What Drives Market Resiliency on the Order-Driven Markets?

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Abstract

Our study investigates the market resiliency of order-driven stock markets. We define resiliency as the feature of the market in which new orders flow quickly to correct liquidity of the market after a

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shock. When an aggressive market order appears, it eliminates a significant ratio of the limit orders

from the order book. The resulting lack of limit orders can cause notable price impact for market

orders. It is crucial for the market players to know the duration of the correction and the possible

long term effects of this kind of shocks. Based on the literature, we build up a vector autoregressive

model to quantify the duration of the correction of market liquidity and explore the size of the

critical market orders which drives to market shocks. This VAR based 'estimation and simulation'

approach let us to use several factors (such as bid-ask spread or market depth) for predicting the

behavior of order book recovery. Using the Budapest Stock Exchange transactional intra-day

dataset, we measure and model the market resiliency in the case of the two most liquid Hungarian

stocks (MOL, OTP). The academic contribution of this paper is to estimate the price impact

formulation and market resiliency after an order imbalance. In contrast to the literature, we consider

the entire order book to define the value of liquidity, by incorporating the indicator of the cost of

round trip (Budapest Liquidity Measure) into the analysis.

Keywords: market liquidity, resiliency, cost of round trip, order-driven market

JEL classification number: C32, C51, G10, G17

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Introduction

This study empirically investigates the resiliency of the order-driven markets concentrating on the structure of the order book. Market resiliency is known as one of the characters of market liquidity. While numerous studies concern with the static dimensions namely tightness, depth or breadth, (e.g. BIS (1999) or Sarr and Lybek (2002)) relatively low numbers of papers discuss how the order book recovers after limit orders disappeared due to market transactions.

When a trader executes a market order, the transactional price may divert from the best available price (bid or ask). Implicit cost of trading refers to the margin between the immediately executed transactional price and the best price. In this paper, we define market resiliency as the recovery process of the order book in response to a temporary order imbalance. We focus on how fast the implicit cost of trading reverts back to the level describes the normal times. Because implicit trading cost may relate to different trade volume levels, we consider the variation of the entire limit order book. A temporary order imbalance can be the result of an aggressive market order or series of market orders which are being fulfilled against large number of limit orders in a very short period.

The theoretical literature of market microstructure (e.g. Foucault (1999), Parlour (1998), Rosu (2009)) often describes order-submission strategy of the investors as a choice between market and limit orders by their time and risk preferences. During these decisions, information has an important role according to the literature. For example, Kyle's (1985) model implies that informed traders should mimic the trading patterns that uninformed traders do, in order not to reveal their information advantage. Following these considerations, it is a reasonable strategy in the order-driven market for both kind of traders to execute small volume market or limit orders when the market is deep enough. With this way of submissions, they do not to cause unwanted impacts on the

price. Furthermore, according to the mentioned studies, small limit orders should be preferred in more volatile times. When a trader realizes that the total volume of the targeted order cannot be executed at the same price level, it should split the order into parts, and submit smaller orders with delays to avoid unwanted market impact. These kinds of order splitting are applied very frequently on the order-driven exchanges. Usually, small volume of order does not have price impact immediately. It can imply biases on the future price if traders believe these are from an informed trader, as before it was shown. However, smaller volume orders do not hurt dramatically the order book, many of limit orders may disappear because of a sequence of order submissions. By this time, the order book also needs a certain time to be refilled.

Market resiliency can be a crucial question for those traders who are forced to use order splitting strategies. Furthermore, understanding the period of lack of liquidity and the recovery process are also important for any trader who does not want to lose money with making transaction at a price that unnecessary diverts to the actual market price.

This paper follows the empirical studies which estimate the dynamics of order-driven markets process of vector-autoregressive models and simulate market resiliency with impulse-response functions (Degryse et al. (2005), Wuyts (2011)). The paper is extending the analysis with focusing on the variation of the order book shape instead of considering only small number of measures about tightness or market depth. Our general question is how the order book changes directly after a shock and how can we describe the typical recovery process in order-driven markets. For this, using order book based stock market data from the Budapest Stock Exchange we develop an empirical model to describe the variation of the order book structure, the order flow and the implicit cost of transactions caused by a typical order shock. We apply the approach of virtual price impact functions investigated and described by Hasbrouck (1991), Lillo et al. (2003), Bouchaud and Potters (2002) and Weber and Rosenow (2005)) to capture the implicit costs of trading. We

remark that this paper does not include the evaluation of the concept of price reverting phenomena, which is also very often linked to market resiliency.

Our main academic contributions are the following. We identify three main components which significantly characterizes the structure of the order book and these components are very similar both on the bid and ask sides. These three components can be featured as a kind of shift, twist and butterfly attributions similarly to the term structure literature, however, these forms determine the structure only at the most frequently used volume levels of transactions. Very far from the best limit quotes there are no really variation in terms of the implicit cost of trade. Despite of the bid and ask sides of the order book do no move independently to each other we can measure these components as independent measures of the book structure. The impulse-response analyses serve also interesting results. According to the simulations based on the estimation of the VAR model, we find that the largest effect of the shocks are on the level of the virtual price impact. After the shock, the implicit cost of trading typically increases almost on every level, and steepens and gets humped in the period of recovery. This means that the book quickly reverts back close the best quotes, and the most significant variation belongs to the middle volume levels. Furthermore, a shock causes a decrease in arriving market orders while limit orders get more frequent shortly after the shock. The responses for the bid and ask are similar but not symmetric.

The rest of the paper is structured as the following. Section 1 briefly reviews the literature of market resiliency. We present in details our research design and the dataset in Section 2. Section 3 investigates the methodology that the paper uses, namely it considers the applied principal component analysis and the vector-autoregressive model. In Section 4 we show our empirical findings and discuss the results. Last section concludes.

1. Literature of Market Resiliency

However there are numerous studies in the market microstructure literature about the virtual price impact or the shape of the order book, and a couple of very progressive papers on market resiliency, relatively low number of researches investigate the variation of the virtual price impact in the case of order book adjustment in order-driven markets.

In the rest of this section, we briefly review the most important theoretical models related to market resiliency and the main empirical contributions and findings regarding to the VAR approach used for studying resiliency and the shape of the order book.

1.1. Resiliency in the Market Microstructure Theory

Market resiliency has a central role in the market microstructure literature from the early periods. The first studies which deal with static market liquidity dimensions (tightness, depth, breadth) and resiliency belong to Black (1971) and Kyle (1985). This enumeration was completed with the dimension of immediacy by Harris (1990), which means the time during which a certain size portfolio can be sold or bought in a certain price range; and with diversity by Kutas and Végh (2005), which shows the homogeneity of the market investors according to motivation, size, information and home country or foreign residency. However, seminal empirical work of Sarr and Lybek (2002) use bit different definitions on liquidity dimensions, the most of the literature and our study use the concepts are established in the research report of BIS (1999).

Kyle (1985)'s model implies among others that informed traders should submit small trades to hide their identity and maintain the market price unchanged. This argument leads to that traders

should split their large market orders in order-driven market and wait between submissions until the order book recovers to normal. The trade-off between submitting market order or limit order was analyzed first by Foucault (1995, 1999) in a game theoretic model. However submitting limit orders may be executed at better price, it bears execution risk and the investor faces with the winner's course problem. Foucault finds that the volatility of the assets determines the choice of investors between market and limit orders. When asset volatility is high, submitting market orders are getting more costly because investors placing limit orders ask for larger compensation of being picked off in markets with high volatility. Parlour (1998) formulates another model about the same decision problem. She models that traders take the impacts of their actions also into account. She argues that there are systematic patterns in the order placements and the transaction prices because of this endogenity, even with symmetric information relations of the traders.

Foucault et al. (2005) identify resiliency as 'by the probability that, after a liquidity shock, the spread reverts to its former level before the next transaction' (on page 1173.). They find that four factors have an effect on market resiliency. These are the proportion of patient traders, order arrival rate, waiting cost of the traders and tick size. Those factors that increase the speed of spread improvement have a positive effect on the resiliency and vice versa. The spread will decrease if the proportion of patient traders increase; or when the waiting cost of a limit order increases; or if the order arrival rate decreases. Furthermore, the reduction in the tick size will decrease market resiliency, and increase the spread. Considering also patient and impatient types of traders, Rosu (2009) explains why it is rational to place limit orders not too close to the best quote and shows how the hump-shape of the limit order book varies. Rosu's model captures market resiliency as the speed with which the bid-ask spread reverts to its minimum value. His model predicts that resiliency increases when the proportion of patient traders increases.

Altogether, the theoretical literature suggests considering market resiliency as generally the process of spread reversion. In contrast to the theory, empirical studies mostly define resiliency as

the correction of the market price (mid-price) (e.g. Garbade (1982), Dong et al. (2003)), which is also testable, however it grabs the dynamics of another dimension. According to our approach, we define market resiliency in a slightly wider sense. In our terms, *market resiliency is the recovery process of the order book in response to temporary order imbalance, involving the dynamic of the order-flow and the limit order book.*

1.2. Brief Overview on Empirical Findings

VAR models are often used in market microstructure research to model resiliency. Among the pioneers, Hasbrouck (1991) analyses the impact of large trades. He founds that the impact arrives with a lag and it is higher when the asset is not frequently traded, or if the trade size is large, or the market spread is wide. Later, Dufour and Engle (2000) extend the model of Hasbrouck (1991), by incorporating the duration between the consecutive trades to exploit information for modelling the price and trade dynamics. Their empirical model proposes that the market is more active when increased ratio of informed traders present in the market. When frequency of the transactions increases the price impact of trades and the speed of price adjustment to trade related information also increase. Moreover, the speed-up positively affects the positive autocorrelation of signed trades. Their results also verify the findings of Easley and O'Hara (1992), namely that the duration between trades can be a good indicator of the appearing market news.

Engle and Patton (2004) take also on the approach of Hasbrouck (1991) by modeling the dynamics of bid and ask prices rather as a system than a single mid-quote variable. According to them, short duration and medium volume trades have the largest impact on quote prices, the spread is mean-reverting, and traders have a greater impact on quotes in both the short and long run for the

infrequently traded stocks. They also find evidence for a strong asymmetric impact of a trade on bid and ask prices in the short run. Similarly, Escribano and Pascual (2006) conclude that an unexpected buy order has a bigger effect on average on the ask quotes, than an unexpected sell trade on the bid quote, since the buyer initiated trades are more informative. The sensitivity of the ask orders exceeds the sensitivity of the bid orders. They also take cognizance that it is worth to model the bid and ask prices together instead of modeling only the mid-price, because it causes serious loss of important information. This consideration supports us to model the bid and ask side of the limit-order book jointly.

Coopejans et al. (2003) analyzes the stochastic dynamics of liquidity and its relation to returns and volatility in a VAR model as well. In their model they use market depth on the ask and the bid side of the limit order book as the indicator of liquidity, where depth is being defined as the number of contracts k ticks away from the mid-quote. They find that the volume and liquidity is concentrated in certain points in time, so strategic order placement has a value, moreover they have shown also, that liquidity is clustering, which means that if liquidity increases on one side of the book, it will increase on the other side as well. Another interesting result was that they have pointed out, that shocks in liquidity elapses quickly, so resiliency is high during shocks, while the shock in volatility has a contemporaneous and persistent effect on liquidity, but the strongest in the first 10 minutes after the volatility shock, which also shows the high natural market resiliency.

Hmaied et al. (2006) estimate a VAR model to examine the relationship between liquidity and volatility on an emerging limit order market (the BVMT), and with the impulse response functions of the VAR model they analyze resiliency. By their results, there is a dynamic relationship between spread, depth and volatility; buyers are likely to be more information motivated than sellers, and they behave differently; and finally, the impulse response analysis shows that shocks are absorbed more quickly for frequently traded stocks than for infrequently traded ones.

Degryse et al. (2005) apply event study methods to analyze the dynamics of the limit order book on the dataset of the Paris Bourse. Related to resiliency an important finding is, that on average, the depths of the order book stay around their mean before and after aggressive orders, while the spreads return to their mean only after about twenty best limit updates. It means that the aggressive orders create a long-term price effect. They also find that there is a strong persistence in the submission of aggressive orders, but usually they take place when the spreads and depths are relatively low. Muranaga (2000) uses event study also to show how the reduction of tick size improves the efficiency of the market, and the liquidity, which he analyzes by bid-ask spread, market impact and resiliency. He also points out by correlation calculation that if the trade frequency increases, the correlation increases as well between the frequency and all of the three liquidity indicators (spread, market impact, resiliency).

Another notable research on this field is carried out by Large (2007). He constructs a continuous-time impulse response function based on intensities. He uses the Hawkes point process to model the timing of orders and cancellations. He estimates the model using the data of LSE SETS electronic limit order book for Barclays. His main finding is that an aggressive market order often happens after unaggressive market orders. Usually aggressive limit orders are replenishing liquidity on the market.

The last type of model we highlight is the model of Nolte (2008). He uses copula techniques applied to multivariate transaction processes. In his model he analyzes jointly the price changes, transaction volumes, bid—ask spreads and inter-trade durations on a tick-by-tick time scale. Nolte uses three NYSE stocks for the estimations. His main result is that one can observe the usual relationship between bid-ask spread and transaction volumes, and their positive effect on the volatility and trade arrival times. He does not find any obvious effects of inter-trade duration on the volatility and the bid-ask spread.

In the paper of Wuyts (2011) a vector-autoregressive model is built, in which he measures the resiliency of the market after an aggressive market order. He defines aggressive order by a market order, which uses at least the first three best prices of the order book. In his VAR model he incorporates different dimensions of liquidity, by having various liquidity indicators in the model. He proposes that in a market, where there isn't a market maker, who would ensure the proper liquidity of the market, the market is resilient. He confirms this statement by showing that each liquidity indicator he analyzes (depth at the best prices, spread, order book imbalances) reverts to a steady-state value within 15 orders after a shock. His other interesting result in respect to our research is the effect of the shock on the ask and the bid side of the order book is not the same. He states that the effect of the shock on the ask side is stronger.

2. Research Design

This section lists our main hypotheses and overviews the descriptive statistics of our dataset.

2.1. Hypotheses Development

According to our approach, we define market resiliency in a slightly wider sense than the literature does. In our terms, *market resiliency is the recovery process of the order book in response to temporary order imbalance, involving the dynamic of the order-flow and the limit order book.*Some of the studies use the term of 'aggressive market order' that executes all limit orders at the best quotes (bid or ask) hence it widens the market spread. The theory usually does not support any reasonable argument why a market order is aggressive, in this study we prefer to use the term shock in the sense of temporary order imbalance. However sometimes depth is also considered together with the adjustment of tightness, there is no particular attention to the variation of the entire order book. We set up our hypotheses to explore more about the formulation of order book after a shock.

Hypothesis 1. (Impact) In the case of temporary order imbalance, a bid (ask) shock at first time negatively affects the level of total cost of trade (makes it more expensive), furthermore it positively affects the steepness and negatively modifies the hump-shape of the order book. At first, we discover the direct effects of a temporary order imbalance. Whenever large volume or series of limit orders are executed, the implicit cost of trading will be higher at lower levels of trade volumes. It may not only increase the implicit cost of trade, but it also may make the structure of the book to generally steeper. Because of the shock may eliminate one or more levels form the order book, the

former hump (if it was presented before) will be closer to the best quote. We presuppose that the curvature of the book mitigates due to the shock.

Hypothesis 2. (Readjustment) At the period of recovery after a temporary order imbalance, the virtual price impact function reverses back on each levels of the order book. Our second hypothesis regards to the period of correction. To reveal the distinguishing marks of the readjustment, we model the level, steepness and hump-shape components of the limit order book. It is not trivial how the total range of the book is getting refilled. In our preliminary argument, the incomplete book starts to put on limit orders at the middle distance from the best (existing) quote. Thus, curvature of the book would be deeper. The verification of the readjustment hypothesis could confirm the suggestions of Rosu (2009)'s model.

Hypothesis 3. (Co-movement) There are co-movements in the recovery process, bid (ask) side adjustment also implies a certain degree of ask (bid) side order-book correction. Our test specifies a kind of cross reaction of the order book. Whenever a market buy (sell) shock comes, there may appear new limit order not only on ask (bid) side, but on the other side of the limit order book. It is a reasonable strategy of players, considering that while frequent series of market buy submissions would certainly divert the price, mix of market buy and limit bid orders from an informed trader is able to hide his identity and thus his extra information about the asset.

Hypothesis 4. (Asymmetry) The bid (ask) side of the limit order book reacts differently to the liquidity shocks than the ask (bid) side. Because of different upper and lower boundaries, the bid and ask side of the order book should look a little bit different. We link the reason of the differences to the distinction between buying or selling an asset. On one hand, traders prefer to submit ask limit orders on very high limit prices to win an occasional rally. On the other hand, investors use to put bid limits not as far from the mid-price (even also in logarithmic scale). We test how traders behave on the bid and ask side during the process of correction.

Hypothesis 5. (Order flow sequence) After a shock, more limit orders arrive and market orders tend to be rare until the book is adjusted. When the order book is refilled, market orders became more frequent and the number of new limit order submissions decreases. If traders act as the microstructure theory suggests, it is worth to use order splitting strategies and taking care on the current structure of the book. In consequence of that, shock would imply a notable drop of market order submission for a short period while limit orders coming. Later on, the frequency of market and limit order submissions should be equalized.

2.2. Data

Our dataset is provided by the Budapest Stock Exchange, and contains the intraday event-based — but aggregated if more than one event has occurred within one second — data of the two most liquid Hungarian stocks, the OTP and the MOL for 2012 and 2013. MOL is a company from the oil industry, and has a market capitalization of 4.6 billion EUR, while OTP is from the banking sector with a capitalization of 4.4 billion EUR. These two stocks are the most frequently traded stocks on the Budapest Stock Exchange, with an average daily traded volume of 3.3 million EUR for the MOL, and 16.5 million EUR for the OTP. The dataset contains the Adverse Price Movement data for the ask and bid sides of the limit order book, for eleven different order sizes, for those seconds of trading, when there was any kind of change in the order book. It means that each row in the dataset is reported, when there is a market buy/sell, limit bid/ask, or a cancellation on the ask/bid side of the book. Therefore we are able to trace back from the dataset what happened on the market, since we had turnover data, and we had information also about the order book, namely that what is the total value of order on the ask/bid side.

In Table 1 we show the daily number of orders in 2013, also for the MOL and for the OTP. In Table 1 the mean and median daily number of orders can be seen, also the standard deviation of the orders, and those two data, that had the maximum and minimum number of a certain order at one day.

MOL, 2013

variables	mean	median	sd	min	max	sum
Market orders	378	330	231	79	2,196	78,952
Limit bid	417	376	229	103	2,084	87,24
Limit ask	414	370	203	111	1,383	86,469
Market buy	190	166	131	27	1,345	39,767
Market sell	188	166	108	43	856	39,319
Cancel bid	176	156	91	57	534	36,798
Cancel ask	171	143	108	35	724	35,714
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OTP, 2013

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variables	mean	median	sd	min	max	sum
Market orders	1,233	1,077	694	324	6,608	257,653
Limit bid	1,289	1,155	610	393	4,939	269,491
Limit ask	1,177	1,114	436	306	3,098	246,027
Market buy	615	519	401	138	3,95	128,54
Market sell	620	552	311	188	2,666	129,686
Cancel bid	542	501	240	173	1,692	113,255
Cancel ask	440	421	161	118	1,101	92,051

Note: The table present the number of orders by day for MOL and OTP in the year of 2013. In the first two columns there are the mean and median values, while in the third column there is the standard deviation of the daily number of trades. The minimum and maximum values show those trading days' orders, which was the smallest and largest number of order values during the certain year. The 'sum' column shows the total number of trades during the year.

Table 1. Number of orders per day

In the table it can be seen, that the number of limit orders for both of the stocks has the highest number of orders compared to the market orders, and cancellations. Another interesting fact is presented in Table 1, namely that the mean and median number of orders are higher for both stocks on the bid side of the book, than on the ask side. The only exceptions are the market orders in case of the OTP stock. Furthermore, the number of orders are higher for the OTP in each order type on both side of the limit order book. This is confirmed by Table 2 as well, which shows the average time elapses between two same types of orders.

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Measure	Market buyM	Iarket sell	Limit bid	Limit ask	Cancel bid	Cancel ask
Mean	122.2	130.6	55.3	65.4	131.8	183.2
Median	38.0	40.0	22.0	28.0	50.0	74.0

MOL, 2013

Measure	Market buy	Market sell	Limit bid	Limit ask	Cancel bid	Cancel ask
Mean	150.3	152.0	68.7	69.4	163.3	168.4
Median	41.0	42.0	25.0	27.0	57.0	58.0

OTP, 2012

Measure	Market buy	Market sell	Limit bid	Limit ask	Cancel bid (Cancel ask
Mean	35.9	34.5	17.2	19.4	43.6	52.3
Median	13.0	13.0	8.0	9.0	21.0	25.0

OTP, 2013

Measure	Market buy	Market sell	Limit bid	Limit ask	Cancel bid Cancel ask
Mean	46.6	46.2	22.2	24.4	53.0 65.2
Median	15.0	16.0	10.0	12.0	23.0 29.0

Note: The following tables shows the mean and median time between two consecutive same types of orders. The time is being measured in seconds.

Table 2. Time elapsed between the same types of orders

Although the average duration between two events in case of the MOL is higher than one minute, if we look at the median value, it is lower than one minute nearly in every case. This means that in every minute there is at least one market buy and sell, one limit bid and ask order and one cancellation on both side of the book. In case of the OTP the same can be stated the difference is that not in every minute, but nearly every 20 seconds. Based on the data of Table 2, ,in the case of the OTP the average duration between two consecutive events is around 2-3 seconds, while in case of the MOL it takes approximately 5-6 seconds. OTP can be considered as more liquid, than the MOL, which is also confirmed by the two commonly used liquidity indicators – the bid-ask spread and trading volume – as it is seen in Table 3.

MOI		1'	1
MOL	mean	median	sd
2012			
midprice(HUF)	17,725.3	17,697.5	1,221.0
bid-askspread(bp)	17.4	15.1	12.7
tradingvolume(HUF)	2,833,020.1	984,845.0	10,341,593.2
2013			
midprice(HUF)	16,666.2	16,537.5	979.3
bid-askspread(bp)	17.4	15.1	12.6
tradingvolume(HUF)	2,983,225.0	1,294,550.0	7,663,352.5
OTP	mean	median	sd
2012			
midprice(HUF)	3,756.4	3,820.5	305.4
bid-askspread(bp)	9.7	8.6	6.2
tradingvolume(HUF)	3,559,022.5	1,172,381.0	9,664,776.4
2013			
midprice(HUF)	4,580.5	4,563.5	265.4
bid-askspread(bp)	9.6	8.4	6.6
tradingvolume(HUF)	3,830,489.9	1,446,219.5	10,899,541.0

Table 3. Spreads and trading volume

The spread is lower in case of OTP also in 2012 and 2013, while the traded volume is always higher in that stock. This means that OTP was more liquid in these two years, than the MOL.

Based on the literature we have also analyzed the seasonal patterns of the different order types. In the analysis we have considered the seasonality of the number of orders instead of the total volume of the orders, since in our analysis the dynamics of the different order submissions, not the volume of the trades. On Figure 1 we show the daily patterns of the different orders.

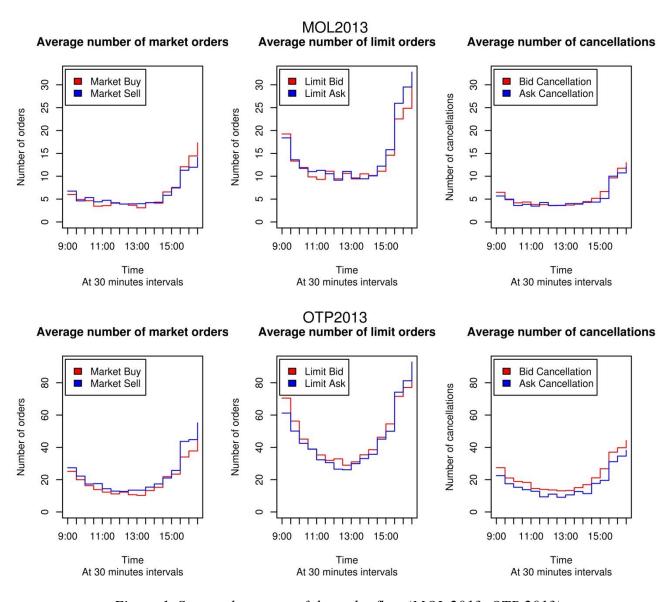


Figure 1. Seasonal patterns of the order flow (MOL 2013, OTP 2013)

In case of the market orders, the most intense order submission can be seen in the last 1-2 hours of trading, which can be the consequence that traders try to close their positions, so they are getting more aggressive by the end of the day. The same is true for the cancellations as well, since those limit orders that were not fulfilled during the trading day, the traders are cancelling as we are closer to the end of the trade. The variation is the highest in case of the limit orders. At the beginning of the trading day, the submission of limit orders is very high, the traders are building up the limit order book that time. The lowest limit order submission can be seen in the middle of the day, while at the end of the day the activity increases again.

MOL, 2013							
variables	mean	median	sd	min	max		
Market orders value	2,983,225	1,294,550	7,663,352	14,550	347,112,000		
Limit bid value	2,121,651	918,250	4,789,842	5	299,700,000		
Limit ask value	2,337,148	1,143,435	5,556,640	14,250	347,112,000		
Cancel bid value	2,723,146	925,050	11,529,551	10	589,650,425		
Cancel ask value	3,164,127	1,172,850	15,891,859	650	964,218,010		
Market buy value	2,971,805	1,294,800	7,499,190	14,550	347,112,000		
Market sell value	3,033,866	1,301,102	7,978,815	14,580	299,700,000		
		OTP, 201	.3				
variables	mean	median	sd	min	max		
Market orders value	3,830,490	1,446,220	10,899,541	4,011	1,187,500,000		
Limit bid value	3,043,887	1,343,484	6,362,034	6	510,786,000		
Limit ask value	3,124,687	1,392,000	7,128,511	196	696,150,000		
Cancel bid value	4,169,558	1,390,038	21,897,398	100	1,689,201,145		
Cancel ask value	4,556,638	1,437,075	30,200,824	750	1,694,071,422		
Market buy value	3,921,925	1,490,382	11,268,167	4,017	1,187,500,000		
Market sell value	3,766,613	1,404,660	10,581,683	4,011	898,900,000		

Note: The table presents the orders values by event in Hungarian forints for the MOL and OTP in the year 2013. In the first two columns there are the mean and median values, while in the third column there is the standard deviation of the orders' value by events. The minimum and maximum values show those orders' value, which was the smallest and largest order during the whole year. The 'sum' column shows the average total value of daily value of orders.

Table 4. Value of orders per event

In Table 4 we show the value of orders by different order types for OTP and MOL, and for 2012 and 2013. Although the number of orders were significantly different for OTP and MOL, the same cannot be said of the value of the orders. The mean and median values of OTP are always higher, than for the MOL, but it is not as notable as for the order numbers. However, we do not report the table consist data from 2012, we remark that the average and median value of orders have increased for 2013.

As it can be seen from the Table/Figure the average value of an order is around 2-3 million HUF (about 7,000-10,000 EUR) for both of the stock, while the median is around 800-1000 thousand HUF (about 2,700-3,300 EUR). We will take this into account in our analysis in case of the identification of the components which significantly characterizes the structure of the order book dynamics.

3. Methodology

In this section we propose a method how to proxy and estimate the variation of the order-book and model the dynamics. At first, we introduce the approach of price impact functions and explain how do we produce and measure these from our dataset. Afterward, we explore the order book structure with principal component analysis and identify the main components of the shape. The last part of this section details and explains the applied vector-autoregressive model.

3.1. Marginal Virtual Price Impact Function Approach

We use the Marginal Supply Demand Curve (henceforth: MSDC) framework applied by Acerbi and Scandalo (2008) to model the limit order book. Following them, we denote the function of MSDC with m(x). MSDC satisfies the following two properties:

- a) m(x) is strictly non-increasing: $m(x_1) \ge m(x_2)$, if $x_1 < x_2$.
- b) m(x) is right-continuous with left limits if x>0 and left-continuous with right limits, if x<0. The $m^{-1}(x)$ inverse function shows the x offered amount (limit bid or ask) at m price level. Negative levels of x indicate ask orders, x>0 values denote bid limits. We refer the best bid limit quote as $b = max\{m(x): x>0\}$ and the best ask as $a = min\{m(s): x<0\}$. The mid-price of the order book forms m=(a+b)/2. Acerbi and Scandolo define the average price of an order s or the Supply-Demand Curve as

$$S(s) = \frac{1}{s} \int_0^s m(x) dx$$

for any $s\neq 0$, and a remark the appropriate interpretation of the integral domain when s<0 ("ask" case). Price impact deeply depends on the market depth which relates to the number and volume of the limit orders.

In our dataset, the current state of the order-book is represented by liquidity premiums (LP) and adverse price movement (APM) indicators. These types of exchange indicators are first used in the studies of Gomber and Schweikert (2002) and Gomber et al. (2011). However the authors investigate Xetra data from the German stock market not Hungarian stock market trade data. Typically disclosed measures of the stock exchanges are the sums of the bid and ask adverse price movements at certain volume levels, and the liquidity premiums. The combined indicators (for example Xetra Liquidity Measure at the Deutsche Börse or Budapest Liquidity Measure at Budapest Stock Exchange) are computed when there is an event on the market. For example, Budapest Liquidity Measure (Kutas and Végh, 2005) is calculated as $BLM(v) = 2LP + APM^{bid}(v) + APM^{ask}(v)$. Among other exchanges, the Budapest Stock Exchange provides information about the BLM. This indicator shows the round trip execution cost of a market order which is defined by the v targeted value of the trade instead of amounts.

Liquidity premium is computed as the relative deviation of the best bid or ask form the midprice with the best bid or ask basis, in basis points

$$LP = |\frac{m_1}{m} - 1| \times 10000$$

where m_I denotes the best limit (bid or ask). Adverse price movement is reported in basis points and it formulates as the expression of

$$APM(v) = \left| \frac{S(s_v) - m_1}{m} \right| \times 10000 = \left| \frac{S(s_v)}{m} - 1 \right| \times 10000 - LP$$

where the volume related to the targeted trade value is given by

$$s_v = \{s: v = \int_0^s m(x) dx\}$$

We remark that if the adverse price movement indicator and the target value is extracted given from the data, then it is easy to calculate backward the targeted amount with the following equation:

$$s_v = \frac{v}{m(1 + LP + APM(v))}.$$

One can interpret liquidity premium as the half of the bid-ask spread in basis points. The liquidity premium may serve as a proxy of *market tightness*. Adverse price movement can be rather used as the proxy of *market depth* at different transactional volume levels.

Family of the price impact functions are often used to explore how a trade impacts on the market price (mid-price). While empirical price impact is defined as the impact of mid-price a certain period later or from the time of the trade, virtual price impact function simply focuses on the immediate effects, namely it measures the implicit trading cost. Furthermore, virtual price impact functions can be interpreted as 'snapshots' of the order book, in the ratio of the current mid-price. We define and compute virtual price impact function immediately (or virtual) price impact function in basis points can be calculated as

$$vPIF(s_v) = \frac{S(s_v)}{m} - 1 = \pm \frac{1}{10000} [LP + APM(v)]$$

where the positive sign denotes the price impact of the ask side, and the negative sign indicates it for the bid side. Similarly to Váradi (2012), we define marginal price impact in basis points as

$$mvPIF(s_v) = \frac{m(s_v)}{m} - 1$$

The levels of the order book are measured by the distance from the current mid-price. This study defines a discrete version of marginal price impact curve (MvPIF) to

$$MvPIF(s_{v_{i-1}}, s_{v_i}) = \frac{M(s_{v_i}, s_{v_{i-1}})}{m} - 1$$

where marginal supply-demand curve is approximated as the quotient of total implicit cost difference and the difference of the trade volumes

$$M(s_{v_i}, s_{v_{i-1}}) = \frac{s_{v_i} S(s_{v_i}) - s_{v_{i-1}} S(s_{v_{i-1}})}{s_{v_i} - s_{v_{i-1}}}.$$

For measuring the gradient of the order-book from the first limit quote level, we introduce a new variable which is a variant of discrete marginal virtual price impact. This measure is a marginal price impact cut at liquidity premium in basis points:

$$\pi(v_{i-1}, v_i) = \frac{1}{10000} [MvPIF(s_{v_i}, s_{v_{i-1}}) - LP].$$

It is easy to show that this indicator can be expressed with liquidity premiums and adverse price movements, such as

$$\pi(v_{i-1}, v_i) = \frac{1}{10000} \begin{bmatrix} \frac{v_i}{1 + \frac{\left(LP + APM(v_i)\right)}{10000}} APM(v_i) - \frac{v_{i-1}}{1 + \frac{\left(LP + APM(v_{i-1})\right)}{10000}} APM(v_{i-1})}{1 + \frac{\left(LP + APM(v_i)\right)}{10000}} - \frac{v_i}{1 + \frac{\left(LP + APM(v_{i-1})\right)}{10000}} \end{bmatrix}.$$

In the course of our analysis, we use the logarithm of this and denote as $\mu = ln(1 + \pi)$. Moreover, $\mu_t \in R^N$ is the vector of marginal price impact function at time t, where each number in the vector belongs to a certain level from the pre-defined trading values. Budapest Stock Exchange declares the following v value levels of the adverse price movement (in EUR): 1,000, 3,000, 5,000, 10,000, 20,000, 40,000, 70,000, 100,000, 200,000, 350,000, 500,000. To illustrate the typical shape of the price impact curve, we present a boxplot on a sample of our data regarding to MOL.

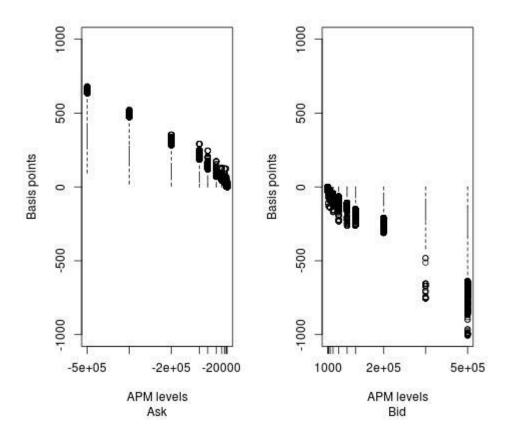


Figure 2. An illustrative figure on the marginal price impact curve, sample on MOL 2012

As one can see, price impact curve generally looks a monotonic function of the value levels. In general, the variation of the implicit cost is larger at level close to the mid-price. It is also useful to have a look at the typical marginal price impact curve means and standard deviations in a log-log plot. However bid and ask sides of the book looks similar, the structure of the bid curve is not the perfect shadow of the ask curve, bid side has a lower constraint and bid curve shows higher volatility in deeper levels than ask curve. Both bid and ask curves present high variability on the levels close to the mid-price.

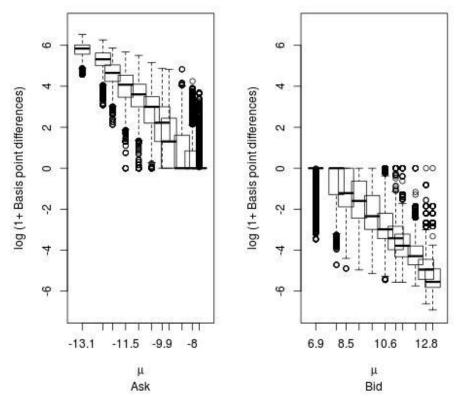


Figure 3. The μ marginal virtual price impact function (boxplot), MOL or OTP

3.2. Principal Components of the Order Book

One of the academic contributions of the paper to the literature is that it extends a vector autoregressive model by new variables that can capture describing the dynamics of the price impact function. In order to be able to quantify the structure of the dynamics of the price impact curve, we use principal component analysis (PCA) with covariance method to identify the substances of the shape. We carry out this analysis for both the OTP and MOL, and for 2012 and 2013 as well. Based on the previous chapter we will measure the price impact by the APM, since it is closely related to the notion of price impact. The APM is a measure that can capture nearly the same information about liquidity as the price impact, since the price impact can be deducted from the value of APM.

Levels Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 -0.168 0.236 -0.3370.455 -0.485 -0.143-0.407 0.471 -0.364 0.202 0.129 0.390 -0.534 -0.4020.459 -0.546 -0.1720.620 -0.512

0.521

-0.307

-0.357

0.440

-0.218

0.210

-0.677

0.333

b8		-0.133	-0.207	-0.261	-0.181	0.472
b9			-0.109	-0.135		0.272
b10						0.175
b11						0.129
sdev	1.083	0.562	0.394	0.306	0.273	0.204

-0.448

-0.304

PCA loadings, MOL 2013 Bid

b1

b2

b3

b4

b5

b6

b7

-0.397

-0.211

-0.112

-0.565

-0.391

-0.205

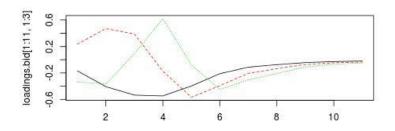
	PCA loadings MOL 2013 Ask						
Levels	Comp.1	Comp.2	Comp.3	Comp.4	Comp.5	Comp.6	
a1	-0.160	0.232	-0.325	0.407	-0.498		
a2	-0.384	0.469	-0.382	0.228			
a3	-0.526	0.410		-0.366	0.514		
a4	-0.554	-0.131	0.627		-0.505		
a5	-0.416	-0.567		0.551	0.392	-0.205	
a6	-0.218	-0.393	-0.450	-0.322		0.682	
a7	-0.116	-0.204	-0.298	-0.369	-0.194	-0.375	
a8		-0.130	-0.199	-0.264	-0.158	-0.490	
a9			-0.109	-0.141		-0.265	
a10						-0.151	
a11							
sdev	1.110	0.589	0.412	0.320	0.286	0.214	

Table 5. MOL 2013 – principal components loadings for (bid and ask sides)

In Table 5 we show the results of the PCA principal component analysis for the MOL in 2013. As it can be seen we analyze the bid and ask side independently in the PCA, since first we have analyzed them together, but the dynamics of the bid and ask side has been separated in different components. Based on this we have done the final analysis separately for the two sides of the limit order book. We analyze the cross-correlation among the components and lagged components jointly in the vector autoregressive model, so we will be able to show the interaction of the bid and ask side.

Figure 4 contains the three figures of the three most important components. The figure visualizes the PCA loadings of first three components, those that had the greatest explanatory power. On the *x* axis the eleven APM levels can be found, while the *y* axis shows the correlations between the APM data.

Despite the analysis contains all eleven levels of APM, we take into account only the levels of 1-7, since as we have pointed out earlier, the mean and median value of the orders are around at 7-10th level, while the median was around at the 3rd level. As a consequence, very far from the best limit quotes there are no really variation in terms of the implicit cost of trade, so these levels do not provide us information about the dynamics of liquidity.



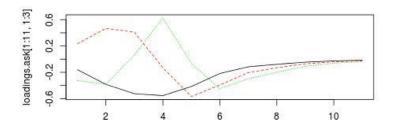
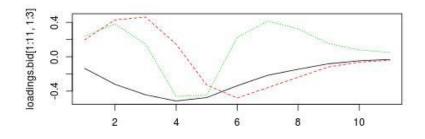


Figure 4.: MOL 2013: graphical illustrations of the three main components (bid, askloadings)

Based on Figure 4 we identify three main components which significantly characterizes the structure of the order book and these components are very similar both on the bid and ask sides. These three components can be featured as a kind of shift, twist and butterfly attributions similarly to the term structure literature, however, these forms determine the structure only at the most frequently used volume levels of transactions. The shift can be seen on the black line, and can be interpreted as the shift in the levels of APM due to sequences of limit orders placed to several levels of the limit order book. Twist can be seen on the red dashed line. Twist in the dynamics of the price impact can be interpreted as limit orders that has been withdrawn from the best price levels, and placed on worse ones. The green dot line is the butterfly, which can be interpreted as the limit orders are being withdrawn, and being placed again on better price levels, and even to price levels that were worse than the level where the orders has been withdrawn from.



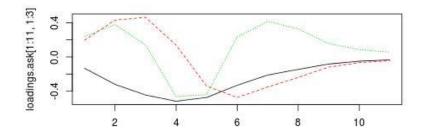


Figure 5.: OTP 2013: illustrations of the three main components (bid, ask) loadings

It is interesting to point out, that the dynamics on the ask and on the bid side are nearly similar in case of the MOL in 2013. We have analyzed also the APM of the OTP stock, and we have found the same patterns as in the case of the MOL. The results can be seen in Figure 5.

3.3. Vector-autoregressive Model of the Book

Here we set up a vector-autoregressive (VAR) model to describe market resiliency as the recovery of market characteristics to normal features. We choose the best bid and ask quotes, the components of the marginal price impact dynamics, and the time elapsed between market events (limit and market submissions separately). Our model has common roots with the model presented by Wuyts (2011). However, in contrast to Wuyts's model, we use the components of marginal price impact dynamics to describe the recovery process of the order book, instead of other market depth proxies. Furthermore, we use the duration of market events by the type of the orders (market or limit) in our model. Lastly, we look for the readjustment of the temporary order imbalance rather than the response for aggressive orders. We define a shock in a narrower sense. To capture market resiliency, we compute the impulse-response functions of bid and ask shocks and evaluates the responses considering marginal price impact dynamics and frequency of market events. We start our model description with the general equations. The main equation in matrix form follows

$$y_t = \sum_{l=1}^{L} Ay_{t-l} + \sum_{m=1}^{M} Bx_{t-m} + Cz_{t-1} + \epsilon_t$$

where y vector denotes the endogenous and x and z vectors mark the exogenous variables. We choose 15 periods (or events) both for both L and M for 15 periods (events), which sufficiently may covers the relevant time period (at least around 30 seconds from the shock). We choose endogenous variables too:

$$y_t = \{\Delta logAsk, \Delta logBid, Book. Components_t, logDUR(Orders_t)\}.$$

where we use logarithmic duration between events as

$$Orders_t = \{Market_t, Limit_t\}$$

and components of the order book dynamics:

 $Book. Components_t$

=
$$\{Bid. Shift_t, Bid. Twist_t, Bid. Butterfly_t, Ask. Shift_t, Ask. Twist_t, Ask. Butterfly_t\}$$

Our dataset does not contain the event type. We use the following set of rules of thumb to identify market events. Market order appears if there is an increase of turnover and decrease of the number of limit orders in the book at the same time. When last traded price is over the mid-price, it indicates buy order vice versa. Increments of the number of limit order in the book signs new limit orders. Changes in the number of bid and ask limit offers clearly shows whether the arrived limit isis bid or ask. Cancellation of limit orders can be identified as the decrease of the number of limit orders when there is no growth in turnover. Number of bid and ask limits in the book provide the type of the order. Because of *logAsk* and *logBid* variables are cointegrated, we put into the equation the difference between *logAsk* and *logBid* as an exogenous variable denoted by z, such as:

$$z_{t-1} = logSpread_{t-1}$$

We note that logSpread variable is lagged. The literature suggests involving the duration of the market events into the vector-autoregressive model. We use two proxies for the time elapsed between two events, one for the market orders, and one for the limit orders. These are not strongly interconnected to each other and may contain extra information of the character of order flows. In the model, we use the following variables as the exogenous variables are:

$$x_t = \{logOrderSize_t, D_t^{AO}, D^{Time}\}.$$

where OrderSize vector forms as

$$OrderSize_t = \{MOSize_t^B, MOSize_t^S, LOSize_t^B, LOSize_t^A, COSize_t^B, COSize_t^A\}$$

The dummies signaling aggressive orders are

$$D_t^{AO} = \{MOAggr_t^B, MOAggr_t^S, LOAggr_t^B, MOAggr_t^A\}$$

and time dummies indicates the 30 minutes sections on a day, such as

$$D^{Time} = \{T_{01}, T_{02}, \dots T_{15}\}$$

for controlling on intraday patterns. For avoiding perfect multicollinearity, we omit T16 dummy from the model. The abbreviations are detailed in Table 6.

Variable name	Timeslot
T_{01}	9:00-9:30
T_{02}	9:30-10:00
T ₁₅	16:00-16:30
T ₁₆	16:30-17:00

Table 6. Time dummies and covered timeslots

4. Empirical Analysis

4.1. Estimation of the VAR Model

We estimate the VAR model for MOL and OTP as well. For MOL we will estimate it for 2012 and 2013, while for OTP for the first quarter of 2013. We estimate the models separately on both of the assets. However, later on we plan to use one-month subsamples and average the results to get more robust results.

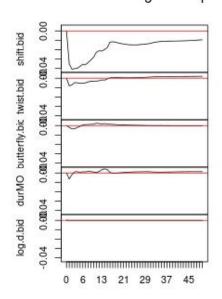
4.2. Impulse-response Dynamics

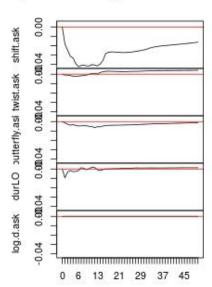
We shock the VAR model through by one-sigma shocks of 'log delta bid' and 'log delta ask' variables separately. This implies two cases. However, there are some interrelations between the bid and ask quotes, which generates common features to them. For the simpler interpretation of our results, we only plot our impulse-response function simulations using only the MOL 2013 dataset. We plot the endogenous variables and analyze the movement of the variables. The bid and ask responses are next to each other.

The first pair of the plots (Figure 6.) represents the original estimations. The most of the variations appear at the change on the level of the marginal price impact. The direct effect of the shock can be identified as the 'level' components suddenly rise, and this holds for some events while new limit orders are submitted to the book. Bid level component clearly indicates that when the limit orders effectively fill the order book, the level of marginal price impact starts to decrease. At the tipping point of the reversion, other endogenous variables behave differently. The level components do not converge exactly to zero in longer terms, but the levels of the bid and ask sides

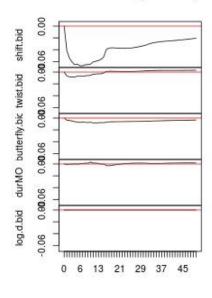
are stabilized. Duration of the market order submissions and duration of the limit order submissions are also notable from the response point of view.

Orthogonal Impulse Response from log.d.bid





Orthogonal Impulse Response from log.d.ask



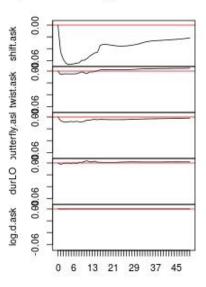
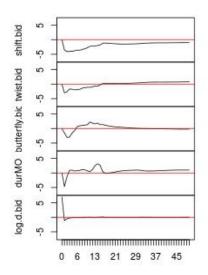
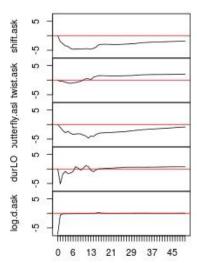


Figure 6. Impulse-response functions caused by bid and ask shocks

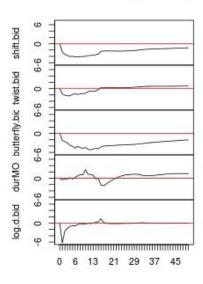
The second pair of plots (Figure 7.) shows the smaller variations in visible form. The bid (ask) shock generates a common jump to ask (bid).

Orthogonal Impulse Response from log.d.bid





Orthogonal Impulse Response from log.d.ask



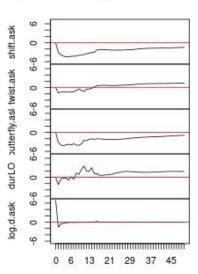


Figure 7. Impulse-response functions caused by bid and ask shocks:

plotting normalized variables

We can group the simulated trajectories into four time periods after the shock which is in period one. The first (2-6th events) is the direct effect of the shock when the implicit cost of trading suddenly increases, and there are quick series of limit orders arrive to the book. In parallel, the order book became more hump shaped (butterfly variables) and less steeper (twist variables). The second (7-15th events) is the time of 'waiting', the speed of both market and limit submissions slows down. By this time, the curvature of the marginal price impact function reduces and the significance of the twist movement decreases. The third period (16-24th events) is about the revival of the market. The market orders arrive more frequently while the shape of the book does not vary significantly. Some smaller pikes can be observed in this period at bid and ask variables, which may signs the echo, or the information effect of the market liquidity shock. The fourth period (25-50) belongs to the steady state. There are no more significant changes of the investigated exogenous variables.

We briefly reflect on our hypotheses. In this version of our paper, we do not go into details about the estimated coefficients of the VAR model. However, many of our hypotheses can be answered properly by the further analysis of the estimations.

According to our first hypothesis, Hypothesis 1, there is a significant impact of a temporary imbalance of the order book, both from the bid and the ask shock. As we identified the first period after the shock, the direct effect culminates when the most important variation, the level of the order book drops gradually back.

Hypothesis 2 refers to the readjustment of the order book. As we can see, the second and the third periods describe well the changes on the order-book structure. However, the change of the curvature (butterfly) component cannot be observed uniformly in these periods at the case of bid and ask shocks, level and twist components move similarly.

In Hypothesis 3, we formulated that there should be some co-movement between the bid and ask side of the order book. However, our econometric model cannot split the effects of a bid (ask)

shock into independent parts (pure bid or pure ask shocks) and their interrelations, it is common whenever a shock is coming for a bid (ask) side, then mid-price moves hence ask (bid) side will change by our definitions. What is more, our impulse-response analysis propose that a bid (ask) shock may invoke extra limit order flow on the ask (bid) side of the book.

Our result regarding Hypothesis 4, is that the asymmetrical responses in bid and ask side can be only weakly verified by our analysis. There are very few differences on the qualitative terms of the bid and ask order-book readjustments, the key distinct points can be the variation of the curvature (hump shape dynamics) and some of the intensity. At the stage of our research, we cannot identify more deviations causing by the different feature of the upper and lower boundary of the ask and bid sides.

Hypothesis 5, the order flow sequence relates to the durations of the market and limit order submissions. As it is described before, limit and market order submissions arrive differently in the particular periods of recovery. A notable finding that in the second period of the adjustment both market and limit order flow get slower. It may mean that market order submitting players slowly realize the lack of liquidity and limit order submitting players became more cautious for a while with giving orders looking the decrease in the speed of market orders.

Summary

In this paper we analyze the resiliency on the Budapest Stock Exchange, based on an event-based database of the Budapest Liquidity Measure. We use the most widespread methodology in the literature, the vector autoregressive analyses, to quantify resiliency. Our contribution to the literature of market resiliency is that we take not only the static variables of the limit-order book into account to measure resiliency, but the dynamic behavior of the book. Using principal component analysis we explore and identify through a principal component analysis three main components that significantly characterize the dynamic structure of the order book. These components can be featured as a kind of variants of shift, twist and butterfly attributions, or rather a level, a linear and a quadratic feature of the marginal virtual price impact, similarly to the term structure of interest rates literature. We also show that these components can be interpreted as same for the ask and bid side of the limit order book, and also same for both of the stocks (MOL and OTP) in our analysis. Although these components' attributes define the dynamics structure of the limit order book at the most frequently used volume levels of transactions, we think that these setting components are able to capture properly the dynamics of the book. We set up a vectorautoregressive model in harmony with the literature suggestions and we extend our model with the separately estimated market and limit order durations and the structure components of the order book (or marginal price impact function). We characterize the process of the correction with four periods: 1) direct effect, 2) waiting, 3) revival and 4) steady state. Movements of the shift, or twist and butterfly components look reasonable. In future research we think it would worth to extend our analysis to illiquid assets as well, and for other period, like for example for the time of the crisis, to show that the dynamics of the limit-order book, and market resiliency is independent from these factors, and our results can be interpreted as a general feature of market liquidity. Further development of the VAR model and impulse-response simulations, and robustness checks are also among in our future plans.

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