Acta Univ. Sapientiae, Informatica, **2**, 2 (2010) 184–193

Score lists in multipartite hypertournaments

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Abstract. Given non-negative integers n_i and α_i with $0 \le \alpha_i \le n_i$ $(i = 1, 2, \dots, k)$, an $[\alpha_1, \alpha_2, \dots, \alpha_k]$-k-partite hypertournament on $\sum_1^k n_i$ vertices is a $(k+1)$-tuple $(U_1, U_2, \dots, U_k, E)$, where U_i are k vertex sets with $|U_i| = n_i$, and E is a set of $\sum_1^k \alpha_i$-tuples of vertices, called arcs, with exactly α_i vertices from U_i, such that any $\sum_1^k \alpha_i$ subset $\cup_1^k U_i'$ of $\cup_1^k U_i$, E contains exactly one of the $\left(\sum_1^k \alpha_i\right)!$ $\sum_1^k \alpha_i$-tuples whose entries belong to $\cup_1^k U_i'$. We obtain necessary and sufficient conditions for k lists of non-negative integers in non-decreasing order to be the losing score lists and to be the score lists of some k-partite hypertournament.

1 Introduction

Hypergraphs are generalizations of graphs [1]. While edges of a graph are pairs of vertices of the graph, edges of a hypergraph are subsets of the vertex set,

Computing Classification System 1998: G.2.2
Mathematics Subject Classification 2010: 05C65
Key words and phrases: hypergraph, hypertournament, multi hypertournament, score, losing score.

consisting of at least two vertices. An edge consisting of k vertices is called a k-edge. A k-hypergraph is a hypergraph all of whose edges are k-edges. A k-hypertournament is a complete k-hypergraph with each k-edge endowed with an orientation, that is, a linear arrangement of the vertices contained in the hyperedge. Instead of scores of vertices in a tournament, Zhou et al. [13] considered scores and losing scores of vertices in a k-hypertournament, and derived a result analogous to Landau's theorem [6]. The score $s(v_i)$ or s_i of a vertex v_i is the number of arcs containing v_i and in which v_i is not the last element, and the losing score $r(v_i)$ or r_i of a vertex v_i is the number of arcs containing v_i and in which v_i is the last element. The score sequence (losing score sequence) is formed by listing the scores (losing scores) in non-decreasing order.

The following characterizations of score sequences and losing score sequences in k-hypertournaments can be found in G. Zhou et al. [12].

Theorem 1 *Given two positive integers n and k with $n \geq k > 1$, a non-decreasing sequence $R = [r_1, r_2, \ldots, r_n]$ of non-negative integers is a losing score sequence of some k-hypertournament if and only if for each j,*

$$\sum_{i=1}^{j} r_i \geq \binom{j}{k},$$

with equality when $j = n$.

Theorem 2 *Given positive integers n and k with $n \geq k > 1$, a non-decreasing sequence $S = [s_1, s_2, \ldots, s_n]$ of non-negative integers is a score sequence of some k-hypertournament if and only if for each j,*

$$\sum_{i=1}^{j} s_i \geq j\binom{n-1}{k-1} + \binom{n-j}{k} - \binom{n}{k},$$

with equality when $j = n$.

Some recent work on the reconstruction of tournaments can be found in the papers due to A. Iványi [3, 4]. Some more results on k-hypertournaments can be found in [2, 5, 9, 10, 11, 13]. The analogous results of Theorem 1 and Theorem 2 for $[h, k]$-bipartite hypertournaments can be found in [7] and for $[\alpha, \beta, \gamma]$-tripartite hypertournaments in [8].

Throughout this paper i takes values from 1 to k and j_i takes values from 1 to n_i, unless otherwise stated.

A k-partite hypergraph is a generalization of k-partite graph. Given non-negative integers n_i and α_i, $(i = 1, 2, \ldots, k)$ with $n_i \geq \alpha_i \geq 0$ for each i, an $[\alpha_1, \alpha_2, \ldots, \alpha_k]$-$k$-partite hypertournament (or briefly k-partite hypertournament) M of order $\sum_1^k n_i$ consists of k vertex sets U_i with $|U_i| = n_i$ for each i, $(1 \leq i \leq k)$ together with an arc set E, a set of $\sum_1^k \alpha_i$-tuples of vertices, with exactly α_i vertices from U_i, called arcs such that any $\sum_1^k \alpha_i$ subset $\cup_1^k U_i'$ of $\cup_1^k U_i$, E contains exactly one of the $\left(\sum_1^k \alpha_i\right)$ $\sum_1^k \alpha_i$-tuples whose α_i entries belong to U_i'.

Let $e = (u_{11}, u_{12}, \ldots, u_{1\alpha_1}, u_{21}, u_{22}, \ldots, u_{2\alpha_2}, \ldots, u_{k1}, u_{k2}, \ldots, u_{k\alpha_k})$, with $u_{ij_i} \in U_i$ for each i, $(1 \leq i \leq k, 1 \leq j_i \leq \alpha_i)$, be an arc in M and let $h < t$, we let $e(u_{1h}, u_{1t})$ denote to be the new arc obtained from e by interchanging u_{1h} and u_{1t} in e. An arc containing α_i vertices from U_i for each i, $(1 \leq i \leq k)$ is called an $(\alpha_1, \alpha_2, \ldots, \alpha_k)$-arc.

For a given vertex $u_{ij_i} \in U_i$ for each i, $1 \leq i \leq k$ and $1 \leq j_i \leq \alpha_i$, the score $d_M^+(u_{ij_i})$ (or simply $d^+(u_{ij_i})$) is the number of $\sum_1^k \alpha_i$-arcs containing u_{ij_i} and in which u_{ij_i} is not the last element. The losing score $d_M^-(u_{ij_i})$ (or simply $d^-(u_{ij_i})$) is the number of $\sum_1^k \alpha_i$-arcs containing u_{ij_i} and in which u_{ij_i} is the last element. By arranging the losing scores of each vertex set U_i separately in non-decreasing order, we get k lists called losing score lists of M and these are denoted by $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$ for each i, $(1 \leq i \leq k)$. Similarly, by arranging the score lists of each vertex set U_i separately in non-decreasing order, we get k lists called score lists of M which are denoted as $S_i = [s_{ij_i}]_{j_i=1}^{n_i}$ for each i $(1 \leq i \leq k)$.

2 Main results

The following two theorems are the main results.

Theorem 3 *Given* k *non-negative integers* n_i *and* k *non-negative integers* α_i *with* $1 \leq \alpha_i \leq n_i$ *for each i* $(1 \leq i \leq k)$, *the* k *non-decreasing lists* $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$ *of non-negative integers are the losing score lists of a* k*-partite hypertournament if and only if for each* p_i $(1 \leq i \leq k)$ *with* $p_i \leq n_i$,

$$\sum_{i=1}^{k} \sum_{j_i=1}^{p_i} r_{ij_i} \geq \prod_{i=1}^{k} \binom{p_i}{\alpha_i}, \tag{1}$$

with equality when $p_i = n_i$ *for each* i $(1 \leq i \leq k)$.

Theorem 4 *Given* k *non-negative integers* n_i *and* k *non-negative integers* α_i *with* $0 \le \alpha_i \le n_i$ *for each* i $(1 \le i \le k)$*, the* k *non-decreasing lists* $S_i = [s_{ij_i}]_{j_i=1}^{n_i}$ *of non-negative integers are the score lists of a* k*-partite hypertournament if and only if for each* p_i*,* $(1 \le i \le k)$ *with* $p_i \le n_i$

$$\sum_{i=1}^{k}\sum_{j_i=1}^{p_i} s_{ij_i} \ge \left(\sum_{i=1}^{k}\frac{\alpha_i p_i}{n_i}\right)\left(\prod_{i=1}^{k}\binom{n_i}{\alpha_i}\right) + \prod_{i=1}^{k}\binom{n_i - p_i}{\alpha_i} - \prod_{i=1}^{k}\binom{n_i}{\alpha_i}, \quad (2)$$

with equality when $p_i = n_i$ *for each* i $(1 \le i \le k)$*.*

We note that in a k-partite hypertournament M, there are exactly $\prod_{i=1}^{k}\binom{n_i}{\alpha_i}$ arcs and in each arc only one vertex is at the last entry. Therefore,

$$\sum_{i=1}^{k}\sum_{j_i=1}^{n_i} d_M^-(u_{ij_i}) = \prod_{i=1}^{k}\binom{n_i}{\alpha_i}.$$

In order to prove the above two theorems, we need the following Lemmas.

Lemma 5 *If* M *is a* k*-partite hypertournament of order* $\sum_1^k n_i$ *with score lists* $S_i = [s_{ij_i}]_{j_i=1}^{n_i}$ *for each* i $(1 \le i \le k)$*, then*

$$\sum_{i=1}^{k}\sum_{j_i=1}^{n_i} s_{ij_i} = \left[\left(\sum_{1=1}^{k}\alpha_i\right) - 1\right]\prod_{i=1}^{k}\binom{n_i}{\alpha_i}.$$

Proof. We have $n_i \ge \alpha_i$ for each i $(1 \le i \le k)$. If r_{ij_i} is the losing score of $u_{ij_i} \in U_i$, then

$$\sum_{i=1}^{k}\sum_{j_i=1}^{n_i} r_{ij_i} = \prod_{i=1}^{k}\binom{n_i}{\alpha_i}.$$

The number of $[\alpha_i]_1^k$ arcs containing $u_{ij_i} \in U_i$ for each i, $(1 \le i \le k)$, and $1 \le j_i \le n_i$ is

$$\frac{\alpha_i}{n_i}\prod_{t=1}^{k}\binom{n_t}{\alpha_t}.$$

Thus,

$$\begin{aligned}\sum_{i=1}^{k}\sum_{j_i=1}^{n_i} s_{ij_i} &= \sum_{i=1}^{k}\sum_{j_i=1}^{n_i}\left(\frac{\alpha_i}{n_i}\right)\prod_1^k\binom{n_t}{\alpha_t}-\binom{n_i}{\alpha_i}\\ &= \left(\sum_{i=1}^{k}\alpha_i\right)\prod_1^k\binom{n_t}{\alpha_t}-\prod_1^k\binom{n_i}{\alpha_i}\\ &= \left[\left(\sum_{1=1}^{k}\alpha_i\right)-1\right]\prod_1^k\binom{n_i}{\alpha_i}.\end{aligned}$$

□

Lemma 6 *If* $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$ $(1 \le i \le k)$ *are* k *losing score lists of a* k*-partite hypertournament* M*, then there exists some* h *with* $r_{1h} < \frac{\alpha_1}{n_1}\prod_1^k\binom{n_p}{\alpha_p}$ *so that* $R'_1 = [r_{11}, r_{12}, \ldots, r_{1h}+1, \ldots, r_{1n_1}]$, $R'_s = [r_{s1}, r_{s2}, \ldots, r_{st}-1, \ldots, r_{sn_s}]$ *(*$2 \le s \le k$*) and* $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$*, (*$2 \le i \le k$*),* $i \ne s$ *are losing score lists of some* k*-partite hypertournament,* t *is the largest integer such that* $r_{s(t-1)} < r_{st} = \ldots = r_{sn_s}$.

Proof. Let $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$ $(1 \le i \le k)$ be losing score lists of a k-partite hypertournament M with vertex sets $U_i = \{u_{i1}, u_{i2}, \ldots, u_{ij_i}\}$ so that $d^-(u_{ij_i}) = r_{ij_i}$ for each i $(1 \le i \le k,\ 1 \le j_i \le n_i)$.

Let h be the smallest integer such that

$$r_{11} = r_{12} = \ldots = r_{1h} < r_{1(h+1)} \le \ldots \le r_{1n_1}$$

and t be the largest integer such that

$$r_{s1} \le r_{s2} \le \ldots \le r_{s(t-1)} < r_{st} = \ldots = r_{sn_s}$$

Now, let

$$R'_1 = [r_{11}, r_{12}, \ldots, r_{1h}+1, \ldots, r_{1n_1}],$$

$$R'_s = [r_{s1}, r_{s2}, \ldots, r_{st}-1, \ldots, r_{sn_s}$$

$(2 \le s \le k)$, and $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$, $(2 \le i \le k)$, $i \ne s$.

Clearly, R'_1 and R'_s are both in non-decreasing order.

Since $r_{1h} < \frac{\alpha_1}{n_1}\prod_1^k\binom{n_p}{\alpha_p}$, there is at least one $[\alpha_i]_1^k$-arc e containing both u_{1h} and u_{st} with u_{st} as the last element in e, let $e' = (u_{1h}, u_{st})$. Clearly, R'_1, R'_s

and $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$ for each i $(2 \le i \le k)$, $i \ne s$ are the k losing score lists of $M' = (M - e) \cup e'$. □

The next observation follows from Lemma 6, and the proof can be easily established.

Lemma 7 *Let* $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$, $(1 \le i \le k)$ *be* k *non-decreasing sequences of non-negative integers satisfying (1). If* $r_{1n_1} < \frac{\alpha_1}{n_1} \prod_1^k \binom{n_t}{\alpha_t}$*, then there exists* s *and* t *(*$2 \le s \le k$*),* $1 \le t \le n_s$ *such that* $R_1' = [r_{11}, r_{12}, \ldots, r_{1h} + 1, \ldots, r_{1n_1}]$, $R_s' = [r_{s1}, r_{s2}, \ldots, r_{st} - 1, \ldots, r_{sn_s}]$ *and* $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$*, (*$2 \le i \le k$*),* $i \ne s$ *satisfy (1).*

Proof of Theorem 3. Necessity. Let R_i, $(1 \le i \le k)$ be the k losing score lists of a k-partite hypertournament $M(U_i, 1 \le i \le k)$. For any p_i with $\alpha_i \le p_i \le n_i$, let $U_i' = \{u_{ij_i}\}_{j_i=1}^{p_i} (1 \le i \le k)$ be the sets of vertices such that $d^-(u_{ij_i}) = r_{ij_i}$ for each $1 \le j_i \le p_i$, $1 \le i \le k$. Let M' be the k-partite hypertournament formed by U_i' for each i $(1 \le i \le k)$.

Then,

$$\sum_{i=1}^{k} \sum_{j_i=1}^{p_i} r_{ij_i} \ge \sum_{i=1}^{k} \sum_{j_i=1}^{p_i} d^-_{M'}(u_{ij_i}) = \prod_1^k \binom{p_t}{\alpha_t}.$$

Sufficiency. We induct on n_1, keeping $n_2, \ldots, n_k$ fixed. For $n_1 = \alpha_1$, the result is obviously true. So, let $n_1 > \alpha_1$, and similarly $n_2 > \alpha_2, \ldots, n_k > \alpha_k$. Now,

$$\begin{aligned} r_{1n_1} &= \sum_{i=1}^{k} \sum_{j_i=1}^{n_i} r_{ij_i} - \left(\sum_{j_1=1}^{n_1-1} r_{1j_1} + \sum_{i=2}^{k} \sum_{j_i=1}^{n_i} r_{ij_i} \right) \\ &\le \prod_1^k \binom{n_t}{\alpha_t} - \binom{n_1 - 1}{\alpha_1} \prod_2^k \binom{n_t}{\alpha_t} \\ &= \left[\binom{n_1}{\alpha_1} - \binom{n_1 - 1}{\alpha_1} \right] \prod_2^k \binom{n_t}{\alpha_t} \\ &= \binom{n_1 - 1}{\alpha_1 - 1} \prod_2^k \binom{n_t}{\alpha_t}. \end{aligned}$$

We consider the following two cases.

Case 1. $r_{1n_1} = \binom{n_1-1}{\alpha_1-1}\prod_2^k \binom{n_t}{\alpha_t}$. Then,

$$\begin{aligned}\sum_{j_1=1}^{n_1-1} r_{1j_1} + \sum_{i=2}^{k}\sum_{j_i=1}^{n_i} r_{ij_i} &= \sum_{i=1}^{k}\sum_{j_i=1}^{n_i} r_{ij_i} - r_{1n_1} \\ &= \prod_1^k \binom{n_t}{\alpha_t} - \binom{n_1-1}{\alpha_1-1}\prod_2^k \binom{n_t}{\alpha_t} \\ &= \left[\binom{n_1}{\alpha_1} - \binom{n_1-1}{\alpha_1-1}\right]\prod_2^k \binom{n_t}{\alpha_t} \\ &= \binom{n_1-1}{\alpha_1}\prod_2^k \binom{n_t}{\alpha_t}.\end{aligned}$$

By induction hypothesis $[r_{11}, r_{12}, \dots, r_{1(n_1-1)}]$, $R_2, \dots, R_k$ are losing score lists of a k-partite hypertournament $M'(U_1', U_2, \dots, U_k)$ of order $\left(\sum_{i=1}^k n_i\right) - 1$. Construct a k-partite hypertournament M of order $\sum_{i=1}^k n_i$ as follows. In M', let $U_1' = \{u_{11}, u_{12}, \dots, u_{1(n_1-1)}\}, U_i = \{u_{ij_i}\}_{j_i=1}^{n_i}$ for each i, $(2 \le i \le k)$. Adding a new vertex u_{1n_1} to U_1', for each $\left(\sum_{i=1}^k \alpha_i\right)$-tuple containing u_{1n_1}, arrange u_{1n_1} on the last entry. Denote E_1 to be the set of all these $\binom{n_1-1}{\alpha_1-1}\prod_2^k \binom{n_t}{\alpha_t}$ $\left(\sum_{i=1}^k \alpha_i\right)$-tuples. Let $E(M) = E(M') \cup E_1$. Clearly, R_i for each i, $(1 \le i \le k)$ are the k losing score lists of M.

Case 2. $r_{1n_1} < \binom{n_1-1}{\alpha_1-1}\prod_2^k \binom{n_t}{\alpha_t}$.

Applying Lemma 7 repeatedly on R_1 and keeping each R_i, $(2 \le i \le k)$ fixed until we get a new non-decreasing list $R_1' = [r_{11}', r_{12}', \dots, r_{1n_1}']$ in which now $'_{1n_1} = \binom{n_1-1}{\alpha_1-1}\prod_2^k \binom{n_t}{\alpha_t}$. By Case 1, R_1', R_i $(2 \le i \le k)$ are the losing score lists of a k-partite hypertournament. Now, apply Lemma 6 on R_1', R_i $(2 \le i \le k)$ repeatedly until we obtain the initial non-decreasing lists R_i for each i $(1 \le i \le k)$. Then by Lemma 6, R_i for each i $(1 \le i \le k)$ are the losing score lists of a k-partite hypertournament. □

Proof of Theorem 4. Let $S_i = [s_{ij_i}]_{j_i=1}^{n_i} (1 \le i \le k)$ be the k score lists of a k-partite hypertournament $M(U_i, 1 \le i \le k)$, where $U_i = \{u_{ij_i}\}_{j_i=1}^{n_i}$ with

$d^+_M(u_{ij_i}) = s_{ij_i}$, for each i, $(1 \le i \le k)$. Clearly,
$d^+(u_{ij_i}) + d^-(u_{ij_i}) = \frac{\alpha_i}{n_i}\prod_1^k \binom{n_t}{\alpha_t}$, $(1 \le i \le k, 1 \le j_i \le n_i)$.

Let $r_{i(n_i+1-j_i)} = d^-(u_{ij_i})$, $(1 \le i \le k, 1 \le j_i \le n_i)$.

Then $R_i = [r_{ij_i}]_{j_i=1}^{n_i} (i = 1, 2, \ldots, k)$ are the k losing score lists of M. Conversely, if R_i for each i $(1 \le i \le k)$ are the losing score lists of M, then S_i for each i, $(1 \le i \le k)$ are the score lists of M. Thus, it is enough to show that conditions (1) and (2) are equivalent provided $s_{ij_i} + r_{i(n_i+1-j_i)} = \left(\frac{\alpha_i}{n_i}\right)\prod_1^k \binom{n_t}{\alpha_t}$, for each i $(1 \le i \le k$ and $1 \le j_i \le n_i)$.

First assume (2) holds. Then,

$$
\begin{aligned}
\sum_{i=1}^{k}\sum_{j_i=1}^{p_i} r_{ij_i} &= \sum_{i=1}^{k}\sum_{j_i=1}^{p_i}\left(\frac{\alpha_i}{n_i}\right)\left(\prod_1^k \binom{n_t}{\alpha_t}\right) - \sum_{i=1}^{k}\sum_{j_i=1}^{p_i} s_{i(n_i+1-j_i)} \\
&= \sum_{i=1}^{k}\sum_{j_i=1}^{p_i}\left(\frac{\alpha_i}{n_i}\right)\left(\prod_1^k \binom{n_t}{\alpha_t}\right) - \left[\sum_{i=1}^{k}\sum_{j_i=1}^{n_i} r_{ij_i} - \sum_{i=1}^{k}\sum_{j_i=1}^{n_i-p_i} s_{ij_i}\right] \\
&\ge \left[\sum_{i=1}^{k}\sum_{j_i=1}^{p_i}\left(\frac{\alpha_i}{n_i}\right)\left(\prod_1^k \binom{n_t}{\alpha_t}\right)\right] \\
&- \left[\left(\left(\sum_1^k \alpha_i\right) - 1\right)\prod_1^k \binom{n_i}{\alpha_i}\right] \\
&+ \sum_{i=1}^{k}(n_i - p_i)\left(\frac{\alpha_i}{n_i}\right)\prod_1^k \binom{n_t}{\alpha_t} \\
&+ \prod_1^k \binom{n_i - (n_i - p_i)}{\alpha_i} - \prod_1^k \binom{n_i}{\alpha_i} \\
&= \prod_1^k \binom{n_i}{\alpha_i},
\end{aligned}
$$

with equality when $p_i = n_i$ for each i $(1 \le i \le k)$. Thus (1) holds.

Now, when (1) holds, using a similar argument as above, we can show that (2) holds. This completes the proof. □

Acknowledgements

The research of the third author was supported by the European Union and the European Social Fund under the grant agreement no. TÁMOP 4.2.1/B-09/1/KMR-2010-0003.

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Received: June 23, 2010 • Revised: October 21, 2010