Chaotic transients in a simple frictional oscillator

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Summary. In this paper, we investigate the model of a single degree-of-freedom periodically forced oscillator that is damped by Coulomb-type friction. During the earlier examination of a similar model - with equal coefficients of sliding and sticking - only periodic steady-state solutions were found. In the present model, the two coefficients of friction are different. We show that this slight modification can lead to the occurrence of chaotic or transient chaotic behaviour in certain parameter ranges, where the ratio of the sticking and sliding coefficient is quite large. The possibility of model verification is highlighted by means of measuring this ratio.

Introduction

The study of piecewise-smooth dynamical systems became very common during the last decade. The need for a better understanding of valve chatter, gear rattle or disc brake squeal has also drawn the interest of the industry towards non-smooth phenomena. For this reason most of the studied examples that can be found are rather complicated. Piecewise-smooth dynamical systems very often show interesting non-smooth bifurcation scenarios and they sometimes also tend to behave chaotically. In this work our motivation was to find the simplest possible dry-friction model that can lead to chaos. We have used two numerical methods that are also applicable to non-smooth systems to prove chaos and we were able to identify a narrow parameter region of transient chaos.

Our mechanical model that originates from [1] and [2] is depicted in Figure 1(a). The corresponding equation of motion can be written in a non-dimensional form as follows:

$$\ddot{x} + x = \cos\left(\Omega\left(t + t_0\right)\right) - Sf\left(\dot{x}\right) \tag{1}$$

$$f\left(\dot{x}\right) \in \begin{cases} 1 & \text{if } \dot{x} > 0 \\ [-S_1/S, S_1/S] & \text{if } \dot{x} = 0 \\ -1 & \text{if } \dot{x} < 0 \end{cases},$$

where $\dot{()}$ denotes non-dimensional time derivative, Ω is the dimensionless excitation frequency and S_1, S are the static and dynamic friction forces respectively. Function $f(\dot{x})$ can be an arbitrary function that models the friction characteristics. Several such functions were introduced in the literature in order to try to handle friction as precisely as possible, however, these formulae may sometimes be rather complicated. For simplicity we choose a piecewise linear function that models the different coefficients for sliding and sticking, otherwise, it is independent of the velocity. A plot of the function can be seen in Figure 1(b). The solution of Eq.(1) can be obtained analytically between two stops.

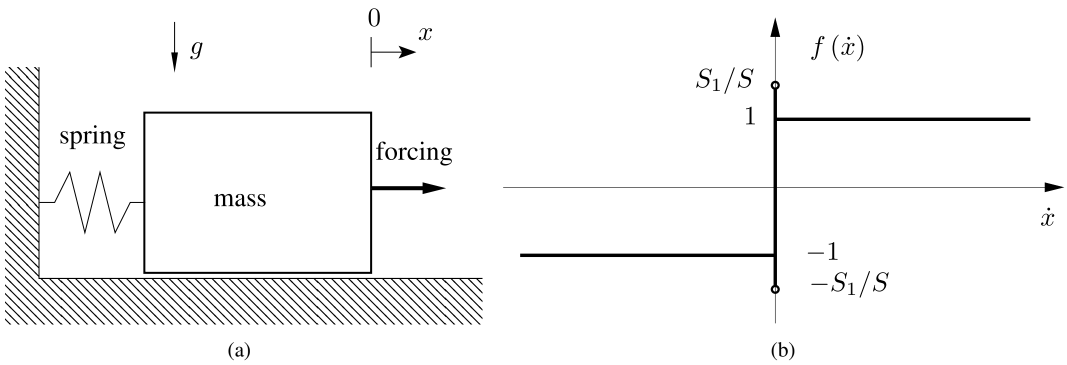

Figure 1: Panel (a) shows the mechanical model and panel (b) depicts the friction characteristics used.

Chaos and transient chaos

For the further analysis we built up a simulation program in Matlab that uses a hybrid approach[3]. Between stops or turnaround points the solution is calculated analytically, however, the exact time instants of these events are determined by means of numerical approximation. In a recent publication of the authors [3] it was showed that this simple system exhibits chaotic behaviour. Moreover, it was proven by two independent methods (exponential distancing of nearby starting trajectories according to the classical definition of the Lyapunov exponent and chaos synchronisation[4]) that the maximal Lyapunov exponent is positive for regions of the bifurcation parameter, namely the ratio of dynamic and static coefficients of friction. With the decrease of this parameter period-doubling was found at $S/S_1 = 0.1737$ followed by

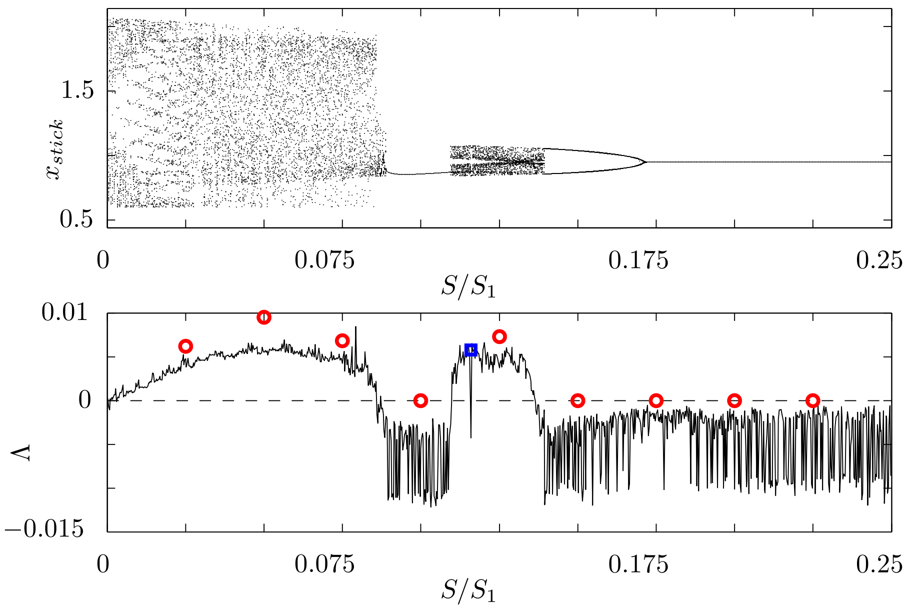

Figure 2: Bifurcation diagram and Lyapunov exponents for $S/S_1 = 0 - 0.25$ with parameters $S_1 = 0.4$ and $\Omega = 0.5$. (Solid curve - direct numerical simulation, red circles - method for non-smooth systems [4], blue square - finite time Lyapunov exponent).

a narrow band of chaotic motion. A broader band of chaos arised below $S/S_1 < 0.089$. Interestingly, a negative peak can be observed in the solid curve of the Lyapunov exponent plot indicating that the long term behaviour is periodic. For this parameter a piecewise Poincarè map was obtained of the sticking displacements. The trajectories of various initial conditions escaped from the repellor to one of the three periodic points and finally turned into a stable three-periodic cycle. Until these escapes the trajectories were driven by an unstable fixed point at $(0.952, 0.952)$. We approximated this map by straight lines and used it to estimate the finite time Lyapunov exponent for the transient phase. We obtained $\lambda_{flow} = 0.005748$, which value fits well in the series of calculated exponents in Fig.2.

Outlook for measurements

Finding material pairs that can show the remarkable ratio $S/S_1 = 1/8 = 0.125$ seems not to be an option. We have experimented with metallic specimens with both ground and polished surface finish but could only obtain $S/S_1 = 0.77$. We have, however, serious expectations towards spherical roller thrust bearings since manufacturers claim that the starting torque of these machine elements can be 8 times higher than the friction torque at normal operation. This would be enough to prove chaos experimentally. For this reason we started the design of a test rig that incorporates two of this kind of bearings and thus performs rotational oscillations. With the help of this equipment not only the ratio of the starting versus rolling torques can be measured but the system in Eq.1 can be reproduced.

Discussion and open points

In this work we presented a simple dry-friction model that was originally introduced by Den Hartog and S. W. Shaw. We extended the friction model to distinguish between static and dynamic friction forces and carried out numerical analyses. We found that for certain parameters the model behaves chaotically. This was proven using two calculation methods to obtain the maximal Lyapunov exponent where we also found the traces of chaotic transients. In this case an estimate of the finite time Lyapunov exponent was calculated with the help of a piecewise-linear map. We plan to carry out numerical bifurcation analyses using these friction models as a proof to our numerical results. Setting up a test bench is under way to test the frictional behaviour of roller bearings that may enable us to measure chaos in this simple mechanical model.

Acknowledgement

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References

[1] Den Hartog J. P. (1930) Forced Vibrations with Combined Coulomb and Viscous Damping. *Trans. ASME*

[2] Shaw S. W. (1986) On the dynamic response of a system with dry friction. *Journal of Sound and Vibration* **108**(2):305-325.

[3] Licskó G. and Csernák G. (2013) On the chaotic behaviour of a simple dry-friction oscillator. *Mathematics and Computers in Simulation* **95**:55-62.

[4] Stefanski A. and Kapitaniak T. (2000) Using chaos synchronization to estimate the largest Lyapunov exponent of nonsmooth systems. *Discrete Dynamics in Nature and Society* **4**(3):207-215.