D6.2 SEARCH DERIVERABLE

Detailed Policy Impact Model

November 2013

SEARCH
SHARING KNOWLEDGE ASSETS: INTERREGIONALLY COHESIVE NEIGHBORHOODS

EUROPEAN COMMISSION
European Research Area
SEVENTH FRAMEWORK PROGRAMME
Funded under Socio-economic Sciences & Humanities

The research leading to these results has received funding from the European Community's Seventh Framework Programme (FP7/2010-2.2-1) under grant agreement n° 266834

Sharing KnowledgE Assets: InteRregionally Cohesive NeigHborhoods (SEARCH) Project

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Deliverable 6.2: Detailed Policy Impact Model

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November 2013

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1. Introduction

The SEARCH project targets the analysis of the impact of the European Neighborhood Policy (ENP) on the integration of EU neighboring countries with the EU. The research has focused on four areas, such as trade flows, people mobility, human capital, technological activities, innovation diffusion and institutional environment.

Work Package 6 is the policy analysis package of SEARCH. This WP synthesizes research results of earlier work packages in order to present an overview of potential EU policy options for strengthening cohesion across the EU-27 and NC16 in the mid to long term. WP 6 employs different research methods ranging from systematic literature analysis via text mining techniques to Delphi methodology and economic modeling. Economic modeling has the advantage that it opens the possibility of ex ante simulating the likely impacts of different kinds of policies. Thus it provides a platform for the comparison of several policy options.

This report provides a detailed description of the economic model that has been developed for estimating the likely impacts of certain policy prescriptions arising from research results of earlier work packages. The specific model construct chosen is the GMR (Geographic Macro and Regional) modeling approach that has been applied earlier for Cohesion policy and EU Framework Program impact analyses at the levels of European regions, the European Union and Hungary.

The particular country chosen for impact analysis is Turkey. This choice is motivated by practical reasons: availability and reliability of data for modeling. Though data collection for Turkey is not a process without difficulties the situation in this respect is relatively more advantageous there as compared to other ENP countries (with the exception of Israel which cannot be considered as a typical ENP country for other reasons). Turkey is an accession country but in several respects its economic, social and cultural features make this country reasonably comparable to many of the ENP countries. In this report we introduce GMR-Turkey. Its applications in actual policy analyses will be reported in working papers and in another deliveries.

This report has the following structure. The second section provides a general overview of GMR-Turkey. Detailed information about modeling structure is given in Section 3. Sensitivity results are reported in Section 4.

2. GMR-Turkey: A general overview

2.1 Policy instruments in GMR-Turkey

The GMR framework is developed and extended in order to test as many as possible policy suggestions generated in earlier work packages of SEARCH. However, not every policy suggestions can be implemented in an economic impact model. Suggestions related to institutions are among them. This explains our choice to focus on prescriptions arising from WPs 2, 3 and 4.

Instruments implemented in GMR-Turkey reflecting SEARCH policy suggestions are categorized into the following classes:

1. General *macroeconomic* (space-neutral) policy instruments (such as policies promoting increasing trade with EU countries, incentives for more intense FDI activity, policies supporting temporary migration, specific government tax and expenditure regulations to foster research activities and innovation collaborations).

2. *Regional/local* (place-based) interventions (such as investment support of SMEs, research subsidies, promotion of more intense local knowledge flows and international scientific networking, physical infrastructure construction, promotion of human capital development by supporting education, place-specific incentives for attracting FDI).

2.2 General features of GMR models

The geographic macro and regional modeling (GMR) framework has been established and continuously improved to better support development policy decisions by ex-ante and ex-post scenario analyses. Policy instruments including R&D subsidies, human capital development, entrepreneurship policies or instruments promoting more intensive public-private collaborations in innovation are in the focus of the GMR-approach.

Models frequently applied in development policy analysis are neither geographic nor regional. They either follow the tradition of macroeconometric modeling (like the HERMIN model - ESRI 2002), the tradition of macro CGE modeling (like the ECOMOD model – Bayar 2007) or the most recently developed DSGE approach (QUEST III - Ratto, Roeger and Veld 2009). They also bear the common attribute of national level spatial aggregation. The novel feature of the GMR-approach is that it incorporates geographic effects (e.g., agglomeration, interregional trade, migration) while both macro and regional impacts of policies are simulated. Why does geography get such an important focus in the system? Why is the system called “regional” and “macro” at the same time?

Geography plays a critical role in development policy effectiveness for at least four major reasons. First, interventions happen at a certain point in space and the impacts might spill over to proximate locations to a considerable extent. Second, the initial impacts could significantly be amplified or reduced by short run (static) agglomeration effects. Third, cumulative long run processes resulting from labor and capital migration may further amplify or reduce the initial impacts in the region resulting in a change of the spatial structure of the economy (dynamic agglomeration effects). Forth, as a consequence of the above effects different spatial patterns of interventions might result in significantly different growth and convergence/divergence patterns.

"Regions" are spatial reference points in the GMR-approach. They are sub-national spatial units ideally at the level of geographic aggregation, which is appropriate to capture proximate relations in innovation. Besides intraregional interactions the model captures interregional connections such as knowledge flows exceeding the regional border (scientific networking or spatially mediated spillovers), interregional trade connections and migration of production factors.

Important regional dimensions that may crucially determine the growth effects of development policies include the following aspects.

- Regional development programs are built on important *local specificities* (industrial structure, research strengths of the region, size and specialization of human capital etc.).
- Models have to capture the effects of policies on *local sources of economic growth* such as technological progress, investment and employment.
- The models also need to be able to follow those cumulative *agglomeration impacts* such as intensifying localized knowledge spillovers and their feedback mechanisms that may arise as a consequence of policies.
- There are certain additional impacts on the regional economy instrumented by *Keynesian demand side* effects or *Leontief-type intersectoral* linkages.
- Most of the infrastructural programs target better physical *accessibility*. Impacts of these policies on regions that are (directly or indirectly) affected also have to be reflected.
- There are different mechanisms through which policies implemented in certain regions affect other territories such as *interregional knowledge spillovers and trade linkages* and as such these effects also need to be incorporated in model structures.

The “macro” level is also important when the impact of development policies is modeled: fiscal and monetary policy, national regulations or various international effects are all potentially relevant factors in this respect. As a result the model system simulates the effects of policy interventions both at the regional and the macroeconomic levels. With such an approach different scenarios can be compared on the basis of their impacts on (macro and regional) growth and interregional convergence.

The GMR-framework is rooted in different traditions of economics (Varga 2006). While modeling the spatial patterns of knowledge flows and the role of agglomeration in knowledge transfers it incorporates insights and methodologies developed in the geography of innovation field (e.g., Anselin, Varga and Acs 1997, Varga 2000). Interregional trade and migration linkages and dynamic agglomeration effects are modeled with an empirical general equilibrium model in the tradition of the new economic geography (e.g., Krugman 1991, Fujita, Krugman and Venables 1999). Specific macroeconomic theories are followed while modeling macro level impacts.

The first realization of the GMR approach was the EcoRET model built for the Hungarian government for ex-ante and ex-post evaluation of the Cohesion policy (Schalk and Varga 2004). This was followed by the GMR-Hungary model, which is currently used by the Hungarian government for Cohesion policy impact analyses (Varga 2007). GMR-Europe was built in the IAREG FP7 project (Varga, Járosi, Sebestyén 2011) and was recently extended (Varga and Törmä 2010) and applied for policy simulations for DG Regional Policy (LSE 2011).

2.3 GMR-Turkey: Geographic and temporal dimensions, policy variables

GMR models reflect the challenges of incorporating regional, geographic and macroeconomic dimensions in development policy impact modeling by structuring the system around the mutual interactions of three sub-models such as the Total Factor Productivity (TFP), Spatial Computable General Equilibrium (SCGE) and macroeconomic (MACRO) model blocks. Following this approach the macroeconomic model of GMR-Turkey calculates policy impacts at the national level while the 26 NUTS 2-level regional models provide results at the regional level. The model system provides policy simulation results for the 2015-2025 time period.

Some of the ENP policies suggested in the SEARCH project can be modeled in the macroeconomic block (such as changes in international trade, in tax regulations or in income subsidies) via policy shocks affecting specific macroeconomic equations. However, most of the policy suggestions target stimulating the regional base of economic growth such as investment support, infrastructure building, human capital development, R&D subsidies, promotion of (intra- and interregional) knowledge flows. In the following sub-section we focus on mechanisms of these latter policies.

2.4 Regional impact mechanisms of the main policy variables

2.4.1 R&D support, interregional knowledge networks and human capital

Figure 1 provides a schematic figure on the way the impacts of policies targeting R&D support, interregional knowledge networks and human capital are modeled in the TFP block.

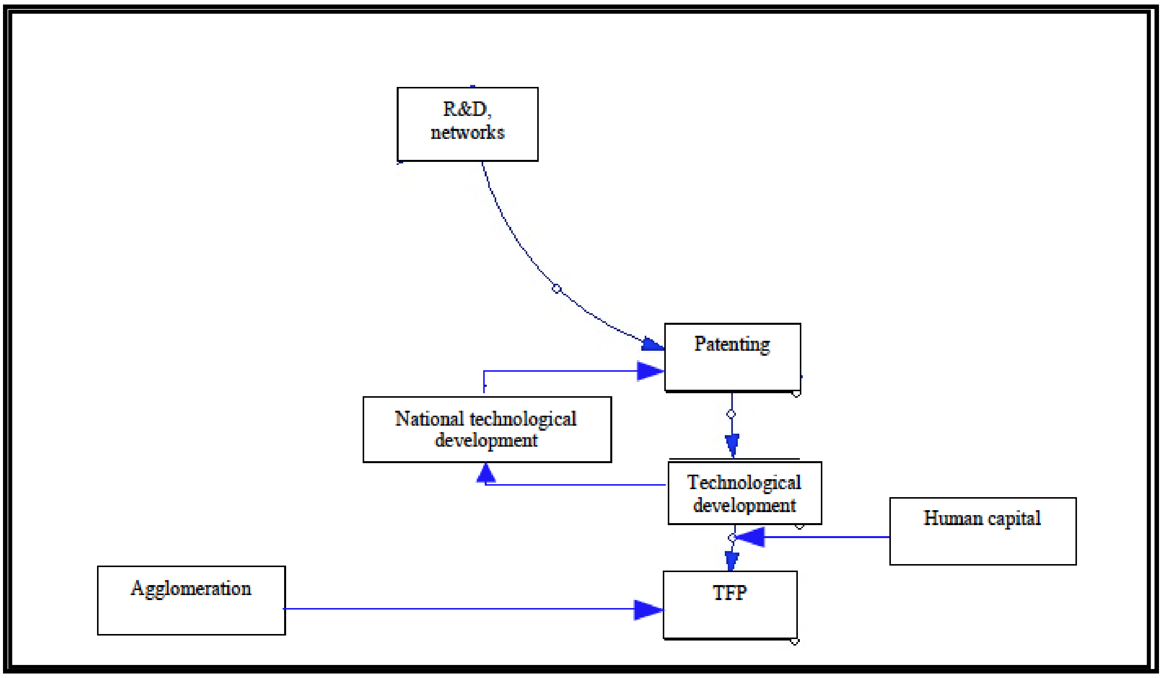


Figure 1: The impact mechanisms of R&D and knowledge networks and human capital promotion

Economically useful new technologies are measured by number of patents in the model. R&D support and interregional networks affect the economy via its impact on patenting. Increasing patenting activity affects positively regions' general technological levels (measured by the stock of patents), which determines productivity measured by Total Factor Productivity. In the model the extent to which technological development affects TFP is influenced by human capital in the region.

The impacts of the promotion of R&D, networking and human capital on economic variables (prices of quantities of inputs and outputs, etc.) are calculated in the SCGE block. Economic impacts of increased productivity are modeled in the SCGE block in the following steps.

1. Short run effects

The impact in the short run results from the interplay between the substitution and output effects. Assuming that the level of production does not change the same amount of output can be produced by less inputs that is the demand for capital (K) and labor (L) decrease as a result of the interventions. However increased TFP makes it also possible to decrease prices to keep firms more competitive, which positively affects demand. This latter effect is called the output effect. The interaction of output and substitution effects might result in the increase of the demand for factor inputs (K and L) but also the impact can be just the opposite. What will actually happen is an empirical question. In case output effect exceeds substitution effect wages will increase in the short run, which together with the relative decrease in prices will result in increasing consumption and higher utility levels.

2. Long run effects
Increased utility levels result in in-migration of labor and capital into the region, which will be the source of further cumulative effects working via centripetal and centrifugal forces. Labor migration increases employment concentration, which is a proxy for positive agglomeration effects in the model. According to findings in the literature localized knowledge spillovers intensify with the concentration of economic activity in the region (e.g., Varga 2000). A higher level of employment thus increase TFP (as shown also in Figure 1), which further reinforces in-migration of production factors following the mechanisms described above. However increasing population also affect the average size of flats negatively which works as a centrifugal force in the model. The balance between centrifugal and centripetal forces will determine the long term cumulative effect of policies at the regional, interregional and macroeconomic levels.

3. Changes resulting from interventions on the quantities and prices of outputs and factors are calculated in the SCGE model both in the short run as well in the long run.

2.4.2 Infrastructure investments

Infrastructure investments increase the level of public capital in the region. It is modeled via a Cobb-Douglas production function where the inputs are labor, private and public capitals. Thus infrastructure investments are modeled as externalities, which eventually affect regional TFP levels. Public investments are also modeled in the macro model via the increase of public capital.

2.4.3 Private investment support

One of the policies suggested is the support of investment by small and medium sized enterprises. The mechanism of this policy instrument affects the model via the increase in private capital, which has further impacts on several other variables both in the region where the intervention occurs and in other regions connected by trade or migration linkages. Private investment support is also modeled in the macro model via the increase of private capital.

2.5 Macroeconomic impacts

The effects of policies are communicated to the macro model by changes in TFP (aggregated from the regional level) and changes in fiscal variables (such as the demand and supply impacts of investment support and physical infrastructure construction). Changing TFP results in an increase of GDP growth rate which, will increase factor demand resulting from their higher marginal productivities. As a result the level of GDP will be higher than what would be observed in its long run equilibrium path. Infrastructure investments and private investment support induce both demand and supply side effects. The demand side (e.g., increased government expenditures) effect on GDP is temporary while the supply side effects (via increased public and private capitals) stabilize in the long run.

2.6 Impact mechanisms in the GMR model

The mutually connected three model-block system is depicted in Figure 2 below. Without interventions TFP growth rate follows the national growth rate in each region. The impacts of interventions run through the system according to the following steps.

1. Resulting from R&D-related interventions as well as human capital and physical infrastructure investments (which increase public capital and eventually impact the level of TPF as well) regional Total Factor Productivity increases.

2. Changing TFP induces changes in quantities and prices of output and production factors in the short run while in the long run (following the mechanisms described above) the impact on in-migration of production factors imply further changes in TFP not only in the region where the interventions happen but also in regions which are connected by trade and factor migration linkages.

3. Increased private investments expand regional private capital which affects further changes in regional variables (output, prices, wages, prices, TFP, etc) in the SCGE model block. The impact of private investment support affects the macro model as well via increased private capital.

4. For each year changes in TFP are aggregated to the national level then this increases TFP in the macro model as time specific shocks. The macroeconomic model calculates the changes in all affected variables at the national level.

5. Changes in employment and investment calculated in the MACRO block are distributed over the regions following the spatial pattern of TFP impacts.

6. The SCGE model runs again with the new employment and capital values to calculate short run and long run equilibrium values of the affected variables.

7. The process described in steps 5 and 6 run until aggregate values of regional variables calculated in the SCGE model get very close to their corresponding values calculated in the MACRO model.

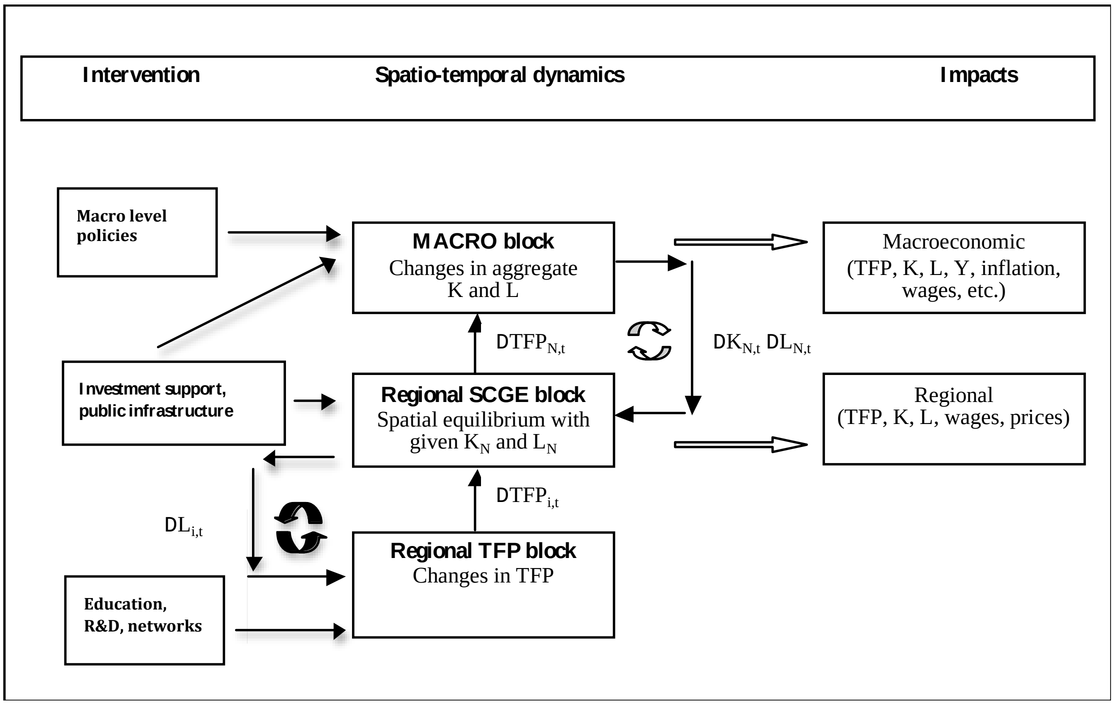


Figure 2: Regional and macroeconomic impacts of the main policy variables in the GMR-Turkey model

3. GMR model blocks

3.1 The TFP block

3.1.1 Estimating TFP for Turkish NUTS 2 regions

TFP is one of the most crucial variables in the GMR model thus a particular care is needed while it is calculated. Below we shortly review the state of the art in Turkey with respect to our knowledge in Total Factor Productivity then we detail its calculation at the regional level.

<u>The Turkish TFP literature</u>

Using Penn World Table (PWT) data and assuming constant returns to scale, Atiyas and Bakış (2013) find that at the national level, the main driver for the growth of GDP is the TFP growth in the post 2000 period. TFP grew more than %3 per annum at this period. TFP growth was very strong in agriculture during the first half of this period, while it was negative in the second half. TFP growth rates for industries and services were positive until 2006, but in the second half TFP growth rates were almost negligible or slightly negative. They attribute this to strong decline of employment in agriculture between 2000-2006, and a return to increasing employment during post 2006 period. On the other hand, industrial employment grew strongly, and that of services followed.

While interesting, these findings contribute little to understanding the drivers behind the TFP growth at a regional level. Turkey is characterized by large regional inequalities, where most of the industries and producer services are located in the Western part of the country. Most of the institutes regarding R&D and technology transfers are as well located in this part, as well as the largest share of skilled workers. Public investments in this part of the country focus more on metropolitan services as well as industry and trade related infrastructure. On the other hand, large infrastructure investments (i.e. irrigation systems, collective roads etc.) were made in the East and Southeastern part of the country, focusing on agriculture and associated industries, which could have contributed to TFP growth in these sectors.

Furthermore, the decline or stagnation of TFP in the post 2006 period is likely to be associated with the global financial crisis and associated volatility. Berument, Dincer, and Mustafaoglu (2011) argue that volatility in trade openness and financial systems had a negative impact on TFP growth in Turkey.

After an intensive literature survey, Vergil and Abasız (2008) discuss that through the end of the import-substitution policy period, TFP decreased sharply during the 1970'es, reaching its lowest value in 1980. TFP has been increasing during the next period, under export oriented industrial growth policy, but with very sharp decreases during phasing to economic recessions, and with sharp increases during phasing to economic boom periods.

Taymaz, Voyvoda, and Yılmaz (2008) have evaluated the productivity growth and TFP in manufacturing industries in Turkey during 1981-2001 period. They find that the contribution of technological progress has been quite low in explaining the productivity growth in manufacturing industries. Furthermore, FDI investments did have a slightly negative role in the productivity growth of local companies. They advocate that the fast growth experienced between 2001-2007 should be sustained by policies directly addressing at technological progress and development of human capital.

One of the first attempts to estimate TFP growth in Turkey at the regional level is by Karadağ (2004). The regions for this study were geographical regions, which do not resemble NUTS classification today. Focusing on manufacturing industry, he has found that TFP grew at an average of 0.5% annually during 1980-2000 period. While manufacturing centers like Marmara Region (Istanbul, Kocaeli, Bursa and Tekirdağ were key industrial locations) and also Aegean Region (where İzmir and Manisa were key industrial locations) experienced much faster TFP growth rates, in the Eastern Anatolia and Southeastern Anatolia regions TFP deteriorated. At a later paper, Karadağ, Önder, and Deliktaş (2005) find that at provincial (today's NUTS3) level, Istanbul experienced a negative TFP growth during the same period, while its immediate neighbor, Kocaeli, experienced the strongest TFP growth. A similar region to Kocaeli is found to be Manisa, neighbor of Izmir, where TFP growth was the second fastest, thanks to development of electronic consumer products industries. This could be partly attributed to growth and relocation of manufacturing industries from core metropolitan areas to the immediate vicinity, and associated off-spring company establishments.

Estimating TFP at the Turkish NUTS2 regional level

The production function is assumed to be a Cobb-Douglas type production function with constant returns to scale for private capital and labor. Public capital stocks have an impact on the efficiency of the private sector, and thus the parameter ε creates the effect of increasing returns to scale, if positive. The subscript i denotes regions as cross-sections and t denotes time as years.

$$Y_{it} = (K_{\text{pricap}}^{\propto})K_{pubcap}^{\varepsilon} \qquad (1)$$

Where Y is gross value added per employee, L is labor, Kpricap is the private capital stock per employee and Kpubcap is the public capital stock in each region at time t.

To measure regional outputs, previous studies have relied on the GDP data provided at provincial or geographic regions level by TURKSTAT. This series is not announced since crisis year of 2001. Instead, TURKSTAT now provides Gross Value Added data at NUTS 2 level, which is available between years 2004-2010, in terms of current TL. This data is deflated to acquire regional output levels by 1998 fixed prices, and used in million TL units. The variable that represents this data is labeled as GVA.

Labor data is also provided by TURKSTAT. This is the employment data on 15 years and older persons, and is used in 1000 units. It is represented as LABOR. Data on capital stocks at regional level do not exist readily, and therefore had to be estimated. Capital stocks are often estimated using the Perpetual Inventory Method (PIM) in the literature. In the case of Turkey, Atiyas and Bakış (2013) can be given as an example at the national level, while Karadağ (2004) can be given as an example at regional level, for a certain economic sector. This method uses the average growth rate of investments and depreciation rate of the capital. To do so, long time series of regional investments are required. That is why, this method is often used in calculating capital stocks at national level.

Gross Regional Investments data is provided by TURKSTAT, under Annual Industry and Service Statistics title, in current TL units. This data covers investments of private sector enterprises and publicly owned enterprises (who are producing goods for the market), in tangible assets. However, the data sets of 2004-2008 period and 2009-2010 period are classified differently, and there are mismatches. The 2009-2010 period do not cover agriculture and financial and insurance sectors, while the previous period does. Furthermore, no data is available for year 2005. Therefore, this data set is indeed more suitable for estimation of a capital stocks in a specific sector like manufacturing industry, rather than estimating aggregate capital stocks.

Public Investments are acquired through former State Planning Organization, which has become the Ministry of Development later. This data, on the other hand, provides budget allocations per regions, and do not necessarily reflect the real amount of investments. On the other hand, it covers not only investments in tangible assets, but also covers expenditures such as project preparations, feasibility studies, etc. This data covers a longer period, from 1998-2011.

Despite the seemingly available regional investment data, they are found to be ineligible to calculate regional capital stocks for the study period, using PIM.

A new data source on national capital stocks is the Penn World Table 8.0, which can be accessed by the web site of University of Groningen (http://www.rug.nl/research/ggdc/data/penn-world-table). Since this data covers a long time period, from 1950 to 2011, and data is comparable for 167 countries, it provides alternatives to estimating regional capital stocks. Since the data was provided at 2005 fixed USD, first, it had to be converted to fixed 1998 TL prices.

Although there were many other alternatives, two viable alternative approaches were evaluated using PWT data on capital stocks. First, by using year 2004 data as the initial year, data is divided into public and private capital stocks components by using shares of Gross Fixed Capital Formation (GFCF) for private and public sectors at national level. Gross Fixed Capital Formation data is provided by TURKSTAT. Then, Gross Regional Investment Data and Public Investment Data from TURKSTAT were used with PIM method to estimate regional capital stocks for years between 2004-2010. The results were not quite satisfactory, since total capital stocks estimated this way diverged from that provided by PWT 8.0. The reason for this can be attached to our evaluation about the Gross Regional Investment data provided above.

As a second alternative, PWT capital stock data was divided into public and private capital stock components for all years between 2004-2010, in the same way described above, using GFCF shares. Then, this data is distributed to NUTS 2 regions, according to electricity consumption shares of regions. Particularly, electricity consumed by the public sector (such as public institutions, irrigation and street lightning) and electricity consumed by private sector (such as used by offices, factories, etc.) were used to calculate shares. We justify using electricity shares to allocate national capital stock data across regions, since, in the literature, electricity data has been directly used as a proxy for capital stocks. Moody (1974), is an early example on using electricity consumption as a proxy for capital services. Schnorbus and Israilevich (1987), has used electricity consumption as a proxy for capital services in Midwest, USA. In the case of Turkey, Pirili and Lenger (2011) used electricity consumption in commercial and industrial facilities as a proxy for private capital stocks.

This method has the advantage that the total capital stocks are the same as provided in PWT 8.0, and regional capital stocks did not fluctuate as much compared to those calculated by the previous method. Furthermore, electricity consumption does not decrease sharply for years of economic recession, due to already installed equipment, and thus this method provides superior results against the other alternative where PIM method would suffer due to sharp decreases in regional investments in years of recession, i.e. during 2008 and 2009. Therefore, this alternative was preferred for calculation of private and public capital stocks. To the best of our knowledge, this is a first attempt to calculate regional capital stocks using this data set. Private capital stocks are labeled as PRICAP, and public capital stocks are labeled as PUBCAP.

Like time series econometrics, which is said non-stationary time series will result in spurious regression and as a result the statistical inference cannot be carried out, an important concern in panel data econometrics is the worry about the non-stationarity, spurious regression and co-integration. Entorf (1997) studied spurious fixed effects regressions when the true model involves independent random walks with and without drifts. Kao (1999) and Phillips and Moon (1999) derived the asymptotic distributions of the least squares dummy variable estimator and various conventional statistics from the spurious regression in panel data[1].

Before estimating the parameters of model, first we test for unit roots in model's variable. We computed two types of tests, namely common unit roots test, Levin, Lin and Chu (2002), and individual unit roots test, Im, Pesaran and Shin (2003), Fisher-type tests using ADF and PP tests (Maddala and Wu (1999) and Choi (2001)). The results show that the null hypothesis of common unit roots is rejected for all variables of the model and variables are stationary.

An important part of panel data modeling is model specification and the choice between, random effects, fixed effects and pooled regression[2]. One common method for testing the endogeneity or exogeneity of regressors is to employ a Hausman (1978) test and compare the fixed and random effects estimates of coefficients (Wooldridge, 2002, p. 288), and Baltagi, 2005, p. 65). We conducted a specification test proposed by Hausman (1978), which is based on the difference between the fixed and random effects estimators. For variance estimation of the error term we used Wallace-Hussain (1969)[3].

We first estimated the random effect model and then conducted Hausman specification test. The estimated model is reported in Table 1. Results show that all parameters are statistically significant. The Hausman test for cross-sections and times random shows that the hypothesis that individual effects are not correlated with the regressors in the model cannot be rejected. Based on the Hausman test, it is concluded that the random effects model is the better choice and there is no error in model specification.

[1] However, it is argued that in panel data econometrics, adding the cross-section dimension to the time series dimension offers an advantage in testing for non-stationarity and co-integration (Kao,1999; Phillips and Moon,1999). Unlike the single time series spurious regression literature, the panel data spurious regression estimates give a consistent estimate of the true value of the parameter as both N and T tend to ∞.This is because, the panel estimator averages across individuals and the information in the independent cross-section data in the panel leads to a stronger overall signal than the pure time series case. It is argued that panel-based unit root tests have higher power than unit root tests based on individual time series (Levin, Lin and Chu(2002).

[2] Mundlak (1961) and Wallace and Hussain (1969) were early proposing using the fixed effects model while Balestra and Nerlove (1966) were suggesting to use the random error component model in empirical works.

[3] Early in the literature, Wallace and Hussain (1969) recommended the within estimator for the practical researcher, based on theoretical considerations but more importantly for its ease of computation. In Wallace and Hussain's (1969, p. 66) words the "covariance estimators come off with a surprisingly clear bill of health".

Table 1: The production function. Estimation results of a two-way random effects model

Variables	Coefficients	Std. Error	T-Statistics	Prob.	R^2	F-Statistics
Const.	0.006	0.1787	1.719	0.0872	0.46	78.47 (0.0000)
$\ln pricap_{it}$	0.314	0.0285	11.004	0.0000		
$\ln pubcap_{it}$	0.064	0.0310	2.0911	0.0379		

TFP for each region is calculated by using coefficient estimates from the production function. As given in Table 1, the coefficient of private capital estimated through the two-ways random effects model above was 0.314, which is similar to the usual assumption of 1/3rd in the literature. The coefficient of labor, under assumption of constant returns to scale, then is 0.686. The coefficient of the public capital is 0.064. Regional TFP for each year and each NUTS 2 region is calculated for years between 2004-2010 by the following equation:

$$TFP_{it} = GVAit \ \ ((Pricap_{it}^{\alpha}) * \ Labor_{it}^{1-\alpha} \ * (Pubcap_{it}^{\varepsilon})) \qquad (2)$$

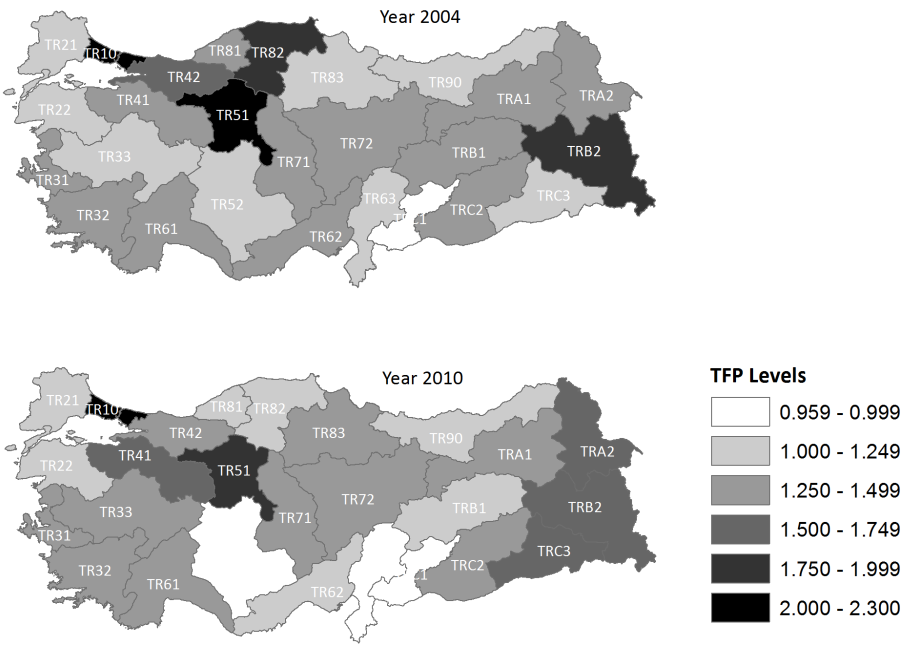
Exploratory information on Turkish regional TFP

Descriptive statistics on the calculated TFP values are provided in the Table 2. The highest TFP value, not surprisingly, belongs to Istanbul. Istanbul has been a region where decentralization of industries to nearby regions has been a long term policy. Both before 2000’s and after 2000’s during Istanbul Metropolitan Plan studies, this view has been shared and implemented up to a level. Despite these approaches, Istanbul still accommodates more than a third of industries. Furthermore, it accommodates most important companies and headquarters in advanced producer services as well as distributor services. However, due to its openness to global economy, it is also influenced from global economic fluctuations. Same effects influence the surrounding region where industries have spilled over. It can be observed in table corroso that Istanbul’s TFP value has contracted at the end of the study period, almost around 7%, despite strong growth in number of patents. Karadağ et al. (2005) have found that the TFP growth in Istanbul’s manufacturing industry during the 1990’s were negative, while its immediate neighbor Kocaeli Province had the highest TFP growth rate. Arguably, were Istanbul not successful in production of knowledge, the negative impact of global recession and the local policies of decentralization of industries on TFP growth could be much more stronger.

Table 2. Descriptive Statistics of the Calculated TFP Values for NUTS 2 Regions in Turkey

	TFP between 2004-2010	TFP at year 2004	TFP at year 2010
N	182	26	26
Mean	1.388	1.397	1.321
Std.dev.	0.297	0.329	0.277
Min.	0.848 (Hatay, TR63)	0.959 (Gaziantep, TRC1)	0.848 (Hatay, TR63)
Max.	2.249 (Istanbul, TR10)	2.249 (Istanbul, TR10)	2.091 (Istanbul, TR10)

Figure 1. Distribution of TFP across Turkish NUTS 2 Regions



As can be followed from Figure 2, highest TFP values are observed in Istanbul, the largest region by population, and Ankara, the second largest region by population, and also accommodating the capital city. This area, including TR42 and TR41 regions between, accommodate most of the manufacturing activities as well as producer services. Although accommodating highly productive manufacturing industries, one can follow that the TFP values are still well below that of Ankara and Istanbul. One particular problem is that these regions are those that are mostly effected in times of economic recession, due to their connectivity to global markets.

Another important factor is the high levels of in-migration, necessitates allocation of capital to provision of basic services and products, rather than diverting to higher technology industries. Despite these shadowing factors, still, this area is an important global production core and the TFP in these regions are highly likely to be more dependent on knowledge production in this area, as well as technology transfers from foreign direct investments. As mentioned above, Karadağ et al. (2005) found that TFP growth in manufacturing industries in the most important province of TR41 Region, Kocaeli Province, was the highest during 1990’s.

An additional important industrial center consists of TR31 (Izmir) Region, and partially TR33 (Manisa) Region. Although these regions accommodate significant agricultural activities, one can follow the advancement of Manisa, where a successful electronic consumer products industry is located. As briefed above, Karadağ et al. (2005) have found that this region had the second fastest TFP growth in manufacturing industries during 1990’s.

Although accommodating important industrial activities and international seaports, TR62, TR63, and TRC1 regions seem to be losers in TFP. This could be particularly attributed to lack of development in capital intensive industries, but also due to high in-migration levels.

Although in the East, TRB2 region, had quite high TFP value at year 2004, it had lower TFP value in 2010, but TRC3 region’s TFP value increased. These are the regions which were discussed in Karadağ (2004), that experienced a deterioration in TFP growth during the 1990’es. These regions are likely to benefit from public infrastructure investments that target agriculture. Particularly, bordering to Iran and Iraq, these regions are likely to be influenced also by cross-border trade activities, while increasing trade relations with Iraq might be beneficiary for TRC3 region, alternating relations with Iran due to global political influences could be a reason for instability in the TRB2 region.

3.1.2 Equations in the TFP block and their estimation

The TFP equation

Table 3 reports the estimation results of the final TFP model. Following Romer (1990) we assumed that the level of TFP depends on two central factors. Knowledge accumulated over the past years and human capital. Accumulated knowledge is measured by cumulative number of patents (CUMPAT) while the level of human capital at regional level is proxied by education capital (CPSTCEDUCAT). Education capital is calculated from regional investment in education following the PIM methodology. The reason why education capital is chosen as a proxy is that this variable will play an important role in policy simulations when the impacts of education investments are simulated. However we run separate regressions with human capital (proxied by data on population with tercier education) and coefficient estimates, test statistics are very similar to the ones reported in Table 3.

Table 3: Regression results – The regional TFP equation

Dependent Variable: LOG(TFP/(LABOR^0.038485))
Method: Panel Least Squares
Sample (adjusted): 2006 2010
Periods included: 5
Cross-sections included: 26
Total panel (balanced) observations: 130
Period weights (PCSE) standard errors & covariance (d.f. corrected)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.058941	0.026744	-2.203905	0.0294
LOG(CUMPAT(-1))*LOG(CPSTCEDUCAT(-1))	0.005256	0.002076	2.532034	0.0126
DUMMYTFPEAST	0.308553	0.043865	7.034119	0.0000
PATHCORE	0.303289	0.072493	4.183670	0.0001

Effects Specification

Period fixed (dummy variables)

R-squared	0.583130	Mean dependent var	0.057734
Adjusted R-squared	0.559211	S.D. dependent var	0.195487
S.E. of regression	0.129788	Akaike info criterion	-1.186267
Sum squared resid	2.055077	Schwarz criterion	-1.009803
Log likelihood	85.10736	Hannan-Quinn criter.	-1.114564
F-statistic	24.37961	Durbin-Watson stat	0.192884
Prob(F-statistic)	0.000000		

It turned out that CUMPAT and CPSTCEDUCAT are highly correlated resulting in high multicollinearity in the estimated equation. The chosen specification is thus the one where the two variables interact with each other. A one-year time lag resulted in the best performing econometric model. The two dummies reflect our suspicion towards the seemingly imprecise estimations of TFP in two eastern regions (DUMMYTFPEAST) and the assumption that the technologically most advanced regions follow different path in TFP. Both hypotheses are supported by the highly significant parameters of the two dummies. The final model is estimated with period fixed effects and with a control for heteroscedasticity via period weights standard errors and covariances.

The Patent equation

The other model in the TFP block is the patent equation. The function of this equation is to estimate the impact of R&D and interregional networking on new knowledge creation. Table 3 reports the regression results.

Table 3: Regression results – The regional patent equation

Dependent Variable: LOG(PAT)
Method: Panel EGLS (Period random effects)
Sample (adjusted): 2007 2012
Periods included: 6
Cross-sections included: 26
Total panel (balanced) observations: 156
Swamy and Arora estimator of component variances
White cross-section standard errors & covariance (d.f. corrected)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-4.245850	0.243584	-17.43077	0.0000
LOG(PUB(-1))*EMPKI(-1)	0.005126	0.000460	11.15532	0.0000
LOG(CUMPATNATIONAL(-1))	0.783791	0.034922	22.44385	0.0000
FP(-1)	0.053764	0.005277	10.18779	0.0000

Effects Specification

	S.D.	Rho
Period random	0.000000	0.0000
Idiosyncratic random	1.131067	1.0000

Weighted Statistics

R-squared	0.502064	Mean dependent var	1.647909
Adjusted R-squared	0.492236	S.D. dependent var	1.571437
S.E. of regression	1.119767	Sum squared resid	190.5894
F-statistic	51.08674	Durbin-Watson stat	0.264074
Prob(F-statistic)	0.000000		

Neither R&D expenditures nor R&D employment (possible proxies for research activities) are available in Turkey at a regional level. As a result we choose to use number of publications (PUB) at the regional level as a close proxy for research efforts. Region size (a proxy for agglomeration effects in regional knowledge creation) is measured by high technology employment (EMPKI). Following again Romer (1990) we assumed that knowledge accumulated at the national level affects regional knowledge production. The impact of interregional knowledge networks is proxied by the number of EU Framework Projects in which the region participates (FP) in each year in the sample. The interaction variable of EMPKI and PUB indicates that the productivity of research is affected by agglomeration which is in accordance with findings on a large European sample (Varga, Pontikakis, Chorafakis).2013). The one-year lag provides the best regression fit. All the variables enter the equation with highly significant parameters with the expected signs. The final model is estimated with period fixed effects and with a control for heteroscedasticity via period weights standard errors and covariances.

3.1.3 The TFP block database

Tables 4-6 provides details on the data sources of the variables used in the production function, the TFP equation and the Patent equation.

Table 4. Variable Descriptions for the Production Function

Variable Name	Description	Source
GVA_{it}	Gross Value Added, in million TL, 1998 fixed prices.	Obtained in current TL terms from TURKSTAT regional data set, and deflated according to 1998 fixed prices, NUTS 2 level.
$LABOR_{it}$	Employmed persons 15 yrs. and older, Thousand persons	TURKSTAT, regional data set, NUTS 2 level.
$PRICAP_{it}$	Private capital stocks, in million TL, 1998 fixed prices	Estimated by using PWT 8.0 data on country level capital stocks and TURKSTAT data on GDP, Gross Fixed Capital Formation of private sector, and regional private electricity consumption at NUTS 2 level.
$PUBCAP_{it}$	Public capital stocks, in million TL, 1998 fixed prices	Estimated by using PWT 8.0 data on country level capital stocks and TURKSTAT data on GDP, Gross Fixed Capital Formation of public sector, and regional public electricity consumption at NUTS 2 level.

Table 5. Variable Descriptions for the TFP Equation

Variable Name	Description	Source
TFPit	Total Factor Productivity	Authors' own calculations
$LABOR_{it}$	Employmed persons 15 yrs. and older, Thousand persons	TURKSTAT, regional data set, NUTS 2 level.
PSTCKit	Patent Stocks calculated by accumulating past 7 years patent registrations.	Turkish Patent Institute data acquired through TUBITAK website.
$PUBCAP_{it}$	Public capital stocks, in million TL, 1998 fixed prices	Estimated by using PWT 8.0 data on country level capital stocks and TURKSTAT data on GDP, Gross Fixed Capital Formation of public sector, and regional public electricity consumption at NUTS 2 level.
CPSTCEDUCATit	Capital Stocks in Education (Private Sector) in million TL, 1998 fixed prices	Estimated by PIM method, using Gross Regional Investment Data from Annual Business Statistics, TURKSTAT Regional Database
DUMMYTFPEAST	Dummy indicating TRB2 and TRC3 Regions.	Authors' own calculation
PATHCORE	Dummy indicating regions where amount of registered patent stocks were ½ s.d. above the mean	Authors' own calculation

Table 6. Variable description of the Patent equation

Variable Name	Description	Source
$PAT_{i,t}$	Number of patent registrations	TPI, accessed through TUBITAK
$PUB_{i,t}$	Number of publications (used as a proxy for regional expenditures on R&D) by affiliation city of author	SCOPUS, own query
$EMPKI_{it}$	Employment in High Tech and Knowledge Intensive Services	EUROSTAT
$PSTCK_{N}$	Total stocks of registered patents at country level	Authors' elaboration on TPI patent data.
FP	Number of FP programs that region participated at the subject year	Authors' elaboration on EU Framework Program Data.

3.3 The SCGE model block

3.2.1 Equations in the SCGE block and their calibration

Spatial Computable Equilibrium (SCGE) models add the spatial dimension to the (usually spaceless) CGE models. This first means that the number of spatial units is larger than one. The term spatial units in SCGE models denotes subnational regions. Additional extension to CGE models that the regions are interconnected by trade linkages and migration, transportation costs are explicitly accounted for and (positive and negative) agglomeration effects are also parts of the model structures.

Features of GMR models are usually determined by data availability to a large extent. At the regional level data are usually not as much detailed as at the national level and the modeler should adjust to this situation. The model distinguishes between short run and long run equilibriums. In short run equilibrium each region is in equilibrium in all the regional markets. However this does not mean that the whole regional system is in equilibrium. In case utilities differ across regions the whole system is not in equilibrium. Utility differences will induce labor migration (followed by the migration of capital). In the long run migration leads to the state where the system reaches the equilibrium state where interregional utility differences disappear.

The supply side

The SCGE model, harmonized with the QUEST III MARO model operates with increasing returns, monopolistic competition characterized with markup pricing. The basic equation of the model is the Cobb-Douglas production function which determines output (Y) resulting from labor (L) and capital inputs. The two capital inputs are private capital (K) and public capital (KPUB)

The C-D production function is characterized by increasing returns to scale thus $(\delta + \gamma + \beta + \beta pub) = \alpha + \beta tot > 1$,

$$Y_{i,t} = A_{i,t} \cdot L_{i,t}^{\delta+\gamma} \cdot K_{i,t}^{\beta} \cdot KPUB_{i,t}^{\beta pub} , \qquad (3)$$

where δ, β and βpub are estimated in Table 1, γ is also estimated econometrically (its value is 0.038485) i stands for region, t for time period. L^{γ} proxies for agglomeration effects in TFP.

A_i plays a crucial role in the system as the SCGE model gets its TFP shocks via this variable. Thus the following relationship exists:

$$A_{i,t}=\frac{TFP_{i,t}}{L^Y{}_{i,t}}$$

where the numerator gets its actual value in the simulations according to the shocks to research, human capital and networking.

Markup pricing is characterized according to the following equations.

Marginal costs is the following:

$$MC = \frac{dTC}{dY} = \frac{w^{\frac{\alpha}{\alpha+\beta}} \cdot r^{\frac{\beta}{\alpha+\beta}}}{A^{\frac{1}{\alpha+\beta}} \cdot \alpha^{\frac{\alpha}{\alpha+\beta}} \cdot \beta^{\frac{\beta}{\alpha+\beta}}} \cdot Y^{\frac{1-\alpha-\beta}{\alpha+\beta}} , \tag{4}$$

Average cost:

$$AC = \frac{TC}{Y} = \ \alpha + \beta \ \frac{w^{\frac{\alpha}{\alpha+\beta}} \cdot r^{\frac{\beta}{\alpha+\beta}}}{A^{\frac{1}{\alpha+\beta}} \cdot \alpha^{\frac{\alpha}{\alpha+\beta}} \cdot \beta^{\frac{\beta}{\alpha+\beta}}} \cdot Y^{\frac{1-\alpha-\beta}{\alpha+\beta}} = \ \alpha + \beta \ \cdot MC , \tag{5}$$

In monopolistic competition price equals average cost:

$$q = \frac{\varepsilon}{\varepsilon-1} MC , \tag{6}$$

where $\frac{\varepsilon}{\varepsilon-1}$ is the markup. It can be proven that $\alpha + \beta = \frac{\varepsilon}{\varepsilon-1}$ where ε equals to the elasticity of substitution as it is applied in the MACRO model.

Labor demand:

$$L = \ \frac{Y}{A}^{\frac{1}{\alpha+\beta}} \cdot \ \frac{r\alpha}{w\beta}^{\frac{\beta}{\alpha+\beta}} . \tag{7}$$

Demand for capital:

$$K = \left(\frac{Y}{A}\right)^{\frac{1}{\alpha+\beta}} \cdot \left(\frac{w\beta}{r\alpha}\right)^{\frac{\alpha}{\alpha+\beta}} . \qquad (8)$$

Output demand:

$$X = \frac{Z}{p} . \qquad (9)$$

here „Z" is income spent ($Z = wL + rK$) with w and r stand for wage and capital rent.

The demand side

Assuming homogenous preferences of the households the utility function is given by equation (10).

$$U_{i,t} = \alpha_H \ln \left(\frac{H_i}{N_{i,t}}\right) + \beta_H \ln x_{i,t} , \qquad (10)$$

where „$x_{i,t}$" stands for consumption „H_i" is housing „α_H" és a „β_H" are paramters.

Households' individual budget is formulated by equation (11)

$$w_{i,t} \frac{L_{i,t}}{N_{i,t}} + r_{i,t} \frac{K_{i,t}}{N_{i,t}} = p_{i,t} x_{i,t} , \qquad (11)$$

where „$N_{i,t}$" is regional population and „$p_{i,t}$" is the general level of prices. Assuming utility maximization equationn (10) and (11) lead to the demand for goods function:

$$X_{i,t} = \frac{\beta_H}{1-\alpha_H} \frac{1}{p_{i,t}} \left(w_{i,t} \frac{L_{i,t}}{N_{i,t}} + r_{i,t} \frac{K_{i,t}}{N_{i,t}} \right) N_{i,t} , \qquad (12)$$

Some of the goods are produced in the region but some of them are traded from other regions. „$s_{i,j,t}$" is the ratio of the share of region i in the market of region j. Assuming iceberg transportation costs the following CES demand function is derived.

$$s_{i,j,t} = \gamma_i \left(\frac{(1+\tau_{i,j}) q_{i,t}}{p_{j,t}} \right)^{-\mu} , \qquad (13)$$

where „μ" is an elasticity parameter of the CES function and „γ_i" is the share parameter. The general price level, „$p_{j,t}$" is calculated as follows:

$$p_{j,t} = \sum_i s_{i,j,t} q_{i,t} \left(1 + \tau_{i,j}\right) , \quad (14)$$

$$p_{j,t} = \left(\sum_i \gamma_i \left(\left(1 + \tau_{i,j}\right) q_{i,t} \right)^{1-\mu} \right)^{\frac{1}{1-\mu}} . \quad (15)$$

Short run equilibrium conitions

For factor markets:

$$L_{i,t}^{(dem)} = L_{i,t}^{(sup)} \; : \forall\, i=1..I \text{ és } \forall\, t=1..T , \quad (16)$$

$$K_{i,t}^{(dem)} = K_{i,t}^{(sup)} \; : \forall\, i=1..I \text{ és } \forall\, t=1..T . \quad (17)$$

The model calculates „$w_{i,t}$" and „$r_{i,t}$" until (16) and (17) is found.

In our model the average interest rate serves as the numeraire:

$$r = \frac{\sum_i r_{i,t} K_{i,t}^{sup}}{\sum_i K_{i,t}^{sup}} = const. \quad (18)$$

Demand for goods produced in region „i" is „$Y_{i,t}$". Taking into account transportation cost (19) describes the equilibrium conditions in the goods market:

$$Y_{i,t} = \sum_j s_{i,j,t} X_{j,t} \left(1 + \tau_{i,j}\right) , \quad (19)$$

Modeling migration

Interregional differences in utilities results in migration:

$$L'_{i,t} = L_{i,t} + LMIGR_{i,t} , \quad (20)$$

where:

$$LMIGR_{i,t} = \Phi \left(e^{\Theta \left(U_{i,t}^{*} + c_i \right)} - e^{\Theta \cdot AVG \left(U_{i,t}^{*} + c_i \right)} \right) L_{i,t} . \quad (21)$$

where „$U^{*}_{i,t}$" is regional utility „c_i" is regional specific constant, „Φ" and „Θ" determines the speed of migration. „AVG" stands for weighted averaging utilities where employment is the weight.

<u>Parameters: estimation and calibration</u>

Parameter	Source
β	Estimated econometrically
δ	Estimated econometrically
γ	Estimated econometrically
α	$\alpha = \gamma + \delta$
ε	$\alpha + \beta = \varepsilon \quad \varepsilon - 1$ according to the relationship in the MACRO model
α_H	$\alpha_H = 0.1$
β_H	Calculated
γ_i	Calibrated: in the baseline the algorithm searches for the value when the model produces the values of all the variables which are equal to the respective observed values.
$\tau_{i,j}$	Calculated based on transportation costs.
μ	Calibrated
ϕ	Calibrated: in the baseline the algorithm searches for the value when the model produces the values of all the variables which are equal to the respective observed values.
θ	Calibrated: in the baseline the algorithm searches for the value when the model produces the values of all the variables which are equal to the respective observed values.
c_i	Calibrated: in the baseline the algorithm searches for the value when the model produces the values of all the variables which are equal to the respective observed values.
σ	Adatokból számítva

3.2.2 The SCGE block database

Table 7: Variable description in the SCGE model, 2010

Variable Name	Description	Source
Y	Regional Gross Value Added	TURKSTAT Regional Data base
L	Employment	TURKSTAT Regional Data base
K	Regional Capital Stocks	Own calculations using Penn World Table 8.0, TURKSTAT National Accounts and TURSTAT Regional Electricity Data
w	Wages	Model calculates
r	Interest Rate	Model calculates
H	Housing Stocks	TURKSTAT Regional Database
N	Population	TURKSTAT Address Based Population Data

3.3 The MACRO model block

The macroeconomic block of GMR is given by a standard, large-scale DSGE (dynamic, stochastic, general equilibrium) model. The role of this model block is to model dynamic economic effects and to provide a framework for the static SCGE block with the dynamics of necessary macro variables. The macroeconomic model we use is the QUEST III model developed by the European Commission which was reestimated on Turkish data. The description of the original model can be found in Ratto et al. (2009).

3.3.1 About DSGE models in general

Modern macroeconomic analysis builds on general equilibrium models which consider market equilibrium as a gravitational point of the economy. These models started to penetrate mainstream macroeconomics as an answer to the Lucas critique which draws the attention to the fact that the efficiency of policy interventions can be counteracted by mechanisms driven by the modified decisions of rational actors expecting these interventions. This critique proved to be a significant theoretical challenge for Keynesian macroeconometric models which, as a result of their inherent structure, cannot account for these adjustments. The answer to these challenges were basically theory-based, and micro-founded structural models which, as a result of their former characteristics, are able to explicitly handle the effects resulting from the change in economic actors' behavior.

The general equilibrium paradigm entered mainstream macroeconomics with RBC (real business cycle) models, which provide a supply-side (basically productivity-based) explanation for business cycles. These models, although, robust to the Lucas critique, are less able to explain that empirical evidence that demand-side shocks have persistent real effects. Subsequent (also called new Keynesian) model developments tried to make the models more realistic by including market imperfections (mainly monopolistic competition) and other frictions (adjustment costs, rigid prices, non-optimizing actors).

Building on these veins of the literature, in the last two decades a kind of synthesis has been established in modern macroeconomics which retains general equilibrium as a sound theoretical basis which drives long run dynamics in the economy, but in the short run the just mentioned frictions and imperfections can generate even large deviations from this long run equilibrium path. During this period DSGE (dynamic stochastic general equilibrium) models step forward as a workhorse of macroeconomics. These models are dynamic because they explicitly take into account intertemporal decisions of economic actors; they are stochastic as the structural relationship and variables of the model can be hit by different shocks driving the economy away from the equilibrium path; they are general equilibrium as they assume market clearing (even if markets are not perfect).

Although DSGE models provide the advantage of explicit microeconomic background and theoretical coherence in contrast to traditional macroeconometric models, partly as a consequence of these characteristics, their empirical fit to the data is problematic as the models do not capture the data-generating process behind observed time series. In spite of this, important development has been done in respect: Smets and Wouters (2003) for example show that a DSGE model based on new Keynesian background can forecast macro time series as precisely as an empirical VAR model.

In the typical DSGE models households decide on consumption, investment and supply differentiated labor, leading to a wage setting power on their side. This labor is employed by the firms, they rent capital and supply differentiated goods to households on a monopolistically competitive market, leading to a price setting power on their side. Both households and firms make decisions in a dynamic environment, maximizing the present value of future utility and profits, through setting the above variables. A basic characteristic of DSGE models is that actors form rational expectations with regards to the future.

Both households and firms face nominal rigidities (rigid prices and wages, indexing) which constrain their wage and price setting power. Capital accumulates endogenously in these models, but investment and capacity utilization is subject to adjustment costs. The preferences of households generally contain habit formation, so that utility is not only dependent on current but also on past consumption (with a specific weight). Most of these models operate with a limited fiscal policy block, and monetary policy is generally integrated through an interest rate (Taylor) rule. This basic structure is then augmented by different shocks which affect the supply side (productivity, labor supply), the demand side (preferences, government expenditures), costs (price- and wage markup, risk premium) or the monetary rule. These shocks are modeled as first order autoregressive processes most of the time. (Tovar, 2008)

The popularity of DSGE models is signaled by the fact that many central bank and economic analyst institute use these models for policy impact analysis or forecasting. Just to mention some: the Federal Reserve in the US (Erceg et al., 2006), the European Central Bank in the Eurozone (Christoffel et al., 2008), the Bank of England in Great Britain (Harrison et al., 2005), or the Hungarian Central Bank (Jakab and Világi, 2008; Szilágyi et al., 2013).

3.3.2 The description of the macro model block

The macroeconomic block of the GMR model is a standard DSGE model which describes the relationship of for macroeconomic sectors (households, firms, government, and foreign sector).

It uses 104 endogenous variables to describe this structure and the dynamics are driven by 23 exogenous shock variables.[4] The model equations are determined by 120 structural parameters, and the standard deviations of the 23 shocks also appear as parameters. In what follows, we describe the equations describing each sector in detail.

Those equations which are finally used in the model are basically defined in growth rates and shares/ratios to the GDP. However, during the derivations, we use levels instead of rates in order to help the understanding. Where appropriate, we move to the declaration system of the technical equations in rates. Due to the many equations and different derivations, we split the numbering of equations into two parts. We use letter ‘A’ to denote equations which are presented only as additional, guiding relationships in the derivations, whereas the letter ‘M’ is used to denote those equations which constitute the final, estimated model.

3.3.2.1 The households

A typical tool of mainstream DSGE models, primarily to indicate real effect of fiscal interventions, is to split the household sector into two parts, namely the ‘Ricardian’ and ‘non-Ricardian’ or in other words non-liquidity constrained and liquidity constrained households. While the former have unconstrained access to financial markets, can borrow and save part of their income, the latter spend their current income solely to consumption.

Ricardian households

The Ricardian households of the model are characterized by the following utility function, which defines utility in function of consumption and leisure. Both factors are equipped with habit formation and we also define preference shocks.

$$U_t^R \; C_t^R, L_t^R \; = \frac{\exp(u_t^C) \;\; C_t^R - h^C C_{t-1}^R \;\; 1-\exp(u_t^L)\omega \; L_t^R - h^L L_{t-1}^R \;\; ^{\kappa} \;\; ^{1-\sigma^C}}{1-\sigma^C} \qquad \text{(A1)}$$

In the above utility function C_t^R denotes the consumption of the representative Ricardian household in period t, L_t^R is the labor supply of the household in period t, u_t^C and u_t^L are exogenous shocks to preferences, h^C and h^L are the habit parameters, σ^C, κ and ω are further preference parameters. The partial derivative of the above utility function according to consumption (C_t^R) is:

$$UC_t^R = \exp(u_t^C) \;\; C_t^R - h^C C_{t-1}^R \;\; ^{-\sigma^C} \;\; 1 - \exp(u_t^L)\omega \;\; L_t^R - h^L L_{t-1}^R \;\; ^{\kappa} \;\; ^{1-\sigma^C} \qquad \text{(A2)}$$

[4] The original model specification estimated for the Eurozone uses 19 exogenous shocks which were augmented by four further effects in order to fit the model into the specific framework of the GMR model.

The partial derivative according to leisure $(1 - L_t^R)$ is:

$$UL_t^R = \exp(u_t^C)\ \ C_t^R - h^C C_{t-1}^R\ \ ^{1-\sigma^C}\ \ 1 - \exp(u_t^L)\omega\ \ L_t^R - h^L L_{t-1}^R\ \ ^{\kappa}\ \ ^{-\sigma^C} \exp(u_t^L)\omega\kappa\ \ L_t^R - h^L L_{t-1}^R\ \ ^{\kappa-1} \quad \text{(A3)}$$

The two relationships above are modified as the model operates with growth rates and shares to GDP. Let's multiply equations (A2) and (A3) both with $P_t^C/(Y_t P_t)/(1 + GY\ \ ^{-\sigma^C}$, where Y_t stands for GDP, P_t^C is the price level of consumption goods, P_t is the price level of GDP (the GDP deflator), and GY is the steady state growth rate of GDP (which is a parameter of the model).

$$NUC_t^R = UC_t^R\ \ \frac{P_t^C}{Y_t P_t(1+GY)}\ \ ^{-\sigma^C} \quad \text{(A4)}$$

$$NUL_t^R = UL_t^R\ \ \frac{P_t^C}{Y_t P_t(1+GY)}\ \ ^{-\sigma^C} \quad \text{(A5)}$$

The two values above define the respective marginal utilities compared to GDP on a nominal basis (utility is monetized on the price level of consumption goods). Substituting the respective marginal utilities into (A4) and (A5):

$$NUC_t^R = \exp(u_t^C)\ \ \frac{C_t^R P_t^C}{Y_t P_t(1+GY)} - h^C \frac{C_{t-1}^R P_t^C}{Y_t P_t(1+GY)}\ \ ^{-\sigma^C}\ \ 1 - \exp(u_t^L)\omega\ \ L_t^R - h^L L_{t-1}^R\ \ ^{\kappa}\ \ ^{1-\sigma^C} \quad \text{(A6)}$$

$$NUL_t^R = \exp(u_t^C)\ \ \frac{C_t^R P_t^C}{Y_t P_t(1+GY)} - h^C \frac{C_{t-1}^R P_t^C}{Y_t P_t(1+GY)}\ \ ^{1-\sigma^C}\ \ 1 - \exp(u_t^L)\omega\ \ L_t^R - h^L L_{t-1}^R\ \ ^{\kappa}\ \ ^{-\sigma^C} \exp(u_t^L)\omega\kappa\ \ L_t^R - h^L L_{t-1}^R\ \ ^{\kappa-1} \quad \text{(A7)}$$

Let's introduce the following notation: $CSN_t^R = (C_t^R P_t^C)/(Y_t P_t)$, which is simply the ratio of Ricardian households' nominal consumption to nominal GDP. Using this definition, (A4) and (A5) can be written in the following form which are at the same time the first equations of the model used in estimation and simulation:

$$NUC_t^R = \exp(u_t^C)\ \ CSN_t^R\ \ 1 - h^C \frac{1}{1+GC_t^R - GY}\ \ ^{-\sigma^C}\ \ 1 - \exp(u_t^L)\omega\ \ L_t^R - h^L L_{t-1}^R\ \ ^{\kappa}\ \ ^{1-\sigma^C} \quad \text{(M1)}$$

$$NUL_t^R = \exp(u_t^C)\ \ CSN_t^R\ \ 1 - h^C \frac{1}{1+GC_t^R - GY}\ \ ^{1-\sigma^C}\ \ 1 - \exp(u_t^L)\omega\ \ L_t^R - h^L L_{t-1}^R\ \ ^{\kappa}\ \ ^{-\sigma^C} \exp(u_t^L)\omega\kappa\ \ L_t^R - h^L L_{t-1}^R\ \ ^{\kappa-1} \quad \text{(M2)}$$

where $GC_t^R = C_t^R / C_{t-1}^R - 1$, is the growth rate of real consumption in the case of Ricardian households. On the basis of equations (M1) and (M2), together with equations (A4) and (A5) define the growth rate of the marginal utility of consumption (in absolute and real terms):

$$\log\ NUC_t^R \ - \log\ NUC_{t-1}^R \ = GUC_t + \sigma^C \ GY_t - GY + \pi_t - \pi_t^C \tag{M3}$$

where $GUC_t = UC_t^R / UC_{t-1}^R - 1$ denotes the rate of change in the marginal utility of consumption, $GY_t = Y_t / Y_{t-1} - 1$ is the growth rate of per capita GDP, π_t is inflation rate (based on the GDP deflator), and π_t^C is the rate of change in the price of consumption goods.

Ricardian households spend their income, over consumption, on investment in physical capital, domestic and foreign bonds, while keeping the remaining income in money. Their budget constraint, written in nominal terms is as follows:

$$1 + t^C \ P_t^C C_t^R + P_t^I I_t + M_t + NB_t + E_t NB_t^F = M_{t-1} + \ 1 + i_{t-1} \ NB_{t-1} + \ 1 + i_{t-1}^F \ \ 1 - rf \frac{E_t NB_{t-1}^F}{P_{t-1} Y_{t-1}} + u_t^F \ \ NB_{t-1}^F + \ i_{t-1}^K - rp_t - t^P (i_{t-1}^K - rp_t + \delta) \ P_{t-1}^I K_{t-1} + \ 1 - t_t^W - ssc \ W_t L_t^R - \frac{\gamma_W L_t^R}{2} \frac{(\Delta W_t)^2}{W_{t-1}} + PR_t P_t \tag{A8}$$

The expenditure (left-hand) side of this budget constraint sums (respectively) consumption, investment in physical capital, money holding, domestic and foreign bonds and lump sum taxes. t^C is the rate of consumption tax (a parameter of the model), M_t is money supply, NB_t is the domestic and NB_t^F is the foreign nominal stock of bonds and E_t is the nominal exchange rate. On the revenue side t^P is the tax rate on capital income, i_t is the domestic and i_t^F is the foreign interest rates on bonds, i_t^K is the nominal return on physical capital. rp_t is the risk premium on physical capital investment, δ is the depreciation rate, t_t^W is the rate of labor income tax, ssc is the rate of social security contributions, $W_t$is the nominal wage, while PR_t is the (real) profit income. There are two non-trivial elements on the right hand side. First, risk premium on foreign bonds, which is a function of foreign debt (the effect of external debt on this element is given by parameter rf) and an exogenous shock (u_t^F). Second, there is an adjustment cost coming from changes in the wage (more details are given in the section on wage setting), which depends on the employment level and wage change (ΔW_t), while its strength is determined by parameter γ_W.

The decision of Ricardian households are also influenced by installations costs linked to physical capital investments: only a part of the total amount of purchasing power spent on physical capital investment (denoted by I_t) is in effect installed as physical capital (J_t), the difference melted in installation costs. This relationship is defined in the following equation:

$$I_t = J_t \left(1 + \frac{\gamma_K}{2}\left(\frac{J_t}{K_t}\right)\right) + \frac{\gamma_I}{2}(\Delta J_t)^2 \quad \text{(A9)}$$

where γ_K and γ_I are parameters determining installation costs. As a result, the accumulation of physical capital is described by the following formula:

$$K_t = J_t + (1-\delta)K_{t-1} \quad \text{(A10)}$$

The decision problem of the households is to maximize (A1) on an infinite time horizon subject to the budget constraint (A8) and further constraints (A9) and (A10). The five decision variables of the household are consumption (C_t^R), purchases of domestic and a foreign bonds (NB_t and NB_t^F), investment in physical capital, (I_t), and the planned level of physical capital (K_t).

Using the (A8) budget constraint in real terms (dividing through by P_t) we obtain the following first order conditions with respect to consumption and domestic bonds respectively (we omit the expectations operator for the sake of clarity):

$$UC_t^R - \lambda_t \frac{(1+t^C)P_t^C}{P_t} = 0 \quad \text{(A11)}$$

$$-\lambda_t + \lambda_{t+1}\beta(1+i_t)\frac{P_t}{P_{t+1}} = 0 \quad \text{(A12)}$$

where λ_t is the Lagrange-multiplier of the budget constraint. Eliminating λ_t from these two equations we get

$$\frac{1}{\beta} = \frac{UC_{t+1}^R}{UC_t^R}(1+i_t)\frac{P_t^C}{P_{t+1}^C} \quad \text{(A13)}$$

which, after taking logarithms, we obtain the (approximate) form of the Euler equation:

$$\frac{1}{\beta} - 1 = GUC_{t+1} + i_t - \pi_{t+1}^C \quad \text{(M4)}$$

The first order condition with respect to foreign bonds in the decision problem of households is:

$$-\lambda_t + \lambda_{t+1}\beta\left(1+i_t^F\right)\left(1 - rf\frac{E_t NB_t^F}{P_t Y_t} + u_t^F\right)\frac{P_t}{P_{t+1}}\frac{E_{t+1}}{E_t} = 0 \quad \text{(A13)}$$

Using (A12) and (A13) we end up with uncovered interest rate parity

$$\frac{1+i_t}{1+i_t^F}\left(1 - rf\frac{E_t NB_t^F}{P_t Y_t} + u_t^F\right) = \frac{E_{t+1}}{E_t} \quad \text{(A14)}$$

Loglinearizing equation (A14) gives the approximate form of uncovered interest rate parity which is directly used by the model:

$$i_t = i_t^F + GE_{t+1} - rf \cdot B_t^F + u_t^F \tag{M5}$$

where GE_t is the growth rate of the nominal exchange rate while B_t^F is the external debt to GDP ratio (in nominal terms: $B_t^F = E_t NB_t^F / P_t Y_t$).

In the optimization problem the partial derivatives with respect to investment and physical capital lead to the following first order conditions respectively:

$$-\xi_t + \xi_t \beta \; 1 - \delta \; + \lambda_{t+1} \beta \; 1 - t^P \; i_t^K - rp_t \; + t^P \delta \; \frac{P_t^I}{P_{t+1}} = 0 \tag{A15}$$

$$-\lambda_t \frac{P_{t-1}^I}{P_t} \; 1 + \gamma_K \frac{J_t}{K_t} + \gamma_I \Delta J_t \; - \lambda_{t+1} \beta \frac{P_t^I}{P_{t+1}} \gamma_I \Delta J_{t+1} + \xi_t = 0 \tag{A16}$$

where ξ_t is the Lagrange-multiplier of the capital accumulation equation (A10) (as an optimization constraint), whereas equation (A9) as a constraint is substituted into equation (A8). Define the present value of the return on physical capital (Tobin-Q) as

$$Q_t = \frac{\xi_t}{\lambda_t} \frac{P_t}{P_t^I} \tag{A17}$$

Using equations (A15)-(A17), and the relationship for λ_{t+1}/λ_t given by first order condition (A12), the following two equations are obtained as drivers of households' investment decisions:

$$\gamma_K \frac{J_t}{K_t} + \gamma_I \Delta J_t - \frac{\gamma_I \Delta J_{t+1}}{1+i_t} = Q_t - 1 \tag{A18}$$

$$Q_t = \frac{1-\delta}{1+i_t} \frac{P_{t+1}^I}{P_t^I} + \frac{1-t^P \quad i_t^K - rp_t \; + t^P \delta}{1+i_t} \tag{A19}$$

Equation (A18) gives investments in function of Q_t. Introduce GI_t for the growth rate of investment and IK_t which denotes the ratio of investment to per capita capital stock. Using these definitions, equation (A18) can be written alternatively as

$$\gamma_K \; IK_t - (\delta + GY + g^{UI} + g^{pop}) \; + \gamma_I \; GI_t - (GY + g^{UI}) \; - \frac{\gamma_I}{1+i_t} \; GI_{t+1} - \; GY + g^{UI} \; = Q_t - 1 \tag{M6}$$

In the above equation GY is the steady state growth rate of GDP, g^{UI} is the steady state growth rate of the productivity of intermediate goods and g^{pop} is the growth rate of population, which values are the parameters of the model. The difference in (M6) compared to (A18) is that investment growth and investment to capital stock ratio is written in their deviations from steady state. In subsequent parts of this description we show that the growth rate of investment in steady state is $GY + g^{UI}$, and the ratio of investment to capital stock per capita in the steady state is $(\delta + GY + g^{UI})$ which is adjusted to the population growth because equations (A18) and (M6) use total capital stock levels.[5]

Liquidity constrained households

The utility function of non-Ricardian households does not contain habit formation in consumption and preference shock to consumption, but apart from these, it is similar to the utility function of the Ricardian households:

$$U_t^{NR}\left(C_t^R, L_t^R\right) = \frac{\left(C_t^R\left(1-\exp(u_t^L)\omega\left(L_t^R - h^L L_{t-1}^R\right)^{\kappa}\right)\right)^{1-\sigma^C}}{1-\sigma^C} \tag{A20}$$

Using the same method as for the Ricardian households, we obtain the marginal utilities analogous to those in (M1) and (M2):

$$NUC_t^{NR} = \left(CSN_t^R\right)^{-\sigma^C}\left(1-\exp(u_t^L)\omega\left(L_t^R - h^L L_{t-1}^R\right)^{\kappa}\right)^{1-\sigma^C} \tag{M7}$$

$$NUL_t^R = \left(CSN_t^R\right)^{-\sigma^C}\left(1-\exp(u_t^L)\omega\left(L_t^R - h^L L_{t-1}^R\right)^{\kappa}\right)^{-\sigma^C}\exp(u_t^L)\omega\kappa\left(L_t^R - h^L L_{t-1}^R\right)^{\kappa-1} \tag{M8}$$

Liquidity constrained households do not optimize, their behavior is described by their budget constraint, which is:

$$\left(1+t^C\right)P_t^C C_t^{NR} + T_t^{LS}P_t = \left(1-t_t^W - ssc\right)W_t L_t^{NR} + TR_t P_t \tag{A21}$$

where in addition to the previous notation C_t^{NR} is the consumption of non Ricardian households, L_t^{NR} is their labor supply, T_t^{LS} is the real value of lump sum taxes and TR_t is the level of transfers.[6] Dividing through (A21) with $Y_t P_t$ we get:

$$\left(1+t^C\right)\frac{P_t^C C_t^{NR}}{Y_t P_t} + \frac{T_t^{LS}}{Y_t} = \left(1-t_t^W - ssc\right)\frac{W_t}{Y_t P_t}L_t^{NR} + \frac{TR_t}{Y_t} \tag{A22}$$

Define $CSN_t^{NR} = P_t^C C_t^{NR}/Y_t P_t$, which is the ratio of the nominal consumption of non Ricardian households to nominal GDP, let $TRW_t = TR_t/Y_t$ be the transfers to GDP ratio and $YWR_t = Y_t/(W_t/P_t)$ be the ratio of GDP and real wage. Define then the nominal share of wages in GDP as:[7]

$$WS_t = L_t\frac{1}{YWR_t} \tag{M9}$$

With these definitions the budget constraint in (A22) can be written in the form:

[5] To define steady state we need per capita variables because these can be constant when population changes.
[6] In the model only liquidity constrained households receive transfers and pay lump sum taxes.
[7] In the model $L_t^R = L_t^{NR} = L_t$.

$$1 + t^{C}\ \ CSN_t^{NR} + TY_t^{LS} = \ \ 1 - t_t^{W} - ssc\ \ WS_t + TRW_t WS_t \qquad \text{(M10)}$$

Aggregation of households

The aggregation of the consumption of Ricardian and non Ricardian households are given by the following relationship where slc is the share of liquidity constrained households (a parameter of the model): $CSN_t = slc \cdot CSN_t^{NR} + (1 - slc) \cdot CSN_t^{R}$ (M11)

3.3.2.2 The firms

The model splits the firms' sector into two parts. Firms producing final consumption goods operate on a monopolistically competitive market and use capital and labor as input. The other sector of firms produces capital (investment) goods), operate on a perfectly competitive market and use domestic and imported final goods as inputs.

Final good producers

Final good producers operate on a monopolistically competitive market. Their production technology is described by the following production function:

$$Y_t^{j} = A_t^{\alpha}(L_t^{j} - LO_t^{j})^{\alpha}(ucap_t^{j}K_t^{j})^{1-\alpha}(K_t^{G})^{1-\alpha_G} \qquad \text{(A23)}$$

where Y_t^{j} is the output of producer j, α is the partial production elasticity of labor, A_t is labor productivity characteristic to the whole economy, L_t^{j} is the labor utilization of producer j, LO_t^{j} is the overhead labor, K_t^{j} is the stock of physical capital, $ucap_t^{j}$ is capacity utilization, K_t^{G} is the level of public (infrastructural) capital and α_G is the additive inverse of the production elasticity of public capital.

The demand for goods produced by the final producers is determined by a nested CES utility function. The elasticity of substitution between domestic and imported goods is σ^{M} and the elasticity of substitution between domestic goods is σ^{d}. All sectors (households, firms, government, and foreign sector) have identical preferences so the following demand function can be written for the goods produced by firm j:

$$Y_t^{j} = \frac{1-s^{M}}{n}\ \ \frac{P_t}{P_t^{j}}^{\ \sigma^{d}}\ \ \frac{P_t^{C}}{P_t}^{\ \sigma^{M}}\ \ C_t + C_t^{G} + I_t^{G} + I_t^{inp} + EX_t \qquad \text{(A24)}$$

where n is the number of final good producers, s^{M} is the share of domestic absorption, P_t^{j} is the price set by firm j, P_t is the aggregate price level, P_t^{C} is the price level of consumption goods and in the last parenthesis we have the consumption demand of households and government, the investment demand of the government, the input demand of capital good firms and the export demand, respectively.

The decision of the firms is constrained by three adjustment costs. They face these costs when changing labor utilization, prices and capacity utilization, defined by the following equations respectively:

$$aL\left(L_t^j\right) = W_t L_t^j u_t^L + \frac{\gamma_L}{2}(L_t^j - L_{t-1}^j)^2 \quad \text{(A25)}$$

$$aP\left(P_t^j\right) = \frac{\gamma_P}{2}\frac{(P_t^j - P_{t-1}^j)^2}{P_{t-1}^j} \quad \text{(A26)}$$

$$aU\left(ucap_t^j\right) = P_t^I K_t^j \left(\gamma_{U1}\left(ucap_t^j - ucap\right) + \frac{\gamma_{U2}}{2}\left(ucap_t^j - ucap\right)^2\right) \quad \text{(A27)}$$

where γ_L, γ_P, γ_{U1} and γ_{U2} are the parameters of the adjustment cost functions, u_t^L is an exogenous shock to the adjustment cost to labor and $ucap$ is the steady state value of capacity utilization.

The profit function of the firm is:

$$PR_t^j = \frac{P_t^j Y_t^j}{P_t} - \frac{W_t L_t^j}{P_t} - \frac{i_t^K P_t^I K_t^j}{P_t} - \frac{1}{P_t}\left(aL\left(L_t^j\right) + aP\left(P_t^j\right) + aU\left(ucap_t^j\right)\right) \quad \text{(A28)}$$

The decision problem of the firms is to maximize profit function (A28) on an infinite time horizon subject to constraints (A23)-(A27). Define the Lagrange function as follows (using the real interest rate (r_t) for discounting):

$$V = \sum_{t=0}^{\infty}\frac{1}{(1+r_t)^t}\left(PR_t^j + \eta_t^j\left(Y_t^j - A_t^\alpha (L_t^j - LO_t^j)^\alpha (ucap_t^j K_t^j)^{1-\alpha}(K_t^G)^{1-\alpha_G}\right)\right) \quad \text{(A29)}$$

then substitute the constraints (A24)-(A27) into the Lagrange function (A29). Differentiating the resulting optimization problem with respect to labor utilization L_t^j, we obtain the following first order condition:

$$-\frac{W_t}{P_t^j} + \alpha\eta_t^j \frac{Y_t^j}{L_t^j - LO_t^j} - \frac{W_t}{P_t^j}u_t^L - \frac{W_t}{P_t^j}\gamma_L\left(L_t^j - L_{t-1}^j\right) + \frac{W_{t+1}}{P_{t+1}^j}\frac{\gamma_L}{1+r_t}\left(L_{t+1}^j - L_t^j\right) = 0 \quad \text{(A30)}$$

Using the notation $YWR_t = Y_t/(W_t/P_t)$ defined previously and the fact that due to the symmetry of the monopolistic competition we can leave superscript j, equation (A30) can be written in the following form:

$$\frac{1+u_t^W}{YWR_t} = \eta_t\alpha\frac{1+lol_t}{L_t} - \frac{1}{YWR_t}\gamma_L\left(L_t - L_{t-1}\right) + \frac{1}{YWR_t}(1 + GY_t - GY)\frac{\gamma_L}{1+r_t}\left(L_{t+1} - L_t\right) \quad \text{(M12)}$$

Differentiating with respect to capacity utilization results in the next first order condition:

$$1-\alpha\ \eta_t^j \frac{Y_t^j}{K_t^j} - \frac{P_t^I}{P_t}\ \gamma_{U1} + \gamma_{U2}\ ucap_t^j - ucap \quad = 0 \tag{A31}$$

Introduce $KSN_t = (P_t^I K_t)/(P_t Y_t)$ which is the physical capital to GDP ratio in nominal terms. Equation (A31) gives the following relationship then:

$$1-\alpha\ \eta_t \frac{1}{KSN_t} = \gamma_{U1} + \gamma_{U2}\ ucap_t - ucap \quad ucap_t \tag{M13}$$

Differentiating with respect to the price we obtain the first order condition for the price markup (the Lagrange multiplier):

$$\eta_t^j - \frac{\sigma^d - 1}{\sigma^d} + \gamma_P \quad \frac{1}{1+r_t}\pi_{t+1}^j - \pi_t^j \quad = 0 \tag{A32}$$

where $\pi_t^j = P_t^j / P_{t-1}^j - 1$. Using the assumption of symmetry and introducing $\tau = 1/\sigma^d$ equation (A32) is modified as follows. First, we assume that a share sfp of firms determine their prices according to equation (A32), in a forward looking way, while the other $(1 - sfp)$ share of firms are indexing their prices according to inflation. Second, in place of inflation itself, we take the deviation of inflation from its steady state value (π – inflation target) into account. Third, the markup is augmented by an exogenous shock (u_t^η), and fourth, we use the discount factor (which is a parameter) instead of real interest rate.

$$\eta_t = 1 - \quad \tau + u_t^\eta \quad - \gamma_P\ \beta\ sfp \cdot \pi_{t+1} + \quad 1 - sfp\ \pi_t - \pi \quad - (\pi_t - \pi) \tag{M14}$$

The behavior of the final goods producer sector is finally described by the production function, which, at the aggregate level, is given in growth rates on the basis of equation (A23):

$$GY_t = \alpha GA_t + \alpha GL_t\ \ 1 + lol \ \ + \ \ 1 - \alpha \quad GK_t + Gucap_t \ \ + (1 - \alpha_G)GKG_t \tag{M15}$$

where GY_t, GA_t, GL_t, GK_t, $Gucap_t$ and GKG_t are the growth rates of GDP, labor productivity, labor utilization, capital stock, capacity utilization and public capital stock respectively, whereas lol is the steady state value of overhead labor (lol_t).

The intermediate goods sector

Intermediate (or investment) goods are produced by a perfectly competitive sector, using domestic and imported final goods as inputs. The production technology is:

$$I_t = A_t^I I_t^{inp} \tag{A33}$$

where A_t^I is the productivity of the sector, I_t^{inp} is the amount of inputs, being a CES aggregate of domestic and imported final goods with σ^M elasticity of substitution (domestic goods are also CES aggregate of goods, with σ^d elasticity of substitution). The price level of investment goods follows simply:

$$P_t^I = \frac{P_t^C}{A_t^I} \tag{A34}$$

where P_t^C is the price level of final (consumption) goods. The nominal investment to GDP share is determined by the investment to capital stock ratio and the capital stock to GDP ratio. In equation (M16) this relationship is adjusted with the deviation of capital growth rate (GK_t) from its steady state level (see equation (M42)):

$$\ln\left(ISN_t\right) = -\ln\left(\frac{1}{KSN_t}\right) + \ln\left(IK_t\right) + GY + g^{AI} - GK_t \tag{M16}$$

Investments are determined implicitly by the following relationship on the basis of bringing the marginal productivity and the marginal cost of physical capital to parity:

$$\eta_t\left(1-t^P\right)\left(1-\alpha\right)\frac{1}{KSN_t} = Q_t - \left(1 - r_t - \delta - rp - u_t^{rp} - g_{t+1}^{AI} + g^{AI}\right)Q_{t+1} + \gamma_{U1}\left(ucap_t - ucap\right) + \gamma_{U2}\left(ucap_t - ucap\right)^2 \tag{M17}$$

3.3.2.3 Labor market and wages

In the model the labor market is also monopolistically competitive. As a consequence, the L_t^j labor demand of firms is a CES aggregate of different types of labor.

$$L_t^j = \left(\int_0^1 L_t^{i,j\,\frac{\theta-1}{\theta}}\,di\right)^{\frac{\theta}{\theta-1}} \tag{A35}$$

Wage setting is carried out by a union, maximizing the weighted average of the utility of the two household types (we assume that labor types are evenly distributed in the whole population). Reservation wage is given by the standard utility maximizing criteria: real wage (on the basis of consumption price level) equals the ratio of the marginal utilities of leisure and consumption (marginal rate of substitution). When determining reservation wage, the value given by optimization is smoothed by a parameter $wrlag$. Taking consumption and wage taxes into account as well as social security contributions, we have the following formula for real (reservation) wages:

$$\frac{W_t}{P_t^C} = \left(\frac{W_{t-1}}{P_{t-1}^C}\right)^{wrlag}\left(\frac{1}{\eta_t^W}\frac{1+t^C}{1-t_t^W-ssc}\frac{\left(1-slc\right)NUL_t^{NR}+slcNUL_t^R}{\left(1-slc\right)NUC_t^{NR}+slcNUC_t^R}\right)^{1-wrlag} \tag{A36}$$

where η_t^W is the wage markup. Wage markup evolves according to an equation analogous to the price markup of consumption goods, where a fraction $(1-sfw)$ of households do not decide on their wage in a forward looking manner but index it to past inflation:

$$\eta_t^W = \frac{\theta-1}{\theta} - \frac{\gamma_W}{\theta} \ \beta \ \pi_{t+1}^W - (1-sfw)\pi_t \ - \ \pi_{t+1}^W - (1-sfw)\pi_t \qquad \text{(A37)}$$

where π_t^W is wage inflation and γ_W is a parameter of the adjustment cost function with respect to wages. We take the combined version of equations (A36) and (A37) into the technical model equations, converted to GDP-shares:

$$1+t^C \quad \frac{1-slc \ NUL_t^{NR}+slcNUL_t^R}{1-slc \ NUC_t^{NR}+slcNUC_t^R}^{1-wrlag} \quad \frac{1-t_t^W-ssc}{1+t^C}\frac{\theta-1}{\theta}\frac{1}{YWR_t}\frac{1}{1+GY_t-GY}^{wrlag} = \frac{\theta-1}{\theta}\frac{1}{YWR_t} \ 1- t_t^W - ssc \ + \frac{\gamma_W}{\theta}\frac{1}{YWR_t} \quad \pi_t^W - \pi - GY \ -(1-sfw)(\pi_{t-1}-\pi) \ -\beta\frac{\gamma_W}{\theta}\frac{1}{YWR_t} \quad \pi_{t+1}^W - \pi - GY \ -(1-sfw)(\pi_t-\pi) \qquad \text{(M18)}$$

3.3.2.4 Government

The role of the government is modelled by a standard monetary policy reaction function and a sophisticated fiscal block, which operates with fiscal reaction functions similar to the monetary policy rule.

Monetary policy

Monetary policy in the model is described by a Taylor rule:

$$i_t = \tau_{lag}^i i_{t-1} + \ 1-\tau_{lag}^i \quad r + \pi_t^T + \tau_\pi^i \ \pi_t^C - \pi_t^T \ + \tau_{Y1}^i \ln \ YGAP_{t-1} \quad + \tau_{Y2}^i \ln \ YGAP_t \ - \ln \ YGAP_{t-1} \quad + u_t^M \qquad \text{(M19)}$$

where τ_{lag}^i is a smoothing parameter, τ_π^i, τ_{Y1}^i and τ_{Y2}^i is the reaction parameters of interest rate to the inflation’s deviation from its target, the output gap and the change in the output gap, respectively. $YGAP_t$ is a proxy for the output gap (see later), $r = 1/(\beta-1)$ is the natural (steady state) real ineterst rate, π_t^T is the inflation target and u_t^M is an exogenous shock from the side of monetary policy.

Fiscal policy

Fiscal policy is described by similar reaction functions as monetary policy. Fiscal policy operates with five elements on the revenue side: (i) wage income tax, (ii) consumption tax, (iii) capital income tax, (iv) lump sum tax, and (v) social security contributions. On the expenditure side we distinguish between (i) transfers, (ii) government consumption and (iii) government investment.

In the case of government consumption, we give a relationship for the change in these expenditures. Government consumption grows in the steady state with the same rate as GDP. Through the output gap we build a counter-cyclical element into the reaction function, and we use the deviation of government consumption from its steady state level among the reaction variables. Finally, we define an exogenous shock and a smoothing behavior. As a result, the following reaction function is written for government consumption:

$$GG_t - GY = \tau_{lag}^{CG} \; GG_{t-1} - GY \; + \tau_{adj}^{CG} \ln \; GSN_{t-1} \; - \ln \; GSN \; + \tau_0^{CG} \ln \; YGAP_t \; - \ln \; YGAP_{t-1} \; + u_t^{CG} \quad \text{(M20)}$$

where GG_t is the growth rate of government consumption, GY is the steady state growth rate of GDP, GSN_t is the nominal share of government consumption in GDP, τ_{lag}^{CG}, τ_{adj}^{CG} and τ_0^{CG} are reaction parameters and u_t^{CG} is the exogenous shock.

We define an analogous reaction function for government investment as in (M20):

$$GIG_t - GY - g^{AI} = \tau_{lag}^{IG} \; GIG_{t-1} - GY - g^{AI} \; + \tau_{adj}^{IG} \ln \; IGSN_{t-1} \; - \ln \; IGSN \; + \tau_0^{IG} \ln \; YGAP_t \; - \ln \; YGAP_{t-1} \; + u_t^{IG} \quad \text{(M21)}$$

where we use the fact that the steady state growth rate of investments is the sum of the steady state growth rate of GDP and that of the productivity of the intermediate sector.

Transfers are linked to employment counter-cyclically. Define $TRW_t = TR_t / W_t$ as the ratio of per employee nominal transfers to nominal wage. The transfer rule is:

$$TRW_t = TRW + \tau^{TR} \; 1 - L_t - (1 - L) \; + u_t^{TR} \quad \text{(M22)}$$

where TRW is the steady state value of transfers, L is the steady state employment, τ^{TR} is a reaction parameters and u_t^{TR} is an exogenous shock.

On the revenue side the rate of social security contributions, the capital income tax and the consumption tax is given (ssc, t^P and t^C respectively), we do not define fiscal rules for these revenue elements. The rate of the labor income tax evolves according to

$$t_t^W = \tau_0^W \; 1 + \tau_1^W \ln(YGAP_t) \quad \text{(M23)}$$

where τ_0^W is the steady state value of the rate of labor income tax and τ_1^W is a reaction parameter. The role of the lump sum tax is to control the public debt, therefore we define the following rule for it:

$$T_t^{LS} - T_{t-1}^{LS} = \tau_1^{LS} \; B_t - B \; + \tau_2^{LS} \; B_t - B_{t-1} \quad \text{(M24)}$$

where B is the target level of the public debt to GDP ratio, τ_1^{LS} and τ_2^{LS} are reaction parameters. The fiscal block is closed by the budget constraint of the government which at the same time defines the dynamics of the public debt:

$$B_t = \ 1 + r_t - GY_t - g^{pop} \ B_{t-1} + GSN_t + IGSN_t + \frac{TRW_t}{YWR_t} L_t - \ t_t^W + ssc \ WS_t - t^P \ 1 - WS_t \ - t^C CSN_t - T_t^{TS} - \varepsilon_t^{GB} \quad \text{(M25)}$$

where WS_t is the nominal wage share in GDP as in equation (M9) and we take into account that GY_t is the growth rate of the GDP per capita. The exogenous disturbance term ε_t^{GB} has a technical role. This variable is not included in the original model estimated for the Eurozone. Its role here is to be able to compensate for the policy interventions appearing on the expenditure side of the government budget on the revenue side. If we were not controlling for this, policy shocks financed by external sources (EU) would lead to spillover effects through increasing deficits and public debt which would bias our results.

Output gap

The output gap is an important variable in the fiscal reaction functions. The model provides an indirect way to measure the output gap. Define the equilibrium employment and capacity utilization as follows:

$$\ln \ L_t^{ss} \ = \rho^{Lss} \ln \ L_{t-1}^{ss} \ + (1 - \rho^{Lss}) L_t \quad \text{(M26)}$$

$$ucap_t^{ss} = \rho^{ucap} ucap_{t-1}^{ss} + (1 - \rho^{ucap}) ucap_t \quad \text{(M27)}$$

These two equations give a moving average representation of what is meant to be the potential employment and capacity utilization. According to the production function (A23) we get the following approximate version for the output gap:

$$\ln \ YGAP_t \ = \ 1 - \alpha \ \ \ln \ ucap_t \ - \ln \ ucap_t^{ss} \ + \alpha \ \ln \ L_t \ - \ln(L_t^{ss}) \quad \text{(M28)}$$

3.3.2.5 The foreign sector

The foreign sector appears in two modules. First, we define equations describing the relationship between domestic and foreign variables and second, we model the joint evolution of the variables describing the rest of the world as a mini-model, which drive exogenously the dynamics of the domestic variables.

As it was introduced previously, domestic final absorption (consumption and investment of households and the government) is a CES aggregate of domestic and foreign final goods where the elasticity of substitution between domestic and foreign goods is σ^M. On the basis of this, the demand for import is determined by a parameter describing the (steady state) import share of domestic absorption together with the relative price of imported and domestic goods. The import demand function deriving from this formula is modified by a smoothing parameter in the effect of the relative price. The import demand thus looks like as follows (in nominal terms, expressed relative to the GDP):

$$IMSN_t = (1 - s^M)\left[\left(\frac{P^C_{t-1}}{P^M_{t-1}}\right)^{\rho^M}\left(\frac{P^C_t}{P^M_t}\right)^{1-\rho^M}\right]^{\sigma^M}\frac{P^M_t}{P^C_t}\left(CSN_t + ISN_t + GSN_t + IGSN_t\right) \quad \text{(M29)}$$

where s^M is the share of domestic absorption and ρ^M is the weight of smoothing in the relative price.

We use an analogous expression for exports, using that in the preferences of the foreign sector the elasticity of substitution between domestic and foreign goods is σ^X:

$$EXSN_t = (1 - s^M)\left[\left(\frac{(E_{t-1})^{\alpha^X s^M} P_{t-1}}{P^X_{t-1}}\right)^{\rho^X}\left(\frac{(E_t)^{\alpha^X s^M} P_t}{P^X_t}\right)^{1-\rho^X}\right]^{\sigma^X}\frac{P^X_t}{P_t}\left(YWY_t\right)^{\alpha^X} \quad \text{(M30)}$$

where YWY_t is the ratio of foreign GDP to domestic GDP and α^X is the weight of this ratio in the demand for export.

We apply markup in the price of both the imported and exported goods, for which the same expression is used as introduced for domestic final goods (see equations (M17 and (A34)). The equation for the export markup is:

$$\frac{P^X_t}{P_t} = 1 + u^{PX}_t + \gamma_{PX}\left[\beta\cdot sfp^X\pi^X_{t+1} + \left(1 - sfp^X\right)\left(\pi^X_{t-1} - \pi\right) - (\pi^X_t - \pi)\right] \quad \text{(M31)}$$

where γ_{PX} is the usual adjustment parameter, sfp^X is the share of exporters who set prices in a forward looking way, π^X_t is the inflation of export-prices and u^{PX}_t is an exogenous shock.

Similarly for the imported goods:

$$\frac{P^M_t}{P_t} = (E_t)^{\alpha^X}\left(1 + u^{PM}_t + \gamma_{PM}\left[\beta\cdot sfp^M\pi^M_{t+1} + \left(1 - sfp^M\right)\left(\pi^M_{t-1} - \pi\right) - (\pi^M_t - \pi)\right]\right) \quad \text{(M32)}$$

The price level of the consumption goods is thus the weighted average of domestic and imported final goods:

$$\frac{P^C_t}{P_t} = \left[s^M + (1 - s^M)\left(\frac{P^M_t}{P_t}\right)^{1-\sigma^M}\right]^{\frac{1}{1-\sigma^M}} \quad \text{(M33)}$$

The current account is given by the following formula, using exports and imports:

$$NTBSN_t = EXSN_t - IMSN_t + u_t^{EX} \qquad \text{(M34)}$$

where u_t^{EX} is an exogenous shock to the current account.

The following equation gives the dynamics of foreign bonds (measured in the domestic currency):

$$B_t^F = \; 1 + i_t - \pi_{t+1} - GY_t - g^{pop} \; B_{t-1}^F + NTBSN_t \qquad \text{(M35)}$$

where B_t^F is the ratio of the stock of foreign bonds to the domestic GDP.

The relationship between domestic and foreign variables is further specified by the uncovered interest rate parity in (M5) and the purchasing power parity as follows:

$$GE_t + \pi_t^F - \pi_t = \ln \; \frac{E_t}{E_{t-1}} \qquad \text{(M36)}$$

where GE_t is the change in the nominal exchange rate and π_t^F is the foreign inflation.

The mini modal describing the dynamics of the foreign sector contains the deviation of foreign interest rate from its steady state level: ${\iota^F}_t = i_t^F - \iota^F$, the deviation of foreign inflation from its steady state level: ${\pi^F}_t = \pi_t^F - \pi^F$, and the deviation of foreign GDP growth from its steady state level: $GYW_t = GYW_t - GYW$, where steady state levels are the parameters of the model. We define the following VAR(1) model for these three variables:

$$\begin{matrix} {\iota^F}_t \\ {\pi^F}_t \\ GYW_t \end{matrix} = \begin{matrix} \rho^{i^F} & \rho^{i^F,\pi^F} & \rho^{i^F,GY^F} \\ \rho^{\pi^F,i^F} & \rho^{\pi^F} & \rho^{\pi^F,GY^F} \\ \rho^{GY^F,i^F} & \rho^{GY^F,\pi^F} & \rho^{GY^F} \end{matrix} \; \begin{matrix} {\iota^F}_{t-1} \\ {\pi^F}_{t-1} \\ GYW_{t-1} \end{matrix} + \begin{matrix} \varepsilon_t^{iW} \\ \varepsilon_t^{PW} \\ \varepsilon_t^{YW} \end{matrix} \qquad \text{(M37)-(M39)}$$

3.3.2.6 Balancing equations and identities

The equations introduced so far are closed by several balance identities – these are enumerated in the following.

The GDP identity (final goods market equilibrium) is defined in nominal terms and in GDP shares:

$$1 = CSN_t + ISN_t + IGSN_t + GSN_t + NTBSN_t \qquad \text{(M40)}$$

The real interest rate:

$$r_t = i_t - \pi_{t+1} \qquad \text{(M41)}$$

The following two equations give the dynamics of private and public capital (their growth rates) respectively:

$$GK_t - \ \ GY + g^{AI} \ \ = IK_t - \ \ \delta + g^{pop} + GY + g^{AI} \ \ + \varepsilon_t^{CAP} \tag{M42}$$

$$GKG_t - \ \ GY + g^{AI} \ \ = IKG_t - \ \ \delta_G + g^{pop} + GY + g^{AI} \ \ + \varepsilon_t^{GCAP} \tag{M43}$$

In equation (M43) GKG_t stands for the growth rate of the per capita public capital stock, IKG_t is the ratio of government investment to public capital and δ_G is the depreciation rate of public capital. The two exogenous shock variables, ε_t^{CAP} and ε_t^{GCAP} is not defined in the original version of the model specified for the Eurozone. Their role is to have a point where we can implement private investment subsidies' and public infrastructure spending's effect on the respective capital stocks.

The definition of the above capital growth rates (in a combined way):

$$GIG_t - GI_t = \ln \ IGSN_t \ \ - \ln \ ISN_t \ \ - \ln \ IGSN_{t-1} \ \ + \ln(ISN_{t-1}) \tag{M44}$$

The identities describing the relationship between investment and capital stock in the two sectors:

$$GI_t - GK_{t-1} = \ln \ IK_t \ \ - \ln \ IK_{t-1} \tag{M45}$$

$$GIG_t - GKG_{t-1} = \ln \ IKG_t \ \ - \ln \ IKG_{t-1} \tag{M46}$$

The growth rate of the private capital stock:

$$GY_t - GK_t + g_t^{AI} = \ln \ \frac{1}{KSN_t} \ \ - \ln \ \frac{1}{KSN_{t-1}} \tag{M47}$$

The definition of disposable income:

$$WSW_t = (1 - t_t^W - ssc)WS_t \tag{M48}$$

The money stock to GDP ratio in function of the interest rate:

$$MRY_t = (1 + i_t)^{\varphi} \tag{M49}$$

The growth rate of consumption, for total consumption, consumption of Ricardian and non Ricardian households respectively:

$$GC_t - GY_t + \pi_t^C - \pi_t = \ln \ CSN_t \ \ - \ln(CSN_{t-1}) \tag{M50}$$

$$GC_t^R - GY_t + \pi_t^C - \pi_t = \ln \ CSN_t^R \ \ - \ln(CSN_{t-1}^R) \tag{M51}$$

$$GC_t^{NR} - GY_t + \pi_t^C - \pi_t = \ln\ CSN_t^{NR}\ - \ln(CSN_{t-1}^{NR}) \quad \text{(M52)}$$

Similarly, the growth rate of exports, imports and government consumption:

$$GEX_t - GY_t + \pi_t^X - \pi_t = \ln\ EXSN_t\ - \ln(EXSN_{t-1}) \quad \text{(M53)}$$

$$GIM_t - GY_t + \pi_t^M - \pi_t = \ln\ IMSN_t\ - \ln(IMSN_{t-1}) \quad \text{(M54)}$$

$$GG_t - GY_t + \pi_t^C - \pi_t = \ln\ GSN_t\ - \ln(GSN_{t-1}) \quad \text{(M55)}$$

The growth rate of employment:

$$GL_t = \ln\ L_t\ - \ln\ L_{t-1} \quad \text{(M56)}$$

The (nominal) ratio of transfers to GDP:

$$TRSN_t = TRW_t \frac{L_t}{YWR_t} \quad \text{(M57)}$$

Net transfers:

$$NTRSN_t = TRW_t \frac{L_t}{YWR_t} - T_t^{LS} \quad \text{(M58)}$$

The growth rate of lump sum tax:

$$GTAX_t - GY_t - \pi_t = \ln\ T_t^{LS}\ - \ln(T_{t-1}^{LS}) \quad \text{(M59)}$$

The growth rate of transfers:

$$GTR_t - GL_t - \pi_t^{WR} = \ln\ TRW_t\ - \ln(TRW_{t-1}) \quad \text{(M60)}$$

The growth rate of capacity utilization:

$$Gucap_t = \ln\ ucap_t\ - \ln(ucap_{t-1}) \quad \text{(M61)}$$

The growth rate of TFP adjusted by capacity utilization:

$$GAU_t = \ 1 - \alpha\ \cdot Gucap_t + \alpha \cdot GA_t \quad \text{(M62)}$$

The growth rate of the ratio of real wage to GDP:

$$GWRY_t = \ln\ YWR_t\ - \ln\ YWR_{t-1} \quad \text{(M63)}$$

The growth rate of the foreign GDP:

$$\ln\ YWY_t\ - \ln\ YWY_{t-1}\ = GYW_t - GY_t \quad \text{(M64)}$$

The change in the output gap:

$$GY_t - GPOT_t = \ln\ YGAP_t\ - \ln(YGAP_{t-1}) \tag{M65}$$

The change in public debt:

$$DB_t = B_t - B_{t-1} \tag{M66}$$

Identities with the price levels and inflations of consumption goods, imports and exports:

$$\pi_t^C - \pi_t = \ln\ \frac{P_t^C}{P_t}\ - \ln\ \frac{P_{t-1}^C}{P_{t-1}} \tag{M67}$$

$$\pi_t^M - \pi_t = \ln\ \frac{P_t^M}{P_t}\ - \ln\ \frac{P_{t-1}^M}{P_{t-1}} \tag{M68}$$

$$\pi_t^X - \pi_t = \ln\ \frac{P_t^X}{P_t}\ - \ln\ \frac{P_{t-1}^X}{P_{t-1}} \tag{M69}$$

The growth rate of nominal wages:

$$-\pi_t^W + GY_t + \pi_t = \ln\ YWR_t\ - \ln(YWR_{t-1}) \tag{M70}$$

The growth rate of real wages:

$$\pi_t^W = \pi_t^{WR} + \pi_t \tag{M71}$$

As the model is written in terms of per capita variables, the following equations give the level growth rates of the main macro variables (GDP, household consumption, investment, government consumption, exports and imports):

$$GY_t^{lev} = GY_t + g^{pop} \tag{M72}$$

$$GC_t^{lev} = GC_t + g^{pop} \tag{M73}$$

$$GI_t^{lev} = GI_t + g^{pop} + \varepsilon_t^{INV} \tag{M74}$$

$$GG_t^{lev} = GG_t + g^{pop} \tag{M75}$$

$$GEX_t^{lev} = GEX_t + g^{pop} + dgex \tag{M76}$$

$$GIM_t^{lev} = GIM_t + g^{pop} + dgim \tag{M77}$$

The exogenous shock variable ε_t^{INV} in equation (M47) has a technical character: it is not used in the original specification for the Eurozone. Its role is to implement private investment subsidies into the model.

The change in the absolute level of import and export prices:

$$\pi_t^{M,lev} = \pi_t^M + dgpm \quad \text{(M78)}$$

$$\pi_t^{X,lev} = \pi_t^X + dgpx \quad \text{(M79)}$$

The household and government consumption to GDP ratios in real terms:

$$\ln\ CY_t\ = \ln\ CSN_t\ - \ln\ \frac{P_t^C}{P_t} \quad \text{(M80)}$$

$$\ln\ GGY_t\ = \ln\ GSN_t\ - \ln\ \frac{P_t^C}{P_t} \quad \text{(M81)}$$

The equations of the model contain several variables also in logarithm. In the description above all logarithms were rewritten in non-logarithmized form, but to be complete with the technical equations, we present here the identities resulting from these dualities. Equation (M9) in logarithms:

$$\ln\ WS_t\ = \ln\ L_t\ - \ln(YWR_t) \quad \text{(M82)}$$

And further:

$$B_t = \exp(\ln\ B_t\ ) \quad \text{(M83)}$$

$$CSN_t^{NR} = \exp(\ln\ CSN_t^{NR}\ ) \quad \text{(M84)}$$

$$GSN_t = \exp(\ln\ GSN_t\ ) \quad \text{(M85)}$$

$$TRSN_t = \exp(\ln\ TRSN_t\ ) \quad \text{(M86)}$$

3.3.2.7 Exogenous processes

The model contains several exogenous shock variables which are determined by the following equations (the content of the different exogenous variables were given previously). Parameters ρ measure the respective persistences while the variables ε_t are the white noises driving the exogenous variables with zero mean and a respective standard deviation σ.

$$GA_t = g^A + \varepsilon_t^Y \quad \text{(M87)}$$

$$lol_t - lol = \rho^{lol}\ \ lol_{t-1} - lol\ \ + \varepsilon_t^{lol} \quad \text{(M88)}$$

$$g_t^{AI} = g^{AI} + u_t^{AI} \quad \text{(M89)}$$

$$u_t^C = \rho^C u_{t-1}^C + \varepsilon_t^C \quad \text{(M90)}$$

$$u_t^{\eta} = \rho^{\eta} u_{t-1}^{\eta} + \varepsilon_t^{\eta} \tag{M91}$$

$$u_t^{PM} = \rho^{PM} u_{t-1}^{PM} + \varepsilon_t^{PM} \tag{M92}$$

$$u_t^{PX} = \rho^{PX} u_{t-1}^{PX} + \varepsilon_t^{PX} \tag{M93}$$

$$u_t^{EX} = \rho^{EX} u_{t-1}^{EX} + \varepsilon_t^{EX} \tag{M94}$$

$$u_t^{CG} = \rho^{CG} u_{t-1}^{CG} + \varepsilon_t^{CG} \tag{M95}$$

$$u_t^{IG} = \rho^{IG} u_{t-1}^{IG} + \varepsilon_t^{IG} \tag{M96}$$

$$u_t^{L} = \rho^{L} u_{t-1}^{L} + \varepsilon_t^{L} \tag{M97}$$

$$u_t^{M} = \varepsilon_t^{M} \tag{M98}$$

$$u_t^{AI} = \rho_1^{AI} u_{t-1}^{AI} + \rho_1^{AI} u_{t-2}^{AI} + \rho_1^{AI} u_{t-3}^{AI} + \rho_1^{AI} u_{t-4}^{AI} + \varepsilon_t^{AI} \tag{M99}$$

$$u_t^{F} = \rho^{F} u_{t-1}^{F} + \varepsilon_t^{F} \tag{M100}$$

$$u_t^{rp} = \rho^{rp} u_{t-1}^{rp} + \varepsilon_t^{rp} \tag{M101}$$

$$u_t^{W} = \varepsilon_t^{W} \tag{M102}$$

$$u_t^{TR} = \rho^{TR} u_{t-1}^{TR} + \varepsilon_t^{TR} \tag{M103}$$

$$\pi_t^{T} - \pi = 0 \tag{M104}$$

3.3.3 The variables and the parameters of the model

The endogenous variables of the model are summarized by Table 1. This table lists the technical variables of the model and the normal and logarithmized forms are denoted according to this.

1. Table – Endogenous variables of the MACRO model

#	Notation	Definition
1.	$\ln(NUC_t^R)$	The ratio of marginal utility of consumption to the GDP for the Ricardian households (nominal)
2.	NUL_t^R	The ratio of marginal utility of consumption to the GDP for the Ricardian households (nominal)
3.	$\ln(NUC_t^{NR})$	The ratio of marginal utility of consumption to the GDP for the liquidity constrained households (nominal)
4.	NUL_t^{NR}	The ratio of marginal utility of consumption to the GDP for the liquidity constrained households (nominal)
5.	CSN_t^{NR}	The consumption to GDP ratio of liquidity constrained households (nominal)
6.	$\ln(CSN_t^R)$	The consumption to GDP ratio of Ricardian households (nominal)
7.	$\ln(CSN_t^{NR})$	The consumption to GDP ratio of liquidity constrained households

		(nominal)
8.	$\ln(CSN_t)$	Consumption to GDP ratio (nominal)
9.	$\ln(ISN_t)$	Investment to GDP ratio (nominal)
10.	$\ln(GSN_t)$	Government consumption to GDP ratio (nominal)
11.	GSN_t	Government consumption to GDP ratio (nominal)
12.	$\ln(IGSN_t)$	Government investment to GDP ratio (nominal)
13.	$\ln(EXSN_t)$	Export to GDP ratio (nominal)
14.	$\ln(IMSN_t)$	Import to GDP ratio (nominal)
15.	$NTBSN_t$	Net export to GDP ratio (nominal)
16.	T_t^{LS}	Lump sum tax to GDP ratio (nominal)
17.	$TRSN_t$	Transfers to GDP ratio (nominal)
18.	$\ln(TRSN_t)$	Transfers to GDP ratio (nominal)
19.	$NTRSN_t$	Net transfers (by lump sum taxes) to GDP ratio (nominal)
20.	$BGYN_t$	Public debt to GDP ratio (nominal)
21.	$\ln(BGYN_t)$	Public debt to GDP ratio (nominal)
22.	$\ln(KSN_t)$	Private capital stock to GDP ratio (nominal)
23.	$\ln(YWY_t)$	Foreign GDP to domestic GDP ratio (nominal)
24.	$\ln(YWR_t)$	The ratio of GDP to nominal wages
25.	WS_t	Wages to GDP ratio (nominal)
26.	$\ln(WS_t)$	Wages to GDP ratio (nominal)
27.	WSW_t	Disposable income to GDP ratio (nominal)
28.	$\ln(MRY_t)$	Money stock to GDP ratio (nominal)
29.	DB_t	Government deficit to GDP ratio (nominal)
30.	B_t^F	External debt to GDP ratio (nominal)
31.	$\ln(E_t)$	Exchange rate (nominal)
32.	i_t	Interest rate (nominal)
33.	i_t^F	Foreign interest rate (nominal)
34.	r_t	Real interest rate
35.	Q_t	Tobin Q
36.	$\ln(L_t)$	Employment rate
37.	L_t^{ss}	Equilibrium employment rate (moving average)
38.	lol_t	The share of overhead labor in employment
39.	$ucap_t$	Capacity utilization
40.	$ucap_t^{ss}$	Equilibrium capacity utilization (moving average)
41.	TRW_t	The ratio of per employee transfers to real wage
42.	η_t	The inverse of markup factor in the final goods sector
43.	GC_t^R	The growth rate of consumption of Ricardian households
44.	GC_t^{NR}	The growth rate of consumption of liquidity constrained households
45.	GC_t	The growth rate of per capita consumption
46.	GC_t^{lev}	The growth rate of consumption
47.	GI_t	The growth rate of per capita investment
48.	GI_t^{lev}	The growth rate of investment
49.	GG_t	The growth rate of per capita government consumption
50.	GG_t^{lev}	The growth rate of government consumption
51.	GIG_t	The growth rate of per capita government investment
52.	GEX_t	The growth rate of per capita exports
53.	GEX_t^{lev}	The growth rate of exports
54.	GIM_t	The growth rate of per capita imports
55.	GIM_t^{lev}	The growth rate of imports
56.	GE_t	The growth rate of the exchange rate
57.	GK_t	The growth rate of private capital stock

58.	GKG_t	The growth rate of public capital stock
59.	GL_t	The growth rate of employment rate
60.	$GTAX_t$	The growth rate of lump sum tax
61.	GA_t	The growth rate of TFP
62.	GAU_t	The growth rate of TFP adjusted by capacity utilization
63.	g_t^{AI}	The growth rate of the productivity of intermediate goods
64.	GTR_t	The growth rate of transfers
65.	GUC_t	The growth rate of the marginal utility of consumption (Ricardian households)
66.	$Gucap_t$	The growth rate of capacity utilization
67.	$GWRY_t$	The growth rate of the ratio of real wage to GDP
68.	GY_t	The growth rate of per capita GDP
69.	GY_t^{lev}	The growth rate of GDP
70.	$GPOT_t$	The change in potential GDP (proxy)
71.	GYW_t	The growth rate of foreign GDP
72.	$\ln(IK_t)$	The ratio if investment to capital stock in the private sector
73.	$\ln(IKG_t)$	The ratio if investment to capital stock in the public sector
74.	$\ln(YGAP_t)$	Output gap
75.	$\ln(P_t^C/P_t)$	The relative price of consumption goods
76.	$\ln(P_t^M/P_t)$	The relative price of import
77.	$\ln(P_t^X/P_t)$	The relative price of export
78.	π_t	Domestic inflation
79.	π_t^F	Foreign inflation
80.	π_t^C	Inflation of consumption goods
81.	π_t^M	Inflation of import goods
82.	π_t^X	Inflation of export goods
83.	$\pi_t^{M,lev}$	Inflation of import goods with trend
84.	$\pi_t^{X,lev}$	Inflation of export goods with trend
85.	π_t^W	Wage inflation
86.	π_t^{WR}	The growth rate of real wages
87.	t_t^W	The tax rate for income tax
88.	π_t^T	Inflation target
89.	$\ln\ CY_t$	Consumption to GDP ratio (real)
90.	$\ln\ GGY_t$	Government consumption to GDP ratio (real)
91.	u_t^C	Shock to consumption preference
92.	u_t^η	Shock to markup
93.	u_t^{PX}	Shock to export prices
94.	u_t^{PM}	Shock to import prices
95.	u_t^{EX}	Shock to current account
96.	u_t^{CG}	Shock to government consumption
97.	u_t^{IG}	Shock to government investment
98.	u_t^L	Shock to leisure preference
99.	u_t^M	Shock to monetary policy
100.	u_t^{AI}	Shock to the productivity of the intermediate goods sector
101.	u_t^F	Shock to foreign risk premium
102.	u_t^{rp}	Shock to risk premium on physical capital
103.	u_t^{TR}	Shock to transfers
104.	u_t^W	Shock to labor demand

The exogenous variables are summarized in Table 2.

2. Table – The exogenous variables of the MACRO model

#	Notation	Definition
1.	ε_t^C	Shock to consumption preference
2.	ε_t^η	Shock to markup
3.	ε_t^{PX}	Shock to export prices
4.	ε_t^{PM}	Shock to import prices
5.	ε_t^{EX}	Shock to current account
6.	ε_t^{CG}	Shock to government consumption
7.	ε_t^{IG}	Shock to government investment
8.	ε_t^L	Shock to leisure preference
9.	ε_t^M	Shock to monetary policy
10.	ε_t^{AI}	Shock to the productivity of the intermediate goods sector
11.	ε_t^F	Shock to foreign risk premium
12.	ε_t^{rp}	Shock to risk premium on physical capital
13.	ε_t^{TR}	Shock to transfers
14.	ε_t^W	Shock to labor demand
15.	ε_t^{lol}	Shock to overhead labor
16.	ε_t^{PW}	Shock to foreign inflation
17.	ε_t^{YW}	Shock to foreign GDP
18.	ε_t^{iW}	Shock to foreign interest rate
19.	ε_t^Y	Shock to TFP
20.	ε_t^{INV}	Shock to private investment
21.	ε_t^{CAP}	Shock to private capital stock growth
22.	ε_t^{CAP}	Shock to public capital stock growth
23.	ε_t^{GB}	Shock to government budget revenues

The parameters of the model are summarized in Table 3.

3. Table – The parameters of the MACRO model

#	Notation	Definition
1.	γ_{U1}	Cost parameter of capacity utilization 1
2.	γ_{U2}	Cost parameter of capacity utilization 2
3.	α^X	The elasticity of exports to foreign GDP
4.	α	The production elasticity of labor
5.	α^G	The additive inverse of the production elasticity of public capital
6.	β	Discount factor
7.	τ_1^{LS}	The reaction of lump sum tax on its deviation from target
8.	τ_2^{LS}	The reaction of lump sum tax on change in public debt
9.	B	The public debt to GDP target
10.	δ	Depreciation rate for the private capital
11.	δ^G	Depreciation rate for the public capital
12.	$dgex$	The empirical trend of the export to GDP ratio
13.	$dgim$	The empirical trend of the import to GDP ratio
14.	$dgpm$	The empirical trend of the import price level
15.	$dgpx$	The empirical trend of the export price level

16.	ι^F	The steady state foreign interest rate
17.	τ_0^{CG}	The reaction of government consumption (growth) on past change in the output gap
18.	γ_I	Adjustment cost parameter of physical capital investments
19.	γ_K	Adjustment cost parameter of physical capital investments
20.	γ_L	Parameter of the adjustment cost function for labor
21.	γ_P	Parameter of the adjustment cost function for price
22.	γ_{PM}	The weight of inflation indexing in the import markup
23.	γ_{PX}	The weight of inflation indexing in the export markup
24.	γ_W	Parameter of the adjustment cost function for wage
25.	π	Inflation target
26.	g^{AI}	The steady state growth rate of the productivity of the intermediate sector
27.	g^{pop}	Population growth rate
28.	π^F	Foreign inflation target
29.	τ_{lag}^{CG}	The smoothing parameter of government consumption
30.	τ_{adj}^{CG}	The reaction of government consumption (growth) on the deviation of G/Y from steady state
31.	GSN	The steady state ratio of government consumption to GDP
32.	g^A	The steady state growth rate of TFP
33.	GY	The steady state growth rate of per capita GDP
34.	GYW	The steady state growth rate of foreign GDP
35.	h^C	Habit parameter in consumption
36.	h^L	Habit parameter in leisure
37.	τ_{lag}^{IG}	The smoothing parameter of government investment
38.	τ_{adj}^{IG}	The reaction of government investment (growth) on the deviation of GI/Y from steady state
39.	τ_{lag}^{i}	The parameter for interest rate smoothing
40.	τ_0^{IG}	The reaction of government investment (growth) on past change in the output gap
41.	$IGSN$	The steady state ratio of government investment to GDP
42.	κ	Parameter of the utility function
43.	L	The steady state employment rate
44.	lol	The steady state share of overhead labor
45.	YWY	The steady state value of the log ratio of foreign and domestic GDP
46.	ω	Parameter of the utility function
47.	ρ^C	Persistence parameter, consumption preference shock
48.	ρ^η	Persistence parameter, markup shock
49.	ρ^{PM}	Persistence parameter, import markup shock
50.	ρ^{PX}	Persistence parameter, export markup shock
51.	ρ^{EX}	Persistence parameter, current account shock
52.	ρ^{CG}	Persistence parameter, government consumption shock
53.	ρ^{IG}	Persistence parameter, government investment shock
54.	ρ^{Lss}	Smoothing parameter in equilibrium employment
55.	ρ^L	Persistence parameter, leisure preference shock
56.	ρ^{lol}	Persistence parameter, overhead labor shock
57.	ρ_1^{AI}	Persistence parameter, intermediate sector productivity shock, lag1
58.	ρ_2^{AI}	Persistence parameter, intermediate sector productivity shock, lag2
59.	ρ_3^{AI}	Persistence parameter, intermediate sector productivity shock, lag3
60.	ρ_4^{AI}	Persistence parameter, intermediate sector productivity shock, lag4
61.	ρ^M	The weight of past prices in import share
62.	ρ^X	The weight of past prices in export share

63.	ρ^{F}	Persistence parameter, foreign risk premium shock
64.	ρ^{rp}	Persistence parameter, physical investment risk premium shock
65.	ρ^{ucap}	Smoothing parameter in equilibrium capacity utilization
66.	ρ^{i^F}	Smoothing parameter of foreign interest rate
67.	ρ^{i^F,π^F}	Effect of foreign inflation on foreign interest rate
68.	ρ^{i^F,GY^F}	Effect of foreign GDP on foreign interest rate
69.	ρ^{π^F,i^F}	Effect of foreign interest rate on foreign inflation
70.	ρ^{π^F}	Smoothing parameter of foreign inflation
71.	ρ^{π^F,GY^F}	Effect of foreign GDP on foreign inflation
72.	ρ^{GY^F,i^F}	Effect of foreign interest rate on foreign GDP
73.	ρ^{GY^F,π^F}	Effect of foreign inflation on foreign GDP
74.	ρ^{GY^F}	Smoothing parameter of foreign GDP
75.	$\rho^{GY^F,GY}$	Effect of the rate of domestic to foreign GDP foreign inflation on foreign GDP
76.	rf	The effect of external debt on foreign risk premium
77.	rp	Risk premium on physical capital
78.	ω^{X}	The share of domestic consumption
79.	sfp	The share of forward looking firms (final consumption goods)
80.	sfp^{M}	The share of forward looking firms (import goods)
81.	sfp^{X}	The share of forward looking firms (export goods)
82.	sfw	The share of forward looking households (wage setting)
83.	σ^{C}	Parameter of the utility function
84.	σ^{X}	Foreign elasticity of substitution between domestic and foreign goods
85.	σ	Domestic elasticity of substitution between domestic and foreign goods
86.	slc	The share of liquidity constrained households
87.	ssc	Social security contribution rate
88.	τ	Inverse of the elasticity of substitution between domestic varieties
89.	t^{P}	Tax rate of capital income
90.	θ	Elasticity of substitution between labor types
91.	τ^{i}_{π}	The reaction of the interest rate on inflation (Taylor rule)
92.	τ^{TR}	The effect of employment on transfers
93.	TRW	The steady state level of transfers (transfer to wage ratio)
94.	ρ^{TR}	Persistence parameter, transfers shock
95.	τ^{i}_{Y}	The reaction of the interest rate on output gap (Taylor rule)
96.	t^{C}	VAT rate
97.	τ^{W}_{0}	Steady state rate of labor income tax
98.	τ^{W}_{1}	The effect of output gap on labor income tax rate
99.	$ucap$	The steady state capacity utilization
100.	$wrlag$	Smoothing parameter in wage setting
101.	φ	The elasticity of money stock to interest rate
102.	$\sigma^{iW}_{\varepsilon}$	The standard deviation of the foreign interest rate shock
103.	$\sigma^{PW}_{\varepsilon}$	The standard deviation of the foreign inflation shock
104.	$\sigma^{YW}_{\varepsilon}$	The standard deviation of the foreign GDP shock
105.	$\sigma^{AI}_{\varepsilon}$	The standard deviation of the intermediate sector productivity shock
106.	$\sigma^{GB}_{\varepsilon}$	The standard deviation of the budget revenue shock
107.	$\sigma^{INV}_{\varepsilon}$	The standard deviation of the private investment shock
108.	σ^{C}_{ε}	The standard deviation of the consumption preference shock
109.	$\sigma^{\eta}_{\varepsilon}$	The standard deviation of the markup shock
110.	$\sigma^{PM}_{\varepsilon}$	The standard deviation of the import price shock

111.	$\sigma_{\varepsilon}^{PX}$	The standard deviation of the export price shock
112.	$\sigma_{\varepsilon}^{EX}$	The standard deviation of the current account shock
113.	$\sigma_{\varepsilon}^{CG}$	The standard deviation of the government consumption shock
114.	$\sigma_{\varepsilon}^{IG}$	The standard deviation of the government investment shock
115.	σ_{ε}^{L}	The standard deviation of the leisure preference shock
116.	$\sigma_{\varepsilon}^{lol}$	The standard deviation of the overhead labor shock
117.	σ_{ε}^{M}	The standard deviation of the monetary policy shock
118.	σ_{ε}^{F}	The standard deviation of the foreign risk premium shock
119.	$\sigma_{\varepsilon}^{rp}$	The standard deviation of the physical capital risk premium shock
120.	$\sigma_{\varepsilon}^{TR}$	The standard deviation of the transfers shock
121.	σ_{ε}^{W}	The standard deviation of the labor demand shock
122.	σ_{ε}^{Y}	The standard deviation of the TFP shock
123.	$\sigma_{\varepsilon}^{INV}$	The standard deviation of the investment growth shock
124.	$\sigma_{\varepsilon}^{CAP}$	The standard deviation of the private capital growth shock
125.	$\sigma_{\varepsilon}^{GCAP}$	The standard deviation of the public capital growth shock
126.	$\sigma_{\varepsilon}^{GB}$	The standard deviation of the government revenue shock

3.3.4 Solving the model

The DSGE model defined by equations (M1)-(M104) is solved by standard algorithms used in the literature, with the help of Dynare, dedicated software for solving and estimating this type of models (see Adjemian et al., 2011). Denote the vector of endogenous variables by $\mathbf{y}$, the vector of exogenous variables by $\boldsymbol{\varepsilon}_t$ and the vector of parameters is $\boldsymbol{\theta}$. The model (M1)-(M104) can be written in compact form as follows, explicitly stating the role of rational expectations:[8]

$$E_t \; F \; \mathbf{y}_{t-1}, \mathbf{y}_t, \mathbf{y}_{t+1}, \boldsymbol{\varepsilon}_t, \boldsymbol{\theta} \quad = \mathbf{0} \qquad \text{(A39)}$$

where E_t is the expectations operator. The solution of the model is a function

$$\mathbf{y}_t = g(\mathbf{y}_{t-1}, \boldsymbol{\varepsilon}_t) \qquad \text{(A40)}$$

which satisfies the system of equations (A42). Instead of exactly finding the function $g(\cdot)$, the standard solution is to take the first or second order approximation to the model. The generally used method follows the algorithm of Uhlig (1999) which constitutes of the following steps (see for example Horváth, 2006):

1. Write the equations of the model. These consist of the first order conditions following from actors' decisions and conditions for market equilibriums. This step is given by the relationships from (M1) to (M104) or in compact form, equation (A39).

[8] For the sake of preciseness, it is due to note that the model, in its form defined by (M1)-(M104) contains one period forward and four periods backward looking (see equation (M99)). Using three auxiliary equations, though, the model can be reformulated as in (A38).

2. Calculating the steady state of the model. This means finding a vector $\mathbf{y} = \mathbf{y}_{t-1} = \mathbf{y}_t = \mathbf{y}_{t+1}$ of endogenous variables such that it satisfies the system in (A39) given that there are no shocks ($\boldsymbol{\varepsilon}_t = \mathbf{0}$):
   $$F\left(\mathbf{y}, \mathbf{y}, \mathbf{y}, \boldsymbol{\theta}\right) = \mathbf{0} \quad \text{(A41)}$$
   On the basis of this, the steady state can be written in function of the model parameters:
   $$\mathbf{y} = s(\boldsymbol{\theta}) \quad \text{(A42)}$$
   It is possible to solve for the steady state a given parameter vector using standard methods (e.g. Newton's method). In the case of our model (M1)-(M104), though, the steady state can be given by simple, logical reasoning (as a consequence of the definitions in growth rates and shares). The determination of the steady state is given in detail in the following subsection.
3. Loglinearizing the model equations around the steady state. This can be done by recasting the equations into Taylor series. As a result, the system of equations in (A39) can be written in the following matrix form:
   $$\mathbf{y}_t = E_t\left(\mathbf{A}(\boldsymbol{\theta})\mathbf{y}_{t+1}\right) + \mathbf{B}\left(\boldsymbol{\theta}\right)\mathbf{y}_t + \mathbf{C}\left(\boldsymbol{\theta}\right)\mathbf{y}_{t-1} + \mathbf{D}(\boldsymbol{\theta})\boldsymbol{\varepsilon}_t \quad \text{(A43)}$$
4. The solution to (A43) is (using (A40)) is the matrix equation
   $$\mathbf{y}_t = \mathbf{F}\left(\boldsymbol{\theta}\right)\mathbf{y}_{t-1} + \mathbf{G}(\boldsymbol{\theta})\boldsymbol{\varepsilon}_t \quad \text{(A44)}$$
   so the exercise is to find the matrices $\mathbf{F}\left(\boldsymbol{\theta}\right)$ and $\mathbf{G}\left(\boldsymbol{\theta}\right)$. This can be done by the method of Blanchard-Kahn (1980) or the method of generalized eigenvalues, among others
5. Using the solution in (A44) we can analyze the model and run simulations.

The steady state

In the steady state of the model the endogenous variables are constant which corresponds to a balanced growth path in the case of a decently specified model. The structure of the model gives simple rules for the steady state values of the different endogenous variables. The steady state growth rate of the domestic GDP (GY), the domestic inflation target (π), the population growth rate (g^{pop}) and the productivity growth of the intermediate sector (g^{AI}) determine the steady state of most of the variables.

The inflation target determines the GDP deflator, and the inflation of consumption goods, intermediate goods, import and export prices:

$$\pi_t = \pi_t^C = \pi_t^I = \pi_t^M = \pi_t^X = \pi \quad \text{(A45)}$$

The following two equations give the import and export inflations with trend (see equations (M78 and (M79)):

$$\pi_t^{M,lev} = \pi + dgpm \quad \text{(A46)}$$

$$\pi_t^{X,lev} = \pi + dgpx \tag{A47}$$

The steady state growth rate of the per capita GDP and the elements of its expenditure side are given by the steady state growth rate of GDP:

$$GY_t = GC_t = GC_t^R = GC_t^{NR} = GG_t = GEX_t = GIM_t = GY \tag{A48}$$

The growth rates of private and public investment are determined by the productivity growth rate of the intermediate sector (see equation (A33)):

$$GI_t = GIG_t = GK_t = GKG_t = GY + g^{AI} \tag{A49}$$

In addition to the per capita growth rates, the level growth rates follow logically:

$$GY_t^{lev} = GC_t^{lev} = GG_t^{lev} = GEX_t^{lev} = GIM_t^{lev} = GY + g^{pop} \tag{A50}$$

$$GI_t^{lev} = GY + g^{AI} + g^{pop} \tag{A51}$$

The respective steady state parameters define the steady state values of the following variables (respectively: employment rate, capacity utilization, government consumption to GDP ratio, government investment to GDP ratio, transfers to wage ratio, public debt to GDP ratio, ratio of foreign and domestic GDP, share of overhead labor):

$$L_t = L \tag{A52}$$

$$ucap_t = ucap \tag{A53}$$

$$GSN_t = GSN \tag{A54}$$

$$IGSN_t = IGSN \tag{A55}$$

$$TRW_t = TRW \tag{A56}$$

$$B_t = B \tag{A57}$$

$$YWY_t = YWY \tag{A58}$$

$$lol_t = lol \tag{A59}$$

Following from the VAR model written for foreign variables (interest rate, inflation, GDP), the steady state of them is defined by the respective steady state parameters:

$$i_t^F = \iota^F \tag{A60}$$

$$\pi_t^F = \pi^F \tag{A61}$$

$$GYW_t = GYW \tag{A62}$$

Following from equations (M26) and (M27):

$$L_t^{ss} = L \tag{A63}$$

$$ucap_t^{ss} = ucap \tag{A64}$$

Using (M28) and the equations right above:

$$YGAP_t = 1 \tag{A65}$$

The subsequent equations follow from those right above and from equations (M56), (M61), (M60), (M63), (M66), (M65) and (M3) respectively.

$$GL_t = 0 \tag{A66}$$

$$Gucap_t = 0 \tag{A67}$$

$$GTR_t = GY \tag{A68}$$

$$GYWR_t = 0 \tag{A69}$$

$$DB_t = 0 \tag{A70}$$

$$GPOT_t = GY \tag{A71}$$

$$GUC_t = 0 \tag{A72}$$

The steady state interest rate using the Taylor rule is:

$$i_t = \frac{1-\beta}{\beta} + \pi \tag{A73}$$

The steady state for the real interest rate is thus (see equation (M41)):

$$r_t = \frac{1-\beta}{\beta} \tag{A74}$$

Using (M14) we get the following steady state for the markup in the final goods sector:

$$\eta_t = 1 - \tau \tag{A75}$$

Using (M31), (M32) and (M33) the steady states of relative prices are:

$$\frac{P_t^X}{P_t} = 1 \tag{A76}$$

$$\frac{P_t^M}{P_t} = E^{\alpha^X} \qquad (A77)$$

$$\frac{P_t^C}{P_t} = \left(s^M + (1 - s^M) \left(E^{\alpha^X} \right)^{1-\sigma^M} \right)^{\frac{1}{1-\sigma^M}} \qquad (A78)$$

where E is the steady state exchange rate which is normalized to 1 during the simulations.

The steady state growth rate of TFP follows the production function (M15):

$$GA_t = \frac{\alpha+\alpha^G-1}{\alpha} GY - \frac{2-\alpha-\alpha^G}{\alpha} g^{AI} \qquad (A79)$$

The TFP growth adjusted for capacity utilization:

$$GAU_t = \alpha \left(\frac{\alpha+\alpha^G-1}{\alpha} GY - \frac{2-\alpha-\alpha^G}{\alpha} g^{AI} \right) \qquad (A80)$$

Using (M12), the steady state for the real wage to GDP ratio is

$$YWR_t = \frac{\alpha}{(1-\tau)(1+lol)L} \qquad (A81)$$

The steady state for the wage share follows from equation (M9):

$$WS_t = L \frac{(1-\tau)(1+lol)L}{\alpha} \qquad (A82)$$

It follows from equation (M17) that

$$KSN_t = \frac{(1-\tau)(1-t^P)(1-\alpha)}{(1-\beta)/\beta+\delta+rp} \qquad (A83)$$

According to equations (M42) and (M43) the steady state of the ratio of investment to capital stock in the private and public sectors respectively is:

$$IK_t = \delta + g^{AI} + GY + g^{pop} \qquad (A84)$$

$$IKG_t = \delta^G + g^{AI} + GY + g^{pop} \qquad (A85)$$

From equation (M16) follows the steady state investment to GDP share:

$$ISN_t = (\delta + g^{AI} + GY + g^{pop}) \frac{(1-\beta)/\beta+\delta+rp}{(1-\tau)(1-t^P)(1-\alpha)} \qquad (A86)$$

The steady state of the external debt stock can be determined using equation (M5):

$$B_t^F = \frac{\iota^F-\pi^F-(1-\beta)/\beta}{rf} \qquad (A87)$$

Equation (M35) determines the share of net exports to GDP:

$$NTBSN_t = \frac{\iota^F - \pi^F - (1-\beta)/\beta}{rf}\left(-(1-\beta)/\beta + GY + g^{pop}\right) \tag{A88}$$

The ratio of consumption to GD follows from equation the GDP identity (M40):

$$CSN_t = 1 - \; GSN + IGSN + (\delta + g^{AI} + GY + g^{pop})\frac{(1-\beta)/\beta + \delta + rp}{(1-\tau)(1-t^P)(1-\alpha)} + \frac{\iota^F - \pi^F - (1-\beta)/\beta}{rf}\left(-(1-\beta)/\beta + GY + g^{pop}\right) \tag{A89}$$

Using (M23) the steady state rate for labor income tax is:

$$t_t^W = \tau_0^W \tag{A90}$$

The steady state share of disposable income in GDP is (M48):

$$WSW_t = (1 - \tau_0^W - ssc)L\frac{(1-\tau)(1+lol)L}{\alpha} \tag{A91}$$

From (M25) follows the steady state lump sum tax:

$$T_t^{LS} = \; \frac{1-\beta}{\beta} - GY - g^{pop} \;\; B + GSN + IGSN + \;\; TRW - \;\; \tau_0^W - ssc \;\; + t^p \;\; L\frac{1-\tau \;\; 1+lol \;\; L}{\alpha} - t^P - t^C CSN_t \tag{A92}$$

The steady state growth rate of lump sum tax (M59):

$$GTAX_t = GY + g^{pop} \tag{A93}$$

The steady state share of transfers to GDP (M57):

$$TRSN_t = TRWL\frac{(1-\tau)(1+lol)L}{\alpha} \tag{A94}$$

The steady state growth rate of the exchange rate according to the purchasing power parity (M36):

$$GE_t = \pi - \pi^F \tag{A98}$$

The share of imports in GDP (M29):

$$IMSN_t = (1 - s^M) \;\; \frac{P_t^C}{P_t^M}^{\;\sigma^M - 1} \;\; (1 - NTBSN_t) \tag{A99}$$

The share of exports in GDP (M30):

$$EXSN_t = (1 - s^M)\ E^{\alpha^X s^M}{}^{\sigma^X} YWY^{\alpha^X} \quad \text{(A100)}$$

Using equations (M70) and (M71):

$$\pi_t^W = GY + \pi \quad \text{(A101)}$$

$$\pi_t^{WR} = GY \quad \text{(A102)}$$

The steady state value of exogenous shocks is zero by definition:

$$u_t^C = u_t^\eta = u_t^{PX} = u_t^{PM} = u_t^{EX} = u_t^{CG} = u_t^{IG} = u_t^L = u_t^M = u_t^{AI} = u_t^F = u_t^{rp} = u_t^{TR} = u_t^W = 0 \text{(A103)}$$

3.3.5 Calibration

An important problem in the case of such large scale models is the determination of model parameters. The model introduced here works with 126 parameters. In order to determine this amount of parameters, the information in even long time series is insufficient. In our case, the quarterly data between 2001Q1 and 2013Q2 are clearly not enough to satisfyingly identify all the parameters. Moreover, as usual in DSGE models, the system converges to a steady state in the long run which is determined by the parameters of the model. It is easier to obtain information from the data (trend-filtered time series) on the parameters describing the adjustment mechanisms towards the steady state, while the parameters which determine the steady stat typically depend on the trend-characteristics of these time series. On the basis of this, it is common in the literature to use basically three different approaches to identify the model parameters.

- Parameter identification with taking 'standard' or 'conventional' values from the literature.
- Parameter identification with 'calibration' which ties the parameter values to the data at hand but without the application of rigorous econometric techniques.
- Parameter identification through estimation when the given parameters are determined by using econometric techniques and in an integrated manner.

Following this distinction above, the standard methods in the literature and especially those applied for the QUEST model specification for the Eurozone, we determine part of the parameters by taking results from other studies (especially the original specification), part of them by calibrating to the steady state and part of them by Bayesian estimation. In what follows, we report the parameter values which were taken from other studies or calibrated, the reason for the utilization of parameter values from other studies and the principles of the calibration. The strategy for the determination of parameters was the following:

1. We take the Eurozone specification of the QUEST model as a starting point. The parameters estimated there are also estimated, steady state growth rates and shares are calibrated according to the Turkish data and all other parameters are used as specified in the original model.
2. Part of the parameters is tuned to the parameters used in the SCGE model block in order to ensure consistency between the model blocks.
3. The remaining parameters are estimated using Bayesian techniques.

3.3.5.1 Parameters taken from the original QUEST specification

As mentioned in the previous points, part of the parameters is used as specified in the version of the QUEST model estimated for the Eurozone. These parameters and their respective values are presented in Table 4.

4. Table – Parameter values taken from the original (Eurozone) QUEST specification

α^X	The elasticity of exports to foreign GDP	0.5000
τ_1^{LS}	The reaction of lump sum tax on its deviation from target	$0.001*\tau_2^{LS}$
τ_2^{LS}	The reaction of lump sum tax on change in public debt	0.0040
g^{AI}	The steady state productivity growth rate of the intermediate sector	0.0000
lol	The steady state share of overhead labor	0.0000
YWY	The steady state value of the log ratio of foreign and domestic GDP	0.0000
ρ^{EX}	Persistence parameter, current account shock	0.9750
ρ^{lol}	Persistence parameter, overhead labor shock	0.9900
θ	Elasticity of substitution between labor types	1.6000
τ_1^W	The effect of output gap on labor income tax rate	0.8000
$ucap$	The steady state capacity utilization	1.0000
φ	The elasticity of money stock to interest rate	0.4000
ρ_1^{AI}	Persistence parameter, intermediate sector productivity shock, lag1	0.2480
ρ_2^{AI}	Persistence parameter, intermediate sector productivity shock, lag2	0.1374
ρ_3^{AI}	Persistence parameter, intermediate sector productivity shock, lag3	0.1048
ρ_4^{AI}	Persistence parameter, intermediate sector productivity shock, lag4	0.0928
σ_ε^{AI}	The standard deviation of the intermediate sector productivity shock	0.0031

In the case of parameters in Table 4, we employed them as used in the QUEST specification for the Eurozone. Some of these parameters need no modification due to their nature. Specifically the equilibrium capacity utilization and overhead labor rates belong to this category which comes into the model as straightforward normalizations. Substitution elasticity between labor types and the two persistence parameters we do not suggest a difference between the mechanisms in the Turkish and Eurozone economies. The steady state value for the (log) ratio of foreign and domestic GDP means normalization on one hand and on the other it implies balanced growth rates in the domestic economy and in the rest of the world (note that this parameter defines the steady state and does not imply any restrictions on the adjustment mechanisms. Setting the productivity growth in the intermediate sector to zero investments and the two capital stocks grow at the same rate in the steady state as the GDP and the inflation of intermediate goods equal that of the final goods. The reaction parameters of the lump sum tax are of technical nature and their goal is to keep the public debt to GDP ratio close to its target level. The reaction of the labor tax rate is not estimated but set to a value used also in the Eurozone specification (where this parameter is not estimated as well). In the case of export-elasticity, as the model works with shares, we assume that the value used in the original specification is valid also for Turkey. The elasticity of the money stock on interest rate has no real relevance because the money stock does not affect any other variables in the model. The parameterization of the shock of the intermediate sector's productivity is taken from the Eurozone specification where these parameters are estimated separately from the other parameters of the model.

3.3.5.2 Steady state parameters

The second group of parameters determines the country-specific steady state of the model. These parameters are calibrated using Turkish data and are listed in Table 6.

5. Table – Parameters calibrated using Turkish data

β	Discount factor	0.9989
B	The public debt to GDP target	1.8160
$dgex$	The empirical trend of the export to GDP ratio	0.0020
$dgim$	The empirical trend of the import to GDP ratio	0.0069
$dgpm$	The empirical trend of the import price level	-0.0006
$dgpx$	The empirical trend of the export price level	-0.0024
π	Inflation target	0.0125
π^F	Foreign inflation target	0.0077
GSN	The steady state ratio of gov. consumption to GDP	0.1085
GY	The steady state growth rate of per capita GDP	0.0078
GYW	The steady state growth rate of foreign GDP	0.0081
$IGSN$	The steady state ratio of gov. investment to GDP	0.0363
L	The steady state employment rate	0.4643
ssc	Social security contribution rate	0.2344
t^P	Tax rate of capital income	0.1902
TRW	The steady state level of transfers (transfer to wage)	0.1764
t^C	VAT rate	0.1924
τ_0^W	Steady state rate of labor income tax	0.1902

The discount factor was set to match the real interest rate implied by the difference between nominal interest rates and inflation in the end of the sample period. In the case of the steady state parameters (rates) we used average values calculated for the period between 2001 and

2013. The trend parameters are obtained by fitting an exponential trend to the time series. Consider the following linear regression on the exponential trend of variable x:

$$\ln x = a + b \cdot t \tag{A104}$$

The trend of the original variable is thus: e^{bt}. The four trend variables are given according to this where we substitute the export to GDP, the import to GDP, the import deflator to GDP deflator and the export deflator to GDP deflator ratios respectively. The domestic inflation target is the actual 5% target of the CBRT, divided for quarters. Foreign inflation target is obtained as a time average from the rest of the world inflation time series. GDP growth rates and government consumption and investment to GDP ratios are also determined as a time average of the respective values from the time series. Steady state employment is the average rate of employment (the ratio of employment to working age population). The steady state labor tax rate is calculated as the time average of the ratio of labor tax revenues to labor income and use the same rate for capital income tax rate (as in the specification for the Eurozone). The VAT tax rate is calculated as the time average of consumption and import tax revenues to total consumption while the steady state social security rate represents SSC revenues ratio to labor income. The ratio of transfers to wages is determined by the other revenues of the government (over consumption and investment expenses) to labor income ratio.

3.3.5.3 Parameters tuned to the SCGE block and other endogenous and technical parameters

Some parameters in the model are set in order to be in line with the respective parameters in the SCGE block of the framework. These parameters are listed in Table 7.

6. Table – Parameters tuned with the SCGE model block

α	The production elasticity of labor	0.6857
α^G	The additive inverse of the production elasticity of public capital	0.9351
δ	Depreciation rate for the private capital	0.0125
δ^G	Depreciation rate for the public capital	0.0050
τ	Inverse of the elasticity of substitution between domestic varieties	0.0506

Using consistent values for the production elasticities is required by the fact that both models build on the aggregate production function as a cornerstone. The production elasticity of labor and that of the public capital stock comes from a separate estimation of the production function, and the received value is in line with other results in the literature. The amortization rates are needed to be tuned because changes in the capital stock (investments) are an important point of communication between the SCGE and the MACRO model blocks. The depreciation rates come from a separate estimation of capital stocks for Turkey. The elasticity of substitution between domestic varieties is linked to the sum of powers in the aggregate production function in the SCGE model. Although this is not the case for the macro block, to ensure the consistency of implied substitution elasticities, we set parameter τ in a way that the steady state markup $(1/(1-\tau))$ be equal to the sum of powers in production function used in the SCGE block.

7. Table – Endogenous and technical parameters

γ_{U1}	Cost parameter of capacity utilization 1	0.0590
g^A	The steady state growth rate of TFP	0.0071
ω	Parameter of the utility function	0.4573
σ_ε^{INV}	The standard deviation of the investment growth shock	0.0000
σ_ε^{CAP}	The standard deviation of the private capital growth shock	0.0000
$\sigma_\varepsilon^{GCAP}$	The standard deviation of the public capital growth shock	0.0000
σ_ε^{GB}	The standard deviation of the government revenue shock	0.0000

Some parameters are a function of other parameters in the model (Table 8). One is the steady state growth rate in TFP, which is determined by equation (M15) on the basis of the steady state growth rates of employment and the two capital stocks as well as the production elasticities (see equation (A80)). The steady state capacity utilization (set to unity) determines the cost function parameter of capacity utilization adjustment on the basis equation (M17). The parameter of the utility function (ω) is determined by the steady state employment and other parameters. Table 8 contains four additional parameters which serve technical purposes. Their role is to implement the required shock into the model when integrating it into the GMR framework. The standard deviations of these exogenous shock variables are set to zero.

3.3.6 Estimation

Those model parameters which are either not taken from the original setting, either not calibrated or not tuned with the SCGE block, are determined by estimation procedures. The estimation splits into two separate parts. First, we estimate the separate VAR model for the variables describing the evolution of the foreign sector (see model equations (M37)-(M39)) and second, the remaining parameters are estimated with Bayesian techniques. These estimation results are reported in what follows.

3.3.6.1 The database

In line with the estimation of the original specification for the Eurozone, the following quarterly time series are used for the estimation of the Turkish version:

- Nominal short term interest rates
- Nominal effective exchange rate
- Nominal wage
- Employment
- Population in working age
- Household consumption
- Government consumption
- Total investment
- Government investment
- Imports
- Exports
- Gross National Product
- Deflator of the Gross Domestic Product
- Deflator of consumption goods
- Deflator of investment goods
- Deflator of imports
- Deflator of exports
- Government revenues from labor tax
- Government revenues from consumption taxes
- Government revenues from social security contributions
- Government transfers

For all of these time series we take the period between 2001Q1 and 2013Q2 as the basis of our estimations. The reason for this is that the monetary policy regime is (first an implicit and later an explicit) inflation targeting suited with the monetary policy setting of the model (see also e.g. Cebi, 2011 and Huseynov, 2010).

Part of the database (the time series for GDP, consumption, government and private investment, government spending, exports and imports) are extracted from the quarterly SNA tables of Turkstat, the national statistical office of Turkey. These data were seasonally adjusted using the X12-ARIMA method. As Turkstat publishes constant and current price data for these series, the respective price indices were calculated from this data.

The time series for the rest of the world (quarterly inflation, GDP growth and interest rate data) is collected from Eurostat and OECD databases. Turkstat publishes the share of different countries in the international trade of Turkey by year. Using this data we weighted the time series according to this information to obtain three time series for rest of the world GDP growth, inflation and interest rate. From the first two series we can recalculate (normalized) GDP volumes and the price index for the foreign sector.

For government data (transfers, consumption and income taxes as well as social security contributions) we use the data available from the Treasury of Turkey. For the tax and social security rates, as they appear only as parameters, we calculate the average rates for the rest of the period, as reported by the Treasury. For the transfers we collected monthly data from the Treasury, where for the 2006-2013 period there is a detailed breakdown on the expenditure side is available, from which we selected 'Social contributions', 'Current Transfers' and 'Capital transfers' as our transfer time series. For the period 1994-2005 there is a broad breakdown not specifying transfers. We calculated the share of different transfers (as above) in 'Primary expenditures' for every month in the 2006-2013 period. Due to strong seasonality, we used an average of monthly shares to estimate transfers for the 1994-2005 period using the 'Primary expenditures' category still reported there. The monthly data were then aggregated to quarters and used in the estimation after seasonal adjustment.

For the labor force (population) we use Turkstat data on the population between 15 and 64. We used linear extrapolation both to extend the data to a quarterly basis (the estimation procedure uses only the trend in population, so there is no loss of information by using this method) and also to estimate data for years not contained in the Turkstat database. For employment, we use direct employment data available from Turkstat, after seasonal adjustment.

Wages (nominal compensation per employees) is estimated from two data sources. First, Turkstat publishes GDP from the income approach containing data on employee compensation. This data, however, is not consistent with the expenditure approach (used for the GDP and components time series) and do not match even approximately (the difference is almost two fold) with the other data source, which is a survey-based statistics on employment and earnings available for only two years, 2006 and 2010. As the latter source reports higher wages and due to its survey method compared to the official statistics based method of the SNA data, we regard it as more reliable in terms of volumes, however, only two yearly data is available from this compared to the quarterly data of the SNA. As a solution to this problem, we used the following estimation method. For the two years for which the survey data is available we calculated the difference between the two data sources (the two values are 1,93 in 2006 and 1,95 in 2010) and then fit a linear trend on these values to capture the dynamics of the relation between the two databases. Using these estimated ratios we calculated the compensation of employees (total) from the SNA data, which was then divided by the number of employees to obtain per employee data.

Data for the nominal effective exchange rate was obtained from the Bank of International Settlements database, whereas interest rates are calculated from the database of the Central Bank of the Republic of Turkey on interbank overnight rates (according to Cebi (2011) these rates finely move together with the yield on short term treasury bills).

The data are prepared in order to match with endogenous variables of the model. In accordance with the procedure used in the original setting, finally 17 observed data series are used corresponding to endogenous variables – these are listed in Table 8.

8. Table – Observed endogenous variables

1.	$\ln CY_t$	Consumption to GDP share (real)
2.	$\ln(E_t)$	Exchange rate (nominal)
3.	$\ln GGY_t$	Government consumption to GDP share (real)
4.	$\ln(IGSN_t)$	Government investment to GDP share (nominal)
5.	$\ln(ISN_t)$	Investment to GDP share (nominal)
6.	$\ln(L_t)$	Employment rate
7.	GY_t	Growth rate of per capita GDP
8.	$\ln(YWR_t)$	GDP to nominal wages ratio
9.	i_t	Domestic interest rate (nominal)
10.	π_t	Domestic inflation
11.	$\ln(P_t^M/P_t)$	Relative price of imports
12.	$\ln(P_t^X/P_t)$	Relative price of exports
13.	TRW_t	Transfer per capita to real wage ratio
14.	i_t^F	Foreign interest rate (nominal)
15.	π_t^F	Foreign inflation
16.	$\ln(YWY_t)$	Foreign GDP to domestic GDP ratio (nominal)
17.	g_t^{AI}	The growth rate of the productivity of intermediate goods

The observed variables listed in Table 8 can be logically calculated from the time series collected in our database. The raw data are transformed as follows: import and export prices are filtered with exponential trend, transfers are filtered by the transfers to wage ratio while the foreign and domestic GDP ratio is filtered with its own trend. The productivity growth of the investment goods sector can be given by the time change of the log deviation in investment deflator. Inflation is the log deviation of GDP deflator and other variables are transformed to per capita data dividing by the trend of working age population.

4.3.6.2 Macro processes of the foreign sector

The internal processes of the foreign sector are captured by three variables: foreign interest rate, inflation and GDP. We estimate a separate VAR model (see equations (M37)-(M39)) written for the cyclic components of these three variables. We used OLS estimation in line with the procedure in the original specification of the QUEST model. The standard deviations of the three shocks related to these three variables are also obtained from this estimation. The estimation results are summarized in Table 9. We note that the smoothing parameter of the foreign interest rate is estimated at 0.9809, but it is set to 0.95 in order to obtain a stable model.

9. Table – Estimated parameters of the foreign VAR block

ρ^{i^F}	Smoothing parameter of foreign interest rate	0.9500
ρ^{i^F,π^F}	Effect of foreign inflation on foreign interest rate	0.1029
ρ^{i^F,GY^F}	Effect of foreign GDP on foreign interest rate	0.0491
ρ^{π^F,i^F}	Effect of foreign interest rate on foreign inflation	0.2182
ρ^{π^F}	Smoothing parameter of foreign inflation	-0.0651
ρ^{π^F,GY^F}	Effect of foreign GDP on foreign inflation	0.3841
ρ^{GY^F,i^F}	Effect of foreign interest rate on foreign GDP	-0.4866
ρ^{GY^F,π^F}	Effect of foreign inflation on foreign GDP	-0.1730
ρ^{GY^F}	Smoothing parameter of foreign GDP	0.6675
$\rho^{GY^F,GY}$	Effect of the rate of domestic to foreign GDP foreign inflation on foreign GDP	-0.0001
$\sigma_{\varepsilon}^{iW}$	The standard deviation of the foreign interest rate shock	0.0008
$\sigma_{\varepsilon}^{PW}$	The standard deviation of the foreign inflation shock	0.0045
$\sigma_{\varepsilon}^{YW}$	The standard deviation of the foreign GDP shock	0.0045

The remaining parameters (those which are not taken from the original specification, not calibrates and not belonging to the foreign VAR block) are estimated with Bayesian techniques. In what follows, we specify the details of the estimation procedure and present the estimation results and diagnostic tests.

3.3.6.3 Estimation specification

First of all, we need to specify the prior distributions for the estimation. In this case we take the original specification of the QUEST model for the Eurozone as a reference point and used the prior distributions specified there. These distributions, in turn, are based in many cases on considerations regarded as standard in the literature. The prior distributions and their parameters are summarized in Table 10 which also show the posterior means. The latter values are used during the model simulations.

10. Table – Prior distributions and posterior means

Notation	Definition	Prior dist.	Prior mean	Prior std.	**Posterior mean**
γ_{U2}	Cost parameter of capacity utilization 2	Beta	0.0500	0.0240	**0.0864**
τ_0^{CG}	The reaction of government consumption (growth) on past change in the output gap	Beta	0.0000	0.0600	**-0.1827**
γ_I	Adjustment cost parameter of physical capital investments	Gamma	30.0000	20.0000	**20.8569**
γ_K	Adjustment cost parameter of physical capital investments	Gamma	15.0000	10.0000	**1.8758**
γ_L	Parameter of the adjustment cost function for labor	Gamma	30.0000	20.0000	**16.8556**
γ_P	Parameter of the adjustment cost function for price	Gamma	30.0000	20.0000	**28.4572**
γ_{PM}	The weight of inflation indexing in the import markup	Gamma	30.0000	20.0000	**0.3658**
γ_{PX}	The weight of inflation indexing in the export markup	Gamma	30.0000	20.0000	**0.3919**
γ_W	Parameter of the adjustment cost function for wage	Gamma	30.0000	20.0000	**1.1005**
τ_{lag}^{CG}	The smoothing parameter of government consumption	Beta	0.0000	0.4000	**-0.2928**
τ_{adj}^{CG}	The reaction of government consumption (growth) on the deviation of G/Y from steady state	Beta	-0.5000	0.2000	**-0.3766**
h^C	Habit parameter in consumption	Beta	0.7000	0.1000	**0.6063**
h^L	Habit parameter in leisure	Beta	0.7000	0.1000	**0.6624**
τ_{lag}^{IG}	The smoothing parameter of government investment	Beta	0.5000	0.2000	**0.0918**
τ_{adj}^{IG}	The reaction of government investment (growth) on the deviation of GI/Y from steady state	Beta	-0.5000	0.2000	**-0.8150**
τ_{lag}^{i}	The parameter for interest rate smoothing	Beta	0.8500	0.0750	**0.8273**
τ_0^{IG}	The reaction of government investment (growth) on past change in the output gap	Beta	0.0000	0.6000	**-0.6682**
κ	Parameter of the utility function	Gamma	1.2500	0.5000	**0.6056**
ρ^C	Persistence parameter, consumption preference shock	Beta	0.8500	0.0750	**0.7697**
ρ^η	Persistence parameter, markup shock	Beta	0.5000	0.0200	**0.2713**
ρ^{PM}	Persistence parameter, import markup shock	Beta	0.8500	0.0750	**0.9829**
ρ^{PX}	Persistence parameter, export markup shock	Beta	0.8500	0.0750	**0.8085**
ρ^{CG}	Persistence parameter, government consumption shock	Beta	0.5000	0.2000	**0.3812**
ρ^{IG}	Persistence parameter, government investment shock	Beta	0.8500	0.0750	**0.7482**
ρ^L	Persistence parameter, leisure preference shock	Beta	0.9500	0.2000	**0.8844**
ρ^{Lss}	Smoothing parameter in equilibrium employment	Beta	0.8500	0.0750	**0.9467**
ρ^M	The weight of past prices in import share	Beta	0.5000	0.2000	**0.2818**
ρ^X	The weight of past prices in export share	Beta	0.5000	0.2000	**0.1300**
ρ^F	Persistence parameter, foreign risk premium shock	Beta	0.8500	0.0750	**0.9049**
ρ^{rp}	Persistence parameter, physical investment risk premium shock	Beta	0.8500	0.0750	**0.9284**

ρ^{ucap}	Smoothing parameter in equilibrium capacity utilization	Beta	0.9500	0.0200	**0.9489**
rf	The effect of external debt on foreign risk premium	Beta	0.0200	0.0080	**0.0158**
rp	Risk premium on physical capital	Beta	0.0200	0.0080	**0.0251**
ω^X	The share of domestic consumption	Beta	0.8000	0.0800	**0.8817**
sfp	The share of forward looking firms (final consumption goods)	Beta	0.5000	0.2000	**0.8430**
sfp^M	The share of forward looking firms (import goods)	Beta	0.5000	0.2000	**0.7278**
sfp^X	The share of forward looking firms (export goods)	Beta	0.5000	0.2000	**0.8118**
sfw	The share of forward looking households (wage setting)	Beta	0.5000	0.2000	**0.7416**
σ^C	Parameter of the utility function	Gamma	2.0000	1.0000	**0.6753**
σ^X	Foreign elasticity of substitution between domestic and foreign goods	Gamma	1.2500	0.5000	**1.8648**
σ^M	Domestic elasticity of substitution between domestic and foreign goods	Gamma	1.2500	0.5000	**2.2879**
slc	The share of liquidity constrained households	Beta	0.5000	0.1000	**0.3210**
τ_π^i	The reaction of the interest rate on inflation (Taylor rule)	Beta	2.0000	0.4000	**1.7181**
τ^{TR}	The effect of employment on transfers	Beta	0.0000	0.6000	**0.0745**
ρ^{TR}	Persistence parameter, transfers shock	Beta	0.8500	0.0750	**0.7677**
τ_{Y1}^i	The reaction of the interest rate on output gap (Taylor rule)	Beta	0.3000	0.2000	**0.1508**
τ_{Y2}^i	The reaction of the interest rate on output gap change (Taylor rule)	Beta	0.3000	0.2000	**0.0682**
$wrlag$	Smoothing parameter in wage setting	Beta	0.5000	0.2000	**0.5339**
σ_ε^C	The standard deviation of the consumption preference shock	Gamma	0.0500	0.0300	**0.0720**
σ_ε^η	The standard deviation of the markup shock	Gamma	0.1000	0.0600	**0.3496**
σ_ε^{PM}	The standard deviation of the import price shock	Gamma	0.0200	0.0150	**0.1081**
σ_ε^{PX}	The standard deviation of the export price shock	Gamma	0.1000	0.0600	**0.0674**
σ_ε^{EX}	The standard deviation of the current account shock	Gamma	0.0050	0.0300	**0.0151**
σ_ε^{CG}	The standard deviation of the government consumption shock	Gamma	0.0500	0.0300	**0.0452**
σ_ε^{IG}	The standard deviation of the government investment shock	Gamma	0.0500	0.0300	**0.1245**
σ_ε^L	The standard deviation of the leisure preference shock	Gamma	0.0500	0.0300	**0.1534**
σ_ε^{lol}	The standard deviation of the overhead labor shock	Gamma	0.0050	0.0030	**0.0110**
σ_ε^M	The standard deviation of the monetary policy shock	Gamma	0.0025	0.0015	**0.0081**
σ_ε^F	The standard deviation of the foreign risk premium shock	Gamma	0.0050	0.0030	**0.0099**
σ_ε^{rp}	The standard deviation of the physical capital risk premium shock	Gamma	0.0050	0.0030	**0.0113**
σ_ε^{TR}	The standard deviation of the transfers shock	Gamma	0.0500	0.0300	**0.0147**
σ_ε^W	The standard deviation of the labor demand shock	Gamma	0.0500	0.0300	**0.1579**

σ_{ε}^{Y}	The standard deviation of the TFP shock	Gamma	0.0500	0.0300	**0.0522**

After the prior distributions are defined we used the Dynare software (Adjemian et al., 2011) to estimate model parameters on the basis of observed variables listed in Table 8. The estimation basically constitutes of two blocks:

1. In the first phase we use the Kalman-filter to determine the likelihood function. The maximum of this likelihood function gives an estimated mode of the posterior distribution which is the starting point of the second phase of the estimation. Generally this first step is done by some optimization procedures one generally used of which is the algorithm of Sims. Dynare provides several such algorithms but none of these was able to come up with a satisfying solution. In turn, we used an alternative in-built application of Dynare which provides an approximation to the maximum of the likelihood function on the basis of a Monte Carlo method. This option does not provide the maximum but robust enough to serve as a starting point for the second phase. In addition, this method calculates the optimal value of the jumping parameter for the Metropolis-Hastings algorithm (see below).
2. In the second phase we provide a numerical approximation to the posterior distributions using Markov Chain Monte Carlo method. In effect we simulate a sample of different parameter values the distribution (statistical characteristics) of which approaches that of the objective distribution (the posterior in our case) when the sample is large enough. A typical method is to use the Metropolis-Hastings algorithm which walks through the possible range of parameter values (defined by the prior distributions) and using the Kalman-filter it draws those parameter ranges which are the most likely (have high likelihood) for the given dataset.

In the second phase of the estimation procedure the size of the simulation is critical. For the final estimation we used a 300 thousand step MH algorithm in two blocks which gives a sample of 600 thousand parameter combinations. Using the jumping parameter determined in the first phase the acceptation rate moves between 30-35% during the MH algorithm which corresponds to the generally accepted rule-of-thumb. Two blocks are required to run convergence tests which helps in the identification of the parameters. To control for the ‘burn-in’ period of the MH algorithm (the period when the MCMC algorithm is not converging), the first 50% of the simulated 600 thousand units sample (in both blocks) is left out from calculating the posteriors and moments.

3.3.6.4 Estimation results

In what follows, we present the estimation results. We show the posterior distributions for the estimated parameters, the convergence tests and the in-sample forecasting performance of the model. Finally we give a brief comparison with alternative specifications.

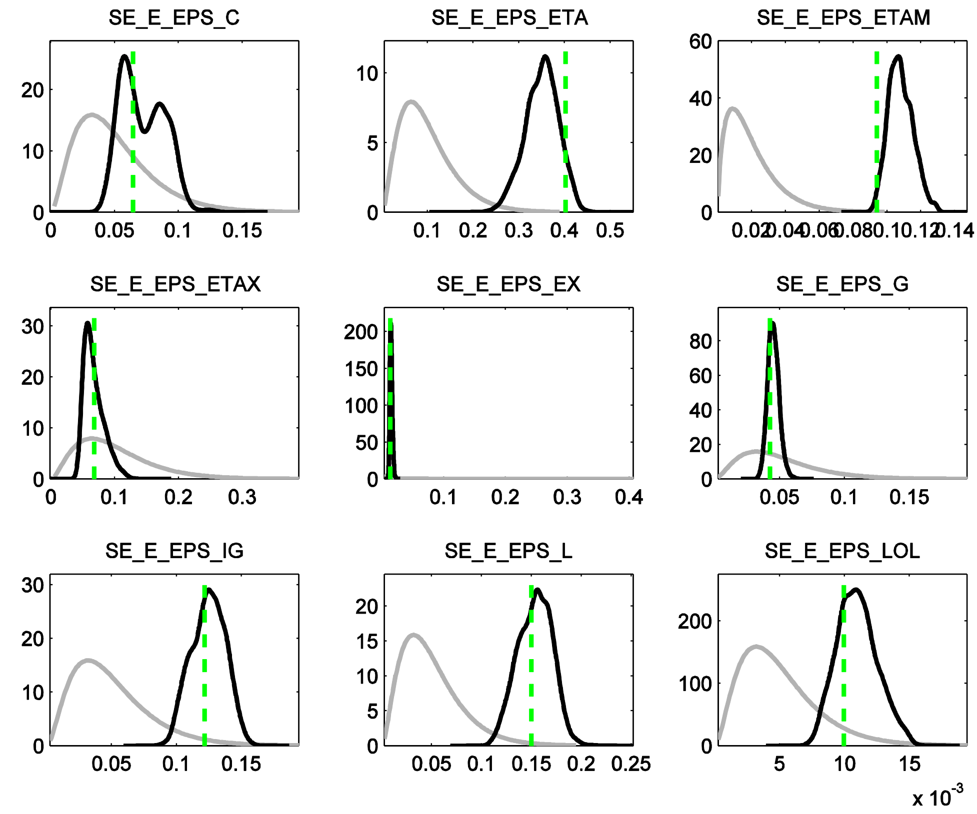
Posterior distributions

Figures 1 present the posterior distributions (black line), the prior distributions (grey line) and the approximated posterior modes given by the first phase of the estimation procedure (dashed lines).[9] The layout of the posterior distributions can serve as a first impact on the quality of estimation results. If the posterior has the same shape and position as the prior we can infer that there is not enough information in the data to identify the given parameter (or, incidentally it may be the case that our prior choice was very accurate). Similarly, a posterior distribution with two or more modi signals that more parameter values are consistent with the model specification and the data. The signal of well identified parameters is the relatively narrow range for the distribution (relative to the prior), the smooth shape of the curve and a different mode compared to the prior (the last one is not a necessary condition as with an accurately chosen prior the modi can be the same).

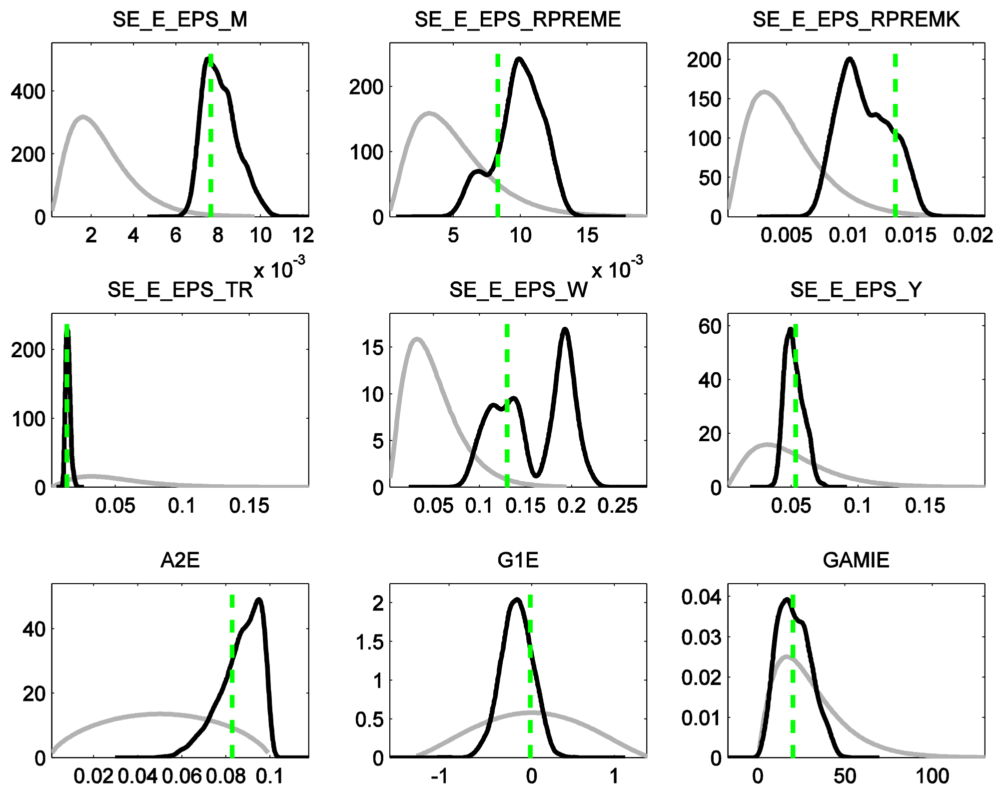
As evidenced by the figures, most of the parameters can be regarded as well identified. Less well identified seems to be the standard deviation of the consumption preference and the labor demand shocks, among the persistence parameters that of the export markup, government investment, transfer shocks, and the parameters defining the share of liquidity constrained households, the foreign risk premium effect and the smoothing parameter of the monetary rule.

The less well identified parameters were left in the estimation on the basis of two considerations. First, a further condition for selection is the overall fit of the model (see later) and the fact that the persistence parameters are either set to zero during the simulations or we do not effectively use them in the absence of shocks.[10] In addition, convergence tests constitute a further selection criterion. However, behind the relatively weekly identified parameters lies partly the quality of the data we could use for the estimation. If we compare our results to other DSGE model estimations for Turkey, we find similar weaknesses in some parameter estimations also taking into account that our model estimates significantly more parameters than the two available reference models (see Cebi, 2011 and Huseynov, 2010).

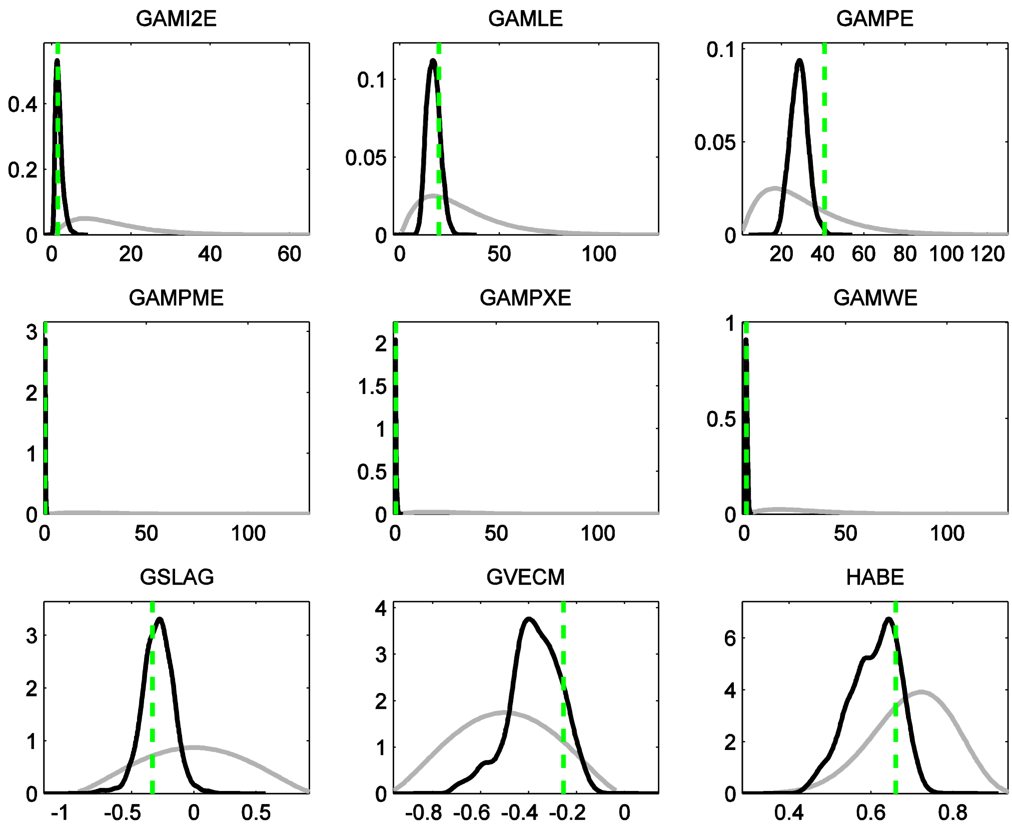
[9] Table 11 gives the concordance between the Dynare codes used in the diagrams and the parameter names used in the model description.
[10] Note that during the simulations only few shocks are used as described in a later section.



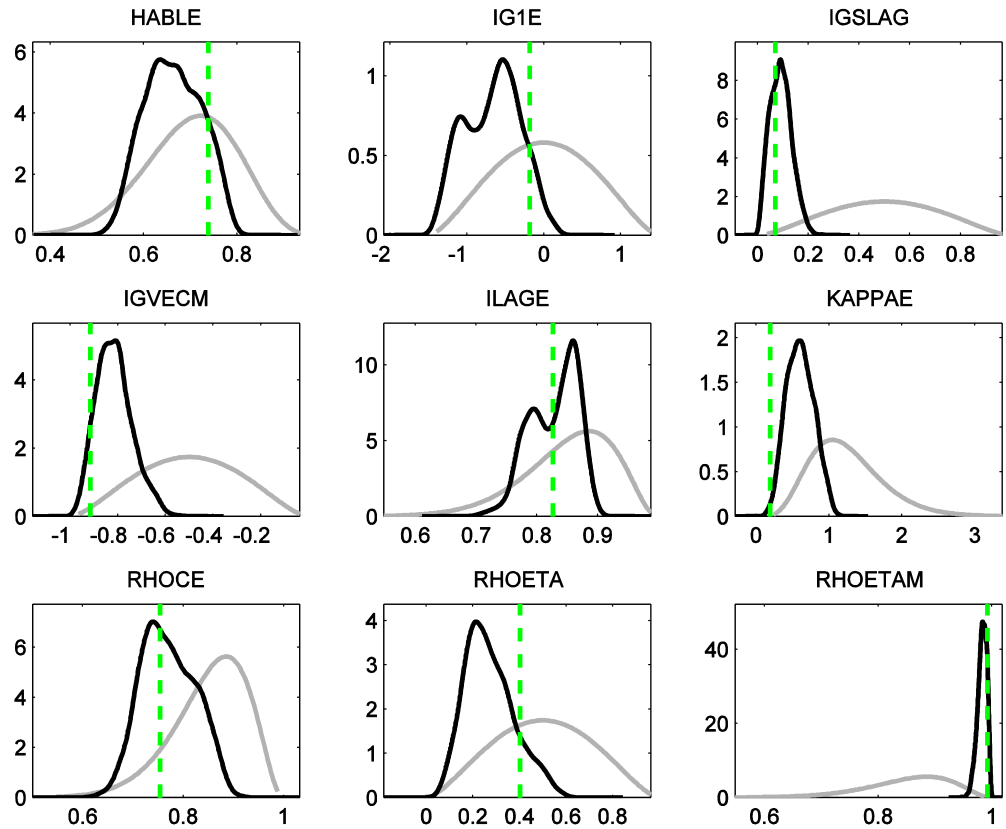
1a. Figure – Prior and posterior distributions



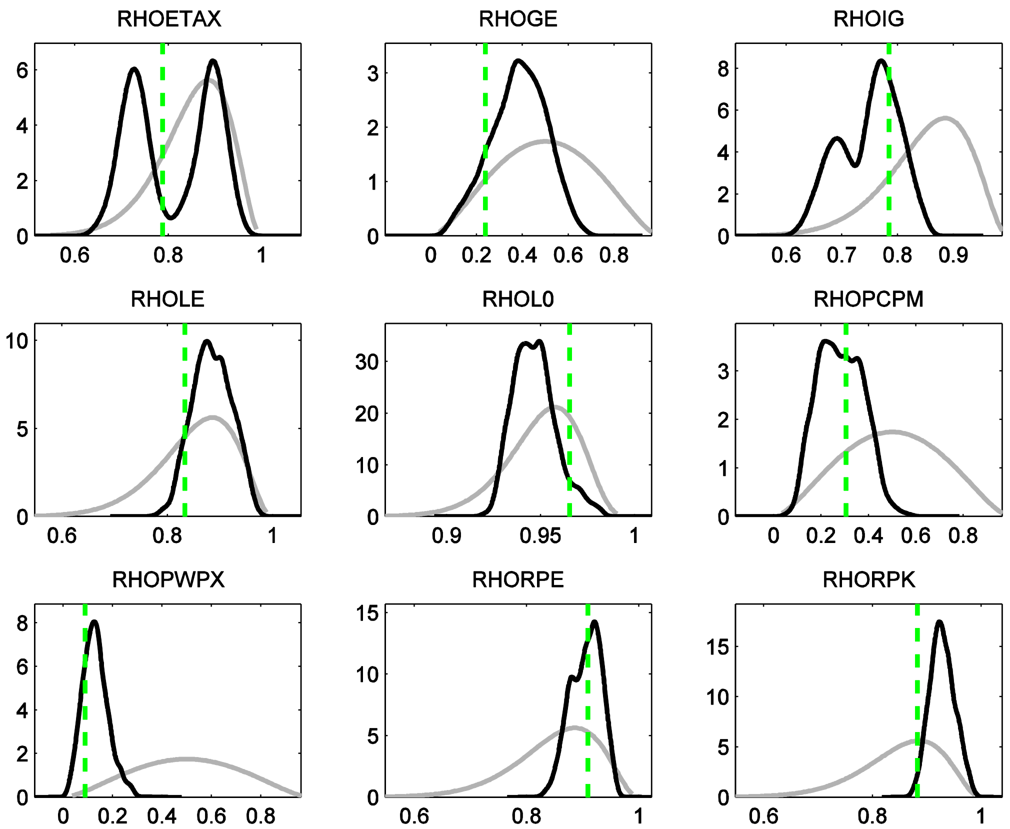
1b. Figure – Prior and posterior distributions



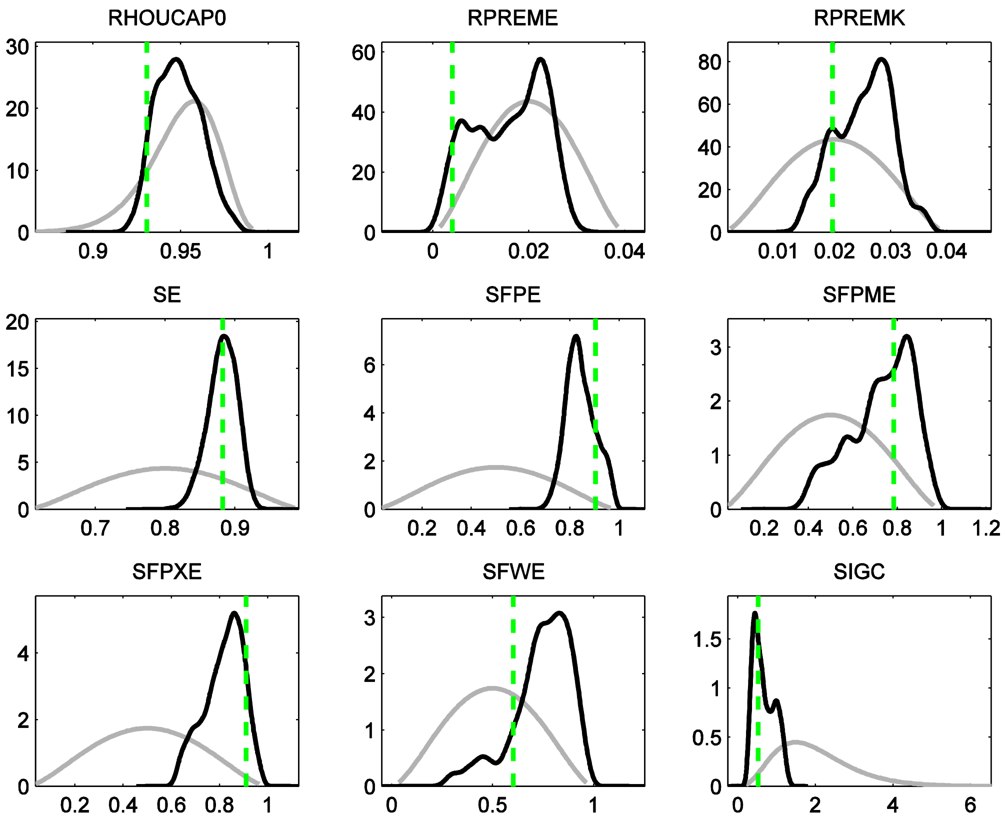
1c. Figure – Prior and posterior distributions



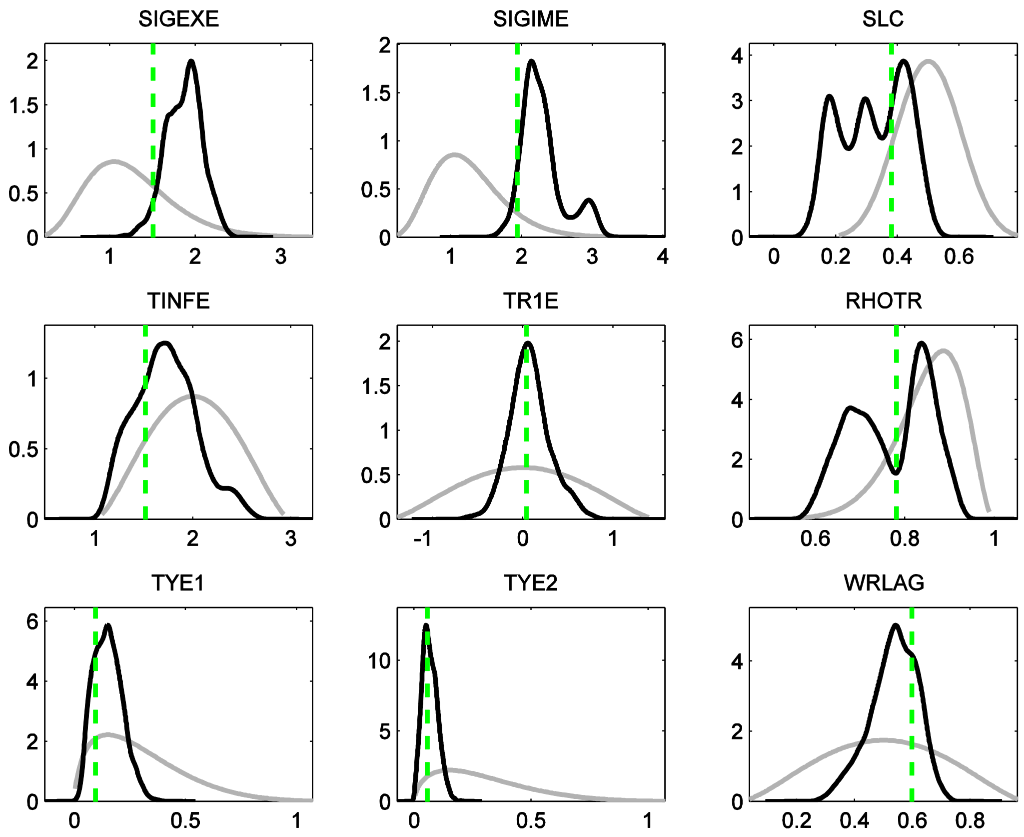
1d. Figure – Prior and posterior distributions



1e. Figure – Prior and posterior distributions



1f. Figure – Prior and posterior distributions



1g. Figure – Prior and posterior distributions

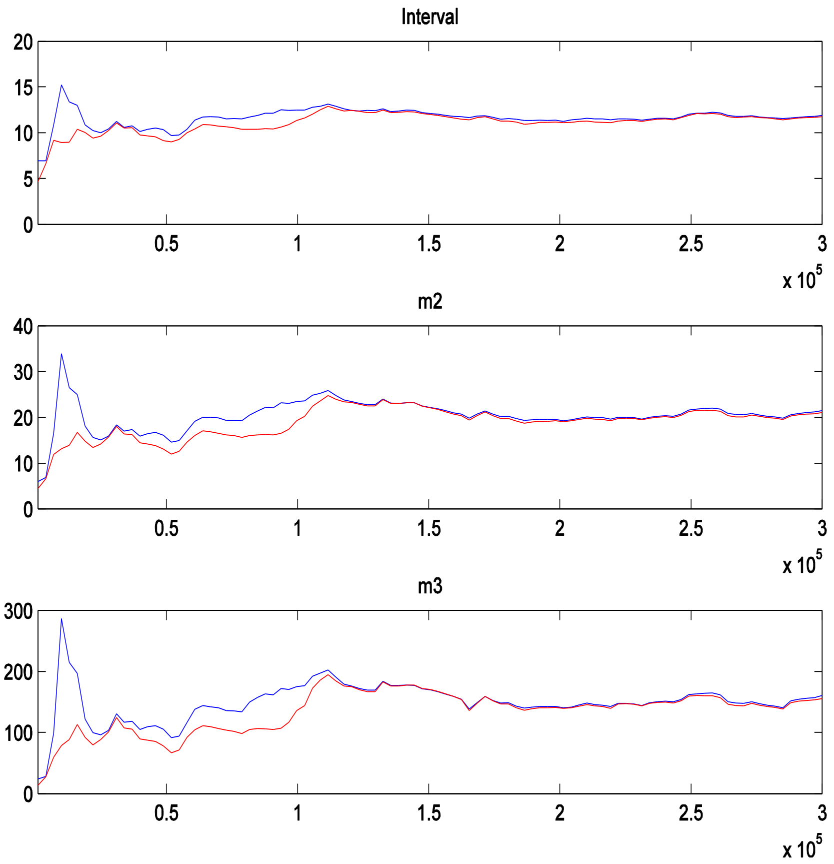
11. Table – Correspondence between notations

Notation	Dynare notation	Notation	Dynare notation	Notation	Dynare notation
γ_{U2}	A2E	ρ^{PX}	RHOETAX	τ_{π}^{i}	TINFE
τ_0^{CG}	G1E	ρ^{CG}	RHOGE	τ^{TR}	TR1E
γ_I	GAMIE	ρ^{IG}	RHOIG	ρ^{TR}	RHOTR
γ_K	GAMI2E	ρ^L	RHOLE	τ_{Y1}^i	TYE1
γ_L	GAMLE	ρ^{Lss}	RHOL0	τ_{Y2}^i	TYE2
γ_P	GAMPE	ρ^M	RHOPCPM	$wrlag$	WRLAG
γ_{PM}	GAMPME	ρ^X	RHOPWPX	σ_ε^C	E_EPS_C
γ_{PX}	GAMPXE	ρ^F	RHORPE	σ_ε^η	E_EPS_ETA
γ_W	GAMWE	ρ^{rp}	RHORPK	σ_ε^{PM}	E_EPS_ETAM
τ_{lag}^{CG}	GSLAG	ρ^{ucap}	RHOUCAP0	σ_ε^{PX}	E_EPS_ETAX
τ_{adj}^{CG}	GVECM	rf	RPREME	σ_ε^{EX}	E_EPS_EX
h^C	HABE	rp	RPREMK	σ_ε^{CG}	E_EPS_G
h^L	HABLE	ω^X	SE	σ_ε^{IG}	E_EPS_IG
τ_{lag}^{IG}	IGSLAG	sfp	SFPE	σ_ε^L	E_EPS_L
τ_{adj}^{IG}	IGVECM	sfp^M	SFPME	σ_ε^{lol}	E_EPS_LOL
τ_{lag}^i	ILAGE	sfp^X	SFPXE	σ_ε^M	E_EPS_M
τ_0^{IG}	IG1E	sfw	SFWE	σ_ε^F	E_EPS_RPREME
κ	KAPPAE	σ^C	SIGC	σ_ε^{rp}	E_EPS_RPREMK
ρ^C	RHOCE	σ^X	SIGEXE	σ_ε^{TR}	E_EPS_TR

ρ^{η}	RHOETA	σ^{M}	SIGIME	σ_{ε}^{W}	E_EPS_W
ρ^{PM}	RHOETAM	slc	SLC	σ_{ε}^{Y}	E_EPS_Y

Convergence tests

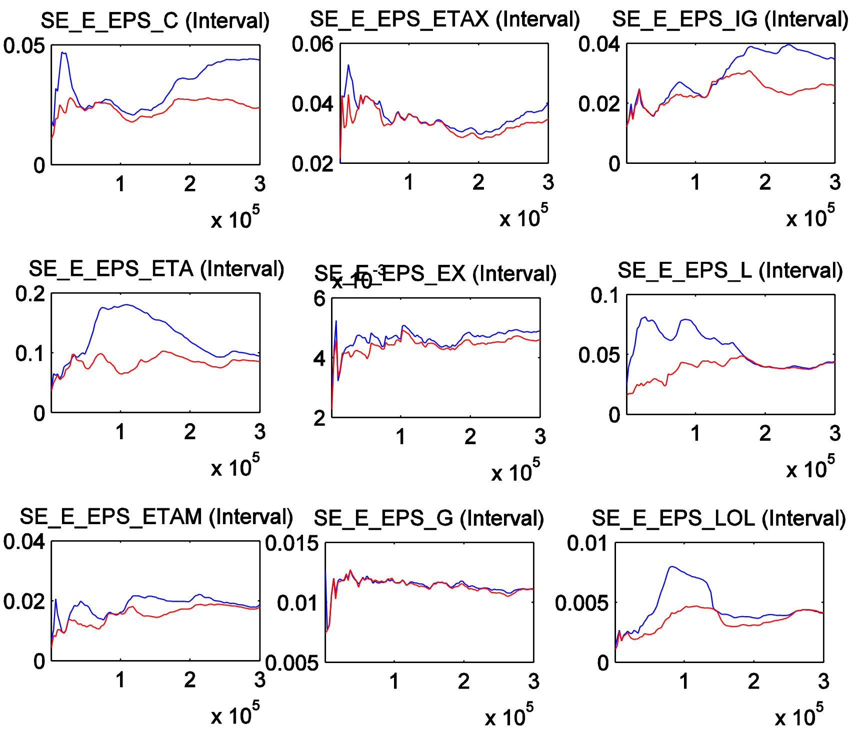
A further test on the quality of estimation results is whether the metropolis-Hastings algorithm converges, so that to what extent the resulting posterior distributions confines with the underlying true distribution. A widely used test for convergence is the diagnostics developed by Brooks and Gelman (1998) which is based on within and between variances. To calculate the test, in each iteration of the MH algorithm we calculate the within variances in each block (then taking their average) and the between variance among blocks. The condition of convergence is that between variance go to zero (i.e. the average values of the different blocks converge to each other) while the within variance stabilizes. These statistics can be calculated for the estimated parameters separately, but an overall value can also be constructed. In addition, the tests can be calculated for any moment of the posterior distribution. The overall convergence test of our estimation is shown in Figure 2.



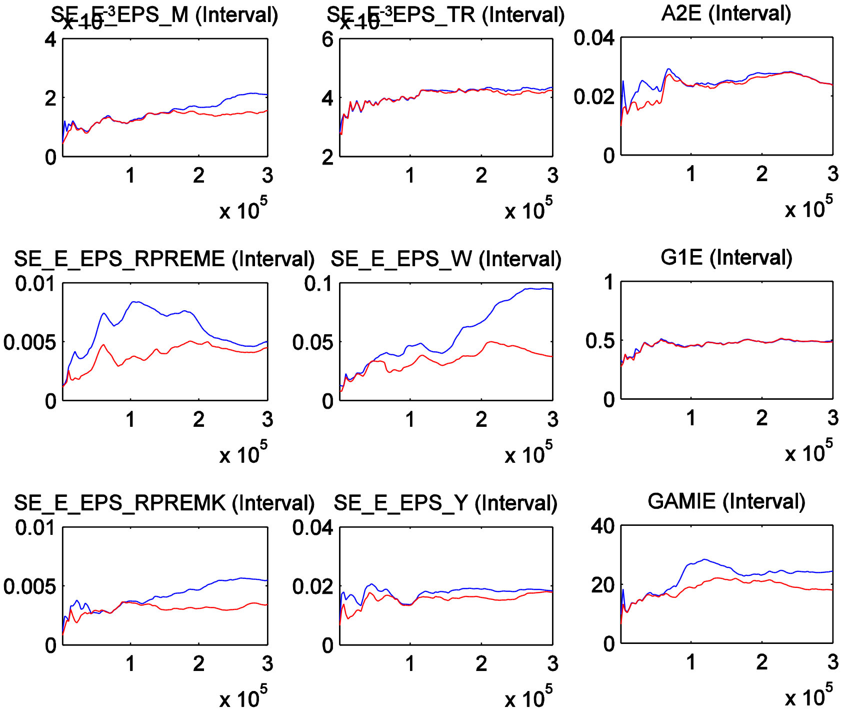
2. Figure – Overall convergence diagnostics

In the case of the convergence test generated by Dynare the red (lower) line represents the within variance while the blue (upper) shows the sum of between and within variances. As a result, converging lines mean convergence among the blocks and stabilizing lines show convergence in the distribution as a whole. The three panels show the first, second and third moment statistics respectively. According to the figure, we can infer that on average the parameters are characterized by good convergence, between variance disappears while within variance stabilizes.

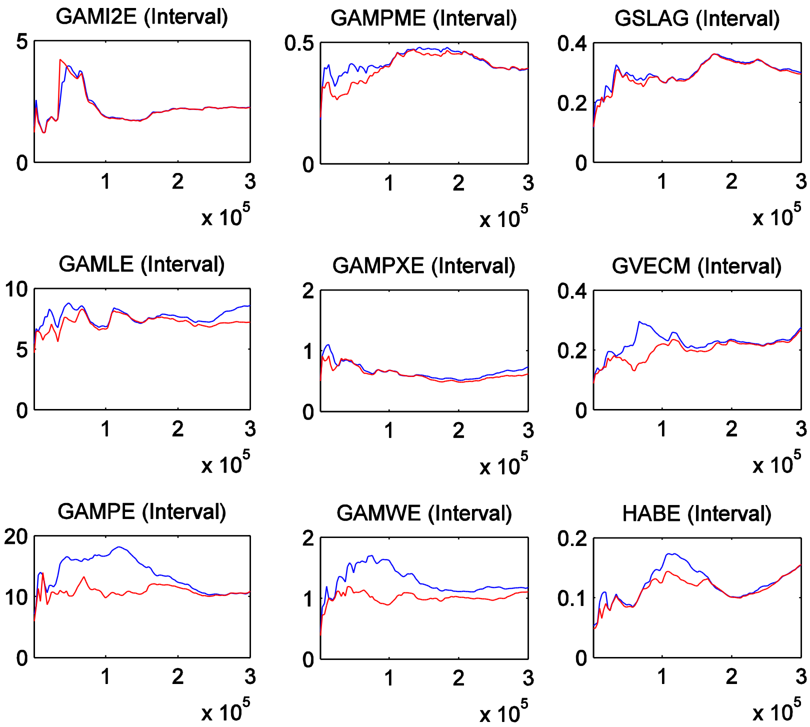
In addition to the overall statistics it is also important to examine the individual convergence tests of the estimated parameters. These are shown in Figures 3. The convergence tests are generally acceptable for most of the parameters, unsatisfying results mostly accord with those parameters for which the posterior distributions sign a less strong identification.



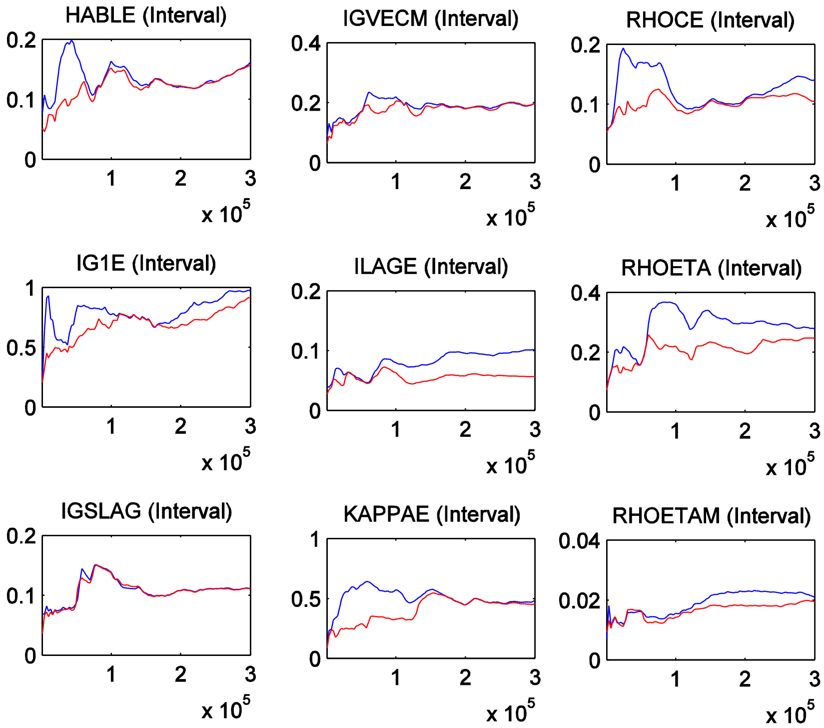
3a. Figure – Convergence tests for separate parameters



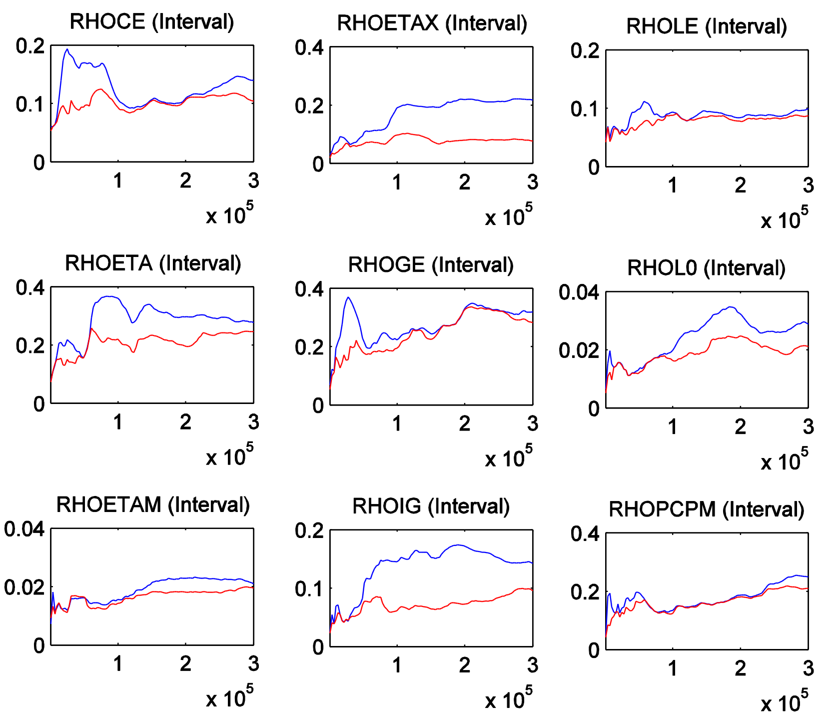
3b. Figure – Convergence tests for separate parameters



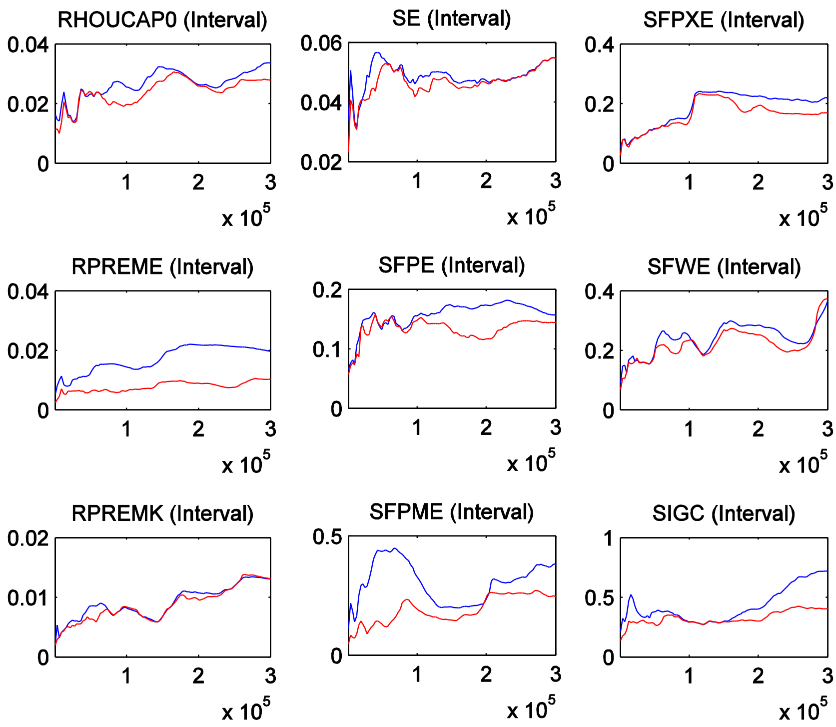
3c. Figure – Convergence tests for separate parameters



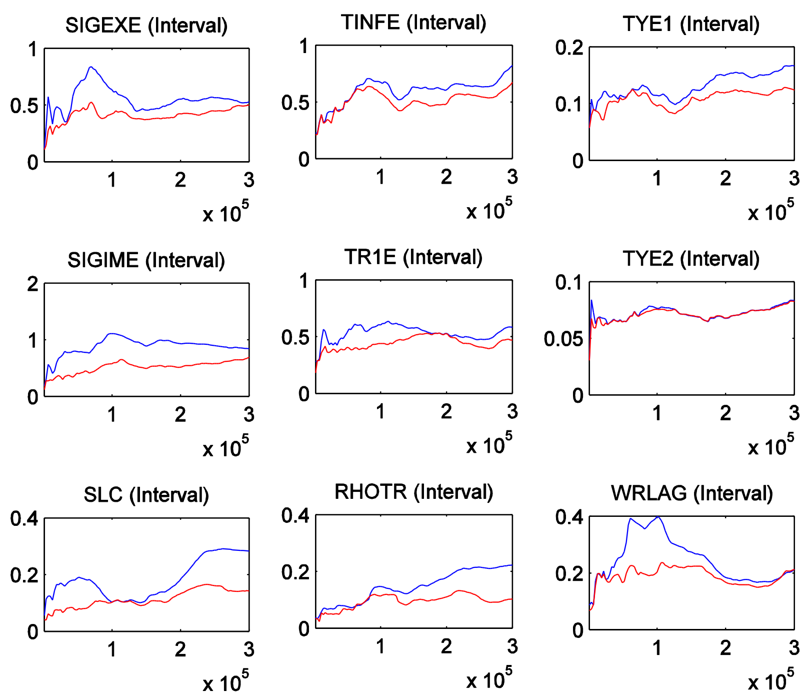
3d. Figure – Convergence tests for separate parameters



3e. Figure – Convergence tests for separate parameters



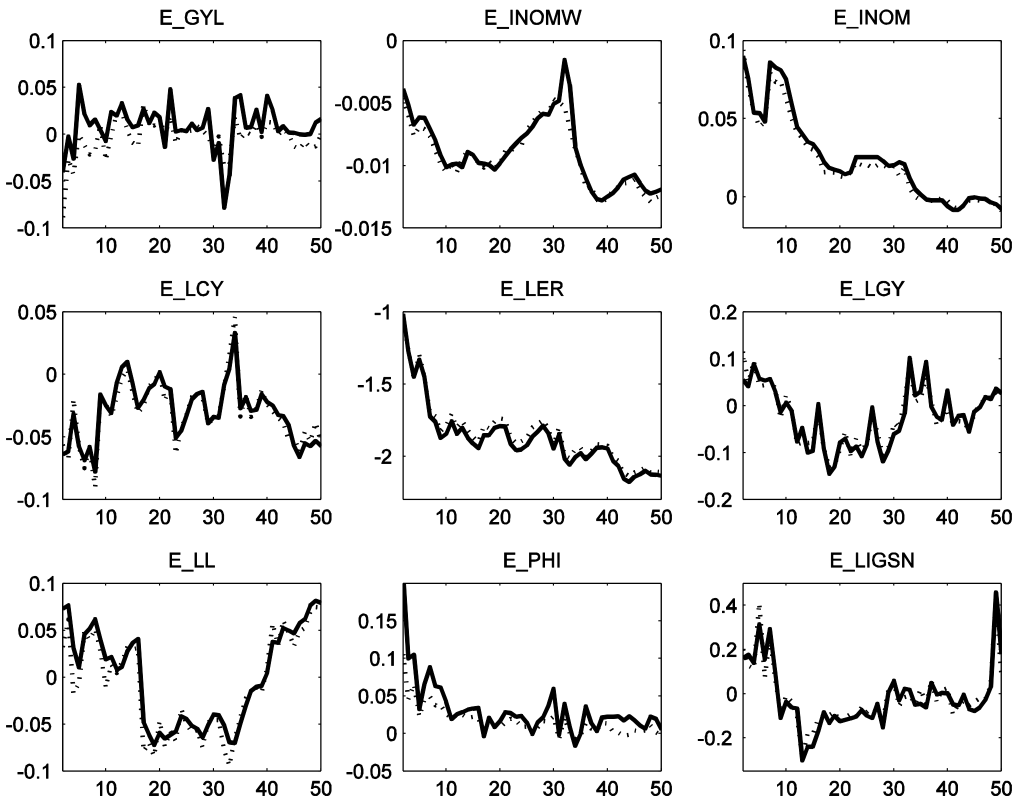
3f. Figure – Convergence tests for separate parameters



3g. Figure – Convergence tests for separate parameters

In-sample forecast

Beyond the individual evaluation of parameter estimates, a good test for the fit of the model is to examine its in-sample forecast performance. In order to do this, we prepared a one period ahead forecast with the Kalman filter for the observed endogenous variables. The nine most important of these are shown on Figure 4. The solid line marks the observed time series (after the transformations discussed previously) while the dashed line is the one period ahead forecast. The results show good in-sample forecast performance in most of the cases, only for inflation do we find a more smoothed forecast than the observed time series.



4. Figure – In-sample forecast for some endogenous variables

Comparison of alternative specifications

As it was mentioned previously, several model specifications were estimated before setting up the model for simulation and we chose the most appealing one. The general fit of the estimated models can be described with the marginal density value: the ratio of these values calculated for two different specifications is called the Bayes factor and show the extent to which a specification is more likely than another given the data. Table 11 summarizes three specifications and four estimations.

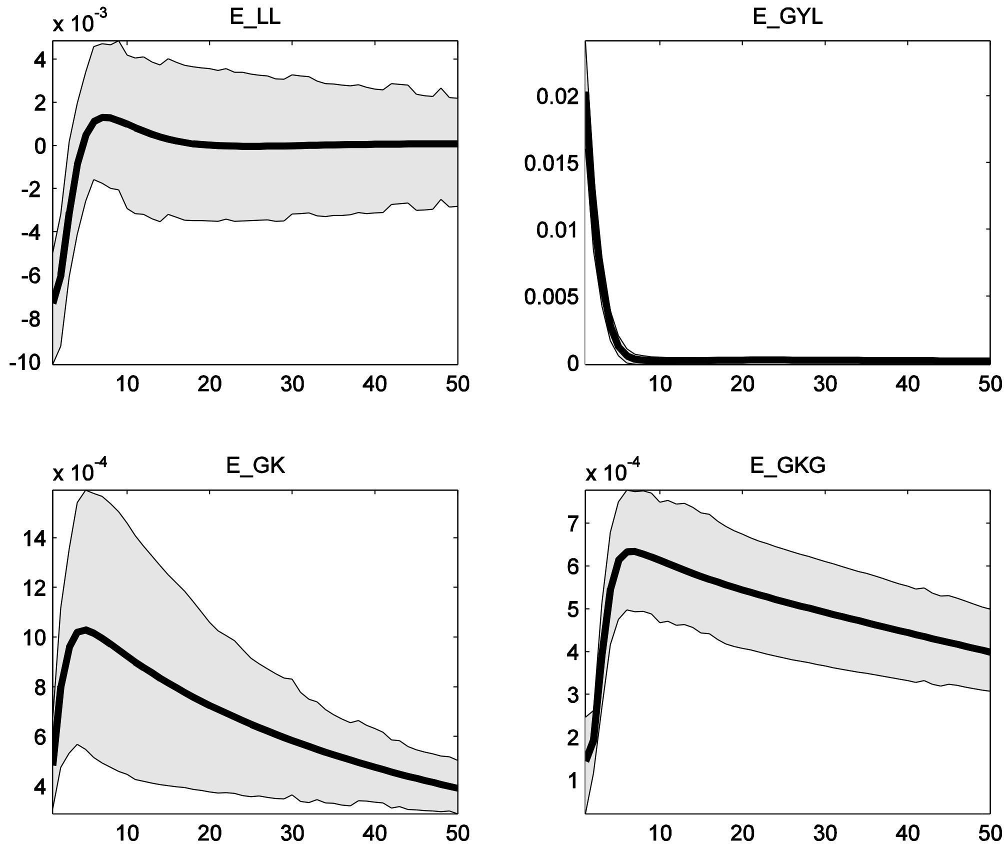
Specification	*1.*	*2.*	*3.*	*4.*
Number of estimated parameters	63	60	58	63
MH iterations	100	100	100	300
Marginal density	***-1494,57***	***-1724,05***	***-1766,94***	***-1476,41***

In the first specification we estimated all parameters (63) which were also estimated in the original specification of the model for the Eurozone. Due to identification problems reported before, we left out three badly identified parameters from the estimation for which we found comparable estimations for Turkey in other studies (Cebi, 2011; Huseynov, 2010). This led to worse estimation results as also indicated by the marginal density value. We then chosen badly identified shock standard errors and set their values according to the original QUEST specification, again leaving them out from the estimation. This third specification also led to worse results. Finally we retained with the first specification (63 parameters estimated) and run a MH algorithm of 300 thousand steps which slightly improved the fit.

Impulse responses

As the simulation of the model is implemented through running impulse responses, it is important to examine the reaction of some focal variables to shocks. In Figures 4 the reaction of four endogenous variables (employment – E_LL, GDP growth – E_GYL, the growth rate of private capital stock – E_GK and the growth rate of public capital stock – E_GKG) are depicted in response to shocks to the TFP growth rate (Figure 5a), to government consumption (Figure 5b) and government investment (Figure 5c). The figures show the deviation of the respective variables from their steady state values while the grey area marks the confidence interval.

On the vertical axes of the impulse responses (in line with the in-built features of Dynare but differing from the standard interpretation) absolute and not percentage deviations are depicted. If we take the endogenous variable x_t the steady state value of which is x^*, then the impulse response is $irf_t = x_t - x^*$. The impulse responses show in each case the fade-out of a one standard deviation shock. In the case of the TFP this is 0.05219, for the government consumption it is 0.0452 and for government investment it is 0.1245. In each case the model uses quarterly growth rates so the magnitudes of the shocks are to be interpreted according to this.

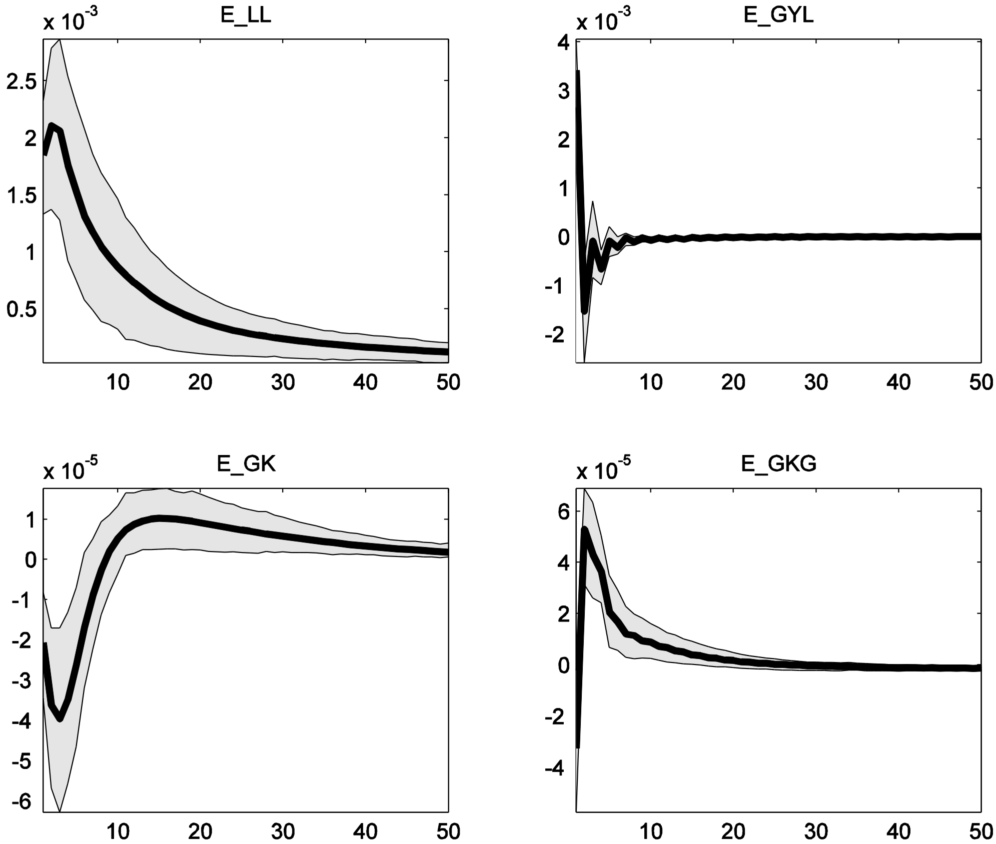


5a. Figure – The reaction of output variables on a shock to TFP growth

A shock to the TFP has a positive effect on GDP growth (which results in a positive shift in GDP levels). According to equation (M87) the growth rate of TFP follows a random walk with drift the persistence of which is zero. This drives the relatively rapid fade-out of the TFP shock. However, it is important to note that the persistence of the TFP shock is endogenized in a way by the other two model blocks (TFP and SCGE blocks), and the macro model only simulates the macroeconomic spillover effects of these exogenous shocks. However, it is less visible on the figure that after the relatively large jump in the beginning, the GDP growth rate persistently remains over the steady state level for a long while.

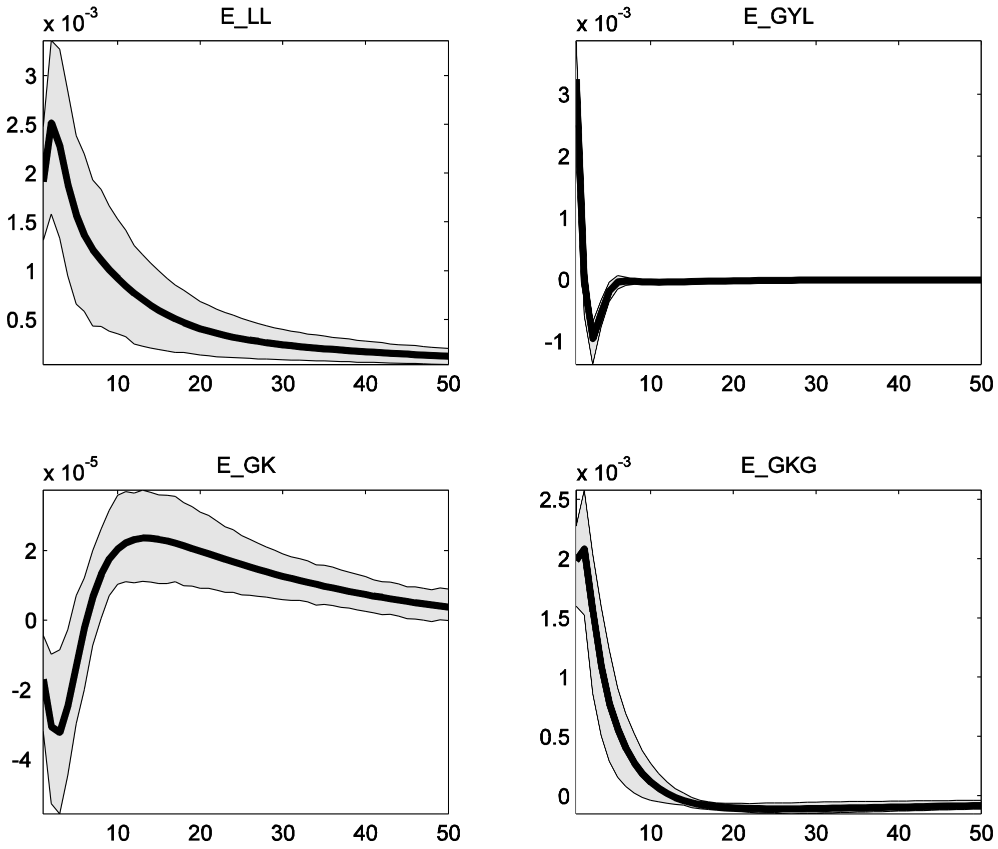
In the first two years the employment effect is negative, which is a general reaction in DSGE models. The reason is that the productivity growth leads to price decreases but due to staggered price setting prices change slowly which makes it optimal for firms to hire less labor. However, this negative effect is balanced in the long run by the increasing labor demand stemming from increased productivity. Public and private capital stocks react similarly to TFP shocks with the reaction of the public capital being more persistent.

It is worth mentioning that the sharp contrast between the fade-out of the GDP and the other three variables is misleading from the picture. It happens that GDP growth is directly and heavily affected by the TFP shock as it enters into the production function. After a sharp decrease, though, GDP growth remains over the steady state for almost the entire period depicted here with a deviation from the steady state corresponding in magnitude to the deviations of the other three variables.



5b. Figure – The reaction of output variables on a shock to government consumption

The shock to government consumption generates a positive employment effect throughout the response horizon, in magnitude similar to that of the TFP shock while its effect on GDP growth is minimal and shows cyclical properties. Public capital moves in a positive direction while due to the crowding out effect private investment decreases.



5c. Figure – The reaction of output variables on a shock to government investment

In the case of a shock to government investment we observe overall a more positive (but in the beginning of the period still negative) effect while the employment increases to the same extent as for government consumption. The effect on GDP growth in magnitude is almost the same as in the case of government consumption, but the cyclical tendency is less prevalent.

It is interesting to see the effect of shocks on levels rather than rates. It is true that only the TFP shock has a persistent level effect on GDP and a smaller effect on capital stocks. Although the government consumption and investment shocks give paths different from steady state, this difference is small (around 0.5% at the most extreme point) and after 50 periods converges back to the steady state path. In the case of employment, the impulse response shows levels by definition.

3.3.7 Integrating the MACRO block into the rest of the GMR model

Tailoring the macro block into the GMR model (in practice with the SCGE block) means basically three steps. The first step is an input interface through which the macro block receives the inputs, the second step is running the macro block which means calculating impulse responses on the input shocks and the third step is providing the SCGE block with the time series generated by the impulse responses.

3.3.7.1 Inputs to the macro block

The macro block requires five time series as an input. These time series are as follows:

1. Time series of TFP levels
2. Time series on shocks (additions, policy interventions) to government consumption constituting of spending on education, R&D support and other demand side stimuli.
3. Time series on shocks (additions, policy interventions) to government investment which corresponds to infrastructural investment.
4. Time series on private investment support.
5. Time series on scheduled repayment of private investment supports.

These time series are available from the SCGE block on an annual frequency, so the input interface of the macro block first converts them into quarterly values and then generates the necessary shock variables from these series which are then the direct inputs to the model.

In the case of the TFP, annual growth rates are converted to quarterly in a way that quarterly rates sum up to annual rates. Then, quarterly growth rates are related to the steady state growth rate in order to obtain those shocks which are the inputted to the macro model.

In the case of government consumption and investment we also split annual data into quarters, assuming even distribution within years. At the same time we have to take into account that government consumption and investment enters into the macro model through growth rates (see equations (M20 and (M21)), so in each quarter we have to convert additional consumption and investment into growth rates. In order to do this we calculate the volume of government investment and consumption throughout the model run and we get the required shocks comparing additional interventions to these volumes.

In the case of private investment support we also split annual interventions to quarters evenly, which (as in the case of government consumption and investment) is inputted into the model after converted into additional growth rates. Repayments of investment support are accounted for as (negative) transfers to the government budget.

3.3.7.2 Running the MACRO model

Running the macro model basically means applying the reduced dynamic matrix equation in (A44). This matrix equation uses transition matrices determined by model parameters with which it is able to generate the time path of endogenous variables as a response to arbitrary shocks to the system. As a result, using the exogenous shock variables (both those originally in the model and those added here to implement policy interventions) we can simulate the effect of government interventions and TFPs given as inputs and we can trace the resulting macroeconomic processes for the endogenous variables of the model.

Implementation of the shocks in the model is done according to the following mechanisms:

- The growth rate of TFP is given by equation (M87) with the help of the exogenous shock variable ε_t^Y. According to this equation TFP follows random walk with drift where the trend is given by the steady stat growth rate of the TFP. Subtracting the steady state TFP growth from the TFP growth rates coming as inputs we obtain that value for ε_t^Y which acts as a shock to the system.
- Government consumption can be influenced by the variable u_t^{CG} in equation (M20). As written earlier, this equation works with growth rates so the additional quarterly consumption inputs (given in levels) are converted into additional growth rates using the value of the consumption expenditures of the previous quarter in order to obtain the required value for u_t^{CG}. As u_t^{CG} is a persistent exogenous variable in the original model setting, which is driven by equation (M95) and shock ε_t^{CG} in it, the persistence parameter ρ^{CG} in equation (M95) is set to zero during the simulations so that we can simulate the clear effect of interventions.
- Simulating government investments is analogous to that of government consumption. Here, we implement the interventions through the exogenous variable u_t^{IG} in equation (M21) as additional growth rate. Similarly to consumption, in equation (M96), driving u_t^{IG} we set the persistence parameter ρ^{IG} to zero. In addition, the higher growth rate of government investment must be inputted also into the growth rate of public capital. This is done through the exogenous shock ε_t^{GCAP} in equation (M43).
- Private investment subsidies are implemented analogously to government investment. The exogenous shock variable ε_t^{INV} in equation (M47) influences the growth rate of private investment whereas the exogenous shock variable ε_t^{CAP} in equation (M42) influences the growth of private capital stock in accordance with the interventions.
- Increasing only the expenditure side of the government budget (consumption and investment) we would observe an additional deficit leading to an increase in public debt. However, the financing source of these expenditures are given in principle, but not accounted for in the model structure. As a result, we have to implement an additional element on the revenue side of the government budget to include the financing of these expenditure elements. This issue is handled through the exogenous shock variable ε_t^{GB} added to equation (M25). As this equation is given relative to the nominal GDP, we

have to trace the nominal GDP level in each period and using this value we can determine that value for ε_t^{GB} which balances the budget expenditures.

- As these revenues are financed from the foreign sector, we also adjust the current account to GDP ratio with the variable ε_t^{GB}. In this setting, we assume synchronized dynamics in the resources and the expenditures of the government budget and as a result, the adjustment of the current account is mostly of technical nature.
- Possible repayments are implemented as negative transfers flowing from the private sector to the government, using the exogenous variable u_t^{TR} in equation (M22). Repayments have to be included here as a ratio to wages so the wages are also traced during the simulation run and we can calculate the value of the shock variable on the basis of this information. As the variable u_t^{TR} is persistent in the model, we set the persistence parameter ρ^{TR} in equation (M103) to zero during simulation run.

Running the (A47) recursive system of equations with the shock variables calculated according to the principles given above, as a result we obtain the time paths of the endogenous variables.

3.3.7.3 Outputs from the MACRO block

The simulated time series of endogenous variables form the macro model is used by the SCGE block. However, only few of the 104 endogenous variables are used: these are the time series for GDP, employment, and government consumption and investment. These outputs are generated by the macro model in a way that for the first year the values are unity and the relative changes are reported for each consecutive year. We use quarterly growth rates for these four variables to calculate output, the cumulative annual growth rates are used to obtain the indices for the output variables for each year.

The macro block generates as output further time series which are not used by the SCGE block. These are the consumption of households, unemployment rate and the deficit to GDP ratio. Household consumption is also given as an index with the first year normalized to one and the other two values are reported naturally in percentages. Due to its special nature, we separately discuss the unemployment rate in what follows.

3.3.7.4 Unemployment

As a general equilibrium model, the macro block does not contain a direct measure for unemployment as the markets, including the labor market, clear in every period. As a result, there is no explicit unemployment in the model, so we can only provide an approximation to it. This approximation is made possible by the variable L_t^{ss} describing equilibrium employment (see equation (M26)). We assume that this value corresponds to labor market equilibrium which is characterized by the natural rate of unemployment. As the variables L_t^{ss} and L_t are employment rates, we can write that

$$F_t = AP_t L_t$$

$$MK_t = AP_t L_t^{ss}$$

where F_t is employment, AP_t is active population and MK_t is the absolute value of labor supply. From these it follows that

$$UR_t = 1 - \frac{F_t}{MK_t} = 1 - \frac{L_t}{L_t^{ss}}$$

where UR_t is the unemployment rate. As L_t^{ss} is interpreted as the employment rate corresponding to the natural rate of unemployment, if $L_t = L_t^{ss}$, or equivalently$UR_t = 0$, then unemployment equals the natural rate. As a consequence, UR_t gives the deviation of unemployment from the natural rate, so for unemployment we can write the following formula with UN_t denoting the natural rate:

$$UR_t = UN_t + UR_t$$

4. Model sensitivity

In what follows we present some sensitivity analyses. These analyses are done on the basis of altering some relevant exogenous parameter of the macro model and then we examine the change in the effect of interventions. Formally this can be considered as follows. Let x_t be the endogenous variable under question, e.g. the growth rate of GDP. Then we record the effect of interventions on variable x_t in the original case (as shown previously, without sensitivity analysis), so we calculate the difference

$$x_t^{SCEN} = x_t^{SCEN} - x_t^{BASE}$$

for those variables which are expressed in percentages (e.g. employment rate) and the ratio

$$x_t^{SCEN} = (x_t^{SCEN} - x_t^{BASE})/x_t^{BASE}$$

for those variable which are expressed in levels (e.g. GDP level). Here, x_t^{BASE} is the time series of the given variable in the baseline, while x_t^{SCEN} is the time series of the same variable in the scenario. Then the same differences and ratios are calculated between baseline and scenario (interventions) for the case when the parameter-setting is altered for the sensitivity analysis (i.e. for the effect of interventions under an alternative parameterization):[11]

$$x_t^{SENS} = x_t^{SENS} - x_t^{BASE,SENS}$$

and

$$x_t^{SENS} = (x_t^{SENS} - x_t^{BASE,SENS})/x_t^{BASE,SENS}$$

where $x_t^{BASE,SENS}$ is the baseline path corresponding to the sensitivity analysis (alternative parameterization) and x_t^{SENS} is the path resulting from the interventions under the alternative parameterization. Then we calculate the difference of the measured effects of interventions between the alternative and the original parameter setting:

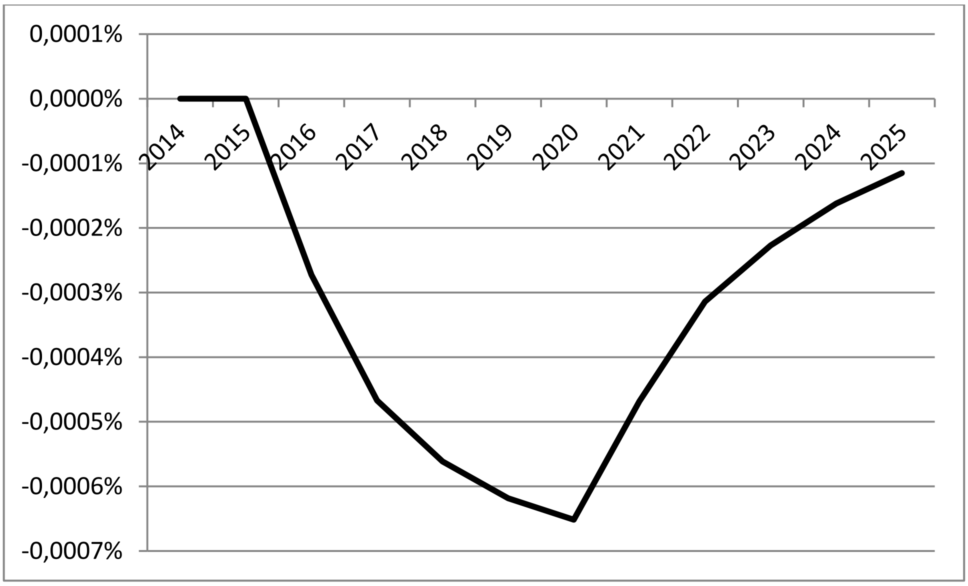
$$x_t = x_t^{SENS} - x_t^{SCEN}$$

In the figures used in the following analyses we present the variable x_t (calculated as before) on the vertical axis. The sensitivity analyses present how the impact of given interventions may change if there are modifications in some of the most important underlying macroeconomic conditions. We present the deviation of GDP levels from the original setting.

[11] We allow for different baseline time paths for different sensitivity analyses

4.1 External demand, current account

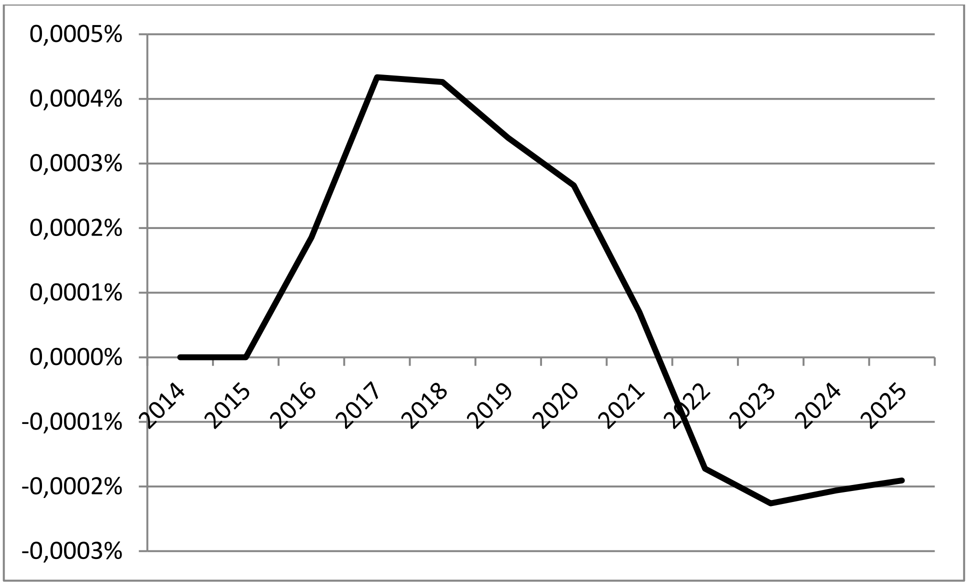
The first analysis presents a setting where the steady state current account moves into a positive direction. This situation can be regarded as a positive shock to external demand. The base version of the macro model works under the assumption that the current account to GDP ratio is zero in the steady state. This assumption is modified in this sensitivity analysis with the steady state current account ratio set to 1%.



The figure above shows how the GDP levels deviate from the original scenario. In the beginning years the effect of interventions on the GDP growth rate is smaller when there is a (permanent) positive external demand to the economy however, in the second half of the examined period this difference turns to positive. Overall, as seen in the picture, the GDP level is below the original scenario in the whole period. Interventions have lower effects when there is a higher external demand. This negative effect comes from the fact that a higher external demand leads to lower consumption share in the GDP which drives aggregate demand, hence GDP below the path observed in the original scenario. However, the path returns to the original setting as the interventions fade out.

4.2 The expenditure structure of government

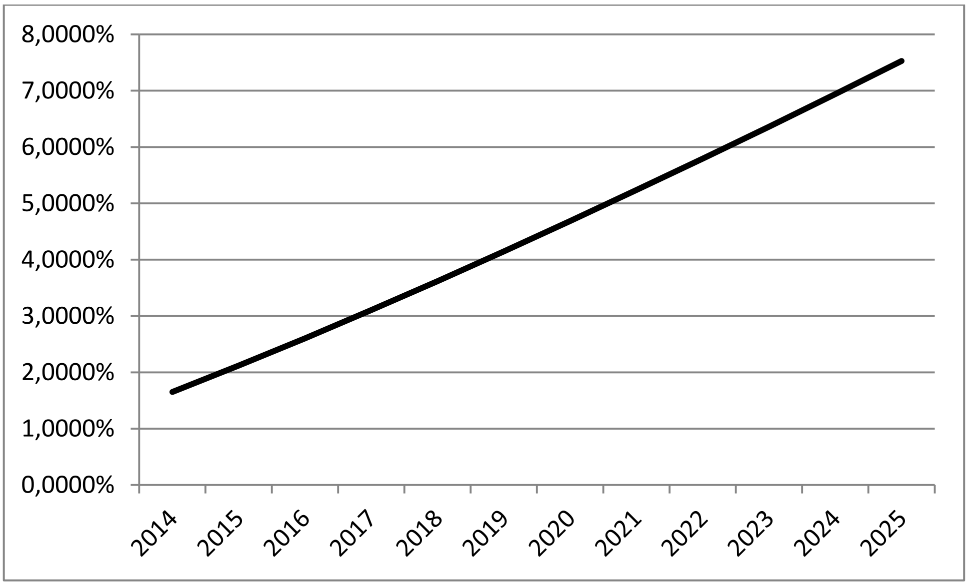
The relatively detailed fiscal block in the macro model makes it possible to examine the effect of changes in the government expenditure structure on the impact of interventions. In this line we restructure the expenditure side in a way that the sum is left unchanged but its structure is modified in advantage of government investments. Formally, the steady state share of government consumption is decreased by 1 percentage point while that of government investment is increased by 1 percentage point.



In the figure above we show again the deviation of GDP levels in the alternative scenario. In the most years there is a positive while later there is a negative deviation. The overall positive effect comes from the fact that restructuring towards investments builds public capital which has a positive effect on GDP. However, the deviation dies out as interventions stop.

4.3 Productivity

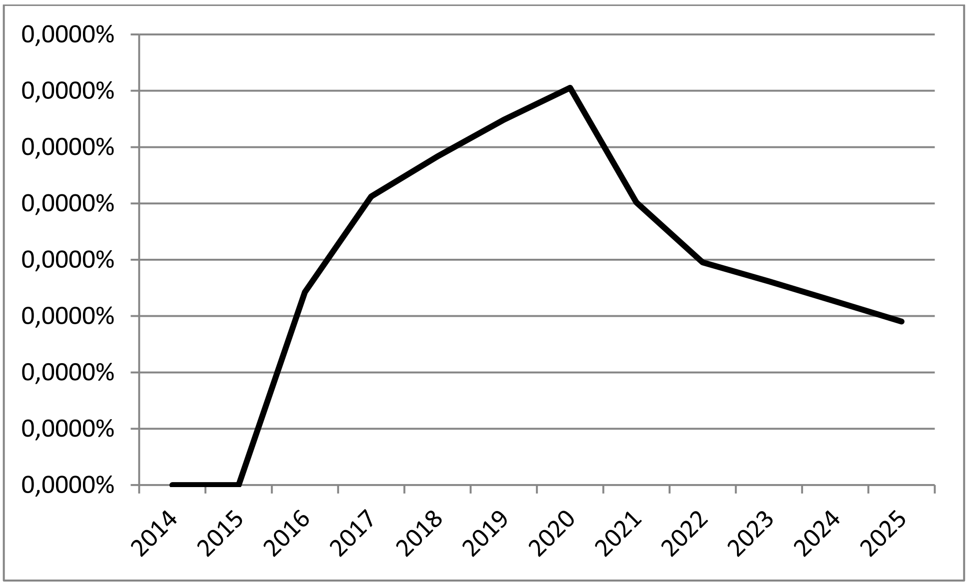
Although productivity (TFP) is a focal endogenous variable in the model, there is a way to simulate changes in the productivity through the productivity of the intermediate sector. The alternative scenario in what follows builds on the assumption that the productivity growth of the intermediate sector is 1% per year (it is zero in our original scenario). This will influence in turn the steady state growth rate of the TFP.



The figure above is clear enough, that as it is expected a better productivity environment is able to amplify the effect of interventions. This positive effect comes from the fact that productivity growth in the intermediate sector makes investments goods cheaper which motivates the actors to invest more in physical capital.

4.4 Inflation

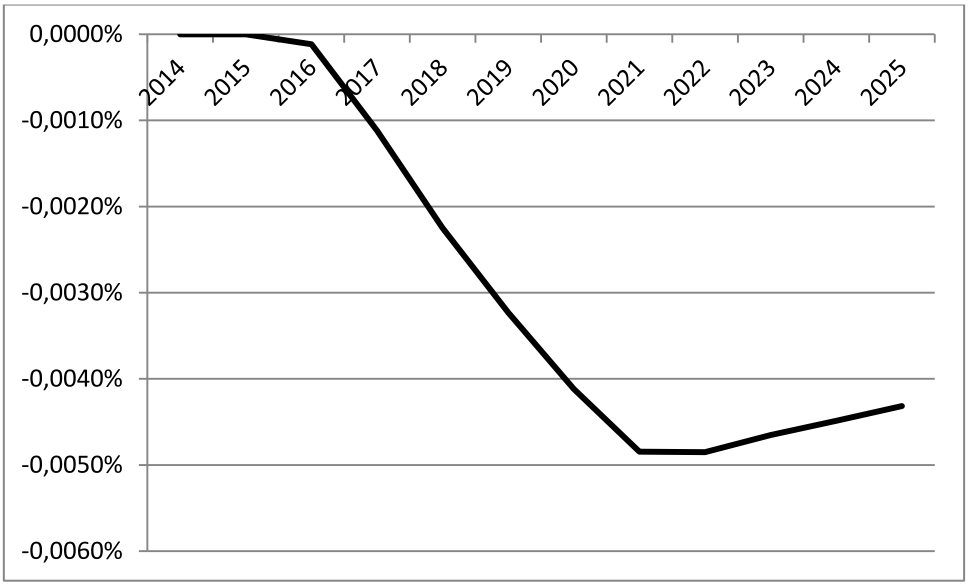
In this sensitivity analysis we ask if a lower inflationary environment can have an effect on the impact of interventions. The lower inflationary conditions are modeled through lowering the inflation target to 4% which is also the steady state inflation level in the model.



As shown by the figure, when the economy faces a lower inflation on general, the impact of interventions may be higher, this difference is very small though.

4.5 Interest rate

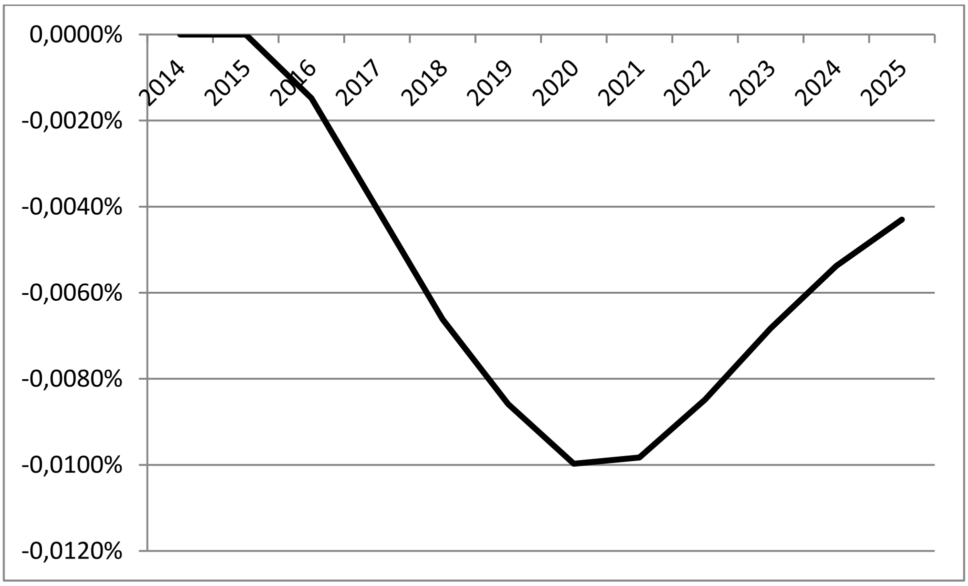
Lower interest rates can be obtained by setting a higher discount factor which results in a lower steady state real interest rate. We modify the discount factor to have a 1 percentage point lower real interest rate.



The impact of interventions is significantly lower in this case. A lower interest rate leads to more investments thus the short run, demand-driven effects of interventions are less pronounced in this case. Later, as investments are spilled over to the supply side, the gap starts to decrease.

4.6 Taylor rule

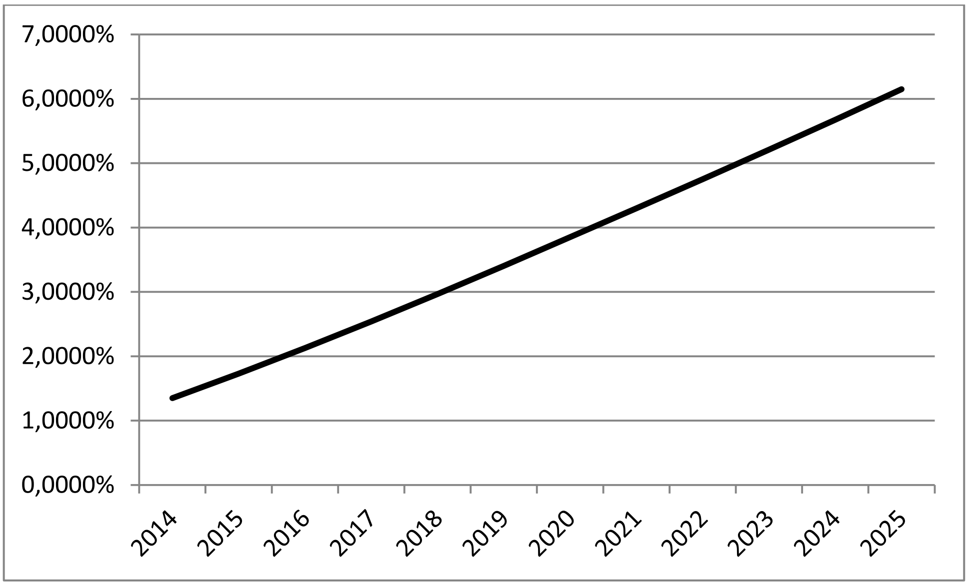
It is interesting to see how the impacts change if the monetary authority reacts differently to economic conditions. In this alternative scenario we assume a more intensive reaction to the output gap.



The lower impact in this case results from the fact that a more sensitive reaction to the output gap means that when GDP rises in response to the interventions, the central bank raises the interest rate more aggressively thus cutting back demand more intensively than in the original scenario.

4.7 GDP growth rate

As the steady state growth rate of the model is exogenous, it is interesting to see how different growth trend affect the impact of interventions. In this analysis we examine how a 1 percentage point increase in the annual GDP growth rate modifies the impacts.



The figure shows that this positive shift in the steady stat growth rate clearly leverages the impact of interventions.

Acknowledgements

The authors wish to express their thanks to the European Commissions's General Directorate Economics and Finance (DG EcFin) for providing us the QUEST Eurozone model. We appreciate specifically the consultation opportunities related to the re-estimation of QUEST on different databases by János Varga (DG EcFin). We are also thankful to Mortaza Baky Hauskee (University of Teheran and University of Pécs) for his contribution at certain stages of the econometric estimations, Áron Kovács (University of Pécs) for his contribution in estimating transportation cost matrices and Serkan ÇİÇEK (Maltepe University, Istanbul) for his assistance in data collection.

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