Sustainability Relations for Innovation, Low-Carbon Principles for "Rubik's Cube" Solution

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Abstract - It is very difficult to calculate in advance the positive and negative long-term impacts of an investment, or a development venture. A serious global problem arises from the fact that numerous environmental-protection oriented private and government ventures are implemented in an incorrect manner significantly impair the conditions of both the environment and the economy (market). There is a high number of innovative energy related investments, waste and water management projects, etc. in Europe, which cause more harm to the society than ever imagined. The various sustainability logics can be synchronised with the 3×3×3 Rubik's Cube's solution algorithms, and the relations of the cube's sides define a planning strategy that provides a new scientific approach for investment planning. We theoretically evaluated the various solution processes, and paralell investment planning levels following the solution levels and stages of the cube. After these various level-evaluations, we made „low-carbon interpretation" summaries.

Keywords - *Rubik's Cube method*, Sustainable planning and practice, Environmental modeling, Solution algorithms, layer-by-layer method, Low-carbon interpretation, sustainable biomass energy productions

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I. INTRODUCTION

In 1980, Ernő Rubik wrote that the cube seems to be alive, as it comes into life while you rotate it in your hands. Rubik's Cube has three rows and three columns, and this can also have a symbolic meaning. If we look at the attributes of the various blocks, the 3×3×3 cube's sides, it's almost immediately obvious that in case of each side, we have system elements, or specific small cubes (mid cubes, outer cubes, and cornercubes) which hide a specific meaning, and keep this meaning in them, regardless of where we rotate them in the system. According to Ernő Rubik, the number „three", through its special meaning, is even able to model life itself. It's able to show the relationship of man and nature, the process of creation, care and destruction, and the relations of cooperation between our resource systems (Rubik et al., 1987).

Our low-carbon optimalisation theory using Rubik's Cube has an important characteristic, which is to analyse a project on various levels (Input, Output, connection) using the real interactions between various project attributes, which helps us spare a lot of time and effort by neglecting the needless analyses. The system connections assigned to the sides of the cubes (edge cube attributes, cornercube attributes) make the direct analysis of some attributes outright unnecessary, meaning not all connections have to „communicate". The „communications" between these system elements can therefore be reached by defining simple border-area connections, or through transferred system connections.

When we analyse the technological applicability during the process of a project development, it's important to note that we don't have to directly consider the market opportunities regarding Outputs, but the correspondence between the two exists, and is included through their interactions [1]. Another similar example is negotiation on the questions of liquidity when analysing a given financial abidement, which isn't directly dependent on the market's demand, but both have an influence on each other, which connection is included even without analysations – assuring proper applicability – by the methodology using Rubik's Cube. The previously mentioned LEDS (Low-Emission Development Strategies) of the UN is based on the above mentioned, which they've wanted to realise since 1992, but the economical interpretation of the program couldn't be defined in the last few decades[3].

Domestic objective system of « low-carbon economy »'s main priorities [5] :

We have to try and improve the effectiveness of all resources, most importantly, energy sources. We have to maintain our energy transformation systems in a much more effective manner, including the local and maximum use of the heat energy by-product of electric energy production.

The intentional consuming must be realised on a very high level – be it environmental protection, or taking social liability – it has to appear on production, trade and personal levels.

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Local production and consuming is preferred! No matter what kind of demand comes up, be it energy, material or service, it has to be satisfied by local supply. Energy sources have to be produced in low emission systems, using as much renewable and alternative energy as possible to reduce CO2 emission to the lowest possible amount.

Every waste has to be maximalised – reccycle it, return it, reduce it, because this spares a lot of resources and energy.

In the case of the „low-carbon-economy”, this complex requirement system is incredibly hard to realise, or integrate its basic theories into investment processes, where the BAU (business as usual) requirements fundamentally disregard sustainability criteria. However, including these criteria, or priorities can happen for the various development and investment projects with the solution of Rubik’s Cube, which models the objective system of the „low-carbon” developments well with its structure interpretable in multi-dimension.

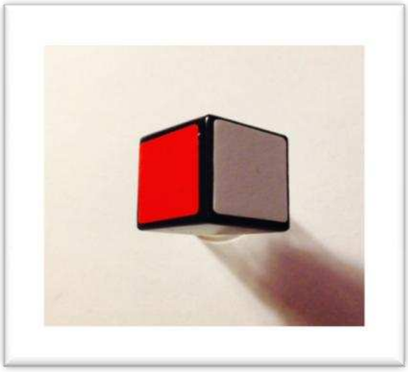
II. Material and Methods

A. Method of problem handling in 1D, 2D and 3D using *Rubik’s Cube*

The management of complex risks is assisted by 1, 2, and 3 viewpoint simultaneous problem handling method using Rubik’s Cube. The base of the cube consists six of immovable smallcubes, which are the basis for the 1D problem handling (Illustration 1). We assign the attributes of project development or investment to these smallcubes, which define the core of development in the process, meaning they define unchangeable fixpoints in the important areas (f.e. technology, regulations, financing, market).

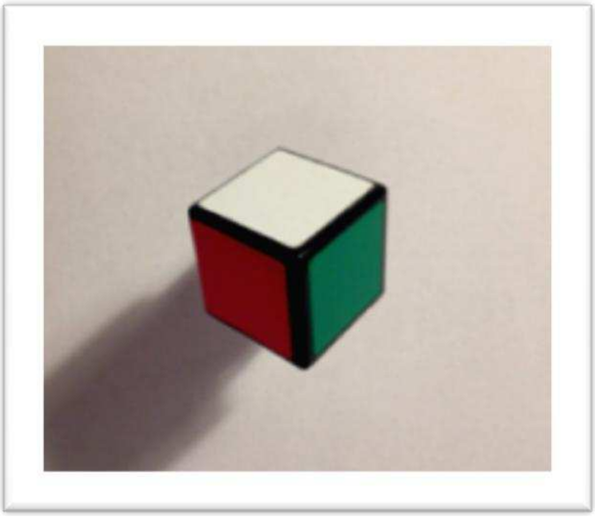
1. Illustration:Skeleton” of Rubik’s Cube, rotateable, but immovable mid cubes , Source: self-made.

The number of edge cubes is 12, which serve as the basis for 2D problem handling by allowing the optimalisation or movement of two attributes simultaneously (f.e. technological regulations and financing). This practically means analysis along the (x,y) axis pair (Illustration 2).



2. Illustration: Edge cube of Rubik’s Cube, which needs to match two colours
Source: self-made.

There are eight cornercubes, which serve as a basis for specialised 3D problem handling with its simultaneously movable or optimalisable three attributes. This basically means analysis along the (x,y,z) axis triple (Illustration 3).



3. Illustration: Cornercube, which needs to match three colours
Source: self-made.

We assume that the six-sided, 3×3×3 Rubik’s Cube has a side and smallcube that harmonizes with each of the elements of project development::

A. Mid cube – it’s a stable/fix element and attribute of all the cube’s sides, and the phases of the development. In the case of the 3×3×3 cube, we define 6 mid cubes, and these stable cubes/fixed attributes also fundamentally outline and define the process of sustainable project planning.

B. Edge cube – it means direct connection between two colours, and attributes. The number of edge cubes, in the case of the 3×3×3 Rubik’s Cube is twelve. These edge cubes define a 2D inherence of attributes, by which the connected attributes also define the syncing and the system criteria together during the evolution of the project.

Cornercube – the most advanced element of coordinating project planning or development, since it optimises three different attributes during the process of development. Between the matching of the three colours, meaning the three sets of attributes, it defines a very direct and complex correlation. White side (Input) cornercubes mean a stable element to project structure, capable of analysing all viewpoints and attributes, while the yellow side (Input) cornercubes realise the selection of sustainability, harmonic energy use and exclusion/correction of detrimental developments.

According to our hipothesis, low-carbon project planning or development method is a paralel protocol using the layer by layer solution method of Rubik's Cube. By assigning the various project attributes to the colours of the cube, we can achieve the realisation of a specialised, sustainable project development process, for which the specifics of development and process are offered by the 1D, 2D, and 3D nature of the various attributes or attribute sets, making them handle better during development programs.

The objective systems of both low-carbon developments and solution method of Rubik's Cube follow the same guiding principle (layer-by-layer), meaning they both try to achieve the state of equilibrium by following the route of logical and low energy consumption (Singmaster, 1981).

Project planning and development is basically a process optimalisation, which is based on the collective handling of different attributes, in a way that the examined segments are placed into the most harmonic constellation compared to each other. In case of a supposed „low-carbon optimalisation protocol", there is a need to create four different determination areas (attribute groups), which can be associated with the 3×3×3 cube's different colored sides. Two opposing sides (white and yellow) would be our project's input and output sides. The attribute groups (in case of renewable energy innovation programs in Europe) which determine our optimalisation can be the following in a demonstrational project: optimalisation of strategic goals system (red side), analysis of market opportunities (green side), the area of actualisation and technological criteria system (blue side), monetary effects (orange side), the attributes summarizing input-side goals (white side), and last, but not least, the attributes summarizing output-side goals (yellow side).

B. Churchman-Ackoff method for the selection of attributes of the main agents

During value estimation processes and choosing dominance sequences, applying the Churchman-Ackoff [2]method is popular, therefore, it's also well-known as a software application (SMART - Simple Multi Attribute Ranking Technic - software). The shortened software application used for the analyses, which analisys was the bases of the ranking of attributes and the preferences order of different Rubik's Cube sides (Input and output sides are the white and yellow colors. Except on these colors we had ranking by the four different colours (green, red, blue, orange) as main groups of the attributes - by Churchman-Ackoff methods.

C. Using multi-variable usefulness functions

Decision makers must take the prevention of environmental problems, and other economic problems into consideration, which can surface due to the cessation of various products and processes. Without including the benefits and setbacks of these consequencesm, there can be no decision. The multi-attribute usefulness theory handles problems where the effect of the decision is defined by two or more variables. We generally assume that all attributes have either discrete or continuous value. For the sake of simplicity, let's assume that discrete attributes were defined in a way that higher usefulness values are matched with higher attribute values, if everything else is unchanged (Churchman – Ackoff, 1957):

Let $\boldsymbol{x} = x_1, x_2, \ldots x_{nb}$ be the attributes,

and $\mathbf{x} = \langle x_1, x_2, \ldots, x_n \rangle$ be the values of the attribute vectors, to define

$U(x_1, \ldots, x_n)$ usefulness function.

D. System of preferences, interpretation of multi-variable usefulness functions

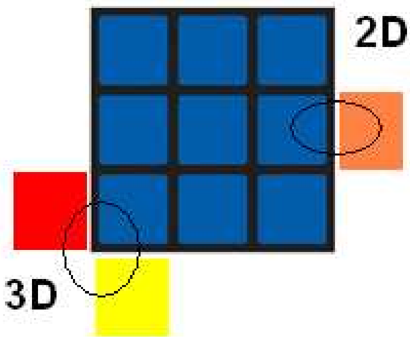
Multi-attribute usefulness theory assumes that usefulness functions have a well-defined structure. The accepted theoretic approach says that we identify regularities in the preferences of behaviour, and by using the so-called representation theses, we can show that the attribute which has a preference system can be defined by a usefulness function as follows:

$$U(x_1, \ldots, x_n) = f[f_1(x_1), \ldots, f_n(x_n)]$$

where f is hopefully a simple function, f.e. an addition. It's obvious that this correspondence is similar to how we used the probability webs to break the summarized probability distribution function. This is important, because we also demonstrate the probability distribution of the various attribute groups of Rubik's Cube in a network-like manner.

III. Discussion

The attributes inherent in the cubes can be interpreted in various ways. (Since the white and yellow sides mean the Input and Output sides of the process respectively [8], their functional interpretation differs from that of the other four sides during the solution). In order to make the connection clear, I'll demonstrate the correspondence on the next illustration (Illustration 4). The 2D marking is for cubes and attributes which are dual-colored (f.e. the blue-orange edge cube on Illustration 4), while the 3D marking is for cubes or connection attribute characteristics, which have three colours. On Illustration 4, the blue-red-yellow cornercube can be called a typed 3D attribute.



4. Illustration: 2D and 3D interpretation of cube side, or main agent
Source: self-made.

2D cube attributes mean that the edge cubes have attributes assigned to them which is influenced by the other main agent (orange). 3D cube attributes mean that (in our case, the) blue cornercube has attributes which are influenced by two other main agents (red and yellow), and vice versa. Marking the elements of the main agents/cube sides with an attribute therefore has to depend on its position on this specific side as well. This means that marking the smallcubes with attributes, and defining their usefullness and dominance in the main agens is possible in regards to this. I did the usefulness analysis with the SMART (Simple Multi Attribute Ranking Technic) attribute evaluation software. Table 1. content shows how the levels of the project planning model were synced with the attribute analyses of the SMART software application.

TABLE I. SYNCING SMART EVALUATION LEVELS TO MODELING LEVELS

Smart Evaluation Level	Level of Model Development	Project Attribute In Question (1D/2D/3D - # of inherent traits)
Level 1	INPUT	„White cross" – defining the starting criteria (1D, 2D)
	INPUT	„White corner" – defining the sustainable development routes, equilibrium-search, non-cooperative optimum (3D)
Level 2	MID CUBE	„Second row" – anchoring of relation points, achieving equilibrium, arranging two-dimensional attributes, positioning fixpoint (1D, 2D)
	MID CUBE	„Yellow cross" – indirect synchronising of input/output sides (1D, 2D)
Level 3	OUTPUT	„Yellow corner" – interpretation of sustainability attributes during the arrangement of outputs (3D)
	OUTPUT	„Yellow side edge-switch" – strict synchronising of input/output sides (2D)
	OUTPUT	„Corner switch" – the phase of setting the final balance, achieving equilibrium, finalising sustainability attributes (3D)

Source: self-made.

A. Interpretation of three-level logical analysis

The three-way layout of the search for optimum using Rubik Theory models [9] (Input, mid cube, Output) alredy showed us that though the planning levels of the project development based on the solution of Rubik's Cube follow the solution logic, but it's advisable to brake the process of search for equilibrium into greater units, meaning some phases (NO1-7) should be merged. During the preparation of the three-level analyses, the first two solution phases (NO1, NO2) were brought to the Input level. The next two solution phases (NO3, NO4) were assigned to the mid cube, and the last three (NO5, NO6, NO7) to the Output (Table 2.).

TABLE II. EVALUATION OF MODELING PROCESS, AND RESULTS

Cube Interpretations (number of rotation algorithm)	Level of Model Development	Low-Carbon Project Attribute In Question
NO1	INPUT	„White cross" – defining the starting criteria
NO2	INPUT	„White corner" – defining the sustainable development routes, equilibrium-search, non-cooperative optimum
NO3	MID CUBE	„Mid row" – anchoring of relation points, achieving equilibrium, arranging two-dimensional attributes, positioning fixpoint
NO4	MID CUBE	„Yellow cross" – indirect synchronising of input/output sides
NO5	OUTPUT	„Yellow corner" – interpretation of sustainability attributes during the arrangement of outputs
NO6	OUTPUT	„Yellow side edge-switch" – strict synchronising of input/output sides
NO7	OUTPUT	„Corner switch" – the phase of setting the final balance, achieving equilibrium, finalising sustainability attributes

Source: self-made.

Therefore, we merged the seven phases connected to the layer by layer (meaning row to row) solution method of the 3×3×3 Rubik's Cube, and defined three analysation levels, which are as follows:

1. Cube level 1: Correction of not allowed differences on Input side (Illustration 5.),
2. Cube level 2: Correction of not allowed differences on mid cube level (Illustration 6.),
3. Cube level 3: Correction of not allowed differences on Output side (Illustration 7.).

(Note: We *defined „not allowed difference" as a not* sustainable attribute in the attribute set)

On the first cube level (or „layer") we marked 21 attributes by defining 9 cubes. These, interpreted together with the neighboring cubes, which are edge cubes (4×2) and cornercubes (4×3), and also a single mid cube, which are defined in the other (yellow, blue, green, and red) main agent dimensions as well (Illustration 5).

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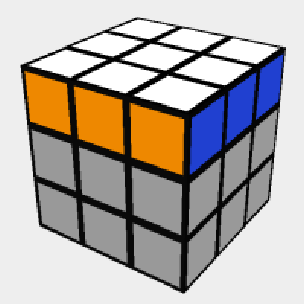
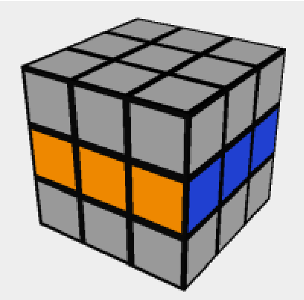


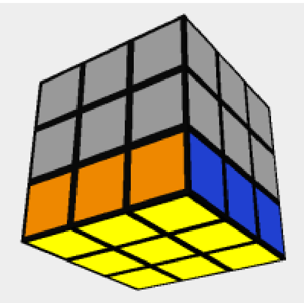
5. Illustration: Interpretation of first cube level (Input side)
Source: self-made

The next level can be specified by defining the middle interpretation zone (1×1×3) of the cube! In this case, we put 12 inner attributes into the interpretation dimension. This means four edge cubes (4×2), and four mid cubes (4×1), meaning we have to calculate with eight cubes as system elements (Illustration 6).



6. Illustration: Interpretation of second cube level, or mid cube connections
Source: self-made

In the last step of defining the interpretation dimensions (levels), similarly to the first step, I marked 21 attributes by defining 9 cubes. These, interpreted together with the neighboring cubes, which are edge cubes (4×2) and cornercubes (4×3), and also a single mid cube, which are defined in the other main agent dimensions (Illustration 7).



7. Illustration: Interpretation of third cube level (Output side)
Source: self-made

We therefore made attribute sets for the SMART evaluation by grouping the mid, edge and cornercubes of the first, second and third rows by solution level. We can say that in the case of cube attributes defineable by values, we have to realise that we can define our entire system with 9+8+9= 26 complex attributes, which have another 54 independent attributes, apart from the main agents (6x9)[4]. We also have to stress that because of the complex attribute handling (1D, 2D, 3D), this analysis can also clearly define the correspondence systems of the various attributes.

B. Churchman-Ackoff process of dominance analysis (SMART software application)

To interpret the attributes of the cubes, and to define the attributes associable to the smallcubes, We chose the SMART (Simple Multi Attribute Ranking System) method, which can handle and illustrate 2D and 3D attributes at the same time. We chose the analysis method as defined in the methodology segment, which method counts as a one of a kind software application in terms of visually illustrating different attributes. The process of the SMART analysis was as follows [7]:

1. Evaluating the results of process of dominance conducted on main agents, input of data,
2. Defining the smallcube attributes of examination levels, and the estimated usefulness values,
3. Creation of SMART charts and illustration in 3D.

Using the results of the Churchman - Ackoff process of dominance analysis, the beginning data of the SMART evaluation is as follows (where I defined the colour/attribute matches according to the results of said process of dominance analysis):

Group1: weighting of attributes	
Summarisation of monetary effects:	90 -----► orange
Adaptation of law and regulations:	70 -----► green
Examination of market opportunities:	60 -----► blue
Technological criteria system:	50 -----► red
Strategic program positioning:	30 -----► not present

In the previous chapter, we answered the question, how we can match the solution algorithms with the different levels for the process of project planning based on Rubik's Cube, and with the Churchman-Ackoff method, we get the four most important attributes from the list of attributes which have an impact on it, namely those we can match to the cube's sides. If white (W) is the Input side, then the most dominant attribute group is matched with the orange (O) side, which gives us our WO base sidepair, from where we continue clockwise around the white side, and the following sidespairs of white-blue (WB), white-red (WR), and white-green (WG) will define the relevant attributes (agents) in the planning process[6].

The assortment criteria is that the most dominant attribute gets placed on the top (in our case, this is orange – O), and the least dominant attribute goes opposite to this side, meaning the bottom (in our case, this is red – R). The reason for this is that the description of their connection profiles (including the contradictions and errors) can be defined best, if it happens via a transaction through two other attributes (left and right sides). In our dominance list, the weakening attributes located in the middle are arranged by their „order of weakening", namely counter-clockwise. The gist of the ranking is that the attribute groups that show stronger dominance are supposedly in better order, while the attribute group of weaker dominance is supposedly further from the point of equilibrium. The attribute group with the strongest relevance will be placed on the top, the weakest relevance attribute group will be opposite to this side, and finally, we define the remaining two groups by their „order of dominance", namely, counter-clockwise. Since the solution of the cube usually happens clockwise in the various algorithms, the parts are optimalised towards the point of equilibrium through the steps of process following the shortest route to solution, which explains why we position the most dominant attribute groups to the green side on the right of our starting orange side, and the third strongest dominant attribute group to the blue side on the left of our starting orange side.

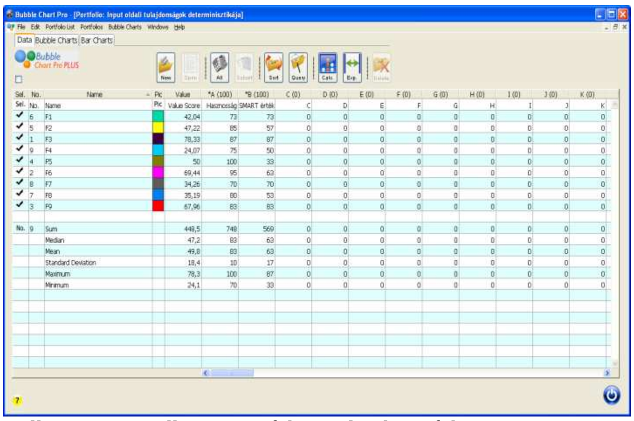
According to the equation above, by portraying the SMART values for the various levels, we obtain clear knowledge on the attributes which have an impact on the different dimensions of usefulness for the main agent. We can wiew the inherent attributes of the main agents (O,R,G,B), their relations to each other, and the usefulness attributes of the Input side in Table 3.

TABLE III. GENERATING SMART INPUT VALUES FOR DATA INSERTION (’►X’ = USEFULNESS)

MID WHITE (1D) – energy rationalisation	Dominance:100	(100) max value
OUTER WHITE (2D) Dominance: Orange/ 90; Green/70; Blue/60; Red/50		
strategy, regulation base connection (WG),		100/70 ► 85
basic technological requirement (WR),		100/50 ► 75
financing expectations (WO),		100/90 ► 95
basic market positioning (WB)		100/60 ► 80
CORNER WHITE (3D) –		
basic criteria of market payoff (WOB),		100/90/60 ► 83,3
eligibility to monetary tools and regulation criteria (WOG),		100/90/70 ► 86,6
syncing basic goals with technological threats and market priorities (WBR),		100/60/50 ► 70
basic criteria designation of matching set of strategies and technologies (WGR),		100/70/50 ► 73,3

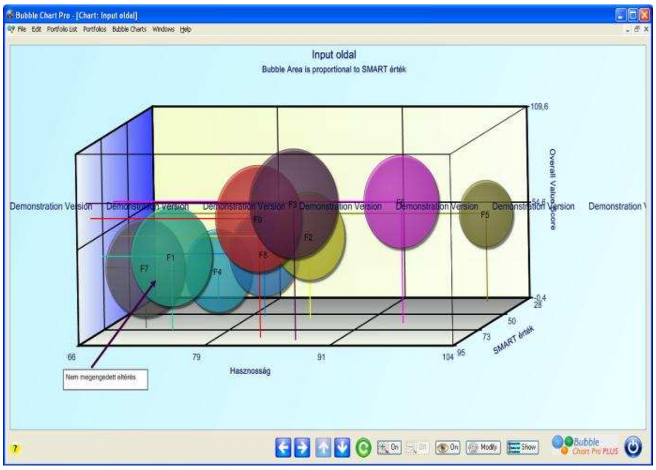
Source: self-made.

After the input of data generated in Table 3., the SMART Bubble Chart Pro (demo version) creates the attribute illustrations via the „Value Score” point rating system, which is useful because the attributes compared to each other can be differentiated visually as well, regardless of that happening by their correctness or their strategic usefulness. Illustration 8 shows the input datachart of the SMART program.



8. Illustration: 2D Illustration of the results chart of the SMART program
Source: self-made based on SMART program analisys

The equilibrium state of the Input side is unstable, evidenced by the F1 attribute, which is the basic attribute designation of matching set of strategies and technologies (WGR/white-green-red). Illustration 8. shows the attributes, and their position in the attribute group. If we click on the sphere, we get the coordinates (x,y,z) for it, which translate to special usefulness-functions. Because of the 3D depiction, both the correspondence of attributes, and the depth of said correspondences can be easily interpreted on Illustration 9.



9. Illustration: Depiction of results and non-equilibrium attributes of SMART program output
Source: self-made based on SMART program analisys

With the aid of the SMART program, we evaluated the Input side (as seen on Illustrations 8. and 9.), the mid cube side-attributes, and the Output side.

IV. RESULTS AND CONCLUSIONS

To summarise the analysis, we can say that we was successful in assigning the project development factors to the side colours of Rubik’s Cube going by the dominance of the attribute groups. The process of dominance analysis conducted with the SMART pilot program, and the summary of its results is as follows:

- The definition of input and output sides of project attributes by picturing them to the white and yellow cube sides was completed,
- In case of the main attributes, orange was the most dominant, while red was the lesast dominant, therefore, the dominance values are the highest for orange, and the lowest for red. If the orange side is in the front, then we find the red side opposite to it, where green goes to the right, and blue goes to the left of orange,
- Going by the dominance, defined in the methodology part, the strongest attribute was assigned to orange, the second strongest assigned to green, the third strongest assigned to blue, while the weakest assigned to red, following the strength of dominance,
- We separated the analysis method (including the various attribute groups) into three different parts – 1) Input ► 2) Mid Cube ►3) Output – for the sake of applicability of the Rubik Logic Theory methods, and the SMART analysis,
- We can set the Rubik Logic Theory optimalisaion for the „selection of technology for base criteria” attribute of the Input side, because this is where the SMART program showed a not allowed difference,

- We can set the Rubik Logic Theory optimalisaion for the „monetary value of the project, and the time needed for payoff" attribute of the Mid cube side, because this is where the SMART program showed a not allowed difference,
- We can set the Rubik Logic Theory optimalisaion for the „market criteria system, balancing of market instruments and provisions" attribute of the Output side, because this is where the SMART program showed a not allowed difference,

Therefore, the results of the analyses on project evaluation of project planning processes for projects which aim at advancement from fossilized energy sources to renewable ones is that the attributes with the most impact are as follows: selection of technology for base criteria (Input side), which has the biggest impact on reaching equilibrium, monetary value of the project, and the time needed for payoff (Mid side, or correspondence attribute group) which if interpreted in a manner more suited to sustainability, will get us closer to the sustainable economical value, and market criteria system, balancing of market instruments and provisions (Output side), which needs proper and balanced planning, for the imbalances it causes may lead to a failed project.

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