varphi\cos\delta + g\cos\alpha\sin\delta + \mu_2\frac{|\mathbf{N}|}{m}\cos\alpha.$$

Writing (10/*b* and *c*) into (10/*a*) and reshaping

$$\cos\varphi - \mu_2 \frac{\sin\alpha + \mu_1\cos\alpha}{\cos\alpha - \mu_1\sin\alpha}\sin\varphi - \mathrm{tg}\,\delta\frac{\sin\alpha + \mu_1\cos\alpha}{\cos\alpha - \mu_1\sin\alpha} = 0 .$$

Using again identity $\mu_1$=tg$\rho$ and introducing notations

$$A = \mu_2 \frac{\sin\alpha + \mu_1\cos\alpha}{\cos\alpha - \mu_1\sin\alpha} = \mu_2\,\mathrm{tg}(\alpha+\rho)\,,$$

$$B = \mathrm{tg}\delta\frac{\sin\alpha + \mu_1\cos\alpha}{\cos\alpha - \mu_1\sin\alpha} = \mathrm{tg}\delta\,\mathrm{tg}(\alpha+\rho),$$

we gain a second-degree equation:

(11) $$(A^2+1)\sin^2\varphi + 2AB\sin\varphi + B^2 - 1 = 0 .$$

From equation (11) angle $\varphi_a$ which belongs to steady-state can be calculated. On the other hand, conditions of real solution give answer to the second question. Coefficients of the equation

$$a = A^2 + 1 = \mu_2^2\,\mathrm{tg}^2(\alpha+\rho) + 1,$$

$$b = 2AB = 2\mu_2\mathrm{tg}\delta\;\mathrm{tg}^2(\alpha+\rho)\,,$$

$$c = B^2 - 1 = \mathrm{tg}^2\delta\;\mathrm{tg}^2(\alpha+\rho) - 1\;.$$

The condition of feasible solution

$$b^2 - 4ac \geq 0 .$$

Solving it for $\delta$

(12) $$\sqrt{\frac{1}{\mathrm{tg}^2(\alpha+\rho)} + \mu_2^2} \geq \mathrm{tg}\delta = \mathrm{tg}\delta_h .$$

What can be read from the result (12). The steady-state characterized by $\dot{\varphi}_r = \omega_0$ can be reached only to a limited $\delta_h$ border where $\delta_h$ is determined by the angles $\alpha$, $\rho$ and the friction coefficient $\mu_2$.

Above the border $\delta_h$ the motion obtains steady-state at only $\delta$=90°. However, the direction of the absolute velocity of the particle is not parallel with axle $z$, namely $\beta \neq -\alpha$ and value of $\dot{\varphi}_r$ never reach value of $\omega_0$. We can follow it in *Fig. 4*, where two particular solutions are shown for a vertical and an inclined ($\delta$=80°) screw. Other parameters for the two solutions are the same ($\mu_1$=0.36, $\mu_2$=0.6, $\alpha$=17.66°, $r$=0.125 m, $\omega_0$=15 1/s, $\varphi_0$=$\pi$/2).

The results for inclined ($\delta > \delta_h$) screw seems to be most interesting in the figure. According to the graph the angle $\beta$ which characterizes direction of absolute motion, the relative angular velocity and acceleration periodically change around a mean value. However, against the previous results the amplitudes of the functions do not decrease and the motion does not reach steady-state. It can be assumed that the continuous change of velocity requires considerable energy. The proof of this assumption needs measuring in the future.

The messages of curves for vertical screw are the same than the previously published results for vertical screws [2], [4]. The solution of differential equation confirms the cited authors who wrote the equilibrium equations resuming steady-state. Naturally equilibrium equations can be gained from system of equations (4) putting the condition of steady-state into them.

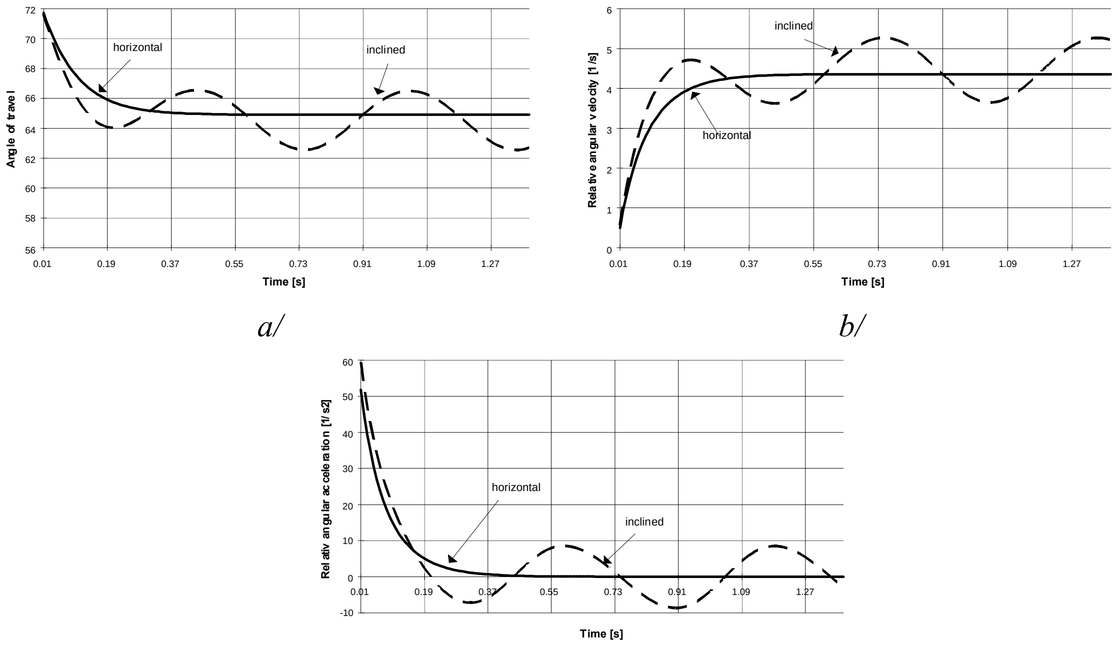


**Fig. 4:** Solution of the differential equation for vertical and inclined screws ($\mu_1$=0.36, $\mu_2$=0.6, $\alpha$=17.66°, $r$=0.125 m, $\omega_0$=15 1/s, $\varphi_0$=$\pi$/2).

For vertical screw $\delta=\pi/2$ and $\ddot{\varphi}_r = 0$, whence

(13/*a*) $$-g\sin\alpha - \mu_1 \frac{|\mathbf{B}|}{m} + \mu_2 \frac{|\mathbf{N}|}{m}\sin\beta = 0,$$

where

(13/*b*) $$\frac{|\mathbf{N}|}{m} = r\omega_0^2 - 2r\omega_0\dot{\varphi}_r + r\dot{\varphi}_r^2,$$

(13/*c*) $$\frac{|\mathbf{B}|}{m} = g\cos\alpha + \mu_2 \frac{|\mathbf{N}|}{m}\cos\beta,$$

(13/*d*) $$\sin\beta = \frac{\omega_0\cos^2\alpha - \dot{\varphi}_r}{\sqrt{\omega_0^2\cos^2\alpha - 2\omega_0\dot{\varphi}_r\cos^2\alpha + \dot{\varphi}_r^2}},$$

(13/*e*) $$\cos\beta = \frac{\omega_0\cos\alpha\sin\alpha}{\sqrt{\omega_0^2\cos^2\alpha - 2\omega_0\dot{\varphi}_r\cos^2\alpha + \dot{\varphi}_r^2}}.$$

From algebraical system of equations the relative angular velocity $\dot{\varphi}_r$ and the direction of absolute velocity, angle $\beta$ can be calculated with any numerical method.

## SUMMARY

Differential equation of particle moving on helical and its particular solutions are applicable to realize and analyze operation of the screw conveyors but in this study the full advantage of the equation have not been taken yet. With assistance of system of equations we can specify the conditions of relative motion and steady-state. From these conditions the greatest and critical revolution of horizontal, vertical and inclined screws can be derived. The placement of steady-state motion which probably effects to the cross-section of the material flow can be calculated

very easily. Moreover, it has been proved that the steady-state can be only obtained to a limited border ($\delta \leq \delta_h$).

Confirmation by measure and clarify the limitation of the theory are the task of the further investigations.

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