# Information Processing Structure of Quantum Gravity

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**Abstract**

The theory of quantum gravity is aimed to fuse general relativity with quantum theory into a more fundamental framework. The space of quantum gravity provides both the non-fixed causality of general relativity and the quantum uncertainty of quantum mechanics. In a quantum gravity scenario, the causal structure is indefinite and the processes are causally non-separable. In this work, we provide a model for the information processing structure of quantum gravity. We show that the quantum gravity environment is an information resource-pool from which valuable information can be extracted. We analyze the structure of the quantum gravity space and the entanglement of the space-time geometry. We study the information transfer capabilities of quantum gravity space and define the quantum gravity channel. We reveal that the quantum gravity space acts as a background noise on the local environment states. We characterize the properties of the noise of the quantum gravity space and show that it allows the separate local parties to simulate remote outputs from the local environment state, through the process of remote simulation. We characterize the information transfer of the gravity space and the correlation measure functions of the gravity channel. We investigate the process of stimulated storage for quantum gravity memories, a phenomenon that exploits the information resource-pool property of quantum gravity. The results confirm the perception that the benefits of the quantum gravity space can be exploited in quantum computations, particularly in the development of quantum computers.

**Keywords:** quantum gravity, quantum computation, quantum Shannon theory.

# 1 Introduction

In general relativity, processes and events are causally non-separable because the causal structure of space-time geometry is non-fixed. In a non-fixed causality structure, the sequence of time steps has no interpretable meaning. In our macroscopic world, events and processes are distinguishable in time and, thus, causally separable because the space-time geometry has a deterministic causality structure. The meaning of time evolution is also non-vanishing and has an interpretable notion in the microscopic world of quantum mechanics. It is precisely the reason why classical and quantum computations are evolved by a sequence of time steps and why the term *time* has an interpretable and plausible meaning in the macro- and microscopic levels. A fundamental difference between the nature of events of general relativity and quantum mechanics is that although the theory of general relativity provides a non-fixed causal space-time structure with deterministic events, in quantum mechanics, the space-time geometry has a fixed, deterministic causality structure whereas the events are nondeterministic. Quantum gravity is provided to fill the gap between these two fundamentally different theories. The theory of quantum gravity combines the results of general relativity with quantum mechanics to construct a more general framework. In quantum gravity, the causal structure is non-fixed, and the events are probabilistic [1–7]. In the quantum gravity space, the computations and the information processing steps are interpreted without the notion of time evolution. This space-time structure allows us to perform *quantum gravity computations* and to build *quantum gravity computers*, which fuse the extreme power of quantum computations and the non-fixed causality structure of general relativity [4]. The space of quantum gravity can be further exploited in quantum communication protocols, in quantum AI, in quantum error correction, and particularly in the development of quantum computers [8–33], [41–51].

Besides the attractive properties of quantum gravity theory, the appropriate characterization of the information processing structure of the quantum gravity space is still missing. In this work, our aim was to provide a model for the information processing structure of quantum gravity. We show that the quantum gravity space acts as an *information resource-pool* and reveal that the quantum gravity space stimulates a noisy map on the local environment states of independent, physically separated local maps. This *background noise* of the quantum gravity space allows the local parties to simulate remote, physically separated processes in the quantum gravity space, in a probabilistic way. We call this process *remote simulation*, an event that can be accomplished only as a coin tossing in a fixed causality structure. We also study the entangled space-time structure of quantum gravity and define the partitions over which the information flow between the separated processes is possible. We characterize the properties of the *quantum gravity channel* and the information transmission capability of the quantum gravity space by the tools of quantum Shannon theory. We introduce the terms *quantum gravity memory* and *stimulated storage*, which allow for the generation and storage of qubit entanglement exploiting the information resource-pool property of the quantum gravity space.

This paper is organized as follows. Section 2 provides the entanglement structure of the quantum gravity space, the information resource-pool property of quantum gravity, and the structure of the quantum gravity channel. Section 3 studies the information flow through the quantum gravity environment and characterizes the correlation measures. Section 4 provides a quantum gravity memory and introduces the term stimulated storage. Finally, Section 5 concludes the paper.

## 2 Information Processing of Quantum Gravity

**Theorem 1** (Entangled structure of the quantum gravity environment). *The space-time geometry (quantum gravity environment $\mathcal{G}_E$) formulates an entangled structure with $E_i B_j$, where $E_i$ is the local environment, and $B_j$ is the remote output of local maps $\mathcal{M}_A$ and $\mathcal{M}_B$, $i \neq j$. The $\rho_{\mathcal{G}_E E_i B_j}$ entangled structure stimulates a non-fixed causality between the local processes $\mathcal{M}_A$ and $\mathcal{M}_B$.*

*Proof*

The proofs throughout this work assume two qubit maps $\mathcal{M}_A$ and $\mathcal{M}_B$, $i = 1,2$, with qubit quantum gravity environment state $\mathcal{G}_E$. Specifically, the utilization of qubit channels is a required condition of the existence of a non-fixed causality structure between independent local completely positive, trace preserving (CPTP) [34–40] maps $\mathcal{M}_A$ and $\mathcal{M}_B$, which follow from the property of the shift-and-multiply unitaries [11].

The local CPTP maps $\mathcal{M}_A$ and $\mathcal{M}_B$ are independent, physically separated maps, with uncorrelated inputs $A_1$ and $A_2$. The local input is denoted by $A_i$, and the local outputs and environments are denoted by $B_i$ and $E_i$, respectively. The remote output is referred to as $B_j$, $j \neq i$. The inputs can convey classical or quantum information, both the same type. A local $\mathcal{M}_i$ can be decomposed into the local logical channel $\mathcal{N}_{A_i B_i}$, which exists between the input $A_i$ and the output $B_i$, and the local complementary channel $\mathcal{N}_{A_i E_i}$, which connects the input $A_i$ with the local environment state $E_i$. Both $\mathcal{N}_{A_i B_i}$ and $\mathcal{N}_{A_i E_i}$ are qubit maps. In particular, for modeling purposes, we also introduce a $C$ qubit state, which identifies the realizations of the two local maps $\mathcal{M}_A$ and $\mathcal{M}_B$ by qubit states $C \in \{|0\rangle, |1\rangle\}$.

Let $p = \frac{1}{2}$ be the probability of each map. Assuming a fixed causality, system $C$ can be modeled as a $d = 2$ dimensional system with density

$$\rho_C = \tfrac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right). \tag{1}$$

If the causality is non-fixed between the two local maps $\mathcal{M}_A$ and $\mathcal{M}_B$, then $C$ can be characterized by the superposition qubit state $C = |+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$, leading to the density

$$\rho_C = \tfrac{1}{2}\left(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|\right). \tag{2}$$

Our investigation here is that the quantum gravity environment $\mathcal{G}_E$, which models the space-time geometry (Theorem 3 will reveal that the local environment states also must be qubit states), does exactly the same controlling mechanism as a superposition qubit state $C = |+\rangle$. However, there is a fundamental difference between systems $C$ and $\mathcal{G}_E$. Although $C$ can be modeled by as a separable qubit state, in the quantum gravity setting, $\mathcal{G}_E$ is a subsystem of an entangled tripartite system $\rho_{\mathcal{G}_E E_i B_j}$, where the quantum gravity environment $\mathcal{G}_E$ is entangled with the two-qubit system $E_i B_j$ via partition $\mathcal{G}_E - E_i B_j$, that is, the system $\mathcal{G}_E$ is non-separable from $E_i B_j$. This injects a fundamental difference between our model and that studied in [11] because, in our model, the simultaneous realizations of the local maps $\mathcal{M}_A$ and $\mathcal{M}_B$ are a consequence of the entangled tripartite qubit system $\rho_{\mathcal{G}_E E_i B_j}$ and a dedicated qubit superposition control system $C$ does not exist.

However, the control state formalism $C = |+\rangle$ still can be utilized to model the vanishing causality of the $\mathcal{M}_A$ and $\mathcal{M}_B$ local maps in our model, as it will be shown in Section 4.

Specifically, taking the Kraus operators of the local channels $\mathcal{N}_{A_1 E_1}$ and $\mathcal{N}_{A_2 B_2}$ of maps $\mathcal{M}_A$ and $\mathcal{M}_B$

$$\mathcal{N}_{A_1 E_1}(\rho) = \sum_i A_i^{A_1 E_1} \rho \left( A_i^{A_1 E_1} \right)^\dagger, \tag{3}$$

$$\mathcal{N}_{A_2 B_2}(\rho) = \sum_j A_j^{A_2 B_2} \rho \left( A_j^{A_2 B_2} \right)^\dagger \tag{4}$$

a CPTP map $\mathcal{M}_{\mathcal{G}}$ can be introduced that describes the parallel realizations of the local channels $\mathcal{N}_{A_1 E_1}$ and $\mathcal{N}_{A_2 B_2}$. This map is defined as follows:

$$\mathcal{M}_{\mathcal{G}}(\rho) = \sum_{i,j} A_i^{\mathcal{G}} \rho \left( A_i^{\mathcal{G}} \right)^\dagger, \tag{5}$$

where the Kraus operator $A_i^{\mathcal{G}}$ is expressed as

$$A_i^{\mathcal{G}} = |0\rangle\langle 0| \otimes A_i^{A_1 E_1} \otimes A_j^{A_2 B_2} + |1\rangle\langle 1| \otimes A_j^{A_2 B_2} \otimes A_i^{A_1 E_1}. \tag{6}$$

The local environment state and remote outputs $E_1$ and $B_2$ of $\mathcal{M}_A$ and $\mathcal{M}_B$ are entangled with the quantum gravity environment state $\mathcal{G}_E$, formulating a mixed tripartite entangled qubit system $\rho_{\mathcal{G}_E E_1 B_2}$, in which $E_1$ is separable from $\mathcal{G}_E B_2$, $B_2$ is separable from $\mathcal{G}_E E_1$, and $\mathcal{G}_E$ is entangled with $E_1 B_2$. Together with the local environment $E_2$ and remote output $B_1$, systems $\rho_{\mathcal{G}_E E_1 B_2}$ and $\rho_{\mathcal{G}_E E_1 B_2}$ formulate the density matrix

$$\rho = \tfrac{1}{2} \rho_{\mathcal{G}_E E_1 B_2} + \tfrac{1}{2} \rho_{\mathcal{G}_E E_2 B_1}. \tag{7}$$

Focusing on the tripartite system $\rho_{\mathcal{G}_E E_1 B_2}$ throughout, the following conditions have to be satisfied for the partitions $\mathcal{G}_E - E_1 B_2$, $E_1 - \mathcal{G}_E B_2$, and $B_2 - \mathcal{G}_E E_1$.

Because the local subsystems $E_1$ and $B_2$ have to be separable from the partitions $\mathcal{G}_E B_2$ and $\mathcal{G}_E E_1$, in this tripartite system, only the quantum gravity environment $\mathcal{G}_E$ can be entangled with $E_1 B_2$, and all other partitions have to be separable with respect to $E_1$ and $B_2$. From these, it clearly follows that the partitions $E_1 - \mathcal{G}_E B_2$ and $B_2 - \mathcal{G}_E E_1$ have to be separable, and $\mathcal{G}_E - E_1 B_2$ has to be entangled.

Without loss of generality, we define a tripartite qubit state that simultaneously satisfies these conditions as

$$\rho_{\mathcal{G}_E E_1 B_2} = \Omega \cdot \xi + \left(1 - \Omega\right)\chi, \tag{8}$$

where

$$\Omega \leq \tfrac{1}{3}. \tag{9}$$

We further evaluate $\rho_{\mathcal{G}_E E_1 B_2}$ in Equation (8) as

$$\begin{aligned}\rho_{\mathcal{G}_E E_1 B_2} = & \\ & \tfrac{1}{2}\Omega\left(\left|000\right\rangle\left\langle 000\right| + \left|110\right\rangle\left\langle 110\right|\right) + \\ & \tfrac{1}{2}\left(\tfrac{1}{2} - \tfrac{1}{2}\Omega\right)\begin{pmatrix}\left|000\right\rangle\left\langle 110\right| + \left|110\right\rangle\left\langle 000\right| + \left|001\right\rangle\left\langle 001\right| + \\ \left|011\right\rangle\left\langle 011\right| + \left|101\right\rangle\left\langle 101\right| + \left|111\right\rangle\left\langle 111\right|\end{pmatrix},\end{aligned} \tag{10}$$

where $\rho_{\mathcal{G}_E E_1}$ is a separable Bell diagonal state, which can be expressed as

$$\begin{aligned}\rho_{\mathcal{G}_E E_1} = & \tfrac{1}{2}\left(\tfrac{1}{2} + \tfrac{1}{2}\Omega\right)\left(\left|00\right\rangle\left\langle 00\right| + \left|11\right\rangle\left\langle 11\right|\right) + \\ & \tfrac{1}{2}\left(\tfrac{1}{2} - \tfrac{1}{2}\Omega\right)\left(\left|00\right\rangle\left\langle 11\right| + \left|11\right\rangle\left\langle 00\right|\right) + \\ & \tfrac{1}{2}\left(\tfrac{1}{2} - \tfrac{1}{2}\Omega\right)\left(\left|01\right\rangle\left\langle 01\right| + \left|10\right\rangle\left\langle 10\right|\right),\end{aligned} \tag{11}$$

and in matrix form as

$$\rho_{\mathcal{G}_E E_1 B_2} = \frac{1}{2}\begin{pmatrix} \Omega & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{1}{2}\Omega & 0 \\ 0 & \frac{1}{2}-\frac{1}{2}\Omega & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}-\frac{1}{2}\Omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{1}{2}\Omega & 0 & 0 \\ \frac{1}{2}-\frac{1}{2}\Omega & 0 & 0 & 0 & 0 & 0 & \Omega & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{1}{2}\Omega \end{pmatrix}, \tag{12}$$

whereas $\rho_{\mathcal{G}_E E_1}$ can be expressed in as

$$\rho_{\mathcal{G}_E E_1} = \frac{1}{2}\begin{pmatrix} \frac{1}{2}+\frac{1}{2}\Omega & 0 & 0 & \frac{1}{2}-\frac{1}{2}\Omega \\ 0 & \frac{1}{2}-\frac{1}{2}\Omega & 0 & 0 \\ 0 & 0 & \frac{1}{2}-\frac{1}{2}\Omega & 0 \\ \frac{1}{2}-\frac{1}{2}\Omega & 0 & 0 & \frac{1}{2}+\frac{1}{2}\Omega \end{pmatrix}. \tag{13}$$

These will be referred via the partitions $\mathcal{G}_E E_1$, $\mathcal{G}_E B_2$, and $\mathcal{G}_E - E_1 B_2$ of $\rho_{\mathcal{G}_E E_1 B_2}$, respectively. In particular, for $\Omega \leq \frac{1}{3}$, the subsystems $\rho_{\mathcal{G}_E E_1}$, $\rho_{\mathcal{G}_E B_2}$, and $\rho_{E_1 B_2}$ remain separable, while $\rho_{\mathcal{G}_E}$ is entangled with $\rho_{E_1 B_2}$; thus, it straightforwardly follows that the system of Equation (10) can be used in the remaining part of the proof.

The separability conditions can be checked by taking the partial transposes $\left(\rho_{\mathcal{G}_E E_1}\right)^{T_{\mathcal{G}_E}}$, $\left(\rho_{\mathcal{G}_E E_1}\right)^{T_{E_1}}$, $\left(\rho_{\mathcal{G}_E E_1 B_2}\right)^{T_{E_1}}$, and $\left(\rho_{\mathcal{G}_E E_1 B_2}\right)^{T_{B_2}}$ of $\rho_{\mathcal{G}_E E_1 B_2}$.

The positivity of $\left(\rho_{\mathcal{G}_E E_1}\right)^{T_{\mathcal{G}_E}}$ and $\left(\rho_{\mathcal{G}_E E_1}\right)^{T_{E_1}}$ trivially follows from Equation (13) because $\rho_{\mathcal{G}_E E_1}$ is a separable Bell diagonal state.

In particular, we will show the partial transpose of $\rho_{\mathcal{G}_E E_1 B_2}$ with respect to $B_2$, which can be expressed as follows:

$$\left(\rho_{\mathcal{G}_E E_1 B_2}\right)^{T_{B_2}} = \frac{1}{2}\begin{pmatrix} \Omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2}-\frac{1}{2}\Omega & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{1}{2}\Omega & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}-\frac{1}{2}\Omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{1}{2}\Omega & 0 & 0 \\ 0 & \frac{1}{2}-\frac{1}{2}\Omega & 0 & 0 & 0 & 0 & \Omega & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{1}{2}\Omega \end{pmatrix}. \tag{14}$$

This partial transpose is non-negative; hence,

$$\left(\rho_{\mathcal{G}_E E_1 B_2}\right)^{T_{B_2}} \geq 0, \tag{15}$$

and similarly, with respect to $E_1$,

$$\left(\rho_{\mathcal{G}_E E_1 B_2}\right)^{T_{E_1}} \geq 0. \tag{16}$$

Tracing out $B_2$ from $\rho_{\mathcal{G}_E E_1 B_2}$, one can check easily that the partial transpose of the resulting matrix $Tr_{B_2}\left(\rho_{\mathcal{G}_E E_1 B_2}\right)$ with respect to $\mathcal{G}_E$ and $E_2$ is positive because $\left(\rho_{\mathcal{G}_E E_1}\right)^{T_{\mathcal{G}_E}} \geq 0$ and $\left(\rho_{\mathcal{G}_E E_1}\right)^{T_{E_1}} \geq 0$.

Specifically, the partial transpose of $\rho_{\mathcal{G}_E E_1 B_2}$ with respect to $\mathcal{G}_E$ is negative; hence,

$$\left(\rho_{\mathcal{G}_E E_1 B_2}\right)^{T_{\mathcal{G}_E}} < 0, \tag{17}$$

which immediately proves that the quantum gravity environment $\mathcal{G}_E$ (the space-time geometry) is entangled with $E_1 B_2$.

The entangled structure of quantum gravity environment $\mathcal{G}_E$ is depicted in Fig. 1. The information transmission is realized through the partition $\mathcal{G}_E - E_i B_j$.

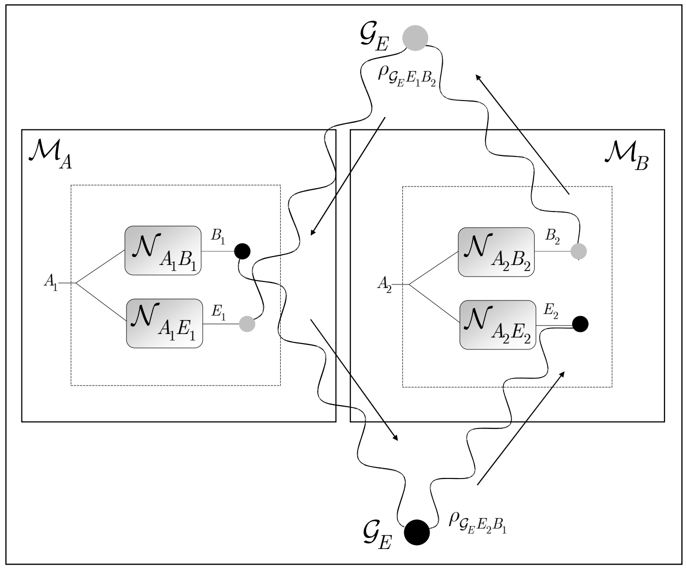


**Figure 1.** The density matrix $\rho = \frac{1}{2}\rho_{\mathcal{G}_E E_1 B_2} + \frac{1}{2}\rho_{\mathcal{G}_E E_2 B_1}$. The local environment state $E_i$ and the remote output $B_j$ of $\mathcal{M}_A$ and $\mathcal{M}_B$ are entangled with the quantum gravity environment state $\mathcal{G}_E$, via the partition $\mathcal{G}_E - E_i B_j$. The entanglement between the local environments and the quantum gravity environment (or space-time geometry) allows the parties to simulate locally the remote outputs from their local environment. (The wavy lines illustrate the entanglement; the arrow refers to the direction of the information flow.)

From the map $\mathcal{M}_{\mathcal{G}}$ of (5), it follows that the entangled structure of the density $\rho$ leads to a non-fixed causality between the local maps $\mathcal{M}_A$ and $\mathcal{M}_B$, which conclude the proof.

■

Note that the strength of the correlation of the local environment state $E_i$ and the remote output $B_j$, $i \neq j$ can be characterized by the amount of information that is transferred through partitions $\mathcal{G}_E E_1$ and $\mathcal{G}_E E_2$. These questions, along with the information transmission capabilities of the quantum gravity environment, will be revealed next.

**Theorem 2** (The information resource-pool property of quantum gravity). *Quantum gravity acts as a background noise in form of a noisy CPTP map* $\mathcal{D}^{E_i \to B_j}$ *on the local environment state* $E_i$, *which allows the parties to simulate the remote output* $B_j$ *as* $B_j = E_i \circ \mathcal{D}^{E_i \to B_j}$ with probability $p > \frac{1}{2}$. *The quantum gravity environment is an information resource-pool for the local parties.*

*Proof*

Theorem 1 has revealed that, in the quantum gravity space, the local environment $E_i$ and the remote output $B_j$ of the local maps $\mathcal{M}_A$ and $\mathcal{M}_B$ together with the quantum gravity environment $\mathcal{G}_E$ formulate an entangled tripartite qubit structure. We step forward from this point and show that the entangled $\mathcal{G}_E - E_i B_j$ structure allows the local parties to simulate the remote output $B_j$ from the local environment $E_j$ with probability $p > \frac{1}{2}$, above the classical limit $p = \frac{1}{2}$ (i.e., a coin tossing), which is precisely the case in a fixed causality structure where the local parties are independent [10].

The quantum gravity setting allows the parties with a probability $p$ to simulate the remote output from the local environment state through the local degrading map, in which the degrading map is a consequence of the quantum gravity environment. The remote output locally will be simulated from the local environment $E_i$ via the local CPTP map $\mathcal{D}^{E_i \to B_j}$, the process of which is called *remote simulation*. It means that Alice can simulate $B_2$ from her local environment state $E_1$ as $B_2 = E_1 \circ \mathcal{D}^{E_1 \to B_2}$, and vice versa, Bob can simulate Alice's output $B_1$ as $B_1 = E_2 \circ \mathcal{D}^{E_2 \to B_1}$ (*Note*: The notation $\circ$ stands for the simulation, and the local degrading map $\mathcal{D}^{E_i \to B_j}$ will be used in the right-hand side in the equations throughout.)

However, in the quantum gravity scenario, the information transmission through the partitions cannot be described by an ideal (i.e., noiseless) map; thus, the local degrading map $\mathcal{D}^{E_i \to B_j}$ can be applied only with success probability $p$. Thus, the remote simulation is a noisy process, that is, it is probabilistic. If the $I$ identity map is realized on $E_i$, then the remote simulation is not possible from $E_i$. This outcome has probability $1 - p$.

In particular, the probabilistic remote simulation process can be characterized by a CPTP map $\mathcal{M}_{\mathcal{D}}$, defined as

$$\mathcal{M}_{\mathcal{D}} = p\mathcal{D}^{E_i \to B_j} + \left(1 - p\right) I \,, \tag{18}$$

and the output of this map is as follows:

$$\begin{aligned} B_j' &= E_i \circ \mathcal{M}_{\mathcal{D}} \\ &= E_i \circ \left( p\mathcal{D}^{E_i \to B_j} + \left(1 - p\right) I \right) \\ &= pB_j + \left(1 - p\right) E_i. \end{aligned} \tag{19}$$

It is trivial that if the parties have no information about each other, then the remote output $B_j$ can be simulated from the local environment $E_i$ only with probability $p = \frac{1}{2}$; hence,

$$\mathcal{M}_{\mathcal{D}} = \tfrac{1}{2}\left( \mathcal{D}^{E_i \to B_j} + I \right) \tag{20}$$

and

$$B_j' = \tfrac{1}{2}\left( B_j + E_i \right). \tag{21}$$

This is precisely the case in a standard scenario, where the quantum gravity effects are not present. The situation changes if we step into the quantum gravity space, which leads to success probability $p > \frac{1}{2}$. To see it, we demonstrate this statement by assuming a case when both local CPTP maps $\mathcal{M}_A$ and $\mathcal{M}_B$ are the so-called entanglement-breaking channels.

The Kraus representation of the $\mathcal{M}_A$ entanglement-breaking channel is evaluated as

$$\mathcal{M}_A\left(\rho_{AA'}\right)=I\left(\rho_A\right)\otimes\mathcal{M}_A\left(\rho_{A'}\right)=\sum_i N_i^{(A')}\rho_{AA'}N_i^{(A')\dagger}, \tag{22}$$

where $\rho_{AA'}$ refers to an entangled input system, and

$$N_i^{(A')}=I_A\otimes\left|\xi_i\right\rangle_{A''}\left\langle\varsigma\right|_{A'}, \tag{23}$$

where $A'$ and $A''$ refer to the input and output systems, and the Kraus-operators $N_i^{(A')}$ are unit rank. The sets $\left\{\left|\xi_i\right\rangle_{A''}\right\}$ and $\left\{\left|\varsigma\right\rangle_{A'}\right\}$ each do not necessarily form an orthonormal set.

Thus, for an entangled input $A_i'$ of an entanglement-breaking channel $\mathcal{N}_{A_iB_i}$, it will destroy every entanglement on its local output $B_i$. Assuming a maximally entangled input system $\left|\Psi\right\rangle_{AA'}=\frac{1}{\sqrt{d}}\sum_{i=0}^{d-1}\left|i\right\rangle_A\left|i\right\rangle_{A'}$, the output of $\mathcal{M}_A$ can be expressed as follows:

$$\mathcal{M}_A\left(\left|\Psi\right\rangle\left\langle\Psi\right|_{AA'}\right)=\sum_x p_X\left(x\right)\rho_x^A\otimes\rho_x^B, \tag{24}$$

where $p_X\left(x\right)$ represents an arbitrary probability distribution, and $\rho_x^A$ and $\rho_x^B$ are the separable density matrices of the output system. The logical channel $\mathcal{N}_{A_iB_i}$ performs a complete von Neumann measurement on its input system $\rho$ and outputs $\sigma=\mathcal{N}_{EB}\left(\rho\right)$; hence, $\mathcal{N}_{A_iB_i}$ is expressed as

$$\mathcal{N}_{A_iB_i}\left(\rho\right)=\sum_x Tr\left\{\Pi_x\rho\right\}\sigma_x, \tag{25}$$

where $\left\{\Pi_x\right\}$ represents a positive operator valued measure (POVM) on $\rho$, and $\sigma_x$ is the output density matrix of the channel [40]. The local $\mathcal{N}_{A_iB_i}$ further can be decomposed into the CPTP map $\mathcal{N}_{A_iB_i}^1$, a measurement operator $\left\{\Pi_x\right\}$, and a second map $\mathcal{N}_{A_iB_i}^2$, which outputs the density matrix $\sigma_x$, together called *conditional state preparation*:

$$\mathcal{N}_{A_iB_i}=\mathcal{N}_{A_iB_i}^1\circ\Pi\circ\mathcal{N}_{A_iB_i}^2, \tag{26}$$

where $\mathcal{N}_{A_iB_i}^1=I$ and $\mathcal{N}_{A_iB_i}^2=I$.

Introducing the notation $\Pi^X$ for the $X$-basis and $\Pi^Z$ for the $Z$-basis, let the local $\mathcal{N}_{A_iB_i}$ channels be defined as follows:

$$\mathcal{N}_{A_1B_1}=I\circ\Pi_1\circ I, \tag{27}$$

where

$$\Pi_1=\tfrac{1}{2}\Pi^X+\tfrac{1}{2}\Pi^Z, \tag{28}$$

and

$$\mathcal{N}_{A_2B_2}=I\circ\Pi_2\circ I, \tag{29}$$

where

$$\Pi_2=\Pi^Z. \tag{30}$$

Let the local $\mathcal{D}^{E_i\to B_j}$ maps of $\mathcal{M}_A$ and $\mathcal{M}_B$ be defined as follows:

$$\mathcal{D}^{E_1 \to B_2} = \Pi^Z \tag{31}$$

and

$$\mathcal{D}^{E_2 \to B_1} = \Pi^Z . \tag{32}$$

Thus, each $\mathcal{D}^{E_i \to B_j}$ performs a projective measurement in the $Z$-basis on the local environment state $E_i$.

Using Equations (31) and (32) along with the local channels $\mathcal{N}_{A_1 E_1}$ and $\mathcal{N}_{A_2 E_2}$, the remote outputs $B_2, B_1$ are evaluated as

$$B_2 = \mathcal{N}_{A_1 E_1} \circ \Pi^Z \tag{33}$$

and

$$B_1 = \mathcal{N}_{A_2 E_2} \circ \Pi^Z . \tag{34}$$

For this setting, the state of $\rho_{\mathcal{G}_E E_i B_j}$ is evaluated as follows:

$$\rho_{\mathcal{G}_E E_i B_j} = \begin{cases} \rho_{\mathcal{G}_E E_2 B_1}, \text{ if } \Pi_1 = \Pi^X \\ \rho_{\mathcal{G}_E E_1 B_2}, \text{ if } \Pi_1 = \Pi^Z \end{cases} . \tag{35}$$

Thus, if $\Pi_1 = \Pi^X$, then Bob simulates Alice's output from his local environment $E_2$ through the partition $\mathcal{G}_E - E_2 B_1$ as $B_1 = \mathcal{N}_{A_2 E_2} \circ \Pi^Z$, whereas for $\Pi_1 = \Pi^Z$, Alice simulates Bob's output from $E_1$ via $\mathcal{G}_E - E_1 B_2$ as $B_2 = \mathcal{N}_{A_1 E_1} \circ \Pi^Z$.

The action of Equations (27)-(32) can be rephrased by the process matrix formalism of [10], as follows. The process matrix $W^{B_1 E_1 B_2 E_2}$ that describes the causality relations of the local maps $\mathcal{M}_A$ and $\mathcal{M}_B$ of $\rho_{\mathcal{G}_E E_i B_j}$ in the quantum gravity scenario can be expressed as

$$W^{B_1 E_1 B_2 E_2} = \tfrac{1}{4}\left( I^{B_1 E_1 B_2 E_2} + \tfrac{1}{\sqrt{2}}\left( Z^{E_1} X^{A_1} Z^{A_2} + Z^{E_2} Z^{A_1} \right)\right), \tag{36}$$

where $X$ and $Z$ are the Pauli operators. By applying the proof of Appendix E from [10], immediately yields that this process matrix identifies a causally non-separable process; and, the $p = \frac{2+\sqrt{2}}{4}$ success probability for the realization of the local degrading map $\mathcal{D}^{E_i \to B_j}$ also straightforwardly follows for $W^{B_1 E_1 B_2 E_2}$.

From these arguments, the main conclusion regarding the information resource-pool property of the quantum gravity environment can be derived. In the quantum gravity setting, the local map $\mathcal{D}^{E_i \to B_j}$ can be realized with probability $p = \frac{2+\sqrt{2}}{4}$; hence, the local map $\mathcal{M}_{\mathcal{D}}$ from Equation (20) can be rewritten as

$$\mathcal{M}_{\mathcal{D}} = \tfrac{2+\sqrt{2}}{4} \mathcal{D}^{E_i \to B_j} + \left(1 - \tfrac{2+\sqrt{2}}{4}\right) I \tag{37}$$

and

$$\begin{aligned} B_j' &= E_i \circ \mathcal{M}_{\mathcal{D}} \\ &= E_i \circ \left( \tfrac{2+\sqrt{2}}{4} \mathcal{D}^{E_i \to B_j} + \left(1 - \tfrac{2+\sqrt{2}}{4}\right) I \right) \\ &= \tfrac{2+\sqrt{2}}{4} B_j + \left(1 - \tfrac{2+\sqrt{2}}{4}\right) E_i. \end{aligned} \tag{38}$$

Thus, from the local environment $E_i$, the remote output $B_j$ can be simulated via the local map $\mathcal{M}_{\mathcal{D}}$ as $B_j = E_i \circ \mathcal{D}^{E_i \to B_j}$ with probability $p > \frac{1}{2}$. In particular, the quantum gravity environment acts as a noisy map on the local environment state and behaves as an information resource-pool for the local parties.
The model of remote simulation in the quantum gravity environment is summarized in Fig. 2.

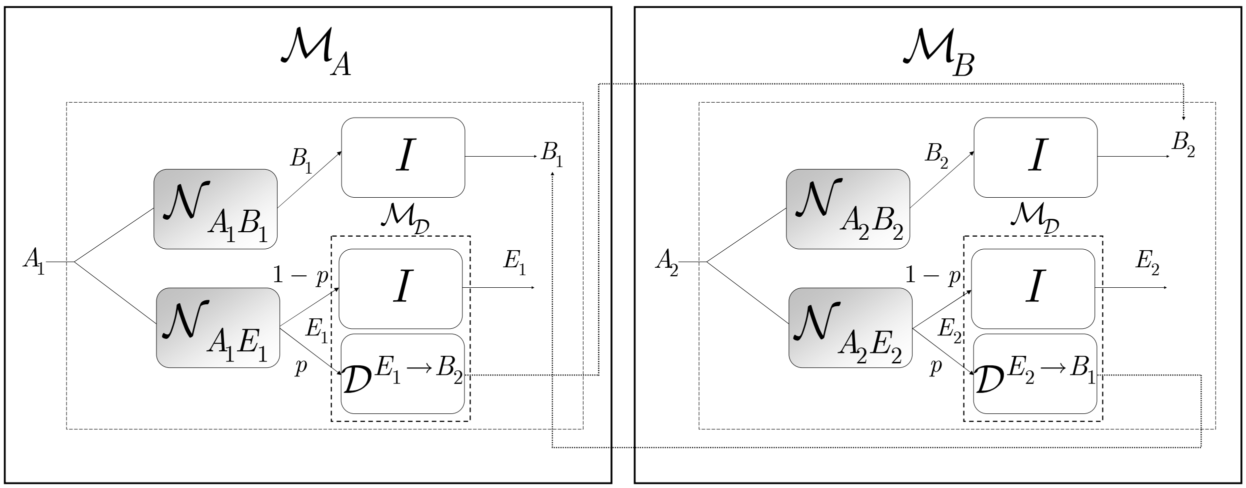


**Figure 2.** The information resource-pool property of quantum gravity. The local CPTP maps $\mathcal{M}_A$ and $\mathcal{M}_B$ are independent, physically separated maps; the inputs $A_1$ and $A_2$ are uncorrelated variables conveying classical or quantum information; and $\mathcal{D}^{E_1 \to B_2}$ and $\mathcal{D}^{E_2 \to B_1}$ are local CPTP maps (called *local degrading maps* or *background noise* of quantum gravity). The local outputs and environment states are referred to as $B_i, E_i$, $i = 1,2$, respectively. The quantum gravity setting allows the parties with a probability of $p > \frac{1}{2}$ to simulate the remote output from the local environment state through the local degrading map $\mathcal{D}^{E \to B}$. Alice can simulate $B_2$ from her local environment state $E_1$ as $B_2 = E_1 \circ \mathcal{D}^{E_1 \to B_2}$, whereas Bob can simulate Alice's output $B_1$ as $B_1 = E_2 \circ \mathcal{D}^{E_2 \to B_1}$. The quantum gravity acts as a noise on the local environments; thus, it behaves as an information resource-pool for the local parties about the remote CPTP maps.

These results confirm that, in the quantum gravity setting, there exists local independent CPTP maps, for which the local environments can be used to simulate the remote outputs with success probability $p > \frac{1}{2}$. The quantum gravity environment, indeed, acts as an information resource-pool for the local parties.

■

In Theorem 3, we reveal the structure of the quantum gravity channel that allows to model the quantum gravity space as an information transmission device between the $E_i$ local environment and the remote output $B_j$.

**Theorem 3** (The structure of the quantum gravity channel) *The local CPTP maps,* $\mathcal{N}_{A_iB_i}$, $\mathcal{D}^{E_i \to B_j}$, $i = 1,2$, $j \neq i$, *formulate the quantum gravity channel* $\mathcal{M}_{A_iB_j}$ *with remote logical channel* $\mathcal{N}_{A_iB_j} = \mathcal{N}_{A_iE_i} \circ \mathcal{D}^{E_i \to B_j}$ *and local complementary channel* $\mathcal{N}_{A_iE_i}$. *The map* $\mathcal{M}_{A_iB_j}$ *is anti-degradable, with local input* $A_i$, *remote output* $B_j$, *and local environment state* $E_i$.

*Proof*

In Theorem 2, we have seen that by exploiting the extra resources of quantum gravity, Alice can simulate Bob's output with probability $p > \frac{1}{2}$, above the standard limit $p = \frac{1}{2}$. Here, we show that it leads to a well-defined channel structure—called the *quantum gravity channel*—between Alice and Bob. The causality structure of quantum gravity space-time geometry leads to an interesting configuration, namely, it brings alive a so-called remote simulation map, which acts locally at the parties, on their local environment states. The quantum gravity channel is referred by the CPTP map $\mathcal{M}_{A_iB_j}$. The dimension of the local input $A_i$ of $\mathcal{M}_{A_iB_j}$ is denoted by $d_{A_i}$, and the dimensions of the local environment $E_i$ and the remote output $B_j$ are referred as $d_{E_i}$ and $d_{B_j}$. The map $\mathcal{M}_{A_iB_j}$ is decomposed into a logical channel $\mathcal{N}_{A_iB_j}$ that exists between the local input $A_i$ and the remote output $B_j$, and into a local complementary channel $\mathcal{N}_{A_iE_i}$, which exists between the local input $A_i$ and the local environment state $E_i$. The logical channel $\mathcal{N}_{A_iB_j}$ is referred as the *remote logical channel* of $\mathcal{M}_{A_iB_j}$ throughout, and it has the decomposition of $\mathcal{N}_{A_iB_j} = \mathcal{N}_{A_iE_i} \circ \mathcal{D}^{E_i \to B_j}$; thus, this channel could exist only with probability $p$.

The structure of the quantum gravity channel $\mathcal{M}_{A_iB_j}$ is summarized in Fig. 3.

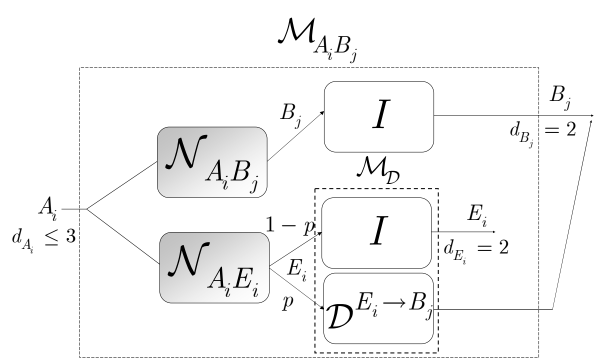


**Figure 3.** The quantum gravity channel $\mathcal{M}_{A_iB_j}$ with remote logical channel $\mathcal{N}_{A_iB_j}$ and local complementary channel $\mathcal{N}_{A_iE_i}$. The input of the channel is $A_i$, and the output is $B_j$, $i = 1,2$, $i \neq j$. The $\mathcal{M}_{A_iB_j}$ remote output channel exists with probability $p > \frac{1}{2}$, and $\mathcal{M}_{A_iB_j}$ is an anti-degradable map; thus, from the local environment state $E_i$, the remote output $B_j$ can be locally simulated by $\mathcal{D}^{E_i \to B_j}$. In $\mathcal{M}_{\mathcal{D}} = p\mathcal{D}^{E_i \to B_j} + (1-p)I$, the map $\mathcal{D}^{E_i \to B_j}$ performs the so-called remote simulation.

Because the gravity channel $\mathcal{M}_{A_iB_j}$ is an anti-degradable qubit channel, without loss of generality, the linear map of $\mathcal{M}_{A_iB_j}: M_2 \to M_2$ can be rewritten as $\mathcal{M}_{A_iB_j}: \frac{1}{2}\left(I + \sum_l w_k \rho_k\right) \to \frac{1}{2}\left(I + \sum_k \left(t_k + \lambda_k w_k\right)\rho_k\right)$, where $t_k$ and $\lambda_k$ formulate the matrix $T_{\mathcal{M}_{A_iB_j}}$ as

$$T_{\mathcal{M}_{A_iB_j}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 \\ t_3 & 0 & 0 & \lambda_3 \end{pmatrix}. \tag{39}$$

From Equation (39), $\mathcal{M}_{A_iB_j}$ can be rewritten as

$$Tr\rho_l \mathcal{M}_{A_iB_j}\left(\rho_k\right). \tag{40}$$

For the input dimension $d_A$ of the qubit gravity channel $\mathcal{M}_{A_iB_j}$ with local environment dimension $d_{E_i} = 2$, a required condition on $d_{A_i}$ immediately follows from Theorem 4 of [40], namely, $d_{A_i} \leq 3$. If $d_{A_i} = 2$, then the remote output $B_j$ can be simulated from the local environment $E_i$, $i \neq j$ because the complementary channel $\mathcal{N}_{A_iE_i}$ *of* $\mathcal{M}_{A_iB_j}$ is degradable, whereas if $d_{A_i} = 3$, then $\mathcal{N}_{A_iE_i}$ is both degradable and anti-degradable.

Furthermore, because $\mathcal{M}_{A_iB_j}$ is qubit channel, for the dimension $d_{B_j}$ of the remote output, the relation $d_{B_j} = 2$ trivially follows. The condition $d_{E_i} = 2$ on the Choi rank is satisfied only if

$$\left(\lambda_1 \pm \lambda_2\right)^2 = \left(1 \pm \lambda_3\right)^2 - t_3^2, \tag{41}$$

and

$$\begin{aligned} \lambda_3 &= \lambda_1 \lambda_2, \\ t_3^2 &= \left(1 - \lambda_1^2\right)\left(1 - \lambda_2^2\right), \end{aligned} \tag{42}$$

where $\left|\lambda_i\right| \leq 1$.

Introducing $u = v = \cos^{-1}\left(\lambda_1\right)$, the matrix in Equation (39) can be rewritten as

$$T_{\mathcal{M}_{A_iB_j}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos u & 0 & 0 \\ 0 & 0 & \cos v & 0 \\ \sin u \sin v & 0 & 0 & \cos u \cos v \end{pmatrix}, \tag{43}$$

where the anti-degradability of the qubit gravity channel $\mathcal{M}_{A_iB_j}$ implies that

$$\sin u > \cos v, \tag{44}$$

which also follows from Theorem 5 of [40]. The Kraus representation of $\mathcal{M}_{A_iB_j}$ is $\mathcal{M}_{A_iB_j}\left(\rho\right) = A_+ \rho A_+^\dagger + A_- \rho A_-^\dagger$, where

$$A_+ = \cos\tfrac{1}{2}v\cos\tfrac{1}{2}uI + \sin\tfrac{1}{2}v\sin\tfrac{1}{2}uZ = \begin{pmatrix} \cos\tfrac{1}{2}\left(v-u\right) & 0 \\ 0 & \cos\tfrac{1}{2}\left(u+u\right) \end{pmatrix}, \tag{45}$$

and

$$A_- = \sin\tfrac{1}{2}v\cos\tfrac{1}{2}uX - i\cos\tfrac{1}{2}v\sin\tfrac{1}{2}uY = \begin{pmatrix} 0 & \sin\tfrac{1}{2}\left(v-u\right) \\ \sin\tfrac{1}{2}\left(u+v\right) & 0 \end{pmatrix}, \tag{46}$$

where $X$, $Y$, and $Z$ are the Pauli operators. One can get the condition $\left|\sin v\right| \geq \left|\cos u\right|$, which is analogous to Equation (44), however, in a slightly different form.

Assume that there are two local maps, $\mathcal{M}_A$ and $\mathcal{M}_B$, in the system, with remote logical channels $\mathcal{N}_{A_1B_2}$ and $\mathcal{N}_{A_2B_1}$. Taking the superset $\mathcal{S}$ of these gravity channels, the result is a convex set because $\mathcal{S}$ formulates a supergravity channel as

$$\mathcal{S} = \tfrac{1}{2}\mathcal{N}_{A_1B_2} \otimes \left|0\right\rangle\left\langle 0\right|_F + \tfrac{1}{2}\mathcal{N}_{A_2B_1} \otimes \left|1\right\rangle\left\langle 1\right|_F, \tag{47}$$

with complementary channel

$$\mathcal{S}^C = \tfrac{1}{2}\mathcal{N}_{A_1E_1} \otimes \left|0\right\rangle\left\langle 0\right|_G + \tfrac{1}{2}\mathcal{N}_{A_2E_2} \otimes \left|1\right\rangle\left\langle 1\right|_G, \tag{48}$$

where $F$ and $G$ are elements of the Stinespring representation.

From the set $\left\{A_k^i\right\}_k$ of Kraus operators of the remote simulation map $\mathcal{D}^{E_i \to B_j}$, that is, $\mathcal{D}^{E_1 \to B_2}$ and $\mathcal{D}^{E_2 \to B_1}$, operator $A_i$ of $\mathcal{D}^{E_i \to B_j}$ is as follows:

$$A_i = A_k^0 \otimes \left|0\right\rangle\left\langle 0\right| + A_k^1 \otimes \left|1\right\rangle\left\langle 1\right|. \tag{49}$$

Applying $\mathcal{D}^{E_i \to B_j}$ on $\mathcal{S}^C$ results in

$$\begin{aligned} \mathcal{S}^C \circ \mathcal{D}^{E_i \to B_j} &= \tfrac{1}{2}\left(\mathcal{N}_{A_1E_1} \circ \mathcal{D}^{E_1 \to B_2}\right) \otimes \left|0\right\rangle\left\langle 0\right|_G + \tfrac{1}{2}\left(\mathcal{N}_{A_2E_2} \circ \mathcal{D}^{E_2 \to B_1}\right) \otimes \left|1\right\rangle\left\langle 1\right|_G \\ &= \tfrac{1}{2}\mathcal{N}_{A_1B_2} \otimes \left|0\right\rangle\left\langle 0\right|_F + \tfrac{1}{2}\left(\mathcal{N}_{A_2B_1}\right) \otimes \left|1\right\rangle\left\langle 1\right|_F \\ &= \mathcal{S}. \end{aligned} \tag{50}$$

Using Lemma 17 from [40], one can readily see that the super gravity channel $\mathcal{S}$ is anti-degradable because applying map $Tr_F$ on Equation (50) leads to

$$\begin{aligned} & Tr_F\left(\mathcal{S}^C \circ \mathcal{D}^{E_i \to B_j}\right) \\ &= Tr_F\left(\mathcal{S}\right) \\ &= Tr_F\left(\tfrac{1}{2}\mathcal{N}_{A_1B_2} \otimes \left|0\right\rangle\left\langle 0\right|_F + \tfrac{1}{2}\left(\mathcal{N}_{A_2B_1}\right) \otimes \left|1\right\rangle\left\langle 1\right|_F\right) \\ &= \tfrac{1}{2}\mathcal{N}_{A_1B_2} + \tfrac{1}{2}\left(\mathcal{N}_{A_2B_1}\right). \end{aligned} \tag{51}$$

These results conclude that the quantum gravity channel $\mathcal{M}_{A_iB_j}$ is anti-degradable and allows the parties to perform the remote simulation of outputs $B_j$ from the local environment state $E_i$ by utilizing the map $\mathcal{D}^{E_i \to B_j}$. This degrading map arises from the extra informational resource-pool property of quantum gravity, and the realization of this map is trivially not possible with probability $p > \frac{1}{2}$ in the standard scenario, where the causality is fixed and non-vanishing.

■

## 3 Information Transfer of Quantum Gravity

**Theorem 4.** (Information transfer of quantum gravity) *The quantum gravity environment allows the transfer of classical and quantum information between the local maps* $\mathcal{M}_A$ *and* $\mathcal{M}_B$. *The information flow is realized through the quantum gravity environment via the partition* $\mathcal{G}_E - E_i B_j$ *of the tripartite system* $\rho_{\mathcal{G}_E E_i B_j}$.

*Proof*

The correlation measure can be settled between subsystems $\mathcal{G}_E E_i$ and $\mathcal{G}_E B_j$. For simplicity, we will use $\mathcal{G}_E E_1$ throughout to characterize exactly the information transmission between the local environment states and the quantum gravity environment state. We derive various correlation measures for the output system $\rho_{\mathcal{G}_E E_1}$.

Specifically, in this special quantum gravity communication scenario, Alice and Bob cannot transmit directly to each other any information. Instead of a direct signaling, the degraded local environment $B_2' = E_1 \circ \mathcal{M}_{\mathcal{D}}$, see Equation (18), and the remote output $B_2$ will characterize the correlation between Alice and Bob's maps $\mathcal{M}_A$ and $\mathcal{M}_B$, despite the fact that all correlations are transmitted via the entangled quantum gravity environment. Thus, in fact, the communication is realized through the quantum gravity environment $\mathcal{G}_E$, via $\mathcal{G}_E E_i$ and $\mathcal{G}_E B_j$. The entangled Hilbert space $\mathcal{G}_E - E_i B_j$, in fact, acts as a communication channel.

Assuming the case that Alice simulates Bob's output, we introduce the CPTP map

$$\begin{aligned} \mathcal{M}\left(B_2\right) &: M_2 \to M_2 \\ &= B_2' = E_1 \circ \mathcal{M}_{\mathcal{D}} \\ &= E_1 \circ \left(p\mathcal{D}^{E_i \to B_j} + \left(1-p\right)I\right), \end{aligned} \tag{52}$$

which gets the remote output $B_2$ as input and outputs Alice's noisy $B_2' = E_1 \circ \mathcal{M}_{\mathcal{D}}$, see Equation (18) and Theorem 3. Thus, it is a noisy evolution on Bob's ideal $B_2$ that results in $B_2'$. We step forward from the results of Theorem 3 to drive the information transmission capabilities of channel $\mathcal{M}\left(B_2\right)$ by quantifying the amount of information that is conveyed by $\mathcal{G}_E E_1$, using the system defined in Equation (8). Hence, the analysis will be made from Alice's viewpoint, via subsystem $\rho_{\mathcal{G}_E E_1}$.

First, we rewrite the Bell-diagonal system $\rho_{\mathcal{G}_E E_1}$ from Equation (13) as

$$\rho_{\mathcal{G}_E E_1} = \frac{1}{4}\left(I \otimes I + \mathbf{r} \cdot \vec{\sigma} \otimes I + I \otimes \mathbf{s} \cdot \vec{\sigma} + \sum_{i=1}^{3} c_i \sigma_i \otimes \sigma_i\right), \tag{53}$$

where $\mathbf{r}$ and $\mathbf{s}$ are the Bloch vectors, $\vec{\sigma} = \left[\sigma_x, \sigma_y, \sigma_z\right]$ with the Pauli matrices $\sigma_i$, and $c_i$ is the real parameter $-1 \leq c_i \leq 1$ [36–37, 50]. For a Bell diagonal state $\mathbf{r} = \mathbf{s} = 0$. For $\mathbf{r} = \left(0, 0, r\right)$

and $\mathbf{s} = \left(0, 0, s\right)$, the input state in Equation (53) can be given in a matrix representation as follows:

$$\rho_{\mathcal{G}_E E_1} = \frac{1}{4}\begin{pmatrix} 1+r+s+c_3 & 0 & 0 & c_1-c_2 \\ 0 & 1+r-s-c_3 & c_1+c_2 & 0 \\ 0 & c_1+c_2 & 1-r+s-c_3 & 0 \\ c_1-c_2 & 0 & 0 & 1-r-s+c_3 \end{pmatrix}. \tag{54}$$

The eigenvalues $u_+, u_-, v_+, v_-$ of $\rho_{\mathcal{G}_E E_1}$ are defined as

$$\begin{aligned} v_+ &= \frac{1}{4}\left(1 - c_3 + \sqrt{\left(r - s\right)^2 + \left(c_1 + c_2\right)^2}\right) \geq 0, \\ v_- &= \frac{1}{4}\left(1 - c_3 - \sqrt{\left(r - s\right)^2 + \left(c_1 + c_2\right)^2}\right) \geq 0, \end{aligned} \tag{55}$$

$$\begin{aligned} u_+ &= \frac{1}{4}\left(1 + c_3 + \sqrt{\left(r + s\right)^2 + \left(c_1 - c_2\right)^2}\right) \geq 0, \\ u_- &= \frac{1}{4}\left(1 + c_3 - \sqrt{\left(r + s\right)^2 + \left(c_1 - c_2\right)^2}\right) \geq 0. \end{aligned} \tag{56}$$

From these eigenvalues, the $-1 \leq c_i \leq 1$ parameters of $\rho_{\mathcal{G}_E E_1}$ can be expressed as

$$c_1 = \left(v_+ - v_-\right), \tag{57}$$

$$c_2 = -\left(v_+ - v_-\right), \tag{58}$$

and

$$c_3 = 1 - 2 \cdot \left(v_+ - v_-\right) = 1 + 2 \cdot c_2. \tag{59}$$

As one can readily check, for these parameters, the relations $\left|c_1\right| + \left|c_2\right| + \left|c_3\right| \leq 1$ and $\max\left\{v_+, v_-, u_+, u_-\right\} \leq \frac{1}{2}$ hold in (54). Some trivial steps then straightforwardly yields that $\Omega$ can be expressed from the eigenvalues $v_+, v_-$ as

$$\Omega = 1 - 2\left(v_+ - v_-\right), \tag{60}$$

from which the correlations in $\rho_{\mathcal{G}_E E_1}$ can be exactly determined in function of $\Omega$.

The $I\left(\rho_{\mathcal{G}_E E_1}\right)$ mutual information function measures the total correlation in $\rho_{\mathcal{G}_E E_1}$. The mutual information function of $\rho_{\mathcal{G}_E E_1}$ can be expressed as follows:

$$I\left(\rho_{\mathcal{G}_E E_1}\right) = S\left(\rho_{\mathcal{G}_E}\right) + S\left(\rho_{E_1}\right) - S\left(\rho_{\mathcal{G}_E E_1}\right). \tag{61}$$

Using the eigenvalues of $\rho_{\mathcal{G}_E E_1}$, $I\left(\rho_{\mathcal{G}_E E_1}\right)$ can be rewritten as

$$I\left(\rho_{\mathcal{G}_E E_1}\right) = S\left(\rho_{\mathcal{G}_E}\right) + S\left(\rho_{E_1}\right) + u_+ \log_2 u_+ + u_- \log_2 u_- + v_+ \log_2 v_+ + v_- \log_2 v_-, \tag{62}$$

where $S\left(\cdot\right)$ is the von Neumann entropy and

$$S\left(\rho_{\mathcal{G}_E}\right) = 1 - \frac{1}{2}\left(1 - r\right)\log_2\left(1 - r\right) - \frac{1}{2}\left(1 + r\right)\log_2\left(1 + r\right), \tag{63}$$

$$S\left(\rho_{E_1}\right) = 1 - \frac{1}{2}\left(1 - s\right)\log_2\left(1 - s\right) - \frac{1}{2}\left(1 + s\right)\log_2\left(1 + s\right). \tag{64}$$

The amount of purely classical correlation $\mathcal{C}\left(\rho_{\mathcal{G}_E E_1}\right)$ in $\rho_{\mathcal{G}_E E_1}$ can be expressed as follows:

$$\mathcal{C}\left(\rho_{\mathcal{G}_E E_1}\right) = S\left(\rho_{E_1}\right) - \tilde{S}\left(E_1 \middle| \mathcal{G}_E\right)$$
$$= S\left(\rho_{E_1}\right) - \min_{E_k} \sum_k pkS\left(\sigma_{E_1|k}\right), \tag{65}$$

where $\rho_{E_1|k} = \frac{\langle k | \rho_{\mathcal{G}_E} \rho_{E_1} | k \rangle}{\langle k | \rho_{\mathcal{G}_E} k \rangle}$ is the post-measurement state of $\rho_{E_1}$, the probability of result $k$ is $p_k = d\left\langle k \middle| \rho_{\mathcal{G}_E} k \right\rangle$, $d = 2$ is the dimension of system $\rho_{\mathcal{G}_E}$, and $q_k$ makes up a normalized probability distribution in the rank-one POVM elements $E_k = q_k \left|k\right\rangle\left\langle k\right|$ [50].

The purely classical correlation can also be expressed by the following formula:

$$\mathcal{C}\left(\rho_{\mathcal{G}_E E_1}\right) = S\left(\rho_{\mathcal{G}_E}\right) - \min\left\{f_1, f_2, f_3\right\}, \tag{66}$$

where the functions $f_1, f_2$, and $f_3$ are defined as follows [36–37, 50]:

$$\begin{aligned} f_1 = &-\frac{1}{4}\left(1 + r + s + c_3\right)\log_2 \frac{1}{2\left(1+s\right)}\left(1 + r + s + c_3\right) \\ &-\frac{1}{4}\left(1 - r + s - c_3\right)\log_2 \frac{1}{2\left(1+s\right)}\left(1 - r + s - c_3\right) \\ &-\frac{1}{4}\left(1 + r - s - c_3\right)\log_2 \frac{1}{2\left(1+s\right)}\left(1 + r - s - c_3\right) \\ &-\frac{1}{4}\left(1 - r - s + c_3\right)\log_2 \frac{1}{2\left(1+s\right)}\left(1 - r - s + c_3\right), \end{aligned} \tag{67}$$

$$f_2 = 1 - \frac{1}{2}\left(1 - \sqrt{r + c_1^2}\right)\log_2\left(1 - \sqrt{r + c_1^2}\right) - \frac{1}{2}\left(1 + \sqrt{r + c_1^2}\right)\log_2\left(1 + \sqrt{r + c_1^2}\right), \tag{68}$$

and

$$f_3 = 1 - \frac{1}{2}\left(1 - \sqrt{r + c_2^2}\right)\log_2\left(1 - \sqrt{r + c_2^2}\right) - \frac{1}{2}\left(1 + \sqrt{r + c_2^2}\right)\log_2\left(1 + \sqrt{r + c_2^2}\right). \tag{69}$$

As follows, the $C\left(\mathcal{M}\left(B_2\right)\right)$ classical capacity of channel $\mathcal{M}\left(B_2\right)$ is

$$C\left(\mathcal{M}\left(B_2\right)\right) = \lim_{n\to\infty} \frac{1}{n} \max_{\forall \rho_{\mathcal{G}_E E_1}} I\left(\rho_{\mathcal{G}_E E_1}\right). \tag{70}$$

From the mutual information $I\left(\rho_{\mathcal{G}_E E_1}\right)$ and the classical correlation $\mathcal{C}\left(\rho_{\mathcal{G}_E E_1}\right)$, the $\mathcal{D}\left(\rho_{\mathcal{G}_E E_1}\right)$ quantum discord is as follows:

$$\begin{aligned} \mathcal{D}\left(\rho_{\mathcal{G}_E E_1}\right) &= I\left(\rho_{\mathcal{G}_E E_1}\right) - \mathcal{C}\left(\rho_{\mathcal{G}_E E_1}\right) \\ &= S\left(\rho_{\mathcal{G}_E}\right) + S\left(\rho_{E_1}\right) + u_+ \log_2 u_+ + u_- \log_2 u_- + v_+ \log_2 v_+ + v_- \log_2 v_- \\ &\quad -\left(S\left(\rho_{\mathcal{G}_E}\right) - \min\left\{f_1, f_2, f_3\right\}\right) \\ &= S\left(\rho_{E_1}\right) + u_+ \log_2 u_+ + u_- \log_2 u_- + v_+ \log_2 v_+ + v_- \log_2 v_- + \min\left\{f_1, f_2, f_3\right\}. \end{aligned} \tag{71}$$

The $I_{coh}\left(\rho_{\mathcal{G}_E E_1}\right)$ coherent information of $\rho_{\mathcal{G}_E E_1}$ can be expressed as

$$
\begin{aligned}
I_{coh}\left(\rho_{\mathcal{G}_E E_1}\right) &= \mathcal{D}\left(\rho_{\mathcal{G}_E E_1}\right) + \mathcal{C}\left(\rho_{\mathcal{G}_E E_1}\right) - 1 = \\
&= I\left(\rho_{\mathcal{G}_E E_1}\right) - \mathcal{C}\left(\rho_{\mathcal{G}_E E_1}\right) + \mathcal{C}\left(\rho_{\mathcal{G}_E E_1}\right) - 1 \\
&= I\left(\rho_{\mathcal{G}_E E_1}\right) - 1 \\
&= S\left(\rho_{\mathcal{G}_E}\right) + S\left(\rho_{E_1}\right) + u_+ \log_2 u_+ + u_- \log_2 u_- + v_+ \log_2 v_+ + v_- \log_2 v_- - 1.
\end{aligned} \tag{72}
$$

The $Q\left(\mathcal{M}\left(B_2\right)\right)$ of the map $\mathcal{M}\left(B_2\right)$ can be given as the maximization of the coherent information $I_{coh}\left(\rho_{\mathcal{G}_E E_1}\right)$ of $\rho_{\mathcal{G}_E E_1}$ as

$$
\begin{aligned}
Q\left(\mathcal{M}\left(B_2\right)\right) &= \lim_{n\to\infty} \frac{1}{n} \max_{\forall \rho_A \rho_B} I_{coh}\left(\rho_{\mathcal{G}_E E_1}\right) \\
&= \lim_{n\to\infty} \frac{1}{n} \max_{\forall \rho_A \rho_B} \left(\mathcal{D}\left(\rho_{\mathcal{G}_E E_1}\right) + \mathcal{C}\left(\rho_{\mathcal{G}_E E_1}\right) - 1\right) \\
&= \lim_{n\to\infty} \frac{1}{n} \max_{\forall \rho_{\mathcal{G}_E E_1}} \left(I\left(\rho_{\mathcal{G}_E E_1}\right) - 1\right) \\
&= \lim_{n\to\infty} \frac{1}{n} \max_{\forall \rho_{\mathcal{G}_E E_1}} \begin{pmatrix} S\left(\rho_{\mathcal{G}_E}\right) + S\left(\rho_{E_1}\right) + u_+ \log_2 u_+ + u_- \log_2 u_- + \\ v_+ \log_2 v_+ + v_- \log_2 v_- - 1 \end{pmatrix}.
\end{aligned} \tag{73}
$$

Because $\rho_{\mathcal{G}_E E_1}$ is a Bell diagonal state with $r = s = 0$, $S\left(\rho_{\mathcal{G}_E}\right) = S\left(\rho_{E_1}\right) = 1$, $Q\left(\mathcal{M}\left(B_2\right)\right)$ is simplified to

$$
Q\left(\mathcal{M}\left(B_2\right)\right) = \lim_{n\to\infty} \frac{1}{n} \max_{\forall \rho_{\mathcal{G}_E E_1}} \left(1 - S\left(\rho_{\mathcal{G}_E E_1}\right)\right). \tag{74}
$$

The results of the correlation measure analysis are summarized in Fig. 4.

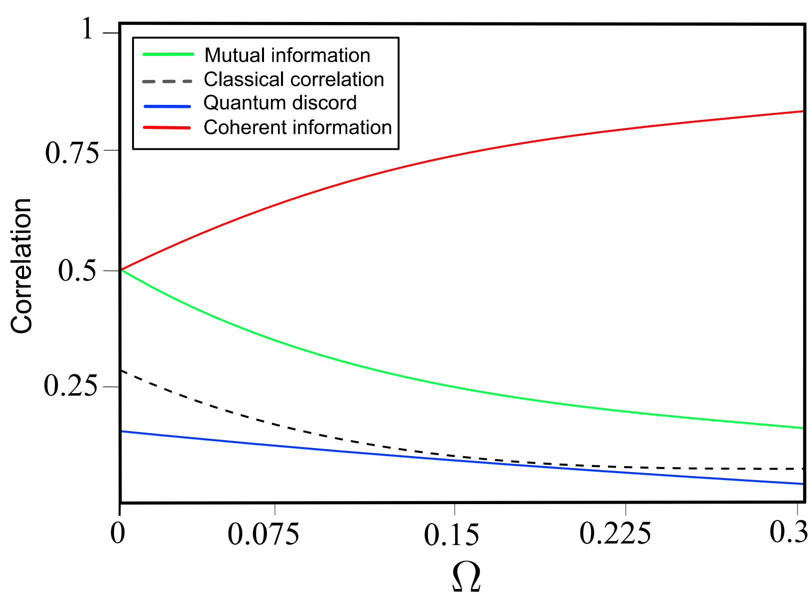


**Figure 4.** The correlation measures between the quantum gravity environment $\mathcal{G}_E$ and the local environment $E_1$, evaluated on $\rho_{\mathcal{G}_E E_1}$, in function of $\Omega$, $\Omega \leq 1/3$. As $\Omega$ increases, the quantum influences become stronger, and the coherent information strongly increases. (The coherent information is shown in the absolute value.)

The quantum gravity environment allows the transfer of classical and quantum information through the entangled partition $\mathcal{G}_E - E_i B_j$ of $\rho_{\mathcal{G}_E E_i B_j}$, which concludes that the local maps $\mathcal{M}_A$ and $\mathcal{M}_B$ can extract classical and quantum information from the quantum gravity environment.

■

# 4 Stimulated Storage in Quantum Gravity Memories

The quantum gravity scenario allows us to build quantum memories with a non-fixed causality. In this section, we propose an example for this statement. Our quantum gravity memory is a quantum SR latch (S—set, R—reset), built from a pair of cross-coupled Toffoli-NOR quantum gates.

In classical computer architectures, the SR latch (flip-flop or bistable multivibrator) is one of the most basic and fundamental storage elements and building blocks of digital electronics devices. An SR latch consists of two cross-coupled NOR gates for the storing of one-bit information, and it operates with two stable states. The SR latch has two control inputs and two signal inputs, which are the back-looped outputs of the neighboring NOR gate (called *cross coupling*). The output of the classical SR latch is controlled by the $S$ and $R$ inputs, which allows only one stable output realization, $Q$, or its complement, $\overline{Q}$. The state transitions of the cross-coupling structure have a fixed causal structure in a classical SR latch.

In particular, in a quantum gravity SR latch, both output realizations are simultaneously allowed as stable state, which makes possible the stimulated storage of a qubit entanglement $\left|\varphi\right\rangle = \frac{1}{\sqrt{2}}\left(\left|Q\overline{Q}\right\rangle + \left|Q\overline{Q}\right\rangle\right)$, utilizing the elements of the standard basis $\left|A_i\right\rangle \in \left\{\left|0\right\rangle, \left|1\right\rangle\right\}$ as inputs. The proposed quantum gravity SR latch exploits the information resource-pool property (see Theorem 2) of the quantum gravity space to preserve the entanglement.

The $C_{Toff}^{NOR}$ Toffoli-NOR qubit gate with control qubit inputs $x$ and $y$ and a target qubit $z$ can be defined as

$$C_{Toff}^{NOR} = G\left(x, y, z\right) = z \oplus \left(\overline{x} \cdot \overline{y}\right), \tag{75}$$

where $G\left(\cdot\right)$ refers to gate, and $\oplus$ stands for the XOR-operation.

The $C_{Toff}^{NOR}$ quantum circuit can be characterized by the density

$$\begin{aligned} C_{Toff}^{NOR} = & \left|000\right\rangle\left\langle 001\right| + \left|001\right\rangle\left\langle 000\right| + \left|010\right\rangle\left\langle 010\right| + \left|011\right\rangle\left\langle 011\right| + \\ & \left|100\right\rangle\left\langle 100\right| + \left|101\right\rangle\left\langle 101\right| + \left|110\right\rangle\left\langle 110\right| + \left|111\right\rangle\left\langle 111\right|, \end{aligned} \tag{76}$$

which in matrix form can be expressed as

$$C_{Toff}^{NOR} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{77}$$

The $C_{Toff}^{NOR}$ structure can be decomposed into $NOT : a \rightarrow \overline{a}$, $CNOT : (a,b) \rightarrow (a, a \oplus b)$, and $\sqrt{X}$ and $\sqrt{X}^{\dagger}$ transformations, where

$$\sqrt{X} = \frac{1}{2}\begin{pmatrix} 1+i & 1-i \\ 1-i & 1+i \end{pmatrix}, \tag{78}$$

and

$$\sqrt{X}^{\dagger} = \frac{1}{2}\begin{pmatrix} 1-i & 1+i \\ 1+i & 1-i \end{pmatrix}. \tag{79}$$

The $C_{Toff}^{NOR}$ Toffoli-NOR quantum circuit is shown in Fig. 5.

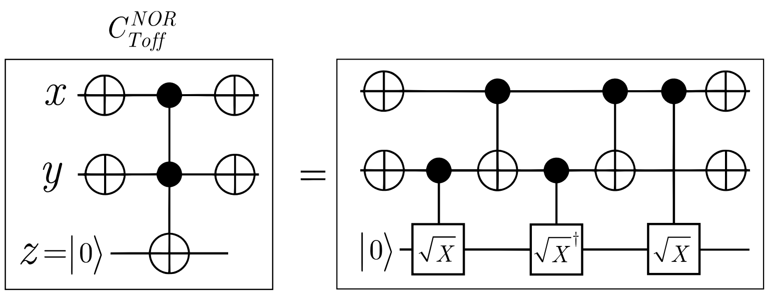


**Figure 5.** The Toffoli-NOR qubit gate. The gate has two control qubit inputs $x$ and $y$ and a target qubit $z$, which is initialized in $|0\rangle$.

The truth table of the $C_{Toff}^{NOR}$ gate is as follows:

| $x$ | $y$ | $z$ | $\overline{x} \cdot \overline{y}$ | $z \oplus (\overline{x} \cdot \overline{y})$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |

**Table 1**. The truth table of the Toffoli-NOR qubit gate.

The $\mathcal{M}_{SR}$ quantum gravity SR latch memory consists of two cross-coupled $C_{Toff}^{NOR}$ circuits, referred by the local maps $\mathcal{M}_S$ and $\mathcal{M}_R$ and defined by the following map:

$$\mathcal{M}_{SR}\left(\rho\right) = \sum\nolimits_{i,j} A_i^{SR}\rho\left(A_i^{SR}\right)^{\dagger}, \tag{80}$$

where $S \in \left\{0,1\right\}$, $R \in \left\{0,1\right\}$. The map of $\mathcal{M}_{SR}$ describes the parallel realizations of the local maps $\mathcal{M}_S$ and $\mathcal{M}_R$.

The Kraus operator $A_i^{SR}$ of Equation (80) is expressed as

$$A_i^{SR} = \left|0\right\rangle\left\langle 0\right| \otimes A_i^{A_1 Q} \otimes A_j^{A_2 \bar{Q}} + \left|1\right\rangle\left\langle 1\right| \otimes A_j^{A_2 \bar{Q}} \otimes A_i^{A_1 Q}, \tag{81}$$

where $A_i \in \left\{0,1\right\}$ is the local input, $Q \in \left\{0,1\right\}$ is the output $\mathcal{M}_R$, $\bar{Q} \in \left\{0,1\right\}$ is the output $\mathcal{M}_S$, and the Kraus operators of $\mathcal{M}_S$ and $\mathcal{M}_R$ are

$$\mathcal{M}_S\left(\rho\right) = \sum\nolimits_i A_i^{A_2\bar{Q}}\rho\left(A_i^{A_2\bar{Q}}\right)^{\dagger}, \tag{82}$$

$$\mathcal{M}_R\left(\rho\right) = \sum\nolimits_j A_j^{A_1 Q}\rho\left(A_j^{A_1 Q}\right)^{\dagger}. \tag{83}$$

The control inputs $R$, $S$ of $\mathcal{M}_S$ and $\mathcal{M}_R$ are entangled with the quantum gravity environment state $\mathcal{G}_E$. In the quantum gravity SR latch, input $R$ is separable from $\mathcal{G}_E S$ and input $S$ is separable from $\mathcal{G}_E R$; however, $\mathcal{G}_E$ is entangled with $SR$, formulating the tripartite system (see Theorem 1)

$$\rho_{\mathcal{G}_E RS} = \kappa \cdot \xi + \left(1-\kappa\right)\chi, \tag{84}$$

where $\kappa \leq \frac{1}{3}$, following the structure of (10).

The main contribution of the $\mathcal{M}_{SR}$ quantum gravity SR latch is that the non-fixed causality of the $\mathcal{G}_E$ quantum gravity structure leads to the simultaneous realizations of the $Q$ and $\bar{Q}$ outputs, which can be used as the stimulation and storage of qubit entanglement, utilizing the resource-pool property of quantum gravity (see Theorem 2).

The active $S$ and $R$ control commands are

$$\left|S\right\rangle : \left|Q\right\rangle = \left|1\right\rangle, \left|\bar{Q}\right\rangle = \left|0\right\rangle \tag{85}$$

and

$$\left|R\right\rangle : \left|Q\right\rangle = \left|0\right\rangle, \left|\bar{Q}\right\rangle = \left|1\right\rangle, \tag{86}$$

and in terms of the control state formalism, the realizations of the local maps is $C = \left|0\right\rangle : \left|S\bar{R}\right\rangle$ and $C = \left|1\right\rangle : \left|\bar{S}R\right\rangle$.

The truth table of the $\mathcal{M}_{SR}$ quantum-gravity SR-latch is as follows:

| $C$ | $S$ | $R$ | $Q$ | $\bar{Q}$ |
|---|---|---|---|---|
| 0 | 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |

**Table 2**. The truth table of the quantum-gravity SR-latch.

Initializing the circuit in $\left|A_1\right\rangle = \left|0\right\rangle, \left|A_2\right\rangle = \left|0\right\rangle$ and by the control state (see (2)) $\left|C\right\rangle = \frac{1}{\sqrt{2}}\left(\left|0\right\rangle + \left|1\right\rangle\right)$, one obtains

$$\left|C\right\rangle = \tfrac{1}{\sqrt{2}}\left(\left|S\bar{R}\right\rangle + \left|\bar{S}R\right\rangle\right), \tag{87}$$

thus, the resulting output of $\mathcal{M}_{SR}$ is evaluated as

$$\left|\varphi\right\rangle = \tfrac{1}{\sqrt{2}}\left(\left|Q\bar{Q}\right\rangle + \left|\bar{Q}Q\right\rangle\right). \tag{88}$$

The $\mathcal{M}_{SR}$ quantum gravity SR latch with quantum gravity control is depicted in Fig. 6. The system is initialized with inputs $A_i \in \{0,1\}$. The outputs $Q$ and $\bar{Q}$ are entangled, stimulated, and kept in a stable state by the quantum gravity space $\rho_{\mathcal{G}_E RS}$.

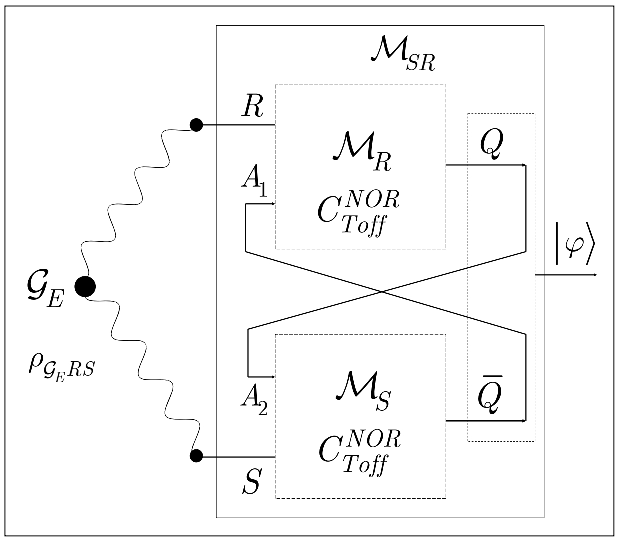


**Figure 6.** Stimulated storage via the $\mathcal{M}_{SR}$ quantum gravity SR latch memory (S—set, R—reset). The $S$ and $R$ inputs are controlled by the quantum gravity environment, formulating the tripartite system $\rho_{\mathcal{G}_E RS}$ with entangled partition $\mathcal{G}_E - RS$. The non-fixed causality of the quantum gravity structure leads to the parallel realizations of maps $\mathcal{M}_S$ and $\mathcal{M}_R$ (Toffoli-NOR gates) and the entanglement of $Q$ and $\bar{Q}$. The resource for the stimulation and storage processes is provided by the quantum gravity environment.

In this example, we showed that the information resource-pool property of the quantum gravity environment can be exploited in quantum memories. We proposed a quantum gravity memory device and introduced the term *stimulated storage*, which allows the stimulation and storage of qubit entanglement, exploiting the information resource-pool property of the quantum gravity environment.

The results indicate that the structure of the quantum gravity space can be further exploited in the development of quantum devices and quantum computers.

# 5 Conclusions

The theory of quantum gravity integrates the fundamental results of quantum mechanics with general reality. This fusion injects and adds several benefits to quantum mechanics, most importantly the non-fixed causality structure of space-time geometry and the existence of causally non-separable processes. In this work, we provided a model for the information processing structure of the quantum gravity space. We analyzed the connection of the gravity environment with the local processes and revealed that the quantum gravity environment is an information transfer device. This property makes the use of quantum gravity space as an information resource-pool available for the parties. We introduced the term remote simulation and showed that the quantum gravity space induces noise on the local environment states, which allows the parties to simulate locally separated remote systems. We investigated the terms of quantum gravity memory and stimulated storage, which allows for the generation and preservation of the entanglement of qubits exploiting the information resource-pool property of quantum gravity. The information processing structure of quantum gravity can be further exploited in quantum computations, in quantum error correction, in quantum AI, in quantum devices, and particularly in the development of quantum computers.

# Acknowledgements

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# Supplemental Information

## S.1 Notations

The notations of the manuscript are summarized in Table S.1.

**Table S.1.** The summary of the notations used in the manuscript.

| **Notation** | **Description** |
|---|---|
| $\mathcal{M}_A, \mathcal{M}_B$ | Independent local CPTP maps in the quantum gravity space. |
| $\mathcal{G}_E$ | Quantum gravity environment (models the space-time geometry). |
| $C \in \{\lvert 0\rangle, \lvert 1\rangle\}$ | Controller state in a fixed causality. Controls the realization sequence of local maps. |
| $C \in \{\lvert +\rangle\}$ | Controller state in a non-fixed causality structure, $\lvert +\rangle = \frac{1}{\sqrt{2}}(\lvert 0\rangle + \lvert 1\rangle)$. Models the vanishing causality between the local maps $\mathcal{M}_A$, $\mathcal{M}_B$ in the quantum gravity space. |
| $B_i, E_i$ | Local output and local environment state of a local CPTP map $\mathcal{M}_i$. |
| $B_j, E_j$ | Remote output and environment state of a remote CPTP map $\mathcal{M}_j$. |
| $\rho_{\mathcal{G}_E E_i B_j}$ | Entangled tripartite qubit system. Defines the entanglement structure of the space-time geometry with local environment $E_i$ and remote output $B_j$. |
| $\rho = \frac{1}{2}\rho_{\mathcal{G}_E E_1 B_2} + \frac{1}{2}\rho_{\mathcal{G}_E E_2 B_1}$ | Density of parallel realizations of local maps $\mathcal{M}_A$, $\mathcal{M}_B$ in a non-fixed causality. |
| $\left(\rho_{\mathcal{G}_E E_1 B_2}\right)^{T_{B_2}}$ | Partial transpose of $\rho_{\mathcal{G}_E E_1 B_2}$, with respect to subsystem $B_2$. If $\left(\rho_{\mathcal{G}_E E_1 B_2}\right)^{T_{B_2}} \geq 0$, then $B_2$ is separable from $\mathcal{G}_E E_1$, while for $\left(\rho_{\mathcal{G}_E E_1 B_2}\right)^{T_{B_2}} < 0$, the partition $B_2 - \mathcal{G}_E E_1$ is entangled. |
| $\mathcal{G}_E - E_i B_j$, $E_i - \mathcal{G}_E B_j$, $B_j - \mathcal{G}_E E_i$ | Partitions of $\rho_{\mathcal{G}_E E_i B_j}$. Partition $\mathcal{G}_E - E_i B_j$ is entangled, $E_i - \mathcal{G}_E B_j$, $B_j - \mathcal{G}_E E_i$ are separable. Parti- |

| | |
|---|---|
| | tion $\mathcal{G}_E - E_i B_j$ models the entangled space-time geometry of the quantum gravity space. |
| $\mathcal{N}_{A_1 B_1}, \mathcal{N}_{A_2 B_2}$ | Local logical channels of maps $\mathcal{M}_A$ and $\mathcal{M}_B$, defined by Kraus operators $\mathcal{N}_{A_1 B_1}(\rho) = \sum_j A_j^{A_1 B_1} \rho \left(A_j^{A_1 B_1}\right)^\dagger$ and $\mathcal{N}_{A_2 B_2}(\rho) = \sum_j A_j^{A_2 B_2} \rho \left(A_j^{A_2 B_2}\right)^\dagger$. |
| $\mathcal{N}_{A_1 E_1}, \mathcal{N}_{A_2 E_2}$ | Local complementary channels of maps $\mathcal{M}_A$, and $\mathcal{M}_B$, defined by Kraus operators $\mathcal{N}_{A_1 E_1}(\rho) = \sum_j A_j^{A_1 E_1} \rho \left(A_j^{A_1 E_1}\right)^\dagger$ and $\mathcal{N}_{A_2 E_2}(\rho) = \sum_j A_j^{A_2 E_2} \rho \left(A_j^{A_2 E_2}\right)^\dagger$. |
| $\mathcal{D}^{E_i \to B_j}$ | Local simulation map. Allows the remote simulation of remote output $B_j$ from the local environment state $E_i$, as $B_j = E_i \circ \mathcal{D}^{E_i \to B_j}$ through the quantum gravity environment. The existence of $\mathcal{D}^{E_i \to B_j}$ is the consequence of the entangled space-time geometry $\rho_{\mathcal{G}_E E_1 B_2}$. |
| $\mathcal{M}_{\mathcal{G}}$ | CPTP map which models the simultaneous realizations of the local channels $\mathcal{N}_{A_1 E_1}$, $\mathcal{N}_{A_2 B_2}$, defined as $\mathcal{M}_{\mathcal{G}}(\rho) = \sum_{i,j} A_i^{\mathcal{G}} \rho \left(A_i^{\mathcal{G}}\right)^\dagger$. |
| $\mathcal{M}_{\mathcal{D}}$ | Local CPTP map, describes the probabilistic simulation via $\mathcal{D}^{E_i \to B_j}$ on the local environment $E_i$ as $\mathcal{M}_{\mathcal{D}} = p\mathcal{D}^{E_i \to B_j} + (1-p)I$. The output of the map is $B_j' = \frac{1}{2} B_j + \frac{1}{2} E_i$. |
| $\Pi^X, \Pi^Z$ | Projective measurement in the $X$ and $Z$ basis. |
| $W^{B_1 E_1 B_2 E_2}$ | Process matrix, describes the causality relations of the local maps $\mathcal{M}_A$ and $\mathcal{M}_B$ of $\rho_{\mathcal{G}_E E_i B_j}$ in the quantum gravity space. |
| $\mathcal{M}_{A_i B_j}$ | The quantum gravity channel. It has a logical channel $\mathcal{N}_{A_i B_j}$, that exists between the local input $A_i$ and the remote output $B_j$, and a local complementary channel $\mathcal{N}_{A_i E_i}$, which exists between the local input $A_i$ and the local environment state $E_i$. |

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|---|---|
| | The logical channel $\mathcal{N}_{A_iB_j}$ is called the remote logical channel of $\mathcal{M}_{A_iB_j}$, $\mathcal{N}_{A_iB_j} = \mathcal{N}_{A_iE_i} \circ \mathcal{D}^{E_i \to B_j}$. The remote logical channel exits with probability $p$. |
| $\mathcal{M}\left(B_2\right)$ | CPTP map $M_2 \to M_2$, which gets as input the remote output $B_2$, and outputs $B_2' = E_1 \circ \mathcal{M}_{\mathcal{D}}$, where $\mathcal{M}_{\mathcal{D}} = p\mathcal{D}^{E_i \to B_j} + \left(1-p\right)I$. |
| $\rho_{\mathcal{G}_E E_1}$ | Bell diagonal state to quantify the correlations that is transmitted via the quantum gravity space $\mathcal{G}_E$. |
| $u_+, u_-, v_+, v_-$ | Eigenvalues of $\rho_{\mathcal{G}_E E_1}$, $\max\left\{v_+, v_-, u_+, u_-\right\} \le \frac{1}{2}$. |
| $c_1, c_2, c_3$ | Parameters defined from the eigenvalues $v_+, v_-$ as $c_1 = \left(v_+ - v_-\right)$, $c_2 = -\left(v_+ - v_-\right)$ and $c_3 = 1 - 2\cdot\left(v_+ - v_-\right) = 1 + 2\cdot c_2$, $\left\|c_1\right\| + \left\|c_2\right\| + \left\|c_3\right\| \le 1$. |
| $I\left(\cdot\right)$ | Mutual information function. |
| $\mathcal{C}\left(\cdot\right)$ | Classical correlation function. |
| $\mathcal{D}\left(\cdot\right)$ | Quantum discord. |
| $I_{coh}\left(\cdot\right)$ | Coherent information. |
| $C\left(\mathcal{M}\left(B_2\right)\right), Q\left(\mathcal{M}\left(B_2\right)\right)$ | Classical and quantum capacity of channel $\mathcal{M}\left(B_2\right)$. |
| $C_{Toff}^{NOR}$ | Toffoli-NOR qubit gate, defined as $C_{Toff}^{NOR} = G\left(x,y,z\right) = z \oplus \overline{x+y}$, where $x$ and $y$ are the control qubit inputs, $z$ is the target qubit. |
| $\sqrt{X}, \sqrt{X}^{\dagger}$ | Square-root $X$ operation and its adjoint. |
| $\mathcal{M}_{SR}$ | Quantum gravity SR-latch memory. Consist of two cross-coupled $C_{Toff}^{NOR}$ circuits, referred by the local maps $\mathcal{M}_S$ and $\mathcal{M}_R$, $\mathcal{M}_{SR}\left(\rho\right) = \sum_{i,j} A_i^{SR} \rho \left(A_i^{SR}\right)^{\dagger}$. |
| $\left\|\varphi\right\rangle = \frac{1}{\sqrt{2}}\left(\left\|Q\bar{Q}\right\rangle + \left\|\bar{Q}Q\right\rangle\right)$ | Entanglement of qubit of outputs $Q$ and $\bar{Q}$ in the quantum gravity $\mathcal{M}_{SR}$. |

## S.2 Abbreviations

**CNOT** **Controlled-NOT**
**CPTP** **Completely Positive Trace Preserving**

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|---|---|
| **GHZ** | **Greenberger–Horne–Zeilinger** |
| **NOR** | **Negation of OR** |
| **POVM** | **Positive Operator Valued Measure** |
| **SR** | **Set-Reset** |