ACCURACY OF ELECTROCARDIOLOGICAL INVERSE SOLUTIONS: A MODEL STUDY

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Abstract. *The principal aim of this work is the investigation of the accuracy of the widely used Tikhonov regularization in electrocardiological inverse solutions by the study of the achievable quality of epicardial potential estimation. Reference epicardial and body surface electrocardiograms were generated in 1003 epicardial and 344 body surface points by the Wei model. The inverse problem has been solved for homogeneous and inhomogeneous chest models by the zero-order Tikhonov regularization formula with various regularization parameters and linear equation system solvers. The results were correlated with the original epicardial potential distribution. The highest correlations (> 0.9) occurred in the first half of the QRS interval and the correlation fell below 0.6 at the end of the QRS. While the result was highly dependent on the chosen regularization parameter and equation system solver, the largest source of error proved to be the inadequate volume conductor model when the correlation fell below 0.4.*

Keywords: Electrocardiological Inverse Solution, Modelling, Tikhonov Regularization, QRS Interval

1. Introduction

The epicardial potential distribution contains essential diagnostic information related to the functioning of the heart. This can be important for example in the assessment of malignant ventricular arrhythmia risk, which is closely related to the action potential heterogeneity of the heart muscle cells [1]. There are two possible ways to obtain the epicardial potential distribution: performing epicardial measurements or estimating the epicardial distribution based on high resolution body surface potential measurements by solving the inverse problem of electrocardiography. Since the former is an invasive procedure, it is not considered to be practical in human diagnostics. Therefore, developing and improving inverse solution methods is very important.

One of the most widely used inverse solution methods is the zero-order Tikhonov regularization which tries to transform the ill-posed problem into a well-posed problem by a special regularization parameter [2]. This method has also been frequently used in electrocardiological inverse computations [3]. The efficiency of the inverse solution methods is characterized mostly by the correlation coefficient of the measured and estimated potential distributions and by the relative error.

The aim of this paper is the study of the achievable epicardial potential distribution quality in the QRS interval by using the Tikhonov method with various regularization parameters and linear equation system solver algorithms. These tests have been performed for both an inhomogeneous and a homogeneous chest model.

2. Methods

Heart and chest model

The reference epicardial and body surface ECG signals were generated in 1003 epicardial and 344 body surface points by the Wei-Harumi forward model [4]. The body surface potential distributions were calculated by solving the forward problem of electrocardiography according to Eq. 1:

$$\mathbf{\Phi}_b = \mathbf{Z}\mathbf{\Phi}_h \qquad (1)$$

where

- $\mathbf{\Phi}_b$ vector of body surface potential distribution (344 x 1)
- $\mathbf{\Phi}_h$ vector of epicardial potential distribution (1003 x 1)
- $\mathbf{Z}$ transfer matrix between the epicardial and thoracic surfaces (344 x 1003, related to the homogeneous chest model) [5].

Tikhonov regularization

Equation 2 represents the inverse solution according to the zero-order Tikhonov regularization:

$$\mathbf{\Phi}_h = \left[\mathbf{Z}^T\mathbf{Z} + \gamma \mathbf{I}^T\mathbf{I}\right]^{-1}\mathbf{Z}^T\mathbf{\Phi}_b \qquad (2)$$

where

- $\mathbf{I}$ identity matrix
- γ regularization parameter.

The regularization parameter is responsible for making the singular matrix to be regular, thus transforming the ill-posed problem into a well-posed one. The optimal value of γ is usually chosen empirically [2].

Solving the linear equation system

After rearranging Eq. 2 and substituting **A**, **x** and **b**, we get the linear equation system of Eq. 3:

$$\mathbf{A}\mathbf{x} = \mathbf{b} \qquad (3)$$

where

$$\mathbf{A} = \mathbf{Z}^T\mathbf{Z} + \gamma \mathbf{I}^T\mathbf{I}$$
$$\mathbf{b} = \mathbf{Z}^T\mathbf{\Phi}_b$$
$$\mathbf{x} = \mathbf{\Phi}_h.$$

We chose six methods offered by the MATLAB library: the default linear equation system solver of MATLAB ("mldivide" command which equaled to the Cholesky factorization in our case) and five iterative methods (conjugate gradients, minimum residual, quasi-minimal residual, least-squares QR and symmetric LQ).

3. Results

We tested the Tikhonov regularization with various γ values. The results are summarized in Fig. 1, using homogeneous chest model during the forward and inverse computations (homogeneous case).

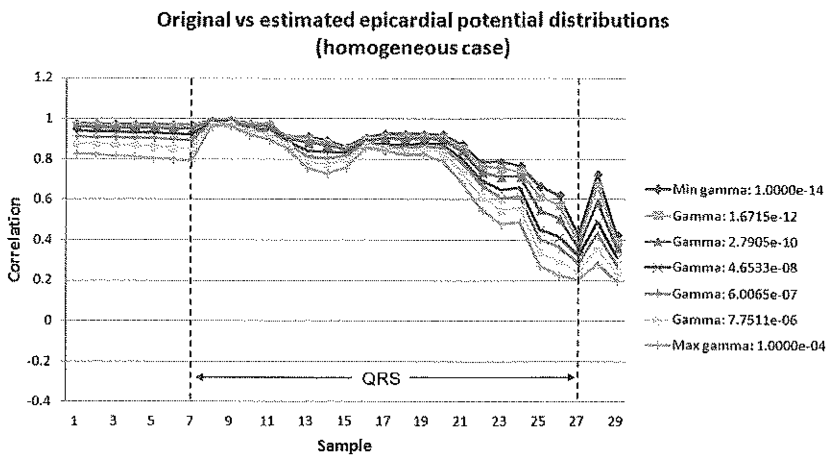

Fig 1. Efficiency (correlations) of the Tikhonov regularization with 7 different γ parameters in homogeneous case.

We performed the computations by also using an inhomogeneous chest model during the forward computations (inhomogeneous case). By using the homogeneous chest model in the inverse computations, we were able to test the sensitivity of the Tikhonov regularization for an inadequate chest model. As expected, this case produced poorer quality results, but using higher regularization parameters (γ > 1e-7) the correlations were not much lower than in the homogeneous case in the first half of the QRS interval (see Fig. 2).

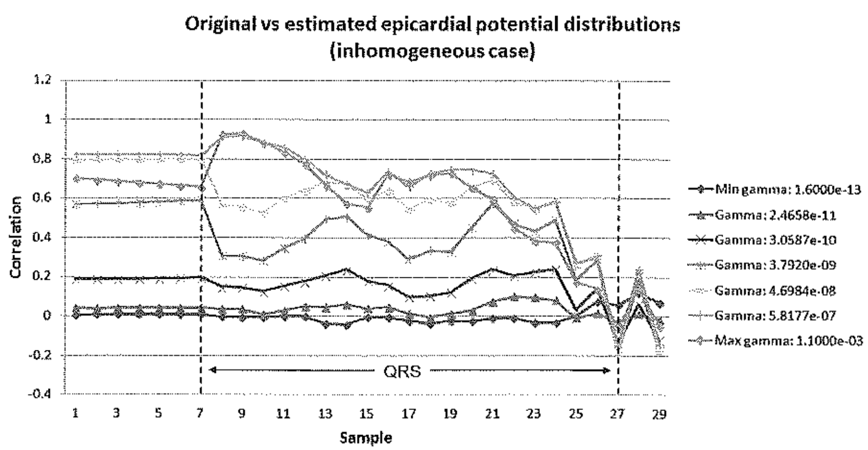

Fig 2. Efficiency (correlations) of the Tikhonov regularization with 7 different γ parameters in inhomogeneous case.

Performing the Tikhonov regularization with a fixed γ value by using different linear equation system solver algorithms, we found significant differences between the accuracy of the algorithms in the homogeneous case as shown in Fig. 3.

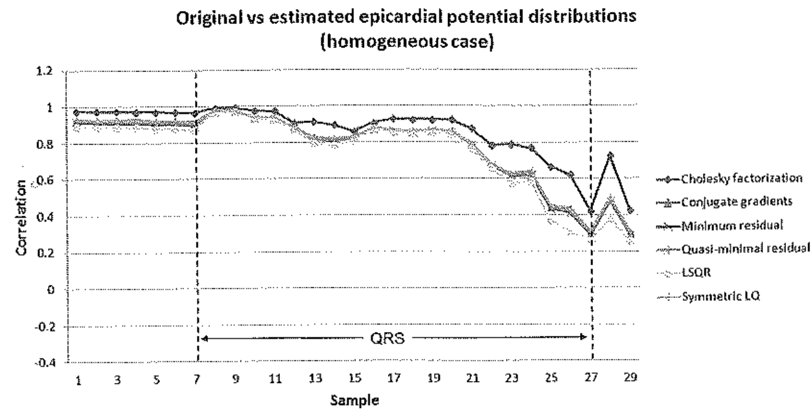

Fig 3. Efficiency (correlations) of the Tikhonov regularization with 6 different linear equation system solver algorithms in homogeneous case. The chosen γ value is 1e-14.

4. Discussion

The results show that the efficiency of the Tikhonov regularization highly depends on the chosen chest model, regularization parameter and the linear equation system solver algorithm. The Cholesky factorization proved to be the best algorithm, while the least-squares QR resulted in the worst correlations. In our experience, using the inadequate volume conductor model (inhomogeneous case) seems to result in the greatest source of error. It is interesting that at the end of the QRS interval none of the configurations could produce correlations higher than 0.8. This is probably due to the high complexity of the epicardial potential maps that show activation in the posterior side of the heart at the end of the QRS.

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