The Interpretation of Sustainability Criteria using Game Theory Models
(Sustainable Project Development with Rubik's Cube Solution)

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I dedicate this book to the memory of my cousin, IT specialist and physicist
Tamás Fogarassy (1968-2013)

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LIST OF DEFINITIONS AND ABBREVIATIONS

AB matrixes
x,y vectors
⟨x,y⟩ dot products of vectors
D(f) domain of function f
R(f) range of function f
φ_k payoff function (k-th objective function)
∈ member of set
L possible set of choice variables' values
x* Nash equilibrium of a player
S_i set of strategy choices for i-th player
S simultaneous set of strategy choices
$S_1 \times S_2 \ldots\ldots \times S_n$ Cartesian product of sets of strategy choices
Γ $(n{:}S_1 S_{\_2},\ldots..S_n;\ \varphi_1\ \varphi_2\ \ldots..\varphi_n)$ personal game defined by S_i sets of strategy choices and φ_i payoff functions
X^T transpose of vector x
A^T transpose of matrix A
u≻w distribution „u" dominates distribution „w"
$D_k(A)$ set „A" compiles k-times continuously derivable functions

Abbreviations
K_M man-made capital
K_H human capital
K_N natural capital
TGÉ total economic value
FGÉ sustainable economic value
SMART Simple Multi Attribute Ranking Technique
NE Nash equilibrium
CIE Cleantech Incubation Europe

ABSTRACT

It is very difficult to calculate in advance the positive and negative long-term impacts of an investment or a development venture. The fact that there are environmental-defense orientated, private and “state” ventures that are incorrectly handled and therefore are detrimental to the state of both the environment and economics (market) is a serious problem all around the world. The number of innovative energetic investments, waste and water management projects, etc. in Europe, which cause more devastation to society than we could even imagine, is also high. In Hungary, the state funding – be it direct or indirect – of such enviro-protection orientated ventures amounts to thousands of billions of HUF every year, and the improper use of these funds can set an entire economic sector back in its development for decades. Therefore, for these funds, proper surveying, guiding in terms of market mechanisms, and sufficient re-structuring (as fast as possible) can not only liberate enormous resources for both society and the national economy, but can also contribute to the advancement of the current state of the environment, the labor market, and welfare statistics. For these aforementioned development strategies and investment programs to be contestable, the need for a “construction mechanism” that can fundamentally overcome these detrimental processes and wrong ways of development throughout the course of planning and actualization arises. In this document, I try to find solutions to these planning and development methodology questions, which are based on both the Rubik Logic and modern Game Theory applications.

A standard form of environmentally/economically detrimental funding is when economical policy doesn’t have sufficient information in terms of the environmental damages and negative environmental effects on the infrastructure to be developed (e.g. electric systems or regional waste management facilities). Because of the low transparency of enviro-orientated developments, both in an economic and in a market sense, if we compare them to “non-government financed” sectors, we are faced with far worse conditions when we aim for an economic advancement which keeps the market balance intact. And, if this balance is not present, supply and demand being either too high or too low is a much more relevant and problematic occurrence.

It is therefore a necessity from society that the investments show a sustainable structure not only in terms of economic indexes, but also in terms of labor, by the usage of market and natural resources. The instability of product paths and the ever-changing economic regulation, unstable economic environments in case of the evaluation criteria preceding the investments result in an unpredictable planning environment, primarily for temporary economies (mainly the ex-members of the Eastern Bloc, now members of the EU), which makes creating an appropriate method of planning impossible, or at the least, through much trouble. Therefore, if we find investment and development programs aimed at risk-avoidance, then in our case, the practice of implementing project management and realization through unique laws and new economic policy products (e.g. Policy Risk Insurance) is the usual way.

The evaluation of project resources is further complicated by the fact that economic indexes are practically useless for implementation in development processes, since the relevance of underground and shadow economies distorts the criteria system of processes and derail it from the market’s measurement system to a degree that they show impossible economic optimums and times required for payoff. Knowing the circumstances, it’s no coincidence that the *Environment Programme* of the *United Nations,* or the programs of the *European Energy Centre* that analyze the criteria system of financing renewable energy both aim at extreme financing practices which block the spread of renewable energy production systems. Because of the high investment risk presently relevant in developing countries, banks loaning for investment try to avoid the return times which exceed 3-4 years, and this can’t mean a foundation for sustainable economic development in case of renewable sources, either. That is why the interventions by the economic policy try to correct the criteria systems of development through the regulation and tax systems, but this usually causes more damage to product paths which aim at enviro-protection than the positive impact it has. As a result, we can find loads of failed climate protection and renewable energy production projects, both

in the European and the American economic regions. Slower but safer developments, which have a return time of more than 8 years, are only possible through state investments, but in this case the flexibility of capital sources can't be guaranteed to the required degree. The investment methods required for the sustainable developments (low-carbon innovations) are therefore not present in the development processes; however, as we've seen in the previous overview, the realization of this is not only dependant on the perfection of project management processes.

The main goal of this dissertation is to make the investments aimed at enviro-protection and positive impact on the climate change sustainably plannable, while being conscious of both economic and political instability. Therefore, my goal is to make a scientifically comprehensive model of mathematic correlations based on project development and management systems which can handle the aforementioned problems with its new approach, regardless of national borders.

To realize the main goal of my research, I thought up and defined sub-goals, which are the following:

- Through the process of cited literature, introduction of the relations of economical value and sustainability in both the classic and modern style.
- The mathematic interpretation of sustainability factors, introduction of the sustainable economic equilibrium or corporate strategies in a Game Theory approach, interpretations of finding classic balance points in non-cooperative Game Theory solutions.
- Introduction of conflict alleviation and compromise search through Game Theory methods, and the evaluation of its adaptability in sustainable cooperative corporate strategies.
- The sustainability interpretation of the structure and solution of Rubik's Cube, and the analysis of relations between sustainability and the Rubik solution algorithms.

The market errors which presently have a detrimental impact on the economic environment require a capable external – meaning it is positioned outside the market's condition systems but has an impact on them nonetheless – condition system, because it is imperative for the actualization of sustainable enviro-investments. The creation of a set of rules through the summary of external effects and the defining and typification of the correct external effects of developing sustainable economic structures (and keeping said set of rules definable through mathematic functions) is a social requirement, because it can define the presently used criteria of sustainability for both market shareholders and political decision-makers.

If we go by what was said above in the defining of goals, I primarily found the verification of the following hypotheses to be important:

- H1: The correspondence systems of project attributes which have an impact on the "shelf-life" and sustainability of enviro-orientated investments, or investments that have a positive impact on climate change can be defined through models.
- H2: When searching for a state of equilibrium using Game Theory methods, it is possible to make functions of the relations between the compared attributes.
- H3: Trial functions with multiple variables may be used to select attribute groups which dominantly affect the successful actualization of the project.
- H4: If the various sustainability logics can be synchronized with the 3×3×3 Rubik's Cube solution algorithms, then the relations of the cube's sides define a planning strategy that provides a new scientific approach for investment planning.

I assume that the *"Layer by layer"* Rubik's Cube solution method, meaning the row to row solution, can be used to model the sustainability criteria, in other words, various project planning processes (e.g. investments in renewable energy) can abide by the sustainability criteria with this method.

According to the hypothesis on the solution algorithms of the Rubik's Cube, the parts rotated next to each other, meaning the project attributes which have an impact on each other, have a relation system which can be defined in mathematical terms, therefore, their point of balance (e.g. Nash's) can also be determined by Game Theory models (games of finite kind, zero sum games, oligopolistic games, etc.).

In this document, I would also like to prove that the hypothesis which states that the solution algorithm of Rubik's Cube, namely the *"Layer by layer"* method, can be used to model the process of project development. Also, the correspondence system of project attributes can be represented by the proper Game Theory models. This way, the various enviro- and climate-friendly investments can be realized in a well-plannable, low-risk economic environment regarding both human resource planning and the preserving and advancement of environmental criteria.

1. INTERPRETATION OF SUSTAINABILITY WITH BASIC GAME THEORY MODELS AND RUBIK'S CUBE SYMBOLISM

During the detailed review of specialized literature, I categorized the used material into three different topics:

- Sustainability dilemmas and tolerance questions in economic evaluation systems.
- Search for equilibrium using non-cooperative Game Theory models, and cooperative strategies in corporate strategy planning.
- The structure of Rubik's Cube, and mathematic algorithms for its solution.

In the sustainability dilemmas part, I introduce the questions of defining strong and weak sustainability, the sustainable indexes of ecological economics and environmental economics, and the new approach on total economic value. I also show why the economy theses which view ecological economics and environmental economics as the same, uniform system aren't true.

In the parts on Game Theory, I introduce the Game Theory themes, both domestic and foreign, which are connected to sustainability and corporate cooperation, and the creation of cooperative strategies. I introduce the Nash equilibrium as a highlighted topic, which, although written 20 years ago, the view on its applicability and usefulness is varied. Nash divided the optimization games into cooperative and non-cooperative types. According to his thesis, the cooperative game is a kind where cooperation between players is simply enforced. According to him, we can only talk about non-cooperative games if agreement between players is impossible to enforce. In the case of the Nash equilibrium, the strategies of the various players are the optimal replies to others' strategies, so there aren't any players who want to break this status quo by choosing new, different cooperative strategies.

The third part of the procession of literature is basically the myth of Rubik's Cube. The Rubik's Cube, which is one of the most impressive games of Hungarian origins worldwide, is a phenomenon and mathematical mystery even today. The cube's fame was followed closely by the mystical meaning of the number three. Many believe the cube to symbolize connections by which man can be linked with nature and natural existence. During the part on Rubik's Cube, the solution methods, "God's number," and the mathematical interpretation of the solution process is demonstrated.

1.1. Sustainability dilemmas, and questions of tolerance

We can find very different approaches on interpretations of sustainability in economics. In trying to define sustainability orientations/strategies, a widely known dilemma is whether we should apply weak or strong sustainability criteria for the various cases.

1.1.1. Definition of strong and weak sustainability

The evolution of environmentally important, different economic thoughts can be nicely and thoroughly followed in Sándor Kerekes' essay, "A fenntarthatóság közgazdaságtani értelmezése" ("Economic interpretation of sustainability," 2005). However, in this dissertation, I tried to introduce the integration of the sustainability concept into the system of economics, using the effects of various resources on each other, by which we can get a clear view on the fact that handling market questions is possible with market instruments, if there is no aggregation of errors.

According to our line of thought, the indirect assumption that development based on market economy in itself means sustainable development structure is revealed (in case of an environment rid of overproduction and production shortage), which however moves on a very different equilibrium point to the Pareto optimal because of the indirect global effects, and direct political and state interventions against them, and is far from the sustainability point because of market errors. However, the desired market balance and the definition of sustainability are not obvious, and even less so if the process moves according to economic policy interests (Spash, 2011).

The problem and solution search is by no means new, since Pearce and Atkinson already defined the generic requirement named the Hicks-Page-Hartwick-Solow rule in the early development phase of global economics in 1993, which states the criteria of weak sustainability.

The Hicks–Page–Hartwick–Solow rule:

$$dK/dt = K = d(K_M + K_H + K_N)/dt \geq 0$$

$$\text{where: } K = K_M + K_H + K_N$$

Pearce and Atkinson differentiate between three types of capital:

- K_M - man-made (or reproducible) artificial capital (roads, industrial establishments, factories, mansions etc.),
- K_H - human capital (amassed knowledge and experience),
- K_N – natural capital, quite widely interpreted, includes all the natural resources (minerals, soil, etc.) and other natural goods required for daily life, like bio-diversity, capacity assimilating pollution, etc.

According to Pearce and Atkinson, if we accept the basic thesis of neoclassic economy, by which we state that different capitals can be interchanged limitlessly, then weak sustainability is as follows:

$$\frac{dK}{dt} = \frac{d(K_M + K_H + K_N)}{dt} \geq 0$$

Therefore, in an economic sense, weak sustainability is when the value sum of capital accessible to society doesn't decrease with time.

Since capital can be defined as savings minus depreciation, and of the three aforementioned capitals, the depreciation of human capital is zero (accepted since the knowledge and experience amassed by humanity doesn't whither), the weak sustainability (Z – sustainability value) can be defined as such, according to Pearce and Atkinson (based on: Kerekes, 2005):

$$Z = \frac{S}{Y} - \frac{\delta_M * K_M}{Y} - \frac{\delta_N * K_N}{Y}$$

Where S stands for savings, Y for Gross National Product (GNP), delta M (δ_M), and delta N (δ_N) for the amortization rates of man-made and natural capitals, respectively. If we don't allow interchange between capitals, then Pearce and Atkinson (1993) state that we arrive at strong sustainability. For the strict sustainability to be fulfilled, it is a criterion that the value of natural capital does not decrease with time:

$$\frac{\delta_N * K_N}{Y} \geq 0$$

According to Pearce and Atkinson (1993), a national economy is sustainable if the savings are more than the depreciation of artificial man-made and natural capitals (Table 1). We can see in Table 1 that the resource sum (the quotient of savings and GNP) is positive, and therefore sustainable, if the depreciation of natural capital is lower than that of the artificial capital, or if the resource reserves made in the same year are vastly superior to the value sum of both resources' destruction ($K \geq K_M + K_N$).

Scientists and ecologists therefore don't accept the interchange of capitals, and by extension, weak sustainability; furthermore, they have problems even with strict sustainability, since it also allows for the compensations between different natural capitals. Most ecologic economists state in relation to strict sustainability that irreversible changes in the environment (e.g. the extinction of species, or the reduction in diversity) must not be caused (Pearce - Atkinson, 1995).

Table 1. Sustainable and non-sustainable economic systems (1992)

Weak sustainability	*S/Y*	-	δ_M/Y	-	δ_N/Y	=	***Z***
Sustainable economy							
Brazil	*20*		*7*		*10*		*+3*
Finland	*28*		*15*		*2*		*+11*
Germany	*26*		*12*		*4*		*+10*
Hungary	*26*		*10*		*5*		*+11*
Japan	*33*		*14*		*2*		*+17*
USA	*18*		*12*		*4*		*+2*
Marginal economy							
Mexico	*24*		*12*		*12*		*0*
Philippines	*15*		*11*		*4*		*0*
Non-sustainable economy							
Ethiopia	*3*		*1*		*9*		*-7*
Indonesia	*20*		*5*		*17*		*-2*
Madagascar	*8*		*1*		*16*		*-9*
Nigeria	*15*		*3*		*17*		*-5*
Mali	*-4*		*4*		*6*		*-14*

Note:
S stands for savings, Y for Gross National Product (GNP), delta M (δ_M) and delta N (δ_N) for the amortization rates of man-made and natural capitals respectively, Z for sustainability value

Source: Pearce - Atkinson: Are national economies sustainable? Measuring Sustainable Development, 1993

The natural and man-made capital ratio basically determines the difference between weak and strong sustainability. The weak sustainability theory says that natural and man-made capitals are interchangeable. The sustainability criterion is also met if the combined value of the two types of capital is not reduced. This criterion is met if, during the destruction of natural resources, at least the same amount of artificial capital is created (Gowdy - Erickson, 2005).

The strong sustainability theory, however, explicitly states that man-made capital and natural capital are not, or only to a very small extent interchangeable, so this criteria creates an absolute limit of sustainability, which at a defined level must be maintained to reach sustainability (Málovics - Bajmócy, 2009).

1.1.2. Ecologic economy versus enviro-economy

According to Turner and his affiliates (1996), the discussion of the relationship between the economic and ecological systems is a very good opportunity for economists, since the optimistic and the pessimistic theories, economics, and marketing strategies can be checked against each other. The essentially growth-oriented, techno-optimistic environmental economics, and the stable (equilibrium) size-oriented, techno-pessimist ecological economics show different approaches to the sustainability concept via their theories.

The basic theories are the same, in that environmental economics and ecological economics are practically uniform in their starting points: both are based on the natural capital being the basis of all kinds of economic activities, therefore, differences of opinion between these schools in relation to sustainability are not primarily defined by weak or strong sustainability or discussions on artificial and natural capitals (Spash, 1999). The difference is rather in the relation system of the environmental economics and the ecological economics approach (Turner et al., 1993).

The basis of environmental economics is the examination of the macroeconomic indicators (GDP, GNP etc.) also well known to neo-classical economics, and together with their errors they show us the characteristics of changes in well-being. In contrast, ecological economics uses a problem-oriented approach during the empirical phase that often does not really concretize system parameters from the economic point of view. In the case of ecological economics, the deeper integration of other social and natural sciences' knowledge is important for the determination of sustainability (Korten, 2011).

In many cases, ecological economics handles the environmental economics' simplistic views and problem-solving suggestions with reservation. It sees the causes of environmental problems in market errors and market failure problems (the accumulation of external effects and their inadequate internalization) more deeply rooted, and wants to advance sustainability by achieving radical institutional changes (Málovics-Bajmócy, 2009).

Ecological economics is outlined along the original concept of sustainability, which heavily relies on three main issue topics: defining the connection system of ethics, economy, and economic value.

Therefore, we can ask the important main questions: What is the economy? Where are the limits of resource systems? Is it possible to talk about socially and / or environmentally responsible business practices? What values do business and political decision-makers base their decisions on? What is tolerable by the various test systems and what isn't?

1.1.3. Relations between total economic value and sustainable economic value

Over the past decades, economics, including environmental economics, made significant progress in the classification of the economic value of the natural environment. The evaluation was based on reshaping the traditional relationship between the evaluator, man, and goods that it evaluates. People are attributing increasing value to commodities that are not precisely definable, such as environmental goods (Korten, 2013). The aggregate of these values can be perceived as the so-called total economic value. Total economic value (TGÉ) can be divided into several components, the two main elements of which are use-related, or non-use-related value components.

Total economic value is therefore:

$$TGÉ = \textit{use-related values} + \textit{non-use-related values}$$

The system of total economic value (TGÉ), which is basically a modern adaptation of economics that also manages environmental values, is still unable to properly handle the issue of transformations related to the time value of money resources. This is helped by a still not too well known approach entitled the sustainable economic value by Molnár in 2005 (Molnár, F. 2005).

Sustainable economic value (abbreviated FGÉ), in contrast to the traditional concept of total economic value (abbreviated TGÉ), basis much on the future concepts and display of feedback and peer effects, as these questions are important to ensure sustainability. In the sustainable economic value concept, economy is present only in a derived form, not like in the traditional total economic value concept. The FGÉ concept takes matters relating to the time value of money into account much more importantly, and therefore does not forget about future generations or values associated with the preservation of property, either (F. Molnár, 2005). Sustainable economic value is therefore an evaluation method that is capable of - taking into account the local information and their calculated use - integrally presenting the social and technical capital elements' change in addition to natural capital, which the total economic value only implements in a very limited fashion (Kiss, 2004).

The search for economic equilibrium is therefore a prerequisite for meeting the sustainability criteria, so the related research and the primary preference model experiments have been given priority over the years. Finding the point of equilibrium (defined as a kind of sustainability economic value) and the accurate detection of correlations based on Game Theory is therefore one of the most popular areas of research. In order to get an exact outline of sustainability in an actual criteria system of equilibrium, we can't make do without knowledge on renewable energy projects, conditions affecting the market environment, and the cooperative, or non-cooperative game theory context (Fogarassy et al., 2007). In the next Game Theory chapters, I wish to present how the market factors that influence economic decisions may determine the definition and change in "economic equilibrium points," in other words - sustainability.

1.2. Theory of non-cooperative games

Economics basically states that rational behavior is based on consistent preferences. If a person's preferences satisfy some basic needs or consistence-criteria, then these preferences can correspond to a well-defined usefulness function. Therefore, rational behavior can be viewed as the maximization of the usefulness function. This leads to the fact that we can call this definition of rational behavior the usefulness theory, says Harsányi (1995).

However, Harsányi also states that there is much evidence that this hypothesis is not valid, and that economics is rather based on the hypothesis that the preferences of people are completely consistent (1995). Most economists view this as a useful and simplifying hypothesis, arguing that economics, which is built upon this hypothesis, can offer mostly good, even if not perfect forecasts of the machinations of the economic system.

Economic policy argues about how the happenings of economy affect the behavior of society. To prove that this is a basic part of everyday life, e.g. the fact that the average family size has been in decline in most countries in the last few decades, is well-based proof. The reason for this is that the economic pros of a larger family were greatly diminished while the cost was heightened due to the effects of urbanization and intensive technological advancement. The preference of a smaller family model and the consumption and living standards of families consisting of 3-4 people therefore took an important role in the process of optimum search (Hobson, 2012).

In order to link not only decision aspects based on economic indexes, but actual factors that have an impact on living standards to the choice preferences of various families, e.g. the criteria system of their purchases, we have to aim at creating a comparison method which can be used for many different attributes that aren't dependent on each other (Vincze, 2009). If in the course of our decisions we have to form a judgment on a system or an object based on more characteristics simultaneously and these characteristics mean a mostly controversial characteristic set, while the sorting principles being connected to this show complicated contexts, then we may turn to mathematical modeling and its software applications for help. Therefore, to start the examination of the multipurpose systems, we should assign the most important criteria. We have to accept that fact at the time of the optimization of more opportunities, that according to Axelrod (2000) we can't optimize all characteristics taken into consideration simultaneously. The simple reason for this is that the optimum of the attribute-representing objective functions usually won't correspond to the same alternative all the time.

We can generally say about the designation of the criteria system that we cannot take each single characteristic that influences our system or the function of its examined object into consideration. That is why we have to select those that are worth the additional investigation from the essential characteristics. The various characteristics have to be totally independent of each other. This is very important during the course of the selection in order to have no overlaps between the single criteria, since these cross characteristics may cause unnecessary examinations and a loss of time during the analysis.

Therefore, during the multipurpose optimization tasks, we may have the following tasks in order for the process of the model creation, and the criteria system of problem solution to take shape (Forgó et al., 2005):

1. Designation of criteria system (and also, major attribute sets)
2. Independence analysis of attribute sets (avoiding overlaps between attributes)
3. Designation of choice variables and parameters in attribute sets (deterministic, or stochastic – in other words, realized with some level of probability – marking)
4. Designation of binding criteria related to set (creation of sets)
5. Designation of possible criteria of the criteria system, and the number of objective functions in set (the number of objective functions is finite)
6. Search of optimum for objective functions

The general form of the *multi-purpose programming* ***task*** according to Molnár et. al. (2010a) is as follows (if we assume that $D(\varphi_k)$ = L and in case of any $X\in$, φ_k (X) is a real number):

$$\boldsymbol{X \in L}$$

$$\boldsymbol{\varphi_k (X) \rightarrow max \qquad (k=1, 2, \ldots.., n)}$$

X – system of choice variables
L - possible set of values for choice variables
$D(\varphi_k)$ *– choice domain of function*
n – number of objective functions
φ_k *= k - th objective function, in other words, the payoff function*

We have to note, however, that the need for multipurpose problem solving is not the requirement of the present, since János Neumann already laid down the function-like necessities of the behaviors attached to rational decisions in 1928, and, in his work written together with Oskar Morgenstern entitled "*Game Theory and economic behavior*," wrote it down in detail already in 1944 (Neumann – Morgenstern, 2007).

1.2.1. Search for points of equilibrium in non-cooperative games

Game Theory fundamentally deals with the solution of multipurpose problems, that is, with so-called strategic games. Game Theory is one of the branches of mathematics with an interdisciplinary character and it primarily tries to tackle the question of what is rational behavior in situations where the possible choices of the participants influence the result of the decisions of all participants. A problem or problem solution can be called a strategic game if the decision makers may have influence on the outcome of the game between the existing conditions and the framework of rules (Mező, 2011a).

We always assume that we may characterize the outcome of the game for all players with an objective function in Game Theory solutions, in other words, the payoff function already mentioned. And for the various players (characters), the bigger the payoff function value, the more beneficial the outcome of the game is. The players' decisions, in other words, the outcome of the decisions onto the final result, are what we call the player's strategy. We know two- or multi-person variants of Game Theory solutions. The Game Theory solution is non-cooperative if the players or characters compete with each other during problem solving, while the game is obviously cooperative if cooperation takes shape between the players. The importance of searching for points of equilibrium is emphasized in Game Theory. If the search aims at the fact that, when including all players' strategies, the benefit of one player won't change in case of him changing his strategy and none of the other players do so either, we call it the Nash equilibrium (Szidarovszky - Molnár, 1986).

The theory of the Nash equilibrium originates from John Nash, who was rewarded with a Nobel-prize for the development of the theory at the same time as János Harsányi, exactly 20 years ago. It was shown through Nash's equilibrium theory that all finite games have at least one point of equilibrium.

Nash divided the optimization games into cooperative and non-cooperative types. According to his thesis, the cooperative game is the kind where cooperation between players is simply enforced. According to him, we can only talk of non-cooperative games if agreement between the players is impossible to enforce. A non-cooperative game in the case of various strategies of players can only be called stable if the so-called Nash equilibrium is present. In the case of Nash equilibrium, the strategies of the various players are the optimal replies to the others' strategies, so there aren't any players who want to break this status quo by choosing new, different cooperative strategies. The game will not be stable if it is not in the Nash equilibrium point, because there is always at least one player in this case to whom his strategy does not mean the best answer in the given situation, and therefore he will be interested in looking for a new strategy for himself (Harsányi, 1995).

As I've already mentioned, the equilibrium situation may also be stable in case of cooperative games if one of the strategic combinations isn't in accordance with the rest of the strategies, because the strategic cooperation will sooner or later be enforced. However, in economic life we mostly face strategy creations that do not take each other into consideration and run beside each other or that do not take the multipurpose decision process or the designation of choice optimum into consideration (Molnár-Kelecsényi, 2009). From the European or economic policy practice, we have a good example for this: the bulk of strategies concerning environment protection or renewable energetic developments, since we often face a strategy creation with a contradictory direction here.

A principle is that developments with an environment protection aim have an opposite direction to that of the priority system of economic development (f. e. the program taking aim at the reduction of greenhouse gas and fossil energy use takes aim at the minimization of the intake, while the other one at the increase of a polluter energy source). A good example for this is two of the EU's main strategies: EU Low-carbon Roadmap 2050 vs. Nuclear Power in France (2014). 58 nuclear power plants operate currently in France, and additional developments are going on, while Germany just decided that by 2020, all (now 8 operating plants) of them will be shut down.

In case of cooperative games, the selected strategy may also be stable even if a strategy combination is not in Nash equilibrium but the players come to an agreement that this strategy combination will be selected. During the course of the presentation of this search dilemma for the Nash equilibrium point – in case of non-cooperative games - I lean fundamentally on István Mező's study (2011b), "Game Theory," while if I differ from this, I'll note it separately in the description.

Definition 1:

According to the *Definition* of the Nash equilibrium:

The point of equilibrium or strategy for an n player $J = (n, S, (\varphi_i)_{i=1}^{n})$ game is a point (strategic *n*), for which

$$\varphi_i(x_1^*, \ldots, x_{i-1,}^* x_i^*, x_{i+1,}^*) \geq \varphi_i(x_1^*, \ldots, x_{i-1,}^* x_i, x_{i+1,}^*) \qquad (1.1)$$

holds true for every $i = 1, \ldots\ldots, n$ player. The point of equilibrium is therefore called a *Nash equilibrium*. [Shortened: $\varphi_k(x_1^*, \ldots, x_{k,}^*, \ldots\ x_{n,}^*) \geq \varphi_k(x_1^*, \ldots, x_k, \ldots x_{n,}^*)\ where\ k = 1,2, \ldots n$].

If 1.1. 's equality is strict, then it is called a strict equality.

If we do not state anything else, we say it's the point of equilibrium is non-strict.

The i-th player can maximize his own payoff if he plays the equilibrium strategy, namely x_i^*, if all the other players do the same. We will need the following definitions to find the state of equilibrium:

Definition 2:

An $n-$ player J game is called a constant sum if the rewards and demerits earned by the player is a constant c value, regardless of strategy.

With formula:

$$\sum_{i=0}^{n} \varphi_i(x) = c \ (x \ \in S).$$

If $c = 0$, the game is called *zero sum game*.

Two-player, zero sum games are useful for demonstrating the definition of the point of equilibrium better. If we take a $(x_1^*, x_2^*) \epsilon \ S$ point of equilibrium, based on (1.1)

$$\varphi_1(x_1^*, x_2^*) \geq \varphi_1(x_1, x_2^*) \text{ for every } x_1 \in S_1 \qquad (1.2)$$

and

$$\varphi_2(x_1^*, x_2^*) \geq \varphi_2(x_1^*, x_2) \text{ for every } x_2 \in S_2.$$

The game is zero sum, therefore:

$$\varphi_1(x_1, x_2) + \varphi_2(x_1, x_2) = 0 \, , \varphi_2(x_1, x_2) = - \, \varphi_2(x_1, x_2),$$

and:

$$-\varphi_1 \ (x_1^*, x_2^*) \geq -\varphi_1 \ (x_1^*, x_2).$$

If we rearrange the formula:

.

and including (1.2) inequality as well, we get:

$$\varphi_1 \ (x_1, x_2^*) \leq \varphi_1 \ (x_1^*, x_2^*) \leq \varphi_1 \ (x_1^*, x_2).$$

This inequality system states that from a (x_1^*, x_2^*) point of equilibrium, if player one leaves with a strategy different from x_i^*, the payoff function can only be either lower or equal. If player two is the one who leaves, the payoff function of player one will either be greater or equal, and since the game is zero sum, this would mean that his "payment" won't be greater.

Definition 3:

Let's look at two games, which only differ in payoff functions:

$$J = (n, S, (\varphi_i)_{i=1}^n) \ and \ J' = \left(n, S, (\varphi_i')_{i=1}^n\right).$$

J and J' are called strategically equivalent, if there's a positive number, and there are $b_i \ numbers, where \ i = 1, \ldots., n)$, that

$\varphi_i'(x) = a\varphi_i(x) + b_i$ for every $x \ \in S \ and \ i = (1, \ldots., n)$.

The following thesis describes the obvious fact, which is also clear on the basis of simple intuition and logic, that strategically equivalent games must always be played in the exact same manner.

Thesis 1:

The points of equilibrium for strategically equivalent games are the same.

Proof: Let $J = (n, S, (\varphi_i)_{i=1}^{n})$ and $J' = \left(n, S, (\varphi_i')_{i=1}^{n}\right)$ be our strategically equivalent games, and let $(x_1^*, \ldots, x_n^*) \in S$ be one of the points of equilibrium for game J. According to the definition for point of equilibrium (1.1), at $i - th$ player's every $x_i \in S_i$ strategy:

$$\varphi_i(x_1^*, \ldots, x_{i-1}^*, x_i, x_{i+1}^*, \ldots x_n^*) \leq \varphi_i(x_1^*, \ldots, x_{i-1}^*, x_i, x_{i+1}^*, \ldots x_n^*)$$

Since J' is strategically equivalent to J, then with the constants $a > 0 \; and \; b_i$:

$$a\varphi_i'(x_1^*, \ldots, x_{i-1}^*, x_i, x_{i+1}^* \ldots, x_n^*) + b_i \leq$$

$$\leq a\varphi_i'(x_1^*, \ldots, x_{i-1}^*, x_i, x_{i+1}^* \ldots, x_n^*) + b_i.$$

which, due to being positive results in

$$\varphi_i'(x_1^*, \ldots, x_{i-1}^*, x_i, x_{i+1}^* \ldots, x_n^*) \leq \varphi_i'(x_1^*, \ldots, x_{i-1}^*, x_i, x_{i+1}^* \ldots, x_n^*)$$

And this means $(x_1^*, \ldots x_n^*)$ is a point of equilibrium for J'.

Thesis 2:

For every constant sum game, there is a zero sum game that is strategically equivalent to it, therefore having an overlap in points of equilibrium.

Proof: Let us simply subtract the constant sum from the value of payoff functions. This results in a zero sum in every possible outcome, and the new game remains strategically equivalent to the old one. Because of our previous thesis, their points of equilibrium are also the same. If we look at constant sum games, it's enough for us to only concern ourselves with the zero sum variants only. Furthermore, a two-player game is zero sum if one player wins exactly as much as the other one loses (or the opposite). Therefore:

$$\varphi_2(x_1, x_2) = -\varphi_1(x_1, x_2)$$

we can use this function, and we don't actually need a payoff function altogether, we simply use $\varphi -$ instead of e.g. φ_1.

1.2.2. Theoretical correspondences of finite games

If an n – player game's S_k $(k = 1,2, \ldots n)$ strategy sets are finite, then we say the game is of a finite kind. Games of a finite kind can typically be either two-player or n - player, meaning more than two players. In the function-like correspondence system of games of finite kind, it can be made obvious that during the solution of multi-purpose problems, the strategic sets, in other words, the defined parameters, or criteria groups are finite, but are always more than two. Since the project development decision process of our analyzed problem, the investments related to environmental defense and renewable energetic developments is n - player $(n \geq 2)$, it's advisable for us to analyze the behavior of multi-person games (Szidarovszky et. al 2013). We do assume however, that the game isn't concluded in a mere moment, but at previously designated $t_0, t_1, t_2, \ldots$ times, where only one player can modify the state of the game due to a previously set consecution. This state can be depicted with a tree graph (refer to Figure 1). This process in search of optimum is quite similar to a 3×3×3 Rubik's Cube's solution process alternatives, since we decide by the cube's randomness, in other words, unsortedness, which shortest combination row (or depth level) we choose to heighten the level of sortedness, or simply, the solution of the cube in the case of *layer by layer* solution method (G. Nagy, 2008).

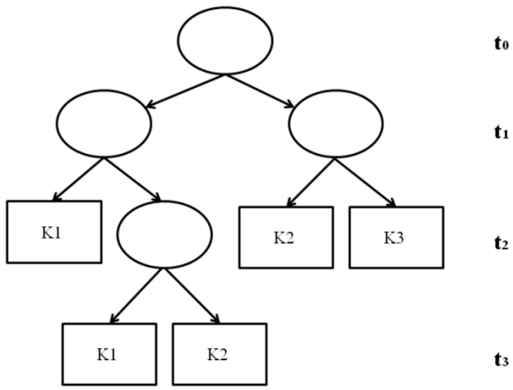


Figure 1: Decision tree graph with pre-determined times (t0, t1, t2, t3) and combination alternatives (K1, K2, K3)

Source: self-made (based on MIEA, 2005)

The state of the system made by the players, in other words, the multi-purpose problem solving, can be illustrated with a problem tree, or tree graph, however, let's assume (according to Forgó et al. 1999) that:

a) the tree has a starting point (state of t_0 time, which goes towards $t_1, t_2 \ldots\ldots, n$),
b) one player is assigned to the starting point and every branching point in accordance with a previously defined rule; this player can choose between the various, finite number of edges, and relocates the system's state from this edge's starting point to the conclusion point,
c) every player knows the game's current progress in every $t_k (k \geq 0)$ time point, including all the states of the game up to it,
d) in all the conclusion points of the tree diagram, all players have a payoff function value known to them.

Furthermore, let's assume that there are no players assigned to some of the peak points of the tree graph and the progression from here happens randomly through predetermined distributions, meaning the players don't have choices, or do, but only symbolically. For the sake of uniformity, let's assign a player to these peak points as well, but their decisions are merely formal. In this case, the payoff functions of the game should be replaced with anticipatory values based on the random distributions. This game can be illustrated with a finite tree graph and is called a perfect information extensive-form game (Molnár - Szidarovszky, 1995; Molnár 1994).

Thesis 3:

Every perfect information extensive-form game that can be depicted with a tree graph has at least one point of equilibrium.

Proof: Let $\boldsymbol{I}$ mark the starting point of the tree graph, while $V_1, V_2, \ldots\ldots V_M$ marks the conclusion points. In our tree graph's case, there is a single route leading from $\boldsymbol{I}$ to every V_k conclusion point. The length of the longest route is called the tree graph's length. We will prove the thesis with the complete induction of the game's tree graph's length, according to Molnár and Szidarovszky (2011a).

If $h(F) = 0$, then the tree graph only consists of the starting point. This means that players only have a single strategy. Furthermore, in this case, this single strategy is obviously the point of equilibrium. Let's say that $h(F) \geq 1$. Let m mark the number of edges originating from the starting point, and let $U_1, U_2, \dots \dots, U_m$ be the conclusion points of the edges originating from the starting point. Furthermore, let $F_k (k = 1,2, \dots, m)$ mark the maximum part-graph with the U_k starting point, excluding the I peak point. It is obvious that F_k's conclusion points are from F's points. Let $\Gamma_k = \left(n; S_1^{(k)} \dots, S_n^{(k)}; \varphi_1^{(k)} \dots, \varphi_n^{(k)}\right)$ mark the original game narrowed to F_k, meaning that to F_k's peak points, we assign players also assigned to them in F, and to its conclusion points, we assign the payoff function values of F.

In this case, it's obvious that $h(F_k) < h(F)$, therefore, according to our induction thesis, Γ_k games have $x^{(k)*} = (x_1^{(k)*}, \dots \dots x_n^{(k)*})$ existing points of equilibrium. Let us say that in the game of graph F, we assigned player i_0 to the starting point.

a) Let us assume that player i_0 can freely decide between the m routes originating from the starting points. In this case, it's obvious that if $i \neq i_0$ then $S_{i_0} = X_{k=1}^{m} S_i^{(k)}$

$$S_{i_0} = X_{k=1}^{m} S_i^{(k)} \, x \, \{1,2, \dots \mathrm{m}\}.$$

Therefore, player i_0, apart from advancement in graph F_k, has to decide the advancement from the starting point as well. Let $\varphi_{i_0} \, mark \, \left(x^{(k)*}\right)$ numbers' highest index, where the strategic n bundle obviously offers the point of equilibrium for the game, in case of $\Gamma_k \, (1 \leq k \leq m)$ games' $x^{(k)*}$ points of equilibrium, supplemented by advancement from starting point I to point U_{k0}.

b) Let us assume that player i_0 randomly makes a choice at the start of the game from a pre-determined $p_1, p_2, \dots \dots, p_m$ probability distribution. In this case, the Cartesian product of Γ_k games' $x^{(k)*}$ points of equilibrium gives the original game's point of equilibrium. To prove this, let's say that in case of $k = 1,2, \dots, m$ and $i = 1,2, \dots, n$:

$$\varphi_i^{(k)} x^{(k)*} \geq \varphi_i^{(k)} (x_1^{(k)*}, x_i^{(k)*} \dots \dots x_n^{(k)*}).$$

with arbitrary $x_i^{(k)} \in S_i^{(k)}$, multiplying the inequality by p_k and in case of $k = 1,2, \dots, m$, we obtain the inequality defining the point of equilibrium for the original game by addition.

The thesis was proven based on Molnár - Szidarovszky (2011b), who stressed during the proof that the multipurpose problem solving is primarily suitable for the solution of smaller tasks, in other words, forming short tree graphs. Here, we get games that can be depicted with zero length tree graphs, in case of which staying in their only point means the only point of equilibrium. However, the number of the short tree graphs may grow so much in case of bigger tasks that it makes searching for points of equilibrium impossible. This can be recognized with experimentation – trial and error.

1.2.3. Games of infinite kind – Game Theory models with one or more points of equilibrium

1.2.3.1. Games with a single point of equilibrium

The simplification of the points of equilibrium for any given game can be proven by e.g. proving the problem of one of the exact fixed point-problems, which is equivalent to the problem of equilibrium. During the proof, I basically follow Szidarovszky's logical process, while if I differ from this, I'll note it separately in the description.

A single-variable, fixed point problem usually means the solution of the $f(x) = x$ equation. It is a known fact that if **f** is a decreasing function of x, there can be no more than one solution. Let's try to generalize this monotonization criteria to a multi-dimension case. Therefore, let **f** be a vector-variable, vector-value function, **f** : $R^n \longrightarrow R^n$. The following example shows that strict monotony by component still won't guarantee exactness (Szidarovszky, 1978a).

Example. Let's look at the

$$x = - x - 2 y$$

$$y = - 2x - y$$

fixed point-problem, given by

$$\boldsymbol{f}(x, y)= \begin{pmatrix} -x - 2y \\ -2x - y \end{pmatrix}$$

imaging. Both components in both their variables are strictly decreasing functions, and yet there are an infinite number of fixed points:

$$y \quad -x.$$

We can show however, that a different type of monotony is sufficient to prove the exactness of fixed points.

Definition. Let D $\subseteq$ R^n be a convex set. We call an *f:* D $\rightarrow$ R^n function *monotone*, if for any *x, y* $\in$ D :

$$(x-y)^T(f(x)-f(y)) \geq 0.$$

function ***f*** is called *strictly monotone*, if for any $x, y \in D$ *and* $x \neq y$:

$$(x-y)^T(f(x)-f(y)) > 0.$$

We can easily see that if ***f*** is monotone, the imaging cannot have two different fixed points. Let us assume in spite of this that **x** and **y** are both fixed points.

$$\mathbf{0} < (x - y)^T(x - y) = (x - y)^T(f(x) - f(y))$$

$$= -(x-y)^T((-f(x))-(-f(y))) \leq \mathbf{0},$$

This is obviously a contradiction.

We can easily check the monotony of the imaging, as shown by the next thesis:

Thesis. Let $D \subseteq R^n$ be a convex set and $\boldsymbol{f}: D \rightarrow R^n$ function continuously differentiable. Let $\boldsymbol{J(x)}$ *mark* $\boldsymbol{f}$'s Jacobi matrix at point $\boldsymbol{x}$.

a) If $J(\mathbf{x}) + J^T(\boldsymbol{x})$ is positive semi-definite in all $\mathbf{x} \in D$ points, $\boldsymbol{f}$ is monotone,
b) If $J(\mathbf{x}) + J^T(\mathbf{x})$ is positive definite in all $\mathbf{x} \in D$ points, $\boldsymbol{f}$ is strictly monotone.

Proof.
For a fixed $\mathbf{x}, \mathbf{y} \in D$, let's introduce the

$$\mathbf{g}(t) = \mathbf{f}(\mathbf{y} + t(\mathbf{x} - \mathbf{y}))$$

scalar variable function. Obviously

$$\mathbf{g}(0) = \mathbf{f}(y) \text{ and } \mathbf{g}(l) = \mathbf{f}(x),$$

therefore

$$f(x) - f(y) = \int_0^1 J\,(y + \mathbf{t}(x - y))\,(x - y)\boldsymbol{dt}.$$

Let's multiply both sides from the left by the $(\mathbf{x} - \mathbf{y})^T$ linear vector, and we get:

$$(\mathbf{x} - \mathbf{y})^T(\mathbf{f}(\mathbf{x}) - \mathbf{f}(\mathbf{y})) = \int_{\mathbf{0}}^{\mathbf{1}} (\boldsymbol{x} - y)^{\boldsymbol{T}}\ J\,(y + t(x - y))\,(x - y)dt$$

$$= \frac{1}{2} \int_0^1 (\boldsymbol{x} - \boldsymbol{y})T\ \{J\,(y + t\,(x - y)) + J^T(y + t\,(x - y))\}\,(x - y)\,dt$$

In the previous step, we used the fact that for any given $u \in D\, R^n$ vector and $n \times n$ type J matrix:

$$\mathbf{u}^T\mathbf{J}\mathbf{u} = \mathbf{u}^T\mathbf{J}^T\mathbf{u},$$

since both sides are scalar, and each other's transposes. If $\mathbf{J} + \mathbf{J}^T$ is positive semi-definite, the right side isn't negative, and if it's positive definite, then it's positive, respectively. Having precise knowledge about imaging is an absolute necessity to use the results above, which isn't always possible, due to the need for knowledge on solving optimum-problems. This is why results which are based on the attributes of strategic sets and payoff functions are very important (Molnár-Szidarovszky, 2011c; Mészáros, 2005).

1.2.3.2. Two-player games of an infinite kind: biomass-management

Let's take a builder depot as an example, which is in contract with a logging firm for firewood (biomass) supply. For the sake of simplicity, I'll only include one type of firewood, meaning one type of product. According to our hypothesis, the distribution of the purchases arriving to the builder depot is exponential, meaning it's a question of incoming needs per timeframe (Molnár-Szidarovszky, 1995):

$$g\,(x) = \alpha e^{-\alpha x}\ \ (\alpha > 0, x > 0).$$

To understand the example, we will be using the following: a_1 is the total profit from selling one unit of product, meaning firewood, if it supplies from its own stock. a_2 will be the total profit of the builder depot, which comes from purchased firewood $(a_2 < a_1)$. b_1 will be the deposit cost of the builder depot per unit. b_2 will be the logging firm's deposit cost per unit.

The strategies of both the builder depot and the logging firm can be described by one number: the size of their stock. If y marks the builder depot's stock while z marks that of the logging firm, then the builder depot's anticipatory profit is as follows:

$$f_1(y,z) = a_1 \int_0^y xg\,(x)dx + \int_y^{y+z} [a_1\,y + a_2(x-y)]g(x)dx + \int_{y+z}^{\infty} (a_1\,y + a_2 z)g(x)dx - b_1 y$$

The first part of the formula refers to when need is lower than y, then the building depot can cover it from its own stock. In this case, his profit per unit is a_1. The second part refers to when $y + z \geq x \geq y$. In this case, the building depot covers y amount from its own stock, and the remaining $x - y$ need is covered by purchase. The third part refers to when $x > y + z$, where the building depot can't cover all needs from its own stock, therefore, it satisfies y from its own stock, and z from purchase, namely the logging company's stock. The fourth part of the formula refers to the cost of deposit. Based on the same correlation, the anticipatory profit of the logging firm is as follows:

$$f_2(y,z) = a_3 \int_y^{y+z} (x-y)g\,(x)dx + \int_{y+z}^{\infty} a_3\,zg(x)dx - b_2 z$$

With this, I defined a two-person game of infinite kind. If we propose that y's permissive values are $y_1, y_2, \ldots \ldots y_{m1}$ and z 's permissive values are $z, z_2, \ldots \ldots z_{m2}$, while we allow mixed strategies as well, then we have a bimatrix game (Molnár et al., 2010b):

$$A = f_1((y_i, z_j))_{i,j=1}^{m1,m2}$$

$$B = f_2((y_i, z_j))_{i,j=1}^{m1,m2}$$

1.2.3.3. Three-player games of infinite kind: production of minerals

The solution optimums for three-person games of infinite kind will be demonstrated with an example based on Molnár's (2010) Enviro-IT studies, taking into account the factors of mining and environmental defense. In the example, we can follow a multi-equilibrium (*three points of equilibrium*) Game Theory solution, during which we'll try to maximize the payoff functions in all three cases.

The production of minerals – literally the process of mining – may and will have an impact on not only the close proximity of the mining area, but also the surrounding bigger regions and ecologically unified areas as well. Usually, nodes can be found below the underground water region; it is therefore necessary to have a water-defense strategy for the planning of the mining process to be realizable. However, the changes in the water system not only require us to form plans and strategies in relation to the close proximity of the mining process, but the drinking water supply system as well, since the mining's water strategy will have an impact on both the drinking water and karst water supplies important to industrial water supplies. Furthermore, karst water plays an important role in thermal and hot spring water supplies as well, which are important factors in health tourism. Therefore, for the mining process of mineral resources, we have to include the factors of the direct water control of mining, the regional strategic objective system of drinking and other water supplies, and the water supply requirements of hot springs for the stable continuation of health tourism. In light of the aforementioned statements, we can define three strategic programs in the process of strategic planning:

A. Water protection related to mining
B. Defense of drinking water reserves
C. Defense of karst water reserves for thermal water supply

Mathematically defining the above mentioned goals is not an easy task, and even after that, the calculation of the Game Theory payoff functions can only be done if the special criteria systems are set up. For the mathematical definitions, we'll introduce the following: let x_i be the i-th mine's extracted water supply and its annual cost. Let xL_i be the i–th mine's profits from the inrush of water. Let xD_i be the i–th mine's loss of water due to water defense. Let xG_i be the i-th mine's sealed water amount. Let $b_i(x_i)$ be x_i water withdrawal's (investment+industrial) cost in the i-th mine. Let $L_i(xL_i)$ be xL_i economical loss due to the inrush of water in the i-th mine, and $d_i(xD_i)$ the water defense cost of water loss due to water defense (investment+industrial) in the i-th mine. Let $g_i(xG_i)$ be the costs of sealing (investment+industrial) in the i-th mine. Let v_{ik} be the amount of water transported from i-th mine to k-th re-feeding point. Let r be the number of artificial re-feeding points, let a_{ik} (V_{ik}) be v_{ik} water amount's transport costs. Let m be the number of water supply requiring points, let y_{ij} be the amount of transported water from mine, or other location (i) to j-th location. Let $s_{ij}(y_{ij})$ be y_{ij} amount of water's transport cost. Let $n = n_1 + n_2$ be the mines' (n_1) and other sources' (n_2) amounts. Let d_j be j-th water supply requiring point's supply needs. Let b be the underground resupply of hot spring waters. Let A_i be the i-th mine's number of inrushes of water.

In our example model, x_i , xL_i , xD_i, xG_i, v_{ik} , y_{ij} are the choice variables. Their values are not dependant due to the applicable circumstantial correlations, therefore, the i-th mine's payoff function can be easily calculated as follows:

$$f_{1i} = b_i(x_i) + L_i(xL_i) + d_i xD_i + g_i xG_i + \sum_k a_{ik}\,(v)_{ik} \;.$$

For the variables, the following must hold true:

$$xL_i + xD_i = x_i$$

meaning the withdrawn water equals to the water lost due to mining and water defense. The mine water's A_i profit equals to x_i withdrawn and xG_i sealed water's sum, as follows:

$$A_i = x_i + xG_i$$

Also,

$$\sum_k v_{ik} \leq x_i \;,$$

meaning we can not transport more water than what we have. The first objective function is the mining objective, meaning the cost of all mines:

$$\sum_k f_{1i} \to min.$$

The water supply goal means that in case of regional water management, we try to satisfy water needs with the best cost efficiency. We consider the possible underground water withdrawal points (including the mines) and the water supply requiring points the points of a network (Salazar et al, 2010). The water supply requirement can be described as follows:

$$f_2 = \sum_{i=1}^{n} \sum_{j=1}^{m} S_{ij}(y_{ij}) \to min;$$

with the following criteria holding true:

$$\sum_{i=1}^{n} = d_j \qquad (1 = 1, 2,.., m);$$
$$\sum_{j=1}^{m} y_{ij} + \sum_{k=1}^{r} v_{ik} \leq x_i \qquad (i=1, 2,..., n_1).$$

The requirement of environmental protection is fulfilled completely, if we pick $\{x_i\}$, $\{y_{ij}\}$, $\{v_{ik}\}$ choices, with which we keep up the required resupplied amount of hot spring waters. Therefore, the environmental protection goal's function is as follows:

$$f_3 = b - h(\ldots., x_{i\ldots.}y_{ij\ldots..}, v_{ij\ldots..}) \rightarrow min$$

We define the underground water amount leaving the system with the h function, regarding i choice variables. If we would only have to take the environmental protection goals into consideration, then we'd pick the choice variables in a way that $h = b$ would hold true. In our model, we define the barring criteria that

$$h(\ldots, x_i \ldots, y_{ij} \ldots., v_{ik}\ldots.) \leq b.$$

Three possible methods are of use to estimate function h. According to Molnár et al. (2010c), all three are based on one of the system's simulation models, which consists of modeling using the partial differential equation system. This equation system describes the water movements underground, and with the help of this, regarding the previously analyzed $\{x_i\}, \{y_{jj}\}, \{v_{ik}\}$ choice variables, and with their set values, function h can be easily calculated.

1. With the help of the system's simulation model, we can calculate function h's various values for many combinations of choice alternatives. The chart model should theoretically not be applicable to solutions, but in reality, it causes IT problems that make it impossible for us to spend time on the alternative.

2. A solution could be if we use the regression analysis in f_3 to estimate h, since with the help of the simulation model we can make a multi-variable linear regression function.

3. To estimate h, we again use regression analysis but using empirical or experienced data. This solution can prove adequate if we have a well-known system at our hands, when we want to determine the best possible development method. In case of a freshly starting system however, empirical data doesn't define the system in its entirety all that well.

In light of the above, we used the second alternative: regression analysis. We have to realize that the choice variables can be categorized into three different groups. The $\boldsymbol{u}_1 = (x_i, xL_i, xD_i, xG_i)$ variables are for mining purposes, $\boldsymbol{u}_2 = (y_{ij})$ are for water supply purposes, while $\boldsymbol{u}_3 = (v_{ik})$ are for environmental protection purposes. The above mentioned three goals are used as payoff-functions, while the $\boldsymbol{u}_1$, $\boldsymbol{u}_2$, $\boldsymbol{u}_3$, vectors are used as strategic vectors, with which we can define a three-player game. Our objective can therefore be written as a three-person game mathematically, where $\boldsymbol{u}_1$, $\boldsymbol{u}_2$, $\boldsymbol{u}_3$, are the appropriate strategic vectors, while, $\boldsymbol{u} = (\boldsymbol{u}_i)_{i=1}^{3}$ is the simultaneous strategic vector:

$$\varphi_i(\boldsymbol{u}) = \varphi_i(\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3,) = c^T_{i1}\,\boldsymbol{u}_1 + \boldsymbol{c}^T_{i2} + \boldsymbol{u}_2 + \boldsymbol{c}^T_{i3} + \boldsymbol{u}_3 = \boldsymbol{c}^T_i + \boldsymbol{u}$$

Objective functions are the strategic vectors.

$$\boldsymbol{A}_1\,\boldsymbol{u}_1 + \boldsymbol{A}_2\,\boldsymbol{u}_2 + \boldsymbol{A}_3\,\boldsymbol{u}_3 \geq \boldsymbol{b}$$

In this case, the coefficients are the vectors and matrixes derived from the previous model coefficients.

In Game Theory, the goal is usually to maximize the value of payoff functions. Therefore, in our case, it is advisable to use the original payoff functions multiplied by (-1).

Let S be $\boldsymbol{A}_1\,\boldsymbol{u}_1 + \boldsymbol{A}_2\boldsymbol{u}_2 + \boldsymbol{A}_3\boldsymbol{u}_3 \geq \boldsymbol{b}$ criteria system's $\boldsymbol{u} = (\boldsymbol{u}_i)_{i=1}^{3}$ set of vectors. In this case, we have to see S as its simultaneous strategic set. The game with S set and φ_i payoff functions has a point of equilibrium, which is a $u^* = (u_i^*) \in S$ vector, for which any S:

$$u^{(1)} = \begin{pmatrix} u_1 \\ u_2^* \\ u_3^* \end{pmatrix} \qquad u^{(2)} = \begin{pmatrix} u_1^* \\ u_2 \\ u_3^* \end{pmatrix} \qquad u^{(3)} = \begin{pmatrix} u_1^* \\ u_2^* \\ u_3 \end{pmatrix}$$

in case of vectors: $\varphi_k(u^{(k)}) \leq \varphi_k(u^*)$

The $\varphi_k(u^{(k)}) \leq \varphi_k(u^*)$ correlation, as we know, technically means that none of the objective functions' values can be raised by single-sidedly changing its strategy, meaning the $\mathbf{u}^*_k$ strategic vector is the maximum of φ_k, if the other $\mathbf{u}^*_i$ $(i \neq k)$ equilibrium strategies are set. Since the $\boldsymbol{A}_1\,\boldsymbol{u}_1 + \boldsymbol{A}_2\boldsymbol{u}_2 + \boldsymbol{A}_3\boldsymbol{u}_3 \geq \boldsymbol{b}$ criteria system and the objective functions are linear, based on the Kuhn – Tucker – theory, **u** can only be a point of equilibrium if the following criteria system holds true:

$$\left.\begin{aligned} c_k^T + v_k^T A_k &= 0^T \\ A_1u_1 + A_2u_2 + A_3u_3 &\geq b \\ A_1u_1 + A_2u_2 + A_3u_3 &\geq b \\ v_k &= 0 \end{aligned}\right\} \qquad (k = 1, 2, 3,),$$

where $\mathbf{V}_k$ is the sufficient vector. The third criteria's left side is never negative; therefore, the problem equals the problem, which means a quadratic programming task for the $\mathbf{u}_k$ choice and $\mathbf{v}_k$ new variables, due to the objective function.

$$\left.\begin{aligned} \boldsymbol{v}_k &\geq \boldsymbol{0} \\ \boldsymbol{A}_k^T\boldsymbol{v}_k &= -\boldsymbol{c}_k \end{aligned}\right\} \qquad \textbf{(k = 1, 2, 3)}$$

$$\boldsymbol{A}_1\boldsymbol{u}_1 + \boldsymbol{A}_2\boldsymbol{u}_2 + \boldsymbol{A}_3\boldsymbol{u}_3 \geq \boldsymbol{b}$$

$$\sum_{k=1}^{3} \boldsymbol{v}_k^T \ (\boldsymbol{A}_1\boldsymbol{u}_1 + \boldsymbol{A}_2\boldsymbol{u}_2 + \boldsymbol{A}_3\boldsymbol{u}_3 - \boldsymbol{b}) \rightarrow \textbf{min.}$$

Using simple transformations, we can rewrite the task's objective function as follows:

If we introduce the

$$H = \begin{pmatrix} 0 & A_2 & A_3 \\ A_1 & 0 & A_3 \\ A_1 & A_2 & 0 \end{pmatrix}, c = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}, b = \begin{pmatrix} b \\ b \\ b \end{pmatrix}, u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}, v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}.$$

hypermatrix and vectors, then the function can be written in a simpler form:

$$\boldsymbol{v}^T\boldsymbol{H}\boldsymbol{u} - \boldsymbol{c}^T\boldsymbol{u} - \boldsymbol{b}^T\boldsymbol{v}$$

This way, we've reduced our original goal to a quadratic programming task, for the solution of which we have more easily applicable methods.

During our calculations, we analyzed three mines, meaning i = 1, 2, 3. For these, $A_1 = 60m^3$/ min, $A_2 = 150\ m^3$/ min, $A_3 = 100\ m^3$ / min, which we got from annual data. The annual investment and operation costs per unit were given by Table 2 (104 HUF / m^3/ min). Therefore, we suggest that the functions in question are all linear.

Table 2: Annual data on profits of the various mines (HUF/m^3/min)

bi, Li, di, gi	WATER WITHDRAWAL			WATER DEFENSE		SEALING	
	invest.	indust.	loss	invest.	indust.	invest.	indust.
Mine 1	28,5	73,5	940	non-realizable		12,0	42,0
Mine 2	32,0	116	1200	35,0	129,0	8,5	25,5
Mine 3	100,0	184	1420	45,0	147,0	22,3	34,7

Source: self-made (based on Molnár (2010c))

Including the three mines, we took six water withdrawal points, and analyzed seven water supply requiring points. The annual investment and industrial costs of water transportation can be seen in Table 3. These costs were also defined in HUF 104 / m^3 / min units, as previously. In each reaction, we show two costs, first of which is the investment cost, second is the industrial unit cost, therefore, S_{ij} the functions are also linear.

The needs of the various water supply requiring points are as follows (m^3 / min): 33, 14, 83, 14, 83, and 28. We also take into consideration two other resupply points, the costs of which (first of which is the investment, second is the industrial) can be seen in Table 4 (HUF / m^3/ min).

Table 3: Analysis of water withdrawal points (HUF / m^3 / min)

S_{ij}	1		2		3		4		5		6		7	
1	6	3	7	8	9	7	11	8	14	8	15	7	1	7
2	9	10	5	4	5	2	8	4	10	4	10	4	14	6
3	9	11	5	2	7	3	10	6	10	6	14	8	18	9
4	261	45	257	40	255	39	262	43	262	43	264	45	268	46
5	272	45	269	43	262	40	259	30	259	30	253	37	256	40
6	264	42	258	32	261	35	258	32	258	32	262	32	265	34

Source: self-made (based on Molnár et. al. (2010c))

Table 4: Costs of water resupply points (HUF /m^3/ min)

a_{ik}	1. RE-FEEDING POINT		2. RE-FEEDING POINT	
Mine 1	25	0	45	53
Mine 2	68	112	72	78
Mine 3	63	94	50	32

Source: self-made (based on Molnár et. al. (2010c))

In the third objective function, b = 30 m^3 / min, and the function h which we got from the regression is as follows:

$$h = 30{,}5 - 0{,}021x_1 - 0{,}012x_2 - 0{,}014x_3 - 0{,}006\sum_i y_{i4} - 0{,}0021\sum_i y_{i5} - 0{,}0042\sum_i y_{i6} + 0{,}07\sum_i v_{i1} + 0{,}14\sum_i v_{i2}.$$

We can solve the model with the quadratic programming task above. The optimal choice variables can be seen in Tables 5 and 6. In the first Table, the optimal x_i, xL_i, xD_i, xG_i, v_{i1}, and v_{i2} values can be found in case of $i = 1,2,3$, while in the second, we will give the y_{ij} ($1 \leq i \leq 6, 1 \leq j \leq 7$) values. From Table 5, based on the exact y_{ij} values, we can also say that according to the optimal program, there is water transportation between nine different places.

Table 5: Optimal choice variables at water withdrawal points (3 places)

i	x_i	xL_i	xD_i	xG_i	v_{i1}	v_{i2}
1	33	33	0	27	0	0
2	33	0	130	0	0	0
3	150	0	100	0	0	25,16

Source: self-made (based on Molnár et. al. (2010c))

Table 6: Choice variables at water withdrawal points (6 places)

i/j	1	2	3	4	5	6	7
1	33	0	0	0	0	0	0
2	0	0	25	14	0	83	28
3	0	14	58	0	2,84	0	0
4	0	0	0	0	0	0	0
5	0	0	0	0	11,6	0	0
6	0	0	0	0	0	0	0

Source: self-made (based on Molnár et. al. (2010c))

Note: The differing results don’t mean contradictions, instead they illustrate the multi-purpose programming tasks’ fundamental problem, meaning the optimal solution is highly dependent on the choice of the applied method. The various methods may designate differing optimum values. And this reflects the attitude of the one who makes the decision.

1.3. Theory of cooperative games

Primarily those market players and companies that can be defined well on the market can already apply Game Theory solutions well in the present practice. Game Theory mathematics, meaning the selection of suitable strategies supported by numbers, may happen in an exact manner. What currently doesn't generally apply to the selection of appropriate strategies is the practice of cooperative aim or strategic choice. The sustainability concepts make it unambiguous that a production/consummation goal system can be planned long-term if the use of resources is planned and in a synchronized manner. Sustainable development may not be realized without this cooperation. The cooperative attribute and the fact that market players form coalitions provides a new approach to the Game Theory approach, as well. This doesn't necessarily mean that the interests of players are the same regarding e.g. the distribution of costs, but during the cooperation, it is compulsory to abide by a collectively defined criteria system (Hardin, 1968).

During cooperative strategies, the players or market players have the natural aim of raising their profits by giving up on their autonomy partially or completely (Solymosi, 2009). By this method, a given group of players or all players cooperate with each other and perhaps form coalitions or even a single coalition. The natural requirement of the cooperation is that the participating players or market players must have a higher share of profits compared to those who do not take part in cooperation at all. In this case, the goal won't be the increase of personal profits, but the maximization of the cooperation's profits. This aim completes the criteria of sustainability, in other words, sustainable development, or economy's weak sustainability (Molnár - Kelecsényi, 2009).

Cooperative games will be defined with the following descriptions. $N = \{1, \ldots\ldots, n\}$ is the *set of players* for which a given S subset is commonly known as a *coalition*: $S \subseteq N$. Let S be the set of subsets or the set of coalitions. The N base set is called *complete coalition* (Szidarovszky, 1978b).

1.3.1. Conflict alleviation methods

Conflict alleviation methods are one of the favored groups of cooperative Game Theory solutions. Of these, we can stress the importance of Nash's axiomatic solution, which used axiom sets to assure that the solution is always placed on the *Pareto-line*. And the Kálai - Smorodinsky solution can give the last possible point which is the achievable minimum or the solution of the conflict by defining the worst outcome point of the conflict (Molnar - Szidarovszky, 1994).

Conflict alleviation methods will be demonstrated with a two-person case. In our example, let S_1 and S_2 be the players' strategy sets, and φ_1 *and* φ_2 the two payoff functions. The set of possible payoffs can therefore be as follows:

$$H = \{\varphi_1(x,y), \varphi_2(x,y) \,|(x,y) \in S_1 \times S_2\}$$

In this case, as always, both players aim at maximizing their payoffs, but their respective payoffs are naturally dependent on the other player's strategy, and a general rule is that raising one player's payoff leads to a drop in the other's (Nowak - May, 1992). Therefore, our task is that we have to find a solution that is acceptable to both parties. Before each solution, we have to state that if there's no cooperation, both parties will get either a lower payoff, or a penalty.

General definitions:

$$\boldsymbol{f}_* = (f_{1*}, f_{2*})$$

this will be our payoff vector, for which we assume that there is a $(f_1, f_2) \in H$, for which $f_1 > f_{1*}$ and $f_2 > f_{2*}$. The conflict is defined mathematically by the $(H, \boldsymbol{f}_*)$ pair. This pair was defined in

Figure 2. We will also assume that set H is closed, convex and bounded, which therefore means that:

$$(f_1, f_2) \in H \; and \; \bar{f_1} \leq f_1, \bar{f_2} \leq f_2$$

for which $(\bar{f_1}, \bar{f_2}) \in H$ is essential, and bounded in both coordinates, therefore

$$sup\,\{f_i|(f_1, f_2) \in H\} < \infty$$

in case of $i = 1,2$.

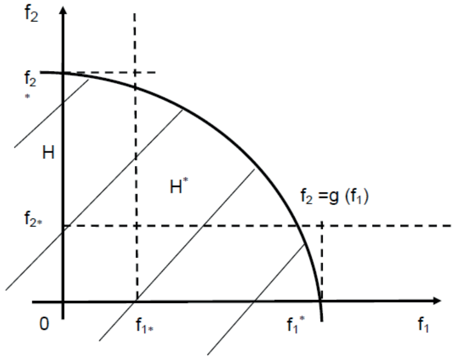


Figure 2: Graph of conflict
Source: based on Molnár-Szidarovszky, 2011c

Furthermore, we also assume that H's borderline is the graph of function $f_2 = g(f_x)$, which is strictly decreasing in point f_1 and concave. The graph of function g is usually called the Pareto line, therefore the optimum criteria for sustainability can only be satisfied here. Between the game and solution dependencies, we have to take note that no rational player will accept a partnership which means a worse payoff than a payoff without any partnerships.

Therefore, the possible payoff set can be narrowed as such:

$$H^* = \{f_1, f_2|f_1 \geq f_{1*}, f_2 \geq f_{2*}, (f_1, f_2 \in H\}$$

1.3.2. Model of oligopolistic games

Oligopolistic game solutions are the most popular for the modeling of economic decision processes. They can be used for both cooperative and non-cooperative strategies; however, in my dissertation, I wish to present the cooperative model to define the sustainable maximization of the usefulness function, due to the importance of the sustainability attribute system.

There may be two relevant problems of economic science in the case of the usefulness function. The first problem was that the consumers aren't only defined by one usefulness function, but by an unlimited number of functions, and they are equivalent to each other. The second came up when the choice was made during insecurity. The solutions used on oligopolistic games can maximize the players' usefulness functions with the highest probability when the criteria are fixed and the strategies used are cooperative (Simonovics, 2003; Ichiishi, 1983).

In the wide scale of engineering (environmental protection) tasks, we can meet with exact problems that have a mathematical model which may be reduced to an oligopolistic game. Multi-player Game Theory may be used to examine many variations of problems (Szilágyi, 2005). In the following example, I'll try to introduce the process of optimum search based on oligopolistic games by concentrating on a single product in the market process (this could be, e.g. green energy on the energy market), and by including market players and groups (manufacturers, transporters, regulators, implementers, etc.), who have an impact on the changes of the product's cost, and the creation of the point of equilibrium.

In case of cooperative games, giving up on independence must result in the raise in profits for players (Simonovics, 2003).

General definitions for cooperative games:

φ_k- payoff function

S_k- strategic sets $(x_1^*, \ldots . x_n^*)$ – strategic attributes

n – a certain positive integer

k – a certain set of S_k

Condition: for $= 1,2,3 \ldots, n$, $x_k^* \epsilon S_k$, and for every given $x_k \epsilon S_k$

$$\varphi_k(x_1^*, \ldots, x_n^*) \geq \varphi_k(x_1^*, \ldots., x_k^*, \ldots., x_n^*).$$

The inequality could be defined in words as such: for the k-th player $(1 \leq k \leq N)$, the equilibrium strategy is the optimal strategy, assuming the other players choose the correct equilibrium strategy. The cooperative game results in a coalition where the coalition itself can always generate profits unlike those who are not members.

Maximum of cooperative game, profits of coalition:

Let's assume that a set M of players k makes the

$$M = \{i_1, i_2, \ldots, i_k\}$$

coalition. Furthermore, let us take the game defined by

$$X = x_{j=1}^{k} S_{ij} \text{ and } Y = x_{l \neq i_j} S_l$$

strategy sets, and

$$\tilde{\varphi}_1(x) = \sum_{j=1}^{k} \varphi_{i_j}(x), \qquad \tilde{\varphi}_2(x) = -\tilde{\varphi}_1(x)$$

payoff functions. It is obvious that function $\tilde{\varphi}_1$'s "max-min" value, meaning

$$v(M) = max_{x_{i_j}} min_{x_{l_{(l \neq ij)}}} \tilde{\varphi}_1(x) \text{ and } v(\emptyset) = 0$$

quantity – in which it exists – is only dependent on set M. The coalition assumes that players not in the coalition aim at minimizing the coalition's profits.

Example for oligopolistic problem handling:

To keep it simple, let's assume that $M = 1, i_1 =. \ldots = i_n = 1$, meaning we only examine a single product, and every group consists of a single unit. Referring to the known max-min function, f can be differentiated into the $[0, \xi]$ interval. Let $I = \{i_1, \ldots . i_r\} \subset \{1, \ldots, N\}$ be a coalition (Molnár – Szidarovszky, 1994).

In this case, $v(I)$ can be as follows:

$$v(I) = max_{x_i} min_{x_j} \sum_{i \notin I} \varphi_i(\boldsymbol{x}), \text{ where max} \rightarrow i \in I \textit{ and } \text{min} \rightarrow j \notin I.$$

To calculate value $v(I)$, first we need to calculate the

$$\min_{x_j} \varphi_i(\boldsymbol{x}) = \psi_I (x_{i_1} \ldots\ldots, x_{i_r}) \text{ where } j \notin I$$

quantity.

We differentiate two possible cases during problem solution:

1) is if $\sum_{j \notin I} L_j \geq \xi$, then obviously

$$\psi_1 (x_{i1} \ldots., x_{ir}) = -\sum_{i \in I} K_i (x_i)$$

2) is if $\sum_{j \notin I} L_j < \xi$, then obviously

$$\psi_1 (x_{i1} \ldots., x_{ir}) = \left(\sum_{i \in I} x_i\right) f \left(\sum_{j \notin I} L_j + \sum_{i \in I} x_i\right) - \sum_{i \in I} K_i (x_i)$$

In our first case, function ψ_1's maximum point is based on functions' K_i's increase, meaning $x_{i1} =, \ldots = x_{ir} = 0$, therefore,

$$v(I) = -\sum_{i \in I} K_i (0)$$

Let's introduce the $L_I = \sum_{i \in I} L_i$ and $\tilde{L}_I = \sum_{j \notin I} L_j$ definitions for our second case.

Then, according to the formula above:

$$\psi_1 (x_{i1} \ldots., x_{ir}) = s_I f(\tilde{L}_I + s_I) - \sum_{i \in I} K_i (x_i) \text{ where } s_1 = \sum_{i \in I} x_i$$

We can solve the above written equality with a programming task. We can use dynamic programming for the numeric solution (Molnár – Szidarovszky, 1994; Simonovics, 2003).

1.3.3. Method of equal compromise

The search for the solutions to environmental problems can be solved in different ways, as I've already mentioned at the beginning of the chapter; therefore, the optimum point, conclusion point or solution can manifest on the Pareto line in multiple coordinates. The method we use is dependent on the attitude of the one who does the optimization, his beliefs, intuition, and the nature of the processed problem (Axelrod, 1984).

Out of all the employable conflict alleviation methods, I chose the method of equal compromise to demonstrate the Game Theory solution, which (in case of two players) assumes that both players reduce their requirements at the exact same pace, to the point where they arrive at a possible solution. One of the method's characteristics is that there is usually a single solution point, and giving it outlines the optimal criteria system for the players (Forgó et al., 2005). This means that they accept the first solution (which is probably the best possible for both of them) as the solution to the conflict, or conclusion point. If both players want the maximum payoff, the (f_1^*, f_2^*) point is needed, which is not possible.

In case of continuously and collectively decreased requirements, the problem's definition (if the solution is (f_1, f_2), the two players always give $f_1^* - f_1$ *and* $f_2^* - f_2$ discounts, therefore, in case of equal discounts:

$$f_1^* - f_1 = f_2^* - g(f_1)$$

Transforming the equation to f_1:

$$f_1 - g(f_1) + (f_2^* - f_1^*) = 0$$

where we see the left side strictly increasing, for $f_1 = f_{1*}$, $f_{1*} - f_2^* + (f_2^* - f_1^*) < 0$ *and for* $f_1 = f_1^*$, $f_1^* - f_{2*} + (f_2^* - f_1^*) > 0$, therefore, there is exactly one solution for the problem (Molnár - Szidarovszky, 1995).

My hypothesis is that the previously introduced non-cooperative and cooperative Game Theory solutions are applicable to mathematically defining sustainability criteria, since they allow the determination of points of equilibrium for economic, production, and strategic creation and planning processes, which create a clear basis for both long-term sustainable resource usage, and avoidance of economy development processes which have a detrimental effect.

During the implementation of the Game Theory method, we refer to the "*Layer by layer method,"* meaning row after row solution of Rubik's Cube, which has the characteristic of being applicable to modeling the process of project development and the attributes that have an impact on each other during the development, through the logical sequence. According to the hypothesis on the solution algorithms of the Rubik's Cube, the parts rotated next to each other, meaning the project attributes which have an impact on each other, have a relation system which can be defined in mathematical terms, therefore, their point of balance (e.g. Nash's) can also be determined by Game Theory models (games of finite kind, zero sum games, oligopolistic games, etc.).

1.4. Rubik's Cube (3×3×3) solution methods and mathematical approaches

Rubik's Cube was invented in 1974 by Hungarian designer and engineer Ernő Rubik. The Hungarian game, first known as the mystical cube, was very popular in the countries of Central Europe, then slowly conquered the entire world, from the USA to China. The primary goal of designing the cube was to create an architecture demonstration tool for students that can make the three dimensions more easily understood by architect students. Via the 3×3×3 cube, some mathematical and logical correspondences can be easily defined, which are exceptionally useful in learning the thought processes in three dimensions. The international interest in Rubik's Cube started in 1980, when it became clear that it is not just a simple toy, but also a specialized system. Each side of the six-color cube consists of 3 rows and 3 columns, which have a special meaning. The connection between the different sides is represented and further developed by the small cubes. Through the subsystems of the cube, by substituting certain attributes, the correspondence between said attributes can be clearly followed during the rotation of the cube. Both the correct and incorrect placements and correspondences can be identified (Rubik's Revenge, 2011).

When Ernő Rubik created the 3×3×3 cube, he needed a whole month of intensive "work" to find the correct method for the cube's solution. During the long time interval, he had many interesting ideas, which later raised the cube from the group of ordinary games and made it into one of the world's most famous and popular logical device. According to Rubik, the "cube" is a perfect example of the unison of designed and natural beauty. The coloring of the cube is the result of a unique and well-thought planning process, by which if anyone looks at it, the first obvious fact is the beauty of the cube (Rubik, 1981). Furthermore, the cube is the perfect system of putting the human mind to the test, correspondence to science, and discipline. Ernő Rubik also holds that the cube can mean the unison of reality and beauty, since these can mean the same thing simultaneously (SZTNH, 2013).

The mystical meaning of the number "three" followed the cube along its story of conquest and success. Many think that the cube symbolizes connections that link man to nature and a natural being. Ernő Rubik called the "mother-child-father" relationship, the "heaven-earth-hell" trio, the processes of "creation-protection-destruction," or "birth-life-death" important symbols for the number three (Rubik et al., 1987).

The cube itself is an imitation of life, or more properly, the pursuit of making said life better. The problem of the cube's solution is also closely related to solving life's problems, thus we can say that our entire life is a puzzle. If you're hungry, you need to find something to eat; but life's daily problems are a bit more complicated than this and are not as obvious. Until you eat, you have only a single problem, but after you're full, other problems quickly arise. The problem of solving the cube is basically your call. You can solve it alone, without help. However, finding happiness in life is not this easy, it is something that cannot be done alone. This is the biggest difference between life and the cube (Goudey, 2003). The above mentioned thought process is basically the same as the sustainability concept defined by *sustainable economic value (FGÉ)* mentioned early in the dissertation, which is able to present – also using local information – the changes in not only the natural capital, but social and technological capitals as well, and in an integrated manner, which can only be partially done using the various economic indexes.

In the case of the logical game that caused the world the greatest headache, scientists were already interested in the early 80's in how they can find "God's number" out of all the possibilities of the quite impressive number of 43,252,003,274,489,856,000 starting positions, meaning exactly what is the maximum number of steps are necessary to solve the cube. The research team who publicized the results checked all the possible positions, using both Google's calculation power, and a bunch of mathematical twists, and found that the possible states or positions is an incredible 43 quintillion ($\sim 43*10^{18}$).

1.4.1. Interpretations of Rubik's Cube solution methods and algorithms

Scientists basically thought that a maximum of 18 steps are required to solve the cube. However, the mathematician Michael Reid created a mathematical formula that made it obvious that the cube can't be solved starting from any given state with a rotation which consists of less than 20 steps (Korf, 1997). This means that the cube can only be solved with at least 20 rotation steps according to theoretical calculations, which means that "God's number" must be at least 20. In 2008, Thomas Rokicki, American scientist, proved (using a group theory method for calculations) that the cube can be solved with 22 rotation steps from any given starting state, which means that the "God's number" being close to 20 is a fact in not only theory, but also technically. However, everyone already knew that the number 22 can't be "God's number," since this was obvious from the calculations (Rokicki, 2008; Gregory, 2007).

As a start, Rokicki and his affiliates divided all the starting configurations using the technique derived from group theory. This meant 2.2 billion groups, each of which consisted of 19.5 billion configurations. The grouping was dependent on the reaction of the configurations to 10 possible rotation movements. The mathematicians working on the project, using the different symmetries of the cube, successfully reduced the groups to 56 million. This reduction was made possible through a very simple methodology, since if we turn the given cube upside down, or to each side, the solution won't become any more complicated, therefore making these equal 'combinations' outright unnecessary. Apart from these simplifications, there were still numerous possible starting configurations, which made the creation of an algorithm for the hastening of the process necessary (Rokicki, 2010).

The new algorithm was very important, since previous methods could only try 4000 cubes every second. This previous algorithm examined a starting movement, then determined if the position which resulted was closer to the conclusion of the overall solution process or not. If this wasn't successful, the algorithm discarded these steps and restarted.

"This method is like visiting our friend in an unknown town, for which he supplies the general direction, when to turn right or left, but he didn't tell us where to start from. If we follow the directions from a random starting point, we will have a slim margin of possible success, meaning arriving at the correct destination, but if we can match it to the correct starting point, we can definitely make the trip," said Joyner (1996a).

Rokicki (2010) realized that those steps which result in a dead end are actually solutions to other starting states, which led him to creating a different algorithm with which he could try a billion cubes every second. This means that the newly created algorithm was able to match movements with the correct starting state at an incredible speed, making the solution of a 19,5 billion series possible in a mere 20 seconds, which may seem like an astounding speed, but still would've required 35 years for an ordinary computer to complete the entire task. In order to shorten the time required, they were searching for an especially effective method. During the process of problem solving, it was quite fortunate that the work was followed by John Dethridge, one of Google's engineers, and offered the free capacities of his IT systems to aid the research. By using the free capacity of the computer empire, he managed to solve the problem in a few weeks.

The result of the astounding and persistent research spanning 15 years therefore proved the assumption made and supported by mathematicians for a long time, that no more than 20 moves is required to solve the 3×3×3 Rubik's Cube from any given starting state. Many researchers agree that these research processes show how simple and pure mathematics can be used to simplify problems which need an enormous calculation (Rokicki, 2010).

Rotational mathematics

The process of thought for the 3×3×3 cube was developed for the simplification of the mathematical definition. As a first step, we must realize that if we (theoretically) fix the innermost element, which keeps all the other ones together, then only the corners and the side edges can change their positions using the rotation movements, the middle element does not move from its position, only rotates around itself. After this, we assume that all the states of the cube are given a number from 1 to 43,252,003,274,489,856,000. We create sets using the numbers, which will define the set of possible cube positions (Davis, 2006).

If we follow the basic theories by Singmaster (1981) and name the rotations based on the initials of the sides which are being rotated, we arrive at the list seen below. We can define a rotation condition that the rotation itself may only happen outward from the inside of the cube, and clockwise. This is also called left hand rule, and means that only six different rotations are possible.

a – rotation of the bottom side

f – rotation of the top side

e – rotation of the foreside

h – rotation of the backside

b – rotation of the left side

j – rotation of the right side

Each one of the cube's rotations (not restricted to these six variants) is called a *transformation*. We can imagine the various rotations as functions, which are defined on the set of stages for the cube. These also make sets, for which the rule of relation is that the value of the given function is the sequential number of the new cube position, defined by executing the correct rotation. In simple terms, it can be written as follows: *f* (64,523) = 578,526,687 (Joyner, 1996b).

If we can define the rotation sequence as the sequence of the exact letters and use it for the analogy of the multiplication process, then the various rotation processes are easy to define. Therefore, in the case of this rotation combination where the left column is rotated twice, followed by the rotation of the top row once, and the back row once, can be written as follows: bbfh, meaning b^2fh. We can mark the rotation sequence with which we turn the cube around, meaning repositioning it to its original state with the number 1. Written as follows: a: bbbb = b·b·b·b = b^4 = 1. Naturally, the rotation 1 is also a transformation, but one which leaves all positions intact. In this correspondence, 1(1)=1, 1(2)=2, … , 1(43,252,003,274,489,856,000) = 43,252,003,274,489,856,000. Based on this, we can interpret the counter-clockwise rotations as well, which are the same as the sequence of three clockwise rotations. Therefore, for example fff =f^3 is such a rotation, which, according to the previous abbreviations, can be defined as 1/f or f^{-1} as well, since the formula 1/f · f = 1 defines that two 90° counter-clockwise rotations of the cube outward from the center, followed by the same re-rotation does not alter the cube's combination. When defining a transaction, we can also say that $a^0 = 1$, $f^0 = 1$, $e^0 = 1$, $h^0 = 1$, $b^0 = 1$, $j^0 = 1$, meaning executing any rotation movement 0 times doesn't alter the cube (Singmaster, 1981).

The set theory interpretation of the cube's solution is also an extremely interesting research field. If we define a set from all the possible rotations, which we name A, and also interpret the process of continuous execution defined by multiplication (·) (which is the composition of rotation functions in another form), then we get an exact group. This can be defined with e.g.: (A, ·). However, we can state that this set has a finite number of elements (meaning that we can imagine a finite number of different rotations) and therefore the group consists of a finite number of elements, since an infinite number of rotations could arrange the cube in an infinite number of combinations. However, there is a bounding criteria that each of the 9×6 sides can at most have six colors, therefore, the cube will strictly have less than 36^6 states, meaning we can see that only a finite number of different rotations is imaginable (Davis, 2006).

Let us define 1 as the neutral element of group (A, ·). However, we can also see that each element is comprised of the sequential execution of *f, e, a h, a b* and/or *j*. In this case, we say that the set *{a, f, e, h, b, j}* generates group (A, ·). The arrangement of the cube, meaning the correct positioning of each color, is written mathematically as follows (assuming we mix it with the x sequence of rotations): e.g. x(1) = 456,358,966,568, where $x = bfj^3f^2\ldots aef^2hj$. If we do not know the value of x, we must find a y rotation, which arranges the cube, meaning y (456,358,966,568) = 1. Thus we can see that x · y = 1, meaning $y = x^{-1}$, which makes our goal to find a transformation that inverts x. It holds true for every rotation that the inverse of the rotation is the same as the rotation done the opposite way. E.g.: the inverse of f is $f^{-1} = f^3$, since $f \cdot f^{-1} = f \cdot f^3 = f^4 = 1$. In case of more complicated rotations, we have to invert the various elements, and do the whole process backwards. E.g.: $(afj^3f^2\ldots bef^2hj)^{-1} = j^{-1}h^{-1}f^{-2}e^{-1}b^{-1}\ldots f^{-2}j^{-3}f^{-1}a^{-1}$, where $f^{-2} = f^2$ and $j^{-3} = j$, as checkable with simple multiplication (Frey, 1982).

The basic rule is that during the arrangement of the cube, our goal is to move the small cubes into a different location or leave them in place, but at a different angle (e.g. let a corner cube do a 120° turn, or rotate an edge cube with its color) while everything else remains untouched. Naturally, it's obvious to everyone that not every imagined movement can actually be done. If we want to rotate a corner cube, we have to know that it can only be moved together with another: we can't rotate just that corner cube alone. This is also the case for the rotation's direction, since the corner cubes don't always rotate the same way. There are rotations which only move edge cubes, while some only move corner cubes. This is due to the rule that edge cubes can't go into a corner cube's position, and vice versa. An interesting fact is that the movements of a 3×3×3 cube's corner cubes are identical to that of the 2×2×2 cube's corner cubes, even though the latter doesn't even have edge cubes at all. We can define this as well using function-form laws for the different solution methods (Snap, 2012; Slocum et al., 1981).

1.4.2. Analysis of solution methods for 3×3×3 Rubik's cube

To solve the 3×3×3 cube, many different methods were made independent of each other in the last few decades, one of which is the very popular *layer by layer* method designed by David Singmaster, which was published in "Notes on Rubik's 'Magic Cube'" in 1981.

"Layer by layer" method:

The core of the method is that the solution of the cube happens row to row, first of which is the top row, the middle row, and finally the low facing side and the third row. With intensive and effective practice, this can result in a solution time lower than a minute. Almost everyone learns this method first. It's important (in case of the solution method) that this has no fix algorithms, which means that two different people who both use the *layer by layer* method can solve the cube using completely different methods (Hardwick, 2014)!

"Corner First" method:

Using another general solution, named *corner first* method, the speed of solution can be decreased well below a minute. Obviously, the speed is dependent on the number of required rotations. The *corner first* method is the basis of one of the fastest, Gilles Roux's method. The point is that as a first step, all corners must be arranged to their position and proper angle. After this, all middle rows can be freely moved in a way that the corners remain intact. With this method, we have a much wider margin of freedom on the cube, compared to the *layer by layer* method (Doig, 2000).

By rotating the middle cubes, the edges can be arranged in a very short time. It's especially popular on solution competitions, since it usually requires the least amount of rotations. A similarly effective method is the *"Edges first method."* This is the inverted version of the previous method, since we arrange the middle rows first, and then the corners. Almost everyone uses this method for "blind," that is blindfolded, solutions. This method has the advantage that it only requires knowledge of a single algorithm, which, if learned completely by the one solving the cube, can help him solve it in a matter of minutes (Ortega, 2013). The method isn't hard, but can't be used to simulate project development, since the solution logic is strictly automatic (Appendix 1).

According to Singmaster and Alexander Frey's 1982 hypothesis, the cube can be solved with 20 rotations using the ideal algorithm, though they couldn't name the algorithm yet (Joyner, 2002a). The proof of minimal rotation requirement was first made in 2007, when Daniel Kunkle and Gene Cooperman used a supercomputer to prove that the minimum number of rotations is 26 or lower. In 2010, Thomas Rocicki and his affiliates proved that "God's number," meaning the minimum number of rotations is 20. This, however, depends on the optimal state from where the 20 rotations result in completion. As a general assumption, the $n \times n \times n$, n=3 Rubik's Cube can be solved with $\Theta(n^2 / \log(n))$ rotations (Rokicki et al., 2010).

Fridrich method:

A very widely known and used method among "cubers" is the *Fridrich method.* The method was developed by Jessica Fridrich and is very similar to the *layer by layer* method but uses a high number of algorithms for the solution. With this method, and lots of practice, the cube can usually be solved in 17 seconds, which is why most of the world's "speedcubers" use this method. The trick is to do a simple F2L (*first two layers*) followed by OLL (*orienting the last layer*), finishing with a PLL (*permuting the last layer*). The *Fridrich method* requires the knowledge of 120 algorithms, and the solution of the cube can be done with 55 rotations (Joyner, 2002b).

Cube enthusiasts stress the importance of the notations of the various rotations. These were developed by Singmaster for the 3×3×3 cube in the early 80's; therefore, these are called Singmaster notations. The relative nature of the notation system allows us to illustrate the solution with disregard to the colors, or fixing the top and bottom sides (Solutions, 2013; Singmaster, 1981):

- *F* (Front)
- *B* (Back)
- *U* (Up)
- *D* (Down)
- *L* (Left)
- *R* (Right)
- *f* (Front two layers): middle row of front
- *b* (Back two layers): middle row of back
- *u* (Up two layers): middle row of top
- *d* (Down two layers): middle row of bottom
- *l* (Left two layers): middle row of left
- *r* (Right two layers): middle row of right
- *x* (rotate): rotate whole cube right
- *y* (rotate): rotate whole cube up
- *z* (rotate): rotate whole cube front

When the (') symbol follows the letters in the description, then we must rotate the front side counter-clockwise, if there's no symbol, we rotate it clockwise. If the letter is followed by 2, then 2 rotations are up next, meaning we rotate the layer by 180 degrees. R means rotation clockwise to the right, R' means rotating it back, or rotating it counter-clockwise. The letters x, y, and z are used when the whole cube is rotated around one of the corners. We can do this towards the directions R, U and F. When the letters x, y, and z are the base symbols, the cube must be rotated to the opposite direction. When they are highlighted in a box, the rotation must be done by 180 degrees. The most notable difference compared to the Singmaster notation is the official standard "*w*," meaning "wide." E.g.: using Rw means the middle row to the right of the front side (Frey, 1982; Rask, 2013).

In case of the *corner first* method, we also use the "MES" extension, where the letters M, E, and S note the rotation of the middle row (Wiki Rubik Kocka, 2013; Demain et al., 2011):

- *M* (Middle): row between L and R, rotation direction L (downward)
- *E* (Equator): row between U and D, rotation direction D (rightward)
- *S* (Standing): row between F and B, rotation direction F

Thistlethwaite algorithm

The optimization of the 3×3×3 cube and reaching the minimum amount of rotations began with the discovery of group theory using computers, the basis of which was laid down by Morwen Thistlethwaite in 1981. The basis of the Thistlethwaite method was to divide the problem into sub-problems, meaning searching for the solution by dividing the cube into subgroups. He mainly used the following categories (Heise, 2002):

$$\boldsymbol{G_0} = < L, R, F, B, U, D >$$

$$\boldsymbol{G_1} = < L, R, F, B, \boldsymbol{U^2}, \boldsymbol{D^2} >$$

$$\boldsymbol{G_2} = < L, R, \boldsymbol{F^2}, \boldsymbol{B^2}, \boldsymbol{U^2}, \boldsymbol{D^2} >$$

$$\boldsymbol{G_3} = < \boldsymbol{L^2}, \boldsymbol{R^2}, \boldsymbol{F^2}, \boldsymbol{B^2}, \boldsymbol{U^2}, \boldsymbol{D^2} >$$

$$\boldsymbol{G_4} = \{\boldsymbol{1}\}$$

The above mentioned groups mean a 3D concept, which can be summarized as follows: $\boldsymbol{G_{i+1} \setminus G_i}$

He found a rotation sequence for every element and group, which resulted in the following groups. Afterwards, he defined the following basic correspondences:

General cube groups of random cube:

$$\boldsymbol{G_0}$$

Position of group rotated to proper side:

$$\boldsymbol{G_1 \setminus G_0}$$

The following group belongs to the direction of proper solution:

$$\boldsymbol{G_1}$$

He then designated the other groups during the process of solution:

$$G_2, G_3, G_4 \ldots.$$

Though the number of G_0 groups is incredibly high (~4,3 x 10^{19}), the number of groups will decrease as we form more and more subgroups. In the beginning, it was assumed that the cube can be solved with a maximum of 85 rotations. Then, using the grouping method, the number decreased substantially, first to 63, then 52, and finally to 45 (Heise, 2002).

Further developments

Daniel Kunkle and Gene Cooperman used a supercomputer to determine the number of rotations to be 26 in 2007, meaning this is how many steps are required to solve the cube from any given position. Thomas Rokicki and his team, as already mentioned above, determined in 2008 that there is no need for 26 rotations, as 23 is sufficient. Then with further research they proved that “God’s number” can’t be below 20, meaning at least 20 rotations are always necessary to solve the 3×3×3 cube (Richard, 2008). The scientific world was faced with these criteria by computer solution programs as well.

1.4.3. Simplified mathematic algorithms of the layer by layer method

We also have to mathematically define the cube solution processes if we want to use the Rubik’s Cube-based optimization with a computer application or to let other users acquire it through software.

In order to make the mathematical definition, we have to be familiar with the classic naming, which (based on Singmaster) is as follows (Singmaster, 1981):

- *F* (Front): side facing the one solving the cube
- *B* (Back): side opposite to front
- *U* (Up): side above front
- *D* (Down): side below front, opposite to up
- *L* (Left): side left to front
- *R* (Right): side right to front

The most known method of modeling the process mathematically is when the solution of Rubik’s Cube happens through the creation of subgroups. One of the most important goals of using a computer program to solve Rubik’s Cube is to create a natural grouping, meaning the formation of subgroups (Joyner, 2002a):

$$G_n = \{1\} < G_{n-1} < \ldots < G_1 < G_0 = G$$

where $G =< R, L, F, B, U, D >$ is the Rubik’s Cube subgroup, which allows the use of the following strategies:

- *let’s mark the given combination of Rubik’s Cube with the element* $g_0 \in G$,
- *defining all elements of the group* $G_{k+1} \,/\, G_k$*:*

$$G_{k+1} \;/\; G_k = \cup_{i=1}^{r_k} g_{k+1,i} G_{k+1}, \;\; where \;\; r_k > 1, \; \forall 0 \leq k < n$$

(note: $m_{-1} = 1, g_{n,1} = 1$*),*

- *(first step) if* $g_0 \in g_{1,i}G_1$ $(where\ \ i\ \in \{1, \ldots, n_1\})$

  $then\ we\ allow\ g_1 = g_{1,i}\ and\ g_1^{'} = g_1^{-1}g_0 (note.:\ g_1^{'} \in G_1)$,
- *(next step) if* $g_k^{'} \in G_k\ is\ defined, and\ if\ g_k^{'} \in g_{k+1,j}G_k\ (where\ j \in \{1, \ldots, n_1\})$

  $then\ we\ allow\ g_{k+1} = g_{k+1,j}\ and\ \ g_{k+1}^{'} = g_{k+1}^{-1}g_k^{'}\ \left(note.: g_{k+1}^{'} \in G_{k+1}\right)$,
- *to summarize, we get* $1 = \ g_n^{-1}g_{n-1}^{-1}g_{n-2}^{-1} \ldots g_1^{-1}g_0,\ \ so$

$$g_0 = g_1 g_2 \ldots g_{n-1} g_n.$$

We hope that using this, we can define the order of the subgroups in G_i, preferably via the shortest route. We can find the solution steps with a relatively simple cube rotation, which in this form isn't too long: $g_0 = g_1 g_2 \ldots g_{n-1} g_n$.

In the case of the 3×3×3 Rubik's Cube, we can calculate the number of possible solutions according to the set group $G =< R, L, F, B, U, D >$. Based on the number of groups, we get the following permutation number: 43252003274489856000 ≅ 4,3 x 10^{19}.

According to the group theory concept, the strategy of the cube's solution is as follows. If we allow $x^y = y^{-1} * x * y$ to mark the correspondence, $[x, y] = x * y * x^{-1} * y^{-1}$ marks the communicator for the elements of x, y group.

Let M_R mark the quarter turn clockwise on the side parallel to the middle row. The solution of the *layer by layer method* can be made from 3 basic states, which are the following (based on Joyner, 2002b):

1. : Solution of top side and top edges,
2. : Solution of middle edges (and bottom edges as quickly as possible),
3. : Solution of bottom corners (and bottom edges, if possible).

Table 7 contains the algorithms which are summarized by Joyner in the work *"Mathematics of the Rubik's Cube"* in 1996. The notations of the Table mark the different sides of the cube with Singmaster's codes. The notation M_R includes the above mentioned rotation in the formula.

Table 7: Modified Joyner mathematical algorithms and rotation order for the solution of 3×3×3 cube

SERIAL NO.	ALGORITHM	ROTATION ORDER
1.	$M_R^2 * U^{-1} * M_R^{-1} * U^2 * M_R * U^{-1} * M_R^2$	3 rotations on edges (UF, UL, UR)
2.	$(M_R * U)^3 * U * (M_R^{-1} * U)^3 * U$	turning top edges upward UF, UB
3.	$(R^2 * U^2)^3$	permutations (UF, UB) (FR, BR)
4.	$(M_R * U)^4$	turning UB, UL and DF, DB upward
5.	$(r^{-1} * D^2 * R * B^{-1} * U^2 * B)^2$	UFR+, BLD++
6.	$[R, U]^3$	permutations (UFR,DFR)(UBR,UBL)
7.	$F^2 * L^2 * U^2 * (F^2 * L^2)^3 * U^2 * L^2 * F^2$	permutations (UF, UB)(UR,UL)
8.	$(D^2 * R^2 * D^2 * (F^2 * R^2)^2 * U)^2$	permutations (UFL,UBR)(DFR,DBL)
9.	$(M_R^2 * U * M_R^2 * U^2)^2$	permutations (UFL,UBR)(UFR,UBR)
10.	$[R * D * R^{-1}, U]$	corners with 3 rotations (BRD, URB, ULB)

Source: self-made, based on Joyner, 2002b

2. CITED SCIENTIFIC SOURCES AND USED METHODS

2.1. The scientific background and evaluated literature

Throughout my research, I thoroughly perused, categorized and critically evaluated literature sources in both electronic and printed formats, which included domestic and international literature. The literary summary was based on the different interpretations of sustainability, the major Game Theory solutions used in economic strategy planning, and the Rubik solutions. For the theoretical basis of the research, the software analysis of the cube's inherent attributes was followed by the creation of the actual professional database. The primary data used for the SMART (Simple Multi Attribute Ranking Technique) 3D analysis was generated through the synthesis of the research results of Cleantech Incubation Europe (CIE).

2.2. The applied methods and their detailed description

To analyze the software aimed at solving Rubik's Cube, I used a SWOT analysis during my research, and I used theoretic process evaluation to evaluate the processes of the Rubik's Cube solution algorithms. For the sustainability interpretations of low-carbon development processes, I used content analysis, for which I employed the aid of the European Union's Low Carbon 2050 strategic guide (http://www.roadmap2050.eu/). The main point of the content analysis was to "ask" the social product named "A practical guide to a prosperous, low carbon Europe" and other professional documents which went through social control during the data collection for the sake of obtaining empirical data. During the research on Game Theory algorithms I was searching for, I used the process of tolerance, meaning I researched the admissible differences between the attributes of the cube, and the parameterization of the Game Theory functions.

To define the criteria system of the low carbon project development concept, I used the Churchman-Ackoff procedure. I examined the estimated usefulness of the factors for the sake of optimizing the ones out of all the factor groups which are most important and most useful to the project. To determine the relative usefulness of all the factors, I created "usefulness-functions" that can properly represent either the equality or the hierarchy of the factor groups.

2.2.1. SWOT analysis

DEFINITION OF METHODOLOGY

The SWOT analysis is a strategic planning tool which helps evaluate the strengths, weaknesses, opportunities and threats that may come up in case of corporate or personal decisions concerning a product, project, or business venture, or any other goal. A SWOT analysis includes the measurement of the system, the person, or the inner and outer environment of the business, thereby helping the decision maker to concentrate only on the most important topics (Start Up guide, 2012).

The answers we seek with the analyses:

Strengths:

- ✓ What pros does the analyzed system have in low carbon innovation practice, analysis of internal attributes?
- ✓ What does it do better compared to the other system?
- ✓ What is the hearsay about the system, its strengths?

Weaknesses:

- ✓ What parts could be improved?
- ✓ What should be avoided?
- ✓ What is the hearsay about the system, its weaknesses?

Opportunities:
- ✓ What opportunities does it have in the future?
- ✓ What trends and market tendencies are known to it?

Threats:
- ✓ What problems may surface during its use?
- ✓ What are the competitors doing?
- ✓ Are unfavorable changes visible in the operation environment?

The above defined questions are answered in the evaluation table below, by giving short answers to them.

	POSITIVE TRAITS	NEGATIVE TRAITS
Internal traits	Strengths	Weaknesses
External traits	Opportunities	Threats

The goal of the SWOT analysis on solution-searching:

In the case of the examined solution-searching software applications, the goal of the SWOT analysis is to determine if the functions of each software are applicable to the input and output system attributes of the low carbon project evaluation model and if they satisfy the user expectations. According to the data at hand, the object of the analysis should be: how, and how much is the low-carbon innovation and incubation system based on the "About low carbon economy" (LCE Ltd. 2011), and "Hubconcepts – Global best practice for innovation ecosystems" (Launonen, 2011) professional guidelines satisfied by the chosen software.

Reason for the choice of method:

The SWOT analysis offers a good opportunity to create an overview comparison, which has no exact attributes definable in easily comparable dimensions. In itself, the SWOT analysis has no meaning; however, if it's part of a complex analysis, it can sufficiently facilitate the thought process.

2.2.2. Theoretic process evaluation

With the solution algorithms of the 3×3×3 Rubik's Cube, the sustainability theories can be synchronized and the relations of the cube's sides define a planning strategy that provides a new scientific approach for investment planning. I theoretically evaluated the various solution processes and investment planning levels parallel to them by following the solution levels and stages of the cube. After these various level-evaluations, I made "low carbon interpretation" summaries.

The structure of the process evaluation is as follows:
- defining sector or level,
- theoretic evaluation,
- evaluation of process and results (interpretations),
- summarizing the evaluation of process and results.

To show the various states of the cube and to attach an explanation to the low carbon interpretations, I used the Online Ruwix Cube Solver program.

Reason for the choice of method:

The process of the cube's solution can be defined with various algorithms, although these only define the technical process of the solution. Since my goal during the analyses I performed was to compare one of the most logical solution methods with the planning process of sustainability instead of merely analyzing the fastest rotation algorithm, I couldn't choose a method more suitable and effective than the theoretic content analysis.

2.2.3. Data collection for multi-dimension "low carbon" development processes with content analysis

Among the various research methods, content analysis can be categorized as a so-called "no intervention" type. The biggest benefit of using this type of examination is that people doing the research are a proper distance away from the actualization of the problem; therefore, they can perform the data collection without any chance of intervention in the process. In this case, there's no problem with our data collection process affecting the respondent. Content analysis is a kind of data collection where we conduct the gathering of information and analysis using the designated document. It is also a kind of social analysis method used for the examination of human messaging (Kérdő, 2008). I analyzed the program documentations through the EU's social inspection mechanism and debates relevant to both the low carbon development concepts and the topic of sustainability. The point of the content analysis was to "ask" the social products named the "A practical guide to a prosperous, low-carbon Europe" and the "National Energy Strategy 2030" as well as the professional document "Hungary's renewable energy plan 2010-2020" (NCST 2010-2020 for short), which went through social control, for the sake of obtaining empirical data.

During the completion of the content analysis, I found it important to "ask" these social products which are mostly goal orientations and documents – as the source of my empirical data – in a way that avoids contradictions and to highlight the preference indicators of green energy or climate-friendly investments as common decision factors.

Reason for the choice of method:

A primary form of data collection in which we try to find an answer with the dissolution of contradictions in the professional documentation or continue the examination by re-defining the contradictions.

2.2.4. Evaluation of Game Theory algorithms by process of tolerance and applicability

The evaluation of the process of tolerance in the sense of engineering means the allowed maximum differentiation from the determined sizes, quantities or qualities. In the case of the Game Theory algorithms, I researched the following: which method is the same as the solution process model of the Rubik's Cube in terms of its attributes and in what scale does it differ from it while staying representative. For the Game Theory algorithms I was searching for, I used the process of tolerance, meaning I was researching the admissible differences between the attributes of the cube and the parameterization of the Game Theory functions (Ligeti, 2006).

I took the Game Theory algorithms one by one and chose the ones linkable to the various rotation algorithms (interpretations) of the modeling process. For the multi-purpose optimization tasks, the process of modeling is as follows (Forgó et al., 2005):

1. Designation of criteria system (and also, major attribute sets)
2. Independence analysis of attribute sets (avoiding overlaps between attributes)
3. Designation of choice variables and parameters in attribute sets (deterministic, or stochastic (in other words, realized with some level of probability) marking)
4. Designation of binding criteria related to a set (creation of sets)
5. Designation of possible criteria of the criteria system and the number of objective functions in a set (the number of objective functions is finite)

6. Search of the optimum for objective functions

Reason for the choice of method:
The modeling process used during linear programming offers a clear view on the criteria system of multi-purpose optimization tasks, which is why I chose this method to realize the three-level strategic objective system planning.

2.2.5. Determining the criteria and cube attributes with the Churchman - Ackoff method

It is important to weight the attributes which have an impact on the development processes and to choose the groups that give the defining conditions of the development of the project and the actualization of the investments. In the method presented by Churchman and Ackoff, there are two almost indistinct processes if we go by the number of attributes. The first one was made for one to seven attributes, while the second one for more than seven attributes.

The method is based on consecutive comparisons and is also usable to select a lower number of attributes, which in our case is four. (The widely known Guilford-method is only preferred for attributes above five.) As a first step, the attributes must be organized based on their supposed importance by "feeling," in other words, a professional estimation. To begin, the most important attribute should be given the number one, and weighting the other attributes should be related to this one. For specification reasons, the most important attribute and its weight have to be benchmarked against the groups made from the other attributes (and the sum of their respective weights). If the highlighted attribute is more important but the relation defined based on the weights doesn't have the same conclusion, the weights must be corrected appropriately. After the correction, the highlighted attribute must be benchmarked against a group with one less element (Simongáti, 2009a, Russell, 2003). We should proceed with this process as long as the highlighted attribute and the group (which is shrinking) made of the other attributes have the same importance. After applying this to the weights as well, we may move on to the second most important attribute, where we have to go through the same process. If we have the respective weights of every attribute, we have to normalize the dominance definition in a way that the sum of the final weight values is one (Churchman - Ackoff, 1957).

The method made for tasks evaluating larger and more numerous attributes differs from the above that in this case, we have to benchmark a randomly chosen attribute against groups consisting of less than 5 attributes. One of the pros of the Churchman - Ackoff methods is that we can get a precise result, albeit with a high time consumption (Simongáti, 2009b).

To define the main agents (using the fundamentals of the Churchman-Ackoff method as a basis), I made the following methodology:

- **Step 0:** Creating a preference order estimated in advance (F_1, $F_2 \ldots F_n$)
- **Step 1:** Assigning usefulness values by importance
    - ❑ By designating the weight of (F_1) as 1, we have to assign the other attributes' weights respectively, relative to that of the first.
    - ❑ E.g. Is F_1 more important, as important, or less important than the others altogether?
    - ❑ Formula: $W_1 > (=,<)\ w_2 + w_3 + \ldots + w_n$?
- Benchmarking and correction of importance:
    - ❑ If F_1 is more important but the inequality defined with weights doesn't reflect this, then w_1 must be corrected in a way that the inequality reflects the relation →Step 2.

- If F_1 isn't that important, then w1 will be lowered as such.
- benchmarking F_1 against $\{F_2, F_3, \dots F_{n-1}\}$ group, and continue until we conclude at group $\{F_2, F_3\}$.

- **Step 2:** benchmarking F_2 against $\{F_3, F_4 \dots F_n\}$ groups as shown in Step 1.
- **Step 3:** continuing the sequence, until reaching benchmark of F_{n-2} against $\{F_{n-1}, F_n\}$.
- **Step 4:** standardization: dividing all weights by Σw_i

Pros of standardization: more reliable result compared to immediate estimation.

Cons of standardization: useless for problems with more than 7 attributes.

During value estimation processes and choosing dominance sequences, applying the Churchman-Ackoff method is popular, therefore, it's also well-known as a software application (Ose L. S., 2008). The shortened software application was not used for the analyses.

Reason for the choice of method:
There are multiple options which are currently preferred (e.g. the Guilford method, or the immediate estimation) for the selection of weighting processes, but since I'd like to introduce the best available mathematical tools in terms of precision, I chose the Churchman-Ackoff method.

2.2.6. Applicability analysis of "usefulness-functions" in multi-dimension evaluation (SMART)

In order to optimize the most important and most useful attributes of the criteria systems (attribute groups) that have an impact on Rubik's Cube low carbon development, we have to evaluate the estimated usefulness of the attributes. To determine the relative usefulness of all the factors, I created "usefulness functions" that can properly represent either the equality or the hierarchy of the attribute groups. These "usefulness functions" designate a single number to all the stages to show the preferability of each stage. By combining the consequences of the actions with its probability, we get the estimated usefulness for each action (Russel - Norvin, 2003).

For the usefulness of state *S* defined by those attributes that have an impact on the decision, we can use *U*(*S*). We interpret the various states as the snapshots of the circumstances, which means that a non-deterministic *A* action has $Result_1(A)$ states as possible consequences, where the index *i* follows the different consequences. According to Russel and Norvin, before executing *A*, the agent assigns a $P(Result_1(A)|Test(A), E)$ probability to each and every consequence, where *E* stands for the world's conclusive facts reachable by the agent, and *Test*(*A*) is a statement that action *A* will be executed in the current state. In this case, we can calculate action $EU(A|E)$'s anticipatory usefulness for given facts with the following formula:

$$EU(A/E) = \sum_i P\ (Result_1(A)/Test(A), E)\ U\ (Result_i(A))$$

The principle of maximum anticipatory usefulness (MVH) states that a rational agent must choose an action that maximizes the anticipatory usefulness of said agent. If we want to determine the best sequence of actions using this equation, we'd have to take each action sequence into consideration and select the best ones (David, 2002; Russel - Norvin, 2003).

The usefulness analyses made during the research program and the evaluation functions were put into the SMART software application. SMART (Simple Multi Attribute Ranking Technique) can also visualize the results of the usefulness analyses and the connections between the various attributes even in 3D; I therefore used primer data from the CIE research program's database.

SMART (Simple Multi Attribute Ranking Technique) is an evaluation technique which can organize attributes which may be dominant for the decision into a sequence. Harvard University, MIT and the University of Southern California cooperated in its development in the last few years. The application can transform the various alternatives' base values into usefulness values with the use of usefulness functions. The most valuable asset of the software is the option of following the correspondence systems in both 2D and 3D (Huhn, 2013). The transformation to usefulness values using usefulness functions happens according to the steps defined by Simongáti (2009b).

Reason for the choice of method:

The SMART program can illustrate the data of the usefulness functions and the evaluation system in 3D extension.

3. SUSTAINABLE PROJECT DEVELOPMENT WITH RUBIK'S CUBE SOLUTION

3.1. Benchmarking of Rubik's Cube solution programs, and their base correspondences using SWOT analyses

The tools of group theory can simplify the calculations of the process of software development by defining subgroups of the hundreds or even millions of layouts that have shared mathematical characteristics. The German mathematician Herbert Kociemba used a cunning method to decrease the 43 quintillion possible rotations of the cube in 1992 (Ajay, 2011). The mathematical basis of the calculation (according to group theory) was how we calculate the variation possibilities, in other words, how many different samples we can observe on the cube:

- 8 corners = 8! positions / each have 3 possible orientations = 3^8
- 12 edges = 12! positions / each have 2 possible orientations = 2^{12}
- Impossibilities:
  - no element substitution (2),
  - no edge orientation (2),
  - no corner orientation (3).
- Meaning 2x2x3 = divided by 12, which totals for = $(8! \times 3^8 \times 12! \times 2^{12}) / 12 \sim= 4.3 \times 10^{19}$

Kociemba had a different approach to the mathematical relations of the cube compared to the usual method of basing it on fix combinations – he made a subgroup that was based on 10 out of the 18 possible rotations of the cube. With the combination of these 10 rotations, he found out that he can reach 20 billion different configurations from a solved cube. This is an important step because this subgroup is small enough to fit an ordinary PC-s memory. Kociemba also developed a program for this, named Cube Explorer, which was further developed by the American mathematician Michael Reid in 1995, who used it to estimate the minimum required rotations to solve the cube, which he defined at 30. Theoretical scientists already considered 20 to be "God's number" (the minimum required rotations), but the proof would have required a supercomputer. Finally, the proof of "God's number" being 20 only happened in July 2010, when Thomas Rokicki, Herbert Kociemba, Morley Davidson and John Dethridge (Rokicki et al., 2010) proudly declared to the world that it's proven – "God's Number for the Cube is exactly 20."

Therefore, Kociemba's Cube Explorer was the first Rubik's Cube solution program which was able to solve a cube from any starting position using around 30 rotations. Thus, after this first software, and also using it as a basis, began the different personal developments for different solution programs all around the world. In order to view the connection network of Rubik's Cube software development, as well as the low-carbon project development methodology based on Rubik's Cube's solution algorithms, I will conduct the SWOT analysis for three different development routes. During low carbon project development, the goal is to make the analyzed development or investment process faster and simpler, even with the use of software. The role of the software can be important if, after assigning the attributes to the cube's respective sides, we can define the starting state of the project even with the disordered state of Rubik's Cube. If we define the degree of disorder with the cube's state, the solution program can easily inform the user how he can reach various levels of order. The solution search using software raises one simple question: is the route appropriate, and can the process of solution search abide by the various professional requirements (global best practice for innovation ecosystems) that lead to the basis of successful project development?

The goal of the detailed introduction of the SWOT analysis in the methodology section was to make it clear to me whether the functions of the software are applicable to project the evaluation model's input and output requirements. The analysis was done by classic SWOT rules, the details of which won't be shown, only the results. For the sake of understanding them, I'll give short descriptions on the various software applications.

Software evaluated using SWOT analyses:

- ✓ RUWIX PROGRAM (KOCIEMBA CUBE EXPLORER DEVELOPMENT)
- ✓ SOLUTION SEARCHING LBL SOFTWARE (GÁBOR NAGY)
- ✓ RUBIKSOLVE PROGRAM (ERIC DIEC)

3.1.1. SWOT analysis of the Ruwix program (Kociemba Cube Explorer development)

The complex solution and demonstration program was developed by the Hungarian Ferenc Dénes, using Kociemba's 2005 solver program as a basis. The software chooses the shortest possible solution from any given starting combination. The average number of rotations is 50-60, which does not prefer *layer by layer* algorithms. In this case, the developers uploaded a lot more algorithms into the optimal solution search program, which finds more right solutions during optimization. The online solution software shares all important information with the user and is great to look at (Figure 3.).

Ruwix is an online Java-based web application that doesn't use any support platforms. The developer endowed it with the necessary functions in order to help users learn Rubik's Cube and the various solution methods for it. It is suitable e.g. to animate the process of solution step by step and display it to the user. The solution search engine can animate the solution and rotation moves from any given combination, which is preferred by users training for Rubik's Cube solving competitions. Using the Ruwix program, users can play with different Rubik products online (2×2×2 cube, 3×3×3 cube, 4×4×4 cube, 5×5×5 cube, etc.), which offer a pleasant game experience in 3D.

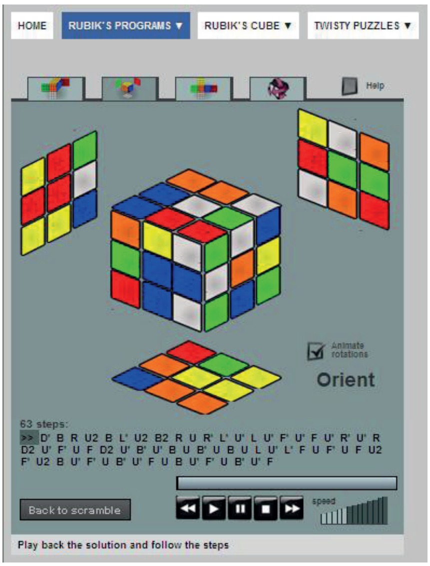


Figure 3: Visual style and shortest solution formula of Ruwix program
Source: Dénes, T. (2005) Ruwix.com

The SWOT evaluation of Ruwix (Table 8), in accordance with the low-carbon project evaluation model's input and output requirements:

Table 8: Ruwix program SWOT Table

	POSITIVE TRAITS	NEGATIVE TRAITS
INTERNAL TRAITS	**STRENGTHS** Exceptional graphics and visual details, some mention it as the world's most advanced solution software. Offers solutions not only to Rubik's Cube, but many other logical games.	**WEAKNESSES** Presently not compatible, since it uses different, faster algorithms than the layer by layer solution, which aren't the best for low-carbon solutions.
EXTERNAL TRAITS	**OPPORTUNITIES** Because of its strengths and the applicability, it would be beneficial to develop low-carbon specifications as well.	**THREATS** Since the program runs in an online format, it isn't possible to add special data to it. Even in case of a low-carbon specification, syncing the free software with the pay-to-use SMART add-on makes it difficult to use.

Source: self-made

3.1.2. SWOT analysis of Solution Searching LBL software for Rubik's Cube

To introduce the Rubik's Cube solution software, I will mostly use a domestic development made by IT technician and engineer Gábor Nagy (University of Debrecen): "Solution searching methods." This description and methodology guide is unique because of its *status space* representation, which was used to work out the problem of multi-level solution search. The other important thing to note about the choice of software was that it prefers the *layer by layer* solution, and as far as I know, this is the only application which uses only this method, because it is considered "too slow." (On another note, any solution search could implement the *layer by layer method*, were it coded with it in the first place.)

The program was developed in 2008 using the Java language and the NetBeans IDE 6.1 development platform. To make the structure of the program clear, we have to understand the respective structures of two packs – the *status space* and *cube* packs.

The pack named *Status Space (*"*Allapotter")* contains two abstract classes and an interface which save the exact various elements and attributes of the status spaces. During the main problem's implementation, these elements are specified by the program to fit the representation of the *status space.* The program checks (for each different status) if a given status is the goal or not. According to the developer's manual, the heuristic result is ensured by the interface named *Heuristic Status (*"*HeurisztikusAllapot"),* which needs to be implemented in the program from the get-go. In the case of the solution search program, we define the *Cube Status (*"*KockaAllapot")* class, or the *cube* pack, as the start, the elements of which describe a given element of the *status space.* This class contains the constructors not included in the 54-element byte packets, which record the various states of Rubik's Cube and all the methods applicable for the different statuses. The *objective status checking function* checks the 3D parts of the cube and, if it finds a color out of place, returns a "false" message, while if it doesn't, the cube is solved and every color is in its place (Nagy, 2008a).

In the program's description, there is also mention that the status of the cube is defined by 54 numbers, which are selected from the [0,5] interval, and the numbers symbolize the various colors (based on Nagy, 2008):

$$H = \{(0,0, \ldots ,0), (1,0, \ldots ,0), \ldots , (5,5, \ldots ,5)\}$$

A ≠ H, since not all elements of H can be real statuses.

$$A = \{a | a \in H_1 \times \ldots \times H_n\}$$

Description of Cube Pack

Using the classes and interface of the "Status Space Pack," the created classes are categorized into the "Cube Pack," which is closely related to Rubik's Cube and its structure. The examples of the "Cube Status" class define the various statuses of the status space, but the class also contains the constructors not included in the 54-element byte packets, which record the various states of Rubik's Cube and all the methods applicable for the different statuses, which are as follows (Nagy, 2008b):

- ✓ "Objective status checking function," which has a return value of either true or false. Using three for loops integrated into each other, it analyzes the 3D block that defines the status of the cube and, if it finds a color out of place, returns a "false" message, while if it doesn't, the cube is solved, and every color is in its place.
- ✓ "Operator" – a function that checks the application master and analyzes if the operator condition is applicable to the given status. This also has a logical return value, which is – for Rubik's Cube – always true.
- ✓ "Apply function," which contains the operator for the given status as a parameter, and its return value is the function of the resulting status. It creates a copy of the cube's status, executes the value copying abiding by the operators, and returns with the copy.
- ✓ The function that benchmarks the given status against a different status, which is the result of a parameter. Has a logical return value, which is true if all elements of the statuses of benchmarked cubes are identical. Otherwise, its value is false.
- ✓ An evaluation function which is exceptionally important for our research.
- ✓ Method to access the "data tags" which register the various states of the cube.
- ✓ Methods related to imaging and burning.

Layer by layer method, and the evaluation function

Due to a choice made by the developer, the program uses a *MOHÓ* search engine (greedy search) to solve the cube; therefore, the evaluation function consists only of the heuristic function, which is implemented by the "Heuristic Method" of the "Cube Status" class, as mentioned above. The method evaluates and scores the various statuses by the sequential row by row, in other words, the *layer by layer* method. Therefore, due to the impact of the heuristic pack, the program uses the *layer by layer* method to find the solution, meaning row by row, though it is a known fact that this isn't the fastest and most effective way to produce the result in solution search. The program doesn't analyze the starting state, since the optimization of the starting side would require a complex evaluation function's implementation, which was deemed unnecessary for this program by the developer, so the program always starts with the yellow side. In terms of the method, this means these are easily checkable layers, or in other words, levels, meaning the heuristic function also begins by checking this so-called level to avoid checks which are not important on the actual level but may be so on lower levels (based on Nagy, 2008c).

These levels are as follows (Figure 4):

Level 0: Cube doesn't abide by level 1's requirements.

Level 1: Edges which also have yellow are in position, with proper orientation, meaning "yellow cross" is complete.

Level 2: Corners which also have yellow are in position, with proper orientation, meaning "upper row" is complete.

Level 3: Middle row is complete.

Level 4: Edges which also have yellow are in position, with proper orientation, meaning "white cross" is complete.

Level 5: Corners which also have yellow are in position, with proper orientation, meaning the cube is in its finished state.

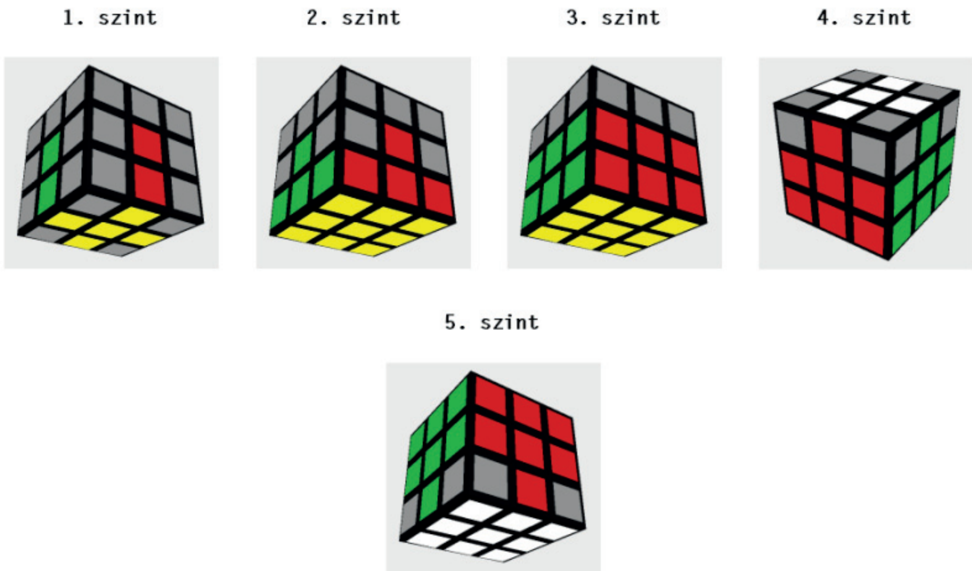


Figure 4: Levels ("szint") of Layer by layer method in the program
Source: self-made (based on Nagy, 2008)

According to the developer's description, we may not be able to continue without heuristics or breaking the level. In this case, the so-called solution algorithms may help when used for the correct statuses, which are series of steps that, though they degrade the heuristics at first, get closer to the goal in the end compared to where we were before applying them. The first level (solution of first row) may be reached even without algorithms, but this is the part of the heuristic function which is implemented with the greatest hardship. According to Nagy, the reason for this is that unlike on higher levels, where we primarily use algorithms apart from 1-2 rotations, at first we use steps which are simple but numerous and give a high number of various alternatives, so translating human knowledge for the program becomes difficult. On higher levels, the use of the heuristic algorithm becomes much less of a problem, as we can assign a few fixed algorithms for virtually any status: we only have to decide which to implement first.

With the heuristics of a status, the programmer defines the return value of the heuristic function, in other words, the "correctness" of the status. His idea was that while we're on lower levels, the heuristics of the status starts from a higher value, while the farther the next level seems during the appropriate checks for each level, the more its value increases. Therefore, the rate of increase is dependent on the positions and/or orientations of the edges and corners required to complete the level. Each of these edges or corners raises the value of the heuristics more or less. The scale therefore depends on how far it is from its proper position, or a position from which it can be moved to its proper position using an algorithm. According to the developer, the value of heuristics will never raise so much within a single level that a lower level's heuristics becomes lower as well. This condition is necessary for the search engine to find the shortest route to the solution, based on the method. One of the consequences for this is that if we reach a certain level with the program, it is sufficient to do the checks only for that given level, since all the others either already stand true, or aren't needed yet. According to this, the scoring in the program is as follows (based on Nagy, 2008):

- ***Determining the level*** is the first step of evaluation/scoring. The higher the level we're on, the lower the number will be. The starting value of heuristics on level 5's evaluation function is "0."
- ***On level 0:*** An edge in its place with proper orientation barely raises heuristics, while the ones far from their position raise it according to their exact "misplacedness." If we have at least two edges in the right position and with proper orientation, we can allow the use of algorithms, but this causes the edges to raise heuristics less, if they're close to being put in their proper position using an algorithm. These algorithms consist of only 3-5 steps, but have other extra effects. For each side, we have to check using three of these algorithms. The reason for this is that the software interprets operators from a fixed point of view, with the yellow side always being on top and the blue side in the front. Because of this, the same sequence of rotations may be built with different operators for the various sides, but we have to be able to choose the correct one. A good example for this would be for us to check three different positions for the yellow-blue edge, from where only an algorithm can put it in its proper position (Figure 5.).

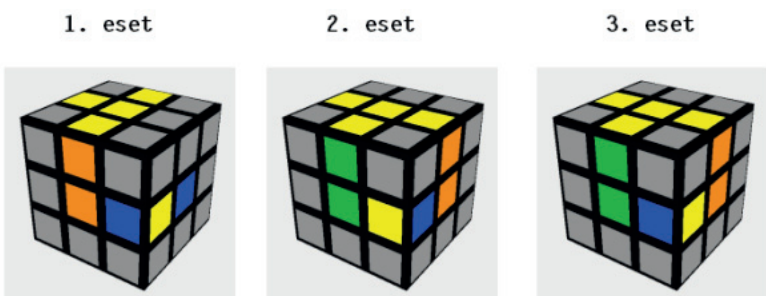


Figure 5: Edges only solvable through algorithms
("eset" stands for case)
Source: self-made (based on Nagy, 2008)

Algorithm 1: UR, LB, UL.
Algorithm 2: UR, LF, UL. Algorithm 3: UR, UR, RR, UL, UL.

Abbreviations are from initials:

F (Front)
B (Back)
U (Up)
D (Down)
L (Left)
R (Right)

- ***On level 1:*** On this level, we can use almost only algorithms to solve a corner. Heuristics may further increase due to the corners' distance of their "algorithm possibilities," apart from the basic increase of the level. On this level, we have to watch 5 different algorithms. Let's go through the blue-yellow-orange corner's five different algorithms via the examples on Figure 6 below:

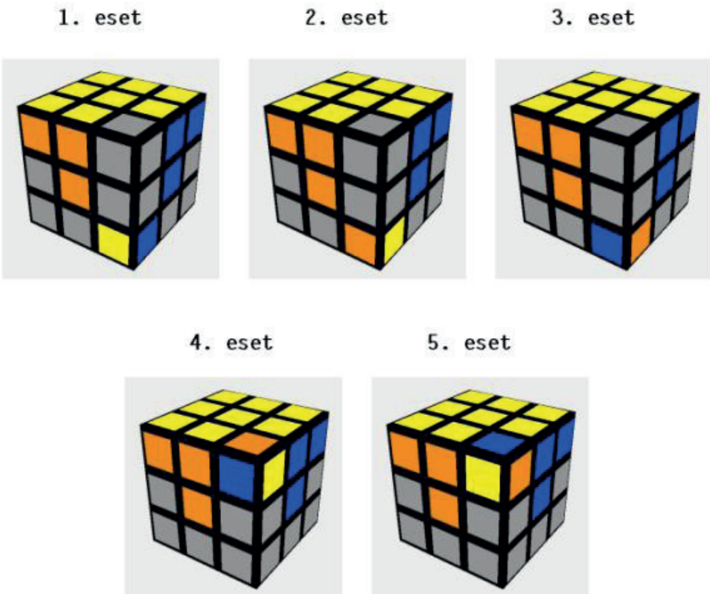


Figure 6: Positions of corners definable via algorithm
("eset" stands for case)
Source: self-made (based on Nagy, 2008)

Algorithm 1: LF, LL, LB. Algorithm 2: FL, LR, FR. Algorithm 3: LF, LR, LB.
Algorithm 4: FL, LL, FR. Algorithm 5: LF, LL, LB.

The solution search program therefore uses the above mentioned seven levels' *MOHÓ* search to solve the cube (Table 9). During the evaluation of the above defined methodology guide, it's obvious that the program is able to solve Rubik's Cube from virtually any starting combination using the *Layer by layer* method. The number of required rotations is dependent on the base combination, but usually needs more than 70 rotations. However, in case of a simpler starting combination, this can decrease to 40-45 rotations (Figure 7).

Table 9: Layer by layer solution algorithms for 3×3×3 Rubik's Cube using software and the MOHÓ search engine (on levels 2., 3., 4., 5.)

Level	Phase	Algorithms
2.	Positions definable with algorithms for second row edges	Algorithm 1: FL, LL, FR, LB, FR, LF, FL. Algorithm 2: LF, LR, LB, FR. LB. FL LF. Algorithm 3: LF, LL, LB, LR.
3.	State fit for edge switch, edge switch on sealing side	Algorithm 1: LF, LL, LL, LB, LR, LF, LR, LB, LR.
	Edge rotation, rotating sealing side to match colors	Algorithm 1: LB, RB, FL, LF, RF, LR, LB, RB, FL, LF, RF, LR, LB, RB, FL, LF, RF, LR
4.	Corner switch	Algorithm 1: LB, LL, RB, LR, LF, LL. RF, LR. Algorithm 2: FR, LR, RR LL, FL, LR EL, LL
5.	Rotating corners to match colors, correction of misplaced corners	Algorithm 1: RB, LL, RF, LL, RB, LR, LR, RF, LB, LR, LF, LR, LB, LR, LR, LF. Algorithm 2: LB, LL, LL, LF, LL, LB, LL, LF, RB, LL, LL, RF, LR, RB, LR, RF.

Source: self-made (based on Nagy, 2008)

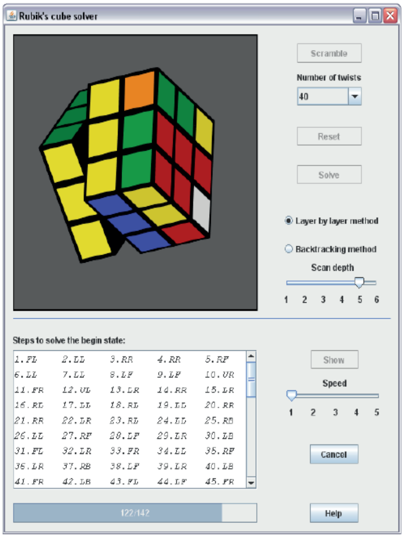


Figure 7: Evaluation screen of Solution Searching LBL software for Rubik's Cube
Source: Solution Searching LBL software for Rubik's Cube

The reason I feel it is necessary to present the Solution Searching LBL software for Rubik's Cube in this much detail is that during the process of solution, it follows rotations by hand almost completely and uses each algorithm of the *layer by layer* method, but doesn't implement any other methods.

The SWOT evaluation of Solution Searching LBL software for Rubik's Cube (Table 10), in accordance to the low-carbon project evaluation model's input and output requirements:

Table 10: Rubik's Cube Solution Search program SWOT Table

	POSITIVE TRAITS	NEGATIVE TRAITS
INTERNAL TRAITS	**STRENGTHS** The steps of conceptual and practical solutions are the same The layer by layer solution is followed through in the program Uses obvious advancement and correction steps Because of the easy programming, it's also easy to develop Every algorithm is also definable in the steps of the low-carbon project evaluation model as well	**WEAKNESSES** The visual interface is not up-to-date Slightly slow processing Not available in online format As of now, it can only solve the 3×3×3 Rubik's Cube
EXTERNAL TRAITS	**OPPORTUNITIES** Visual interface Easy to sync with the SMART evaluation software plug-in The definition of the low carbon domain requires no additional software development Because of the easy programming, it may prove to be a cheap newcomer on the market	**THREATS** Quite an old development The program may seem slow because it can't be accelerated properly because of a set of certain configurations "Easy to copy"

Source: self-made

3.1.3. SWOT analysis of Rubiksolve program

This is one of the most well-known solution programs on the web. The developer, Eric Dietz, has been interested in the mathematics and programming opportunities of Rubik's Cube since his childhood. His first program that solves Rubik's Cube was published and shared with the members of the Rubik "fun" community in 2002. In 2005, he used Kociemba's 3×3×3 method to popularize his own online program. In 2007, he developed a solver program which he further developed by lowering the amount of required rotations with the use of newer algorithms. The one that's currently running, which uses Kociemba's algorithm, was finalized in 2010, meaning it needs less than 25 rotations to finish the cube from any given starting combination. Eric Dietz always used Kociemba's algorithms for the solution, two of which can be seen on Figures 8 and 9, or by clicking the link below (Dietz, 2010).

The program only handles 2×2×2, 3×3×3 and 4×4×4 cubes' solution algorithms; its portfolio has no other Rubik games. It illustrates every detail in 2D, and offers no special visual enjoyment either. The illustrations that explain rotations can be interpreted easily, therefore, in the last few decades, tens of thousands of players learned to solve Rubik's Cube with this program's guides.

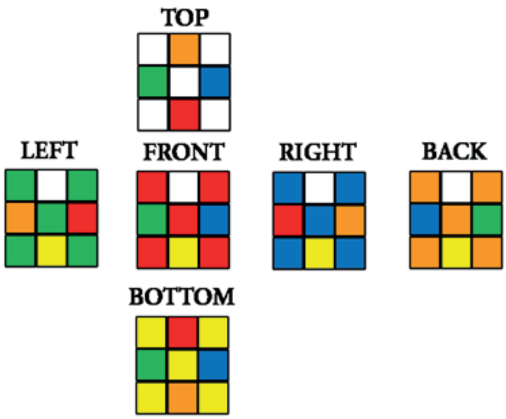
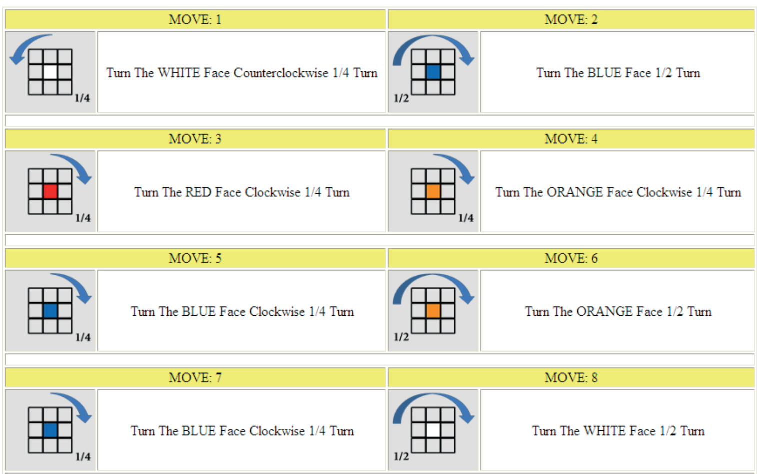


Figure 8: Notations of sides on the program's solution interface (flip state)
Source: based on Dietz, 2010

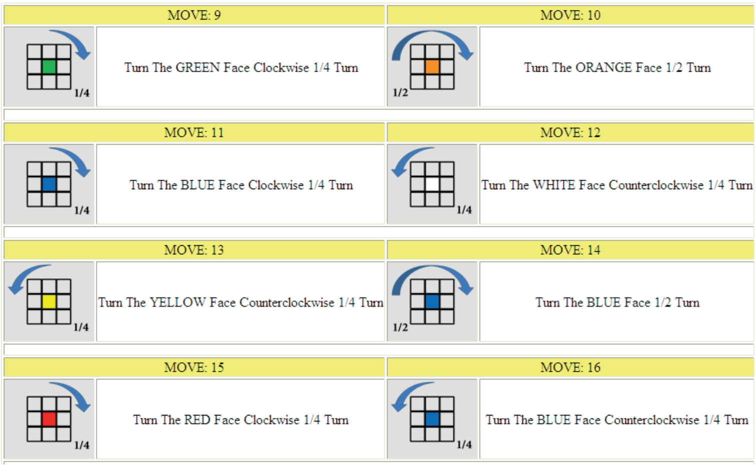
Because of the reduced number of algorithms, we won't find the same levels as for the previously introduced Solution Searching LBL software. The program doesn't implement the *layer by layer method* as a solution process, but some algorithms of the various methods are the same, meaning the same algorithms are sometimes used in different solution searching programs.

The program works quite fast and only needs a few seconds to display the solution formula for the combinations put in. As a comparison, Ruwix and Solution Search need several tens of seconds, or even minutes to display the solution formula (Figure 8.).

Level 1



Level 2



Level 3

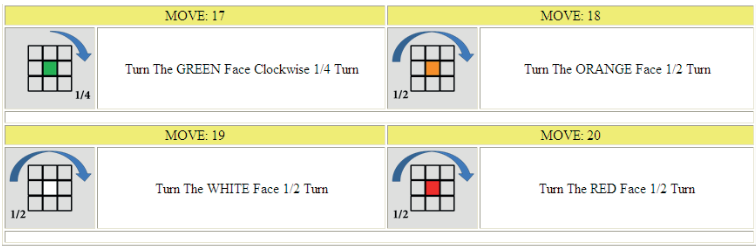


Figure 9: Rubiksolve's solution formula in 20 steps
Source: http://mk2.rubiksolve.com/

The SWOT evaluation of Rubiksolve (Table 11.), in accordance with the low-carbon project evaluation model's input and output requirements:

Table 11: Rubiksolve program SWOT Table

	POSITIVE TRAITS	NEGATIVE TRAITS
INTERNAL TRAITS	**STRENGTHS** Fast, constantly developed, can use layer by layer method	**WEAKNESSES** 2D, can't interpret layer by layer logic at the input, other user functions are missing
EXTERNAL TRAITS	**OPPORTUNITIES** Easy plug-in options offer good compatibility with low-carbon usage	**THREATS** Since it focuses on fast solutions, not all details can be understood by the users

Source: self-made

The introduced Ruwix Solver and Rubiksolve applications are both the further developed versions of Kociemba's Cube Explorer, which was the basis of most Rubik's Cube fans' software development work and ideas since 2005. After reviewing the different solution programs, we can say that there is an option to bring in technically any new algorithm, but of course, the goal of all the developers was to give the competitors a program that offers the solutions with the highest possible procession speed and the lowest number of combinations necessary. In the case of the Rubiksolve program, this is below 25 steps.

The Rubik's Cube Solution Search program completes the cube with the seven solution levels defined by MOHÓ's search engine. During the evaluation of the methodology manual, we made it clear that this one is able to get to the completed stage, meaning the one side – one color state from any starting stage with the layer by layer method. Also, the process may be stopped at any given stage. The number of rotations varies by the starting stage, but usually it takes more than 70 rotations to complete the cube. However, from an easier starting point, thus can be reduced to a mere 40-45 rotations.

Also, by analyzing the SWOT evaluations, it can be said that the swift strengths/weaknesses/opportunities/threats Table prefers the Hungarian-developed Rubik's Cube Solution Search program, which was optimized for the layer by layer algorithms. This Java-based application proved to be best in its functionality for the low carbon project evaluation model's input and output expectations, also noted by the structural trait that the software's "State Area" pack designates almost the same solution levels that the hand-solved algorithms do. (The other evaluated software types designate almost completely different levels.)

3.2. The principles and sustainability relations of the layer by layer solution method

The various sustainability logics can be synchronized with the 3×3×3 Rubik's Cube's solution algorithms and the relations of the cube's sides define a planning strategy that provides a new scientific approach for investment planning. I theoretically evaluated the various solution processes and parallel investment planning levels following the solution levels and stages of the cube. After these various level-evaluations, I made "low-carbon interpretation" summaries. To show the various states of the cube, and to attach an explanation to the low-carbon interpretations, I used the Online Ruwix Cube Solver program.

Cube soul

In 1980, Ernő Rubik wrote that the cube seems to be alive, as it comes into life while you rotate it in your hands. Rubik's Cube has three rows and three columns, and this can also have a symbolic, or even mystical, meaning. If we look at the attributes of the various blocks, the 3×3×3 cube's sides, it is almost immediately obvious that in the case of each side, we have system elements, or specific small cubes (middle cubes, edge cubes, and corner cubes), which hide a specific meaning and keep this meaning in them, regardless of where we rotate them in the system. According to Ernő Rubik, the number "three," through its special meaning, is even able to model life itself. It is able to show the relationship of man and nature, the process of creation, care and destruction, and the relations of cooperation between our resource systems (Rubik, 1981). We may think that the solution to the "mystical cube game" problem may properly portray the biggest question of one of today's hardest problems – the proper and effective use of energy. Nowadays, the entire energy consumption system seems like a huge puzzle in which we don't seem to be able to find the correct pieces. However, we suggest that the 3×3×3 Rubik's Cube's solution method may help us find the relations between the various pieces and the relevant inclusion of system attributes in both a 2D and 3D interpretable manner; therefore, it may give correct pointers on interpreting the supply and demand sides of energy consumption (Fogarassy, 2013).

One of the most widely known and most used methods of solving Rubik's Cube is the "layer by layer" method, but we must also note that it's the basis for the more advanced methods like Fridrich, Corner first, etc. The gist of the method is to complete the cube during the solution process row by row. That means that at first, we form a color cross on the first row, then insert the correct corners, then comes the middle row, and finally, the lower middle cube goes into its position, followed by the lower corner cubes (Fogarassy et al., 2012).

Most amateurs use the layer by layer method, since this is the easiest to learn, and this is one of the few methods that has both a professionally based algorithm and introduction guides. All other advanced solution methods began from this one. I introduced the process of solution according to the outline provided by the www.rubikkocka.hu official website. However, in the current document, I also included UNFCCC's basic development theories, namely "Low-Emission and low-carbon Development Strategies" (LEDS) – which has close ties to basic sustainability criteria – for the official solution method cited in this document. We made the assumption that since the Rubik's Cube's number "three" offers indirect answers to many of our world's currently unsolved questions through its mystical logic, it is correct to also assume that those who can complete the cube can think "Rubically" in general, or more specifically, about the questions of strategic planning and economic equilibrium search. In the next part of this document, I present the methodical steps on solving the cube that can be taken as a compilation theory during strategic development following the solution of the cube, usable for e.g. the advancement from fossilized to renewable energy support systems.

3.2.1. Process evaluation of layer by layer solution method for 3×3×3 Rubik's Cube

The layer by layer method is fundamentally a structured arrangement system which defines cornerstones and stages for the process of completion (white cross, second row, yellow cross, etc.). Even though these stages can be achieved by different routes, or one might say that everyone does it at their own personal leisure, it is technically impossible to advance to the next stage without going through the various stages and phases. In the case of sustainability principles and low-carbon development concepts, abiding by the steps of development phases is important because, even though the circumstances and the makings may define different routes in the search for equilibrium, the arrangement logic must be the same whenever we search for the equilibrium points – be it Hungary, or China, etc. I relied on the methodical guideline of the www.rubikkocka.hu official website and the solution designs of Singmaster (1980) during the defining of the row by row solution phases. However, because of the low-carbon methodology correspondences, the process which is demonstrated and interpreted in this document differs greatly from these guides. To illustrate the various stages and different solution levels of the cube, I used the Online Ruwix Cube Solver program.

3.2.1.1. White cross, multi-level syncing of starting criteria

The special characteristic of the *layer by layer* method is that it always considers the white side as the starting side and the white middle cube (the cube which only ever has one color) as the starting point. Naturally, any color can be the starting point of the solution process, meaning the same rotation logic can be used starting from any level without any changes. Therefore, after we have our white middle cube, as a first step, we find all the four edge cubes (edge cubes are the ones with two colors) which have white as one of their colors. We rotate these one by one next to the white middle cube. The other cubes may be rotated anywhere for now, let's consider them grey! If all white cubes are in place, let's position them by rotating the white side to match at least two above the same color middle cube! Therefore, it is a general demand for at least two (or optimally all four) elements to be positioned correctly on the bottom side as well, as seen on Figure 10. This is the first step in the process of the cube's solution, also known as "White Cross."

Figure 10: White cross with matching edge cubes on the side
Source: self-made

It is extremely important for the White Cross to be oriented on the starting side, while the middle cubes match on all sides transversely. If the white edge cubes don't take this position, we can't proceed with the solution according to the method. Bringing the white edge cubes up to the starting point can be done in various ways from various positions, but all follow the same logical sequence. Usually, we have to bring up the bottom row's edge cubes to the starting side. The process of rotating from bottom to top can be seen on Figure 11. The two different cases show two different cube states. On the upper part of Figure 11 (1), we do a 180º rotation on the top row to bring the cube to its place from the bottom. On the lower part (2), we do a 90º rotation upward, followed by another 90º rotation of the right column upwards. This is how the white-green edge cube goes to its place.

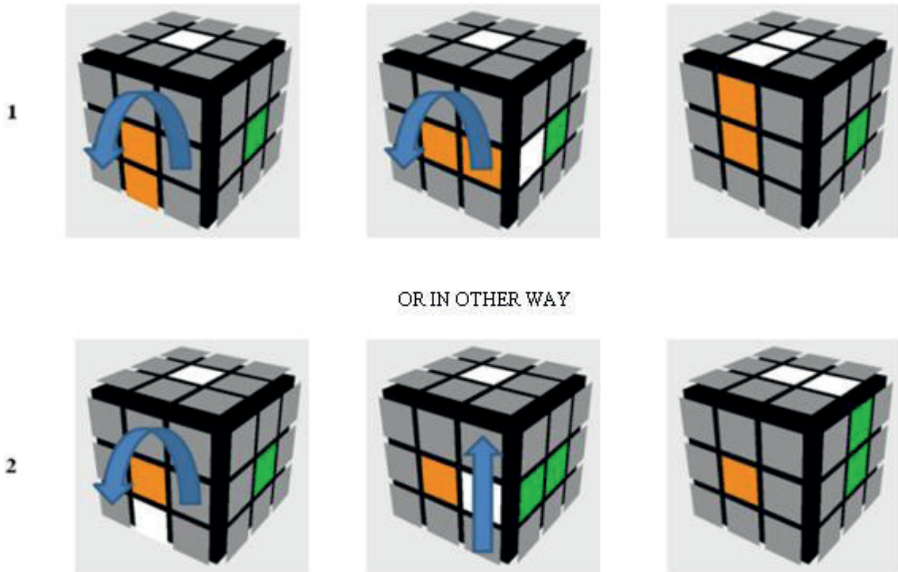


Figure 11: Rotating edge cube to its position from bottom row in two ways
Source: self-made.

If the white cube is lodged between two completed edges, we use the rotation seen in Figure 12. At first sight, this brings the edge cube to the incorrect position, but from here, we can easily relocate it to its proper position.

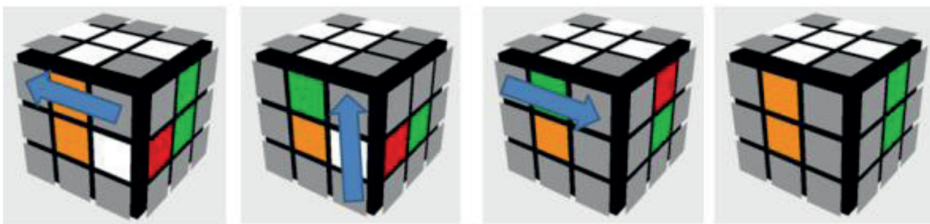


Figure 12: Rotating edge cube to its position from middle row
Source: self-made.

If only two cubes match the middle cube by the time we make the cross, we can exchange the other two sides by finding the pieces we want to switch and rotate that side two times, thereby positioning the white on the bottom. After this, we rotate the cube to its own color and then rotate this side two times. Now, we have the cubes which were in the wrong position on the bottom.

Afterwards, we arrange this cube to its own color, and rotate this side two times, meaning 180º (Figure 11, upper part). This method works even if two neighboring cubes have to be switched, or if two opposing ones need to be exchanged. If all four colors are in place (white and edge cubes match the four color middle cubes, as seen on Figure 10), we can move on to the next step, which is the solution of white corners. However, let's first view what this phase means in the process of searching for sustainability.

"***LOW CARBON INTERPRETATION" NO. 1:***

Our objective system is defined by defining the boundary conditions of the starting state (or the Input side) and the complete or partial rearrangement of the system (fossilized energy provision system's complete or partial change). This is where we define the development program itself, the condition framework, and the boundaries of the project or task. We define what kinds of correspondence systems have an impact on the creation of our process, project, or concept. This will be our white middle cube, which will represent the unchangeable objective system, that is the fixed point of our starting state. In our case, according to professional opinions, we can define Energy rationalization as our fixed point. We also need four comparison points, which have a strong impact on the project environment. These can be the 2D interpretations of the strategic sub-connection, the basic technological requirement, the financing requirements, and the basic market positioning. These attributes, which correlate with the various edge cubes and fixed attributes of points of impact (orange, blue, red, and green middle cubes), give the starting 2D attributes of the development.

Example: If I switch the energy supply system immediately and completely to the new, cleaner technology (strategy 1) or I wait until the life cycle of the current technology runs out (strategy 2), then I have two different strategic goals. In version 1, I induce an immediate and final intervention with decisive costs, while in version 2, the exchange of fossilized energy supply systems will happen gradually, take a longer time, and distribute the cost of the investment in a longer timeframe. The causality of this process is what should be examined. If we don't sync the operation criteria of the "old and outdated," and the "new and clean" technologies, the solution of the cube and the continued sustainable planning of the project can not advance. In this case, the next step of the project can't be completed, or if it continues, it will take a wrong turn in development. Therefore, it is not enough to define the starting basis (solution of white side) with regards only to the obvious facts which fundamentally define the starting criteria; we also have to sync it to the fixed points of the next level. We can interpret this in practice as the white side (or basics of the project) also being solvable while they're not in sync with the first row, or the fixed points of the second planning level, equal to the middle cubes (orange, blue, red, and green). This project/cube state can be seen in Figure 13. From this state, the project won't be sustainable and is doomed to fail.

Figure 13: Incorrect solution of white side, meaning starting point of project designed incorrectly
Source: self-made.

3.2.1.2. Algorithms of solving white corners, search for equilibrium at starting state

After making the White Cross, the next step is to organize the corners to their respective positions (Figure 14). If this is done correctly, the corners match the colors of the sides. Corner cubes are the ones that have three colors (e.g. white, orange, and green). The cube has 8 of these altogether, therefore, our task is to rotate those corner cubes that have a white color to the corners of the White Cross.

Figure 14: Correct positions of white corners, and solution of first row
Source: self-made.

First, we have to find the four corner cubes and then put them into their correct positions using algorithms (rotation combinations) (a) and (b). Both (a) and (b) rotation combinations need the White Cross to be positioned facing upwards. We have the easiest solutions if the bottom row has white corner cubes. First, let's see what colors we can find next to the white color. Let's place this color as close as we can to its own middle cube, by rotating the bottom row. This corner cube is now positioned to the left or right of the middle cube. We take the bottom row towards the way it's aligned and then match the top row as well. To finish the rotation, we rotate the bottom row back and then the top row back as well. The two rotation combinations can be seen in Figures 15 and 16.

(a) The corner cube's white is oriented towards the right. We rearrange it to the white front.

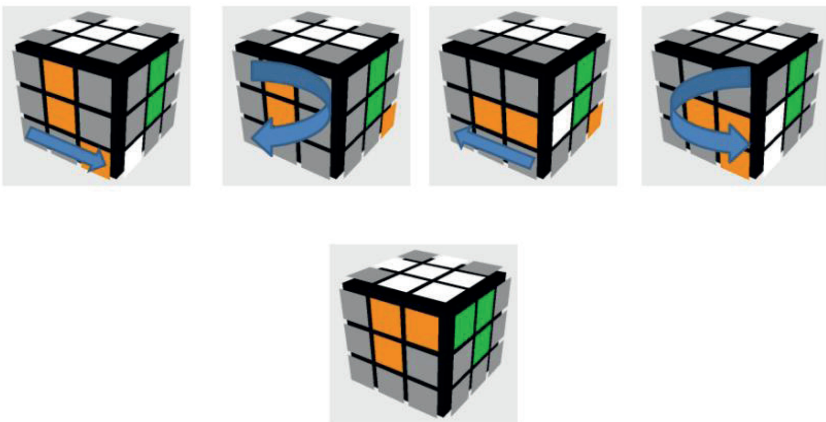


Figure 15: Right-oriented corner cube's rotation to correct position from bottom row
Source: self-made.

(b) The corner cube's white is oriented towards the left. We rearrange it to the white front.

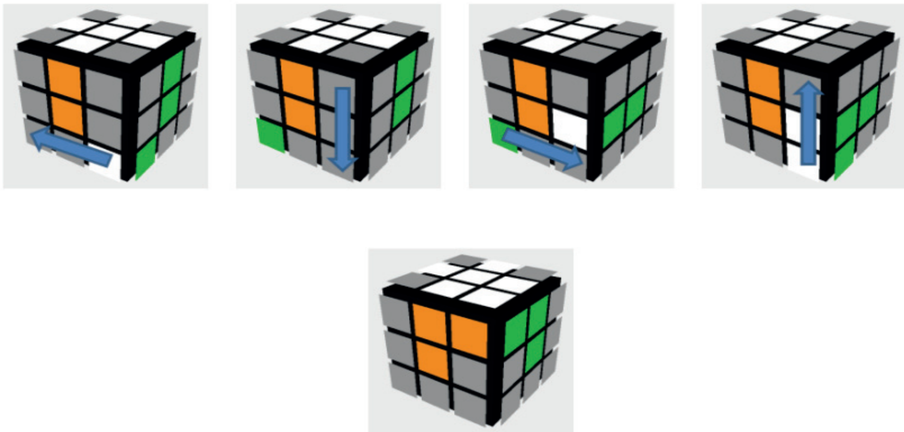


Figure 16: Left-oriented corner cube's rotation to correct position from bottom row
Source: self-made.

Using solution (b) is simple, as seen in Figure 16. We merely have to rotate the corner cube "out of the way," then replace it with the cube that is in the position we want to move it. After rotating the corner cube backwards, we rotate the now neighboring white edge cubes (right column) and corner cubes back to the top row, rotating the corner to its final position.

(c) Solution if the white color of the corner cube faces downward

At first glance, the most complicated position is if the corner cubes face downward with the white color. In this case, the color can be rotated upwards to the starting side with a 180º rotation of the right column, after which we can easily arrange the edge cubes to match it (Figure 17). If the corner cube is in the wrong upwards position, it has to be rotated to the bottom row, and we have to apply one of the previous rotations. We may use different combinations of the previously introduced rotations, depending on personal depth perception and simple skillfulness (left-handed, right-handed).

If there are no more white colored cubes in the bottom row, we've completed our starting white side. But we must be cautious, since one of the cube's sides can be completed even if the corner cubes seem in place but don't match sideways. The corner cube might be in position while the white side is facing outwards. Neither of these positions is suitable for proceeding with the second row, since the misplaced cubes can't be rotated into their positions ideally in either case.

(d) We use multiple versions if the corner cube is on top but is not orientated correctly

Let's turn the cube so that the corner cube faces us from the right side, then rotate the right side of the cube to face us. This time, our corner cube went to the bottom row. Let's rotate the bottom row counter-clockwise, meaning backwards, and the right side to face away from us. With this process, we result in one of (a), (b) or (c) combinations, where we can put the corner cube into its proper position!

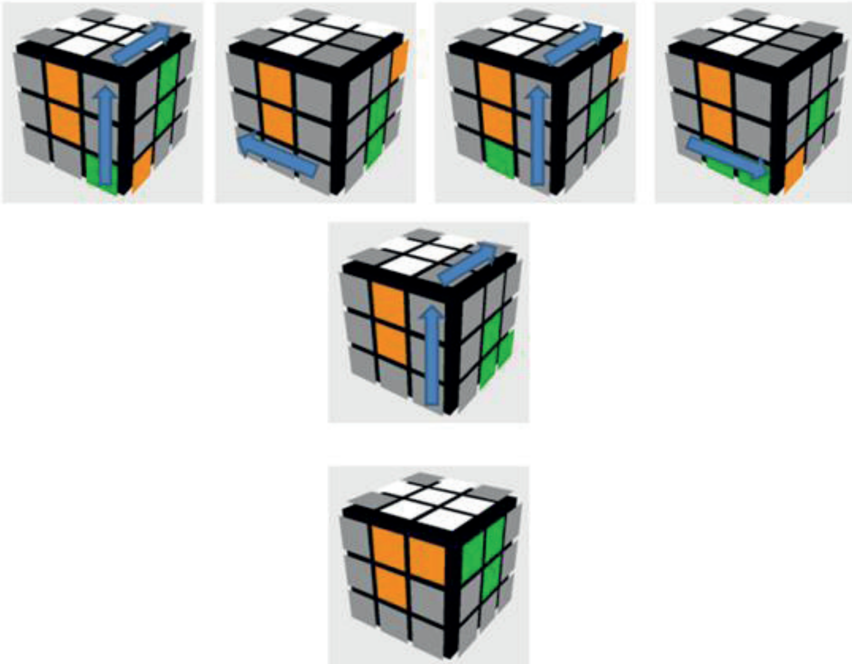


Figure 17: Rotating downward facing corner cube to its place
Source: self-made.

"LOW CARBON INTERPRETATION" NO. 2:

The goal is to define the project's sustainable development course and the finalization of the fixed points of the starting state. Syncing the definitive criteria and definition of the correspondence systems can be done with the corner cube defining the three attributes at once. All attributes are independent, but the process of their sync can be realized via the shortest route and the most effective way. It's important to note that the corner cube in the top row can also be positioned with the white color facing outwards. This can be seen on Figure 18.

Figure 18: Top row corner cube in place, but facing outwards
Source: self-made.

This is also a position from which the solution can not be continued with the second row, since the cubes in wrong positions won't be rotatable to their correct positions at a later time. This shows us that we can also find project attributes in the process of project development which seem to be in place at first glance, but are not in a state of equilibrium. We can't develop our program further, or if we continue to try, the project will take a turn for the worse. In the present cycle of project development (and solution), the search for this starting point of equilibrium is underway. The state of equilibrium we're searching for is called a Nash equilibrium. Writing the function during the process of project development's phase of planning of the first layer can be used for e.g. defining regulation policies and financing policies. In case of cooperative games, the state of equilibrium can be stable even if a strategy combination isn't a Nash equilibrium, if the players agree to choose it.

By the definition for the Nash equilibrium:

At the equilibrium point of a $J = (n, S, (\varphi_i)_{i=1}^{n})$ n-member game or strategy, we classify a point (strategic n), where

$$\varphi_i(x_1^*, \ldots, x_{i-1,}^* x_i^*, x_{i+1,}^*) \geq \varphi_i(x_1^*, \ldots, x_{i-1,}^* x_i, x_{i+1,}^*)$$

holds not strictly true for every $i = 1, \ldots\ldots, n$ player. Therefore, the point of equilibrium is called a Nash equilibrium. Following the completion of the first layer, only the connection with a Nash equilibrium can be further developed, meaning that we can only rotate the cube further from this position. The first layer always correlates with the second layer's middle cube, and can only be the same color. The true point of equilibrium for the first layer and the middle cube is what we may call a Nash equilibrium.

Example: The syncing of technology developments connected to the objective system and the boundary conditions of monetary effectiveness may happen directly or indirectly (by making it abide by the regulation conditions – standards, norms), with the use of a rotation that has impact on three attributes. A good example to this would be how American standards aren't applicable to European user environments, meaning that in this case, the principle of preferring local acquisition over global acquisition means a sustainable and proper point of equilibrium.

3.2.1.3. Solution of the middle row by rotating edge cubes to position (using 3 algorithms)

It is obvious, as seen in Figure 19, that the middle cubes will also be in position after completion of the first row, which makes our next task correctly positioning the side edge cubes. Comparing the first rows' solution algorithms to our next ones, I have to say that we need to implement longer rotation sequences, which assumes 7 rotations for repositioning each edge cube. It is interesting though that the solution of the middle row can be much more easily automated (e.g. with a software application). Using heuristic algorithms doesn't cause a problem here: we can define a fixed algorithm for every state and only have to decide which to implement first.

Figure 19: Two rows solved by positioning edge cubes
Source: self-made.

Therefore, our second row is complete by positioning the edge cubes. There are three (a), (b), and (c) possible positions for the edge cubes, which have the following solutions:

In case of solutions (a) and (b), we need an edge cube on the bottom side of the cube next to the yellow middle cube, which has no yellow color. The reason for this is that edge cubes which don't have yellow all belong to the middle row. If we find the edge cubes which belong in the middle row, we can match them to their respective colors one by one, meaning rotating them right below their middle cubes. If we hold this side to face us, we have to look at what's the edge cube's other color. The matching color will either be to the right (Figure 20) or to the left (Figure 21).

The colors of the middle cube and the bottom cube will match, and in the next step, we'll look at where our edge cube is missing from. (That color must be either to our right or our left.) We rotate the bottom row away from the color of the middle cube which matches the color of our edge cube. After realizing where we have to rotate our edge cube, we turn that side to face us, and re-rotate the edge cube to its original position. This leaves us with two white cubes, which we rotate back to the white side.

If we look at the cube now, we can see that the corner cube on the opposite side (which has white in it) was matched with its edge cube (meaning the one we originally picked out). From this position, we have an easy task: we simply position the corner cube to its place (as was written in the previous part pertaining to the positioning of the white corner cube).

a) Process of rotating from the right (Figure 20)

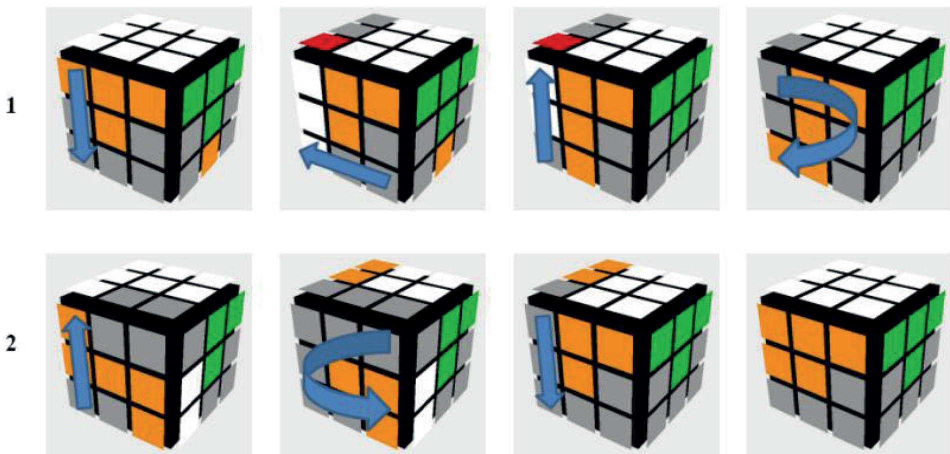


Figure 20: Rotating edge cube to its place, if the missing cube faces rightward
Source: self-made.

b) Process of rotating from the left (Figure 21)

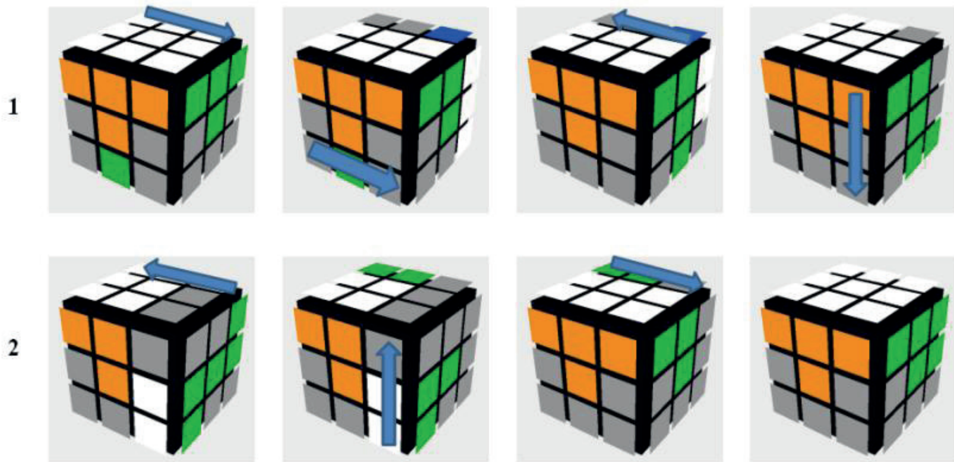


Figure 21: Rotating edge cube to its place, if the missing cube faces leftward
Source: self-made.

c) The edge cube is in the second row, but in a wrong position or orientation (Figure 22)

Using solution (c) might be required because, even though the edge cube is in position, it is e.g. in a wrong orientation color-wise. In this case, we have to go through either solution (a) or (b), with which we achieve that our edge cube, which was previously in the middle row either positioned or orientated wrong, is now in the bottom row, from where we can rotate it back into its proper position using either algorithm (a) or (b).

"*LOW CARBON INTERPRETATION" NO. 3:*

During the process of project planning, our goal with positioning the middle row's edge cubes is to further arrange the correspondence systems of the various attribute sets that have an impact and to find the various points of equilibrium defined by the attributes directly influencing each other, meaning the attributes inherent in the edge cube's two colors and the matching colored opposite edge cube, which is paired with a different color. Without syncing the variables indirectly affecting each other and the attributes they represent, the state of equilibrium isn't optimal (since more than one state or point of equilibrium is present). This state can be defined by the previously introduced multi-variable continuous functions:

Let φ_i be two objective's payoff function and the $\boldsymbol{u}_1$, $\boldsymbol{u}_2$, vectors be strategic vectors by which we can define a two-person game of infinite kind, with at least two points of equilibrium:

$$\varphi_i(\boldsymbol{u}) = \varphi_i(\boldsymbol{u}_1, \boldsymbol{u}_2,)$$

The main reason of multiple points of equilibrium is that the cross-affecting attributes can be optimized multiple ways (we can optimize the edge cube or its represented attributes to both the left and the right, but this is only a stable equilibrium if we can continue the solution of the cube). The cube's wrong state of equilibrium can be seen in Figure 22.

Figure 22: Rotating edge cube to position, if the missing cube faces rightward
Source: self-made.

Example: We can directly sync the most economical technological solutions to high quality and innovation, but if the effect of market changes on the financing system (change in interest rate), risks of foreign currency, and global effects are disregarded, the project can't be realized or only with major redesign and changes (no innovation, or lower quality).

3.2.1.4. Algorithm of Yellow Cross and tuning Output side

Rotating the "Yellow Cross" is the most important phase prior to the solution of the cube. With this rotation, we start to sync the white and yellow sides. By the time we finish the rotation, the yellow colored edge cubes are on the front side facing outwards. In the case of the "Yellow Cross," it is not important for the yellow edge cubes to be color matched, meaning their sides don't have to match the colors of the various middle cubes (Figure 23.).

Figure 23: Yellow Cross
Source: self-made.

After the repositioning, we hold the two not color-matched cube parts to face us rightward (Figure 24) in a way that the yellow middle cube faces upward. We rotate a block of 6 cubes from the bottom upwards, making sure the side that faces us contains exactly two columns of white (excluding the left column). We remake this into an inverted L shape (Figure 24, upper part, last cube). This is done by rotating the top row clockwise, repositioning the two whites in the right column to the bottom, and finally rotating the top row clockwise.

As we get our inverted L, we take the middle column (the L's vertical line) to the bottom, turn the cube to make the white side face upwards, rotate the missing corner from the left, and turn the completed column down.

a) If two neighboring edge cubes are in the wrong position, the rotation sequence is as follows:

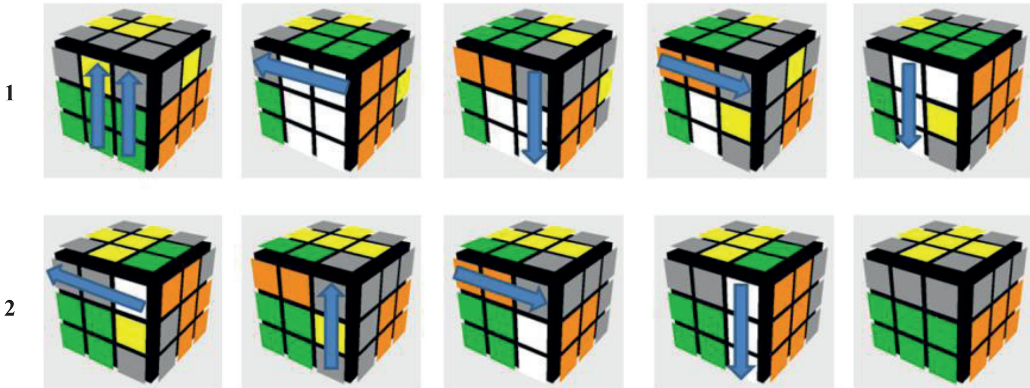


Figure 24: Repositioning edge cubes on yellow side
Source: self-made.

b) If we find the edge cubes on opposite sides, the rotation sequence is as follows:

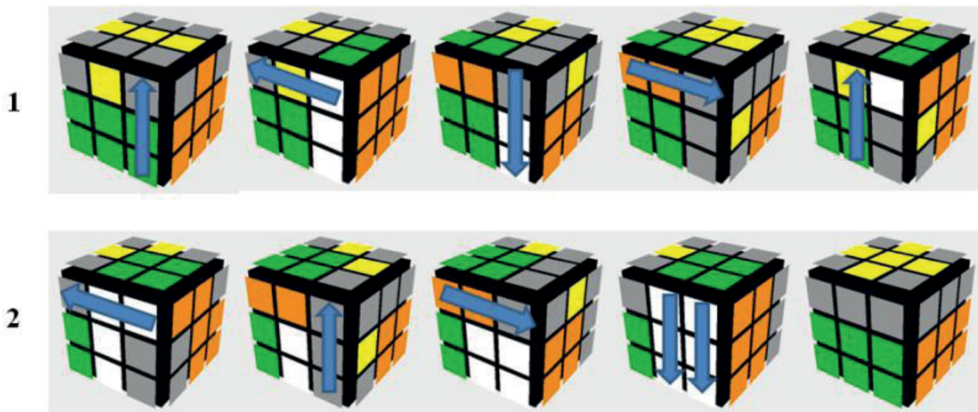


Figure 25: Repositioning edge cubes on yellow side, if they're on opposite sides
Source: self-made.

The process of the solution is as follows: we hold the one of the two wrongly positioned cubes in front of us, and the other opposite to it, as seen in version (b) (Figure 25). We bring a white column up on the right column, rotate the top row (clockwise), and bring the remaining two whites (the right column) down. We rotate the top row counter-clockwise, and by rotating the middle column backwards, we bring up three whites. In this case, we get an inverted L. This has to be completed into a block of six. This can be done by rotating the top row (clockwise), bringing up two whites to the right column by rotating it backwards, then rotating the top row counter-clockwise. The completed block of six has to be rotated back to the other three whites downwards.

c) The front side has no yellow edge cubes

We might not find an edge cube with yellow on the front side. In this case, we follow either algorithm (a) or (b), which results in one or two edge cubes being positioned on the front side. After this, we use the rotation algorithms of either (a) or (b) to reposition the edges.

"***LOW CARBON INTERPRETATION" NO. 4:***

Basically, the solution of the Yellow Cross is the syncing of the Output expectances (yellow side) and the Input side (white side), including all details of the development objective system. The goal here is primarily syncing the trends of Input and Output indirectly. This indirect syncing is important because this phase still offers opportunities for some corrections, or the modification of smaller, flexible attributes, depending on how the points of equilibrium are sorted. The indirect coordination is possible due to disregarding the top row's sync with the middle cubes during the solution of Yellow Cross, which means they're not color matched by the time we finish the rotation phase. After the solution of the middle row, the yellow edge cubes might be in various positions in the top row. If we can't find any yellow colored cubes on the front side (excluding the yellow middle cube) (Figure 26, state "D"), the repositioning takes more time, since we have to apply an algorithm that doesn't help us advance in the solution but only in the rearrangement. After this rearrangement happens, we can begin using the selected algorithm. The above mentioned circumstance clearly illustrates that we may find a state in which the sealing side of the cube is not as sorted as expected because no edge cubes are in their proper position. This can be said about project development as well, since there might be times when we have to rearrange the project outputs compared to what the expected outputs originally were. This can easily happen, since during actualization we can face situations when the realization of a development or investment is months or even years late, which is enough time for the economic environment (market, regulations) to generate new changes related to the requirements. One of the more defining moments of the economic rearrangement process of the 2010's was the phenomenon which caused failed "giga-developments" not only in Hungary, but all around the entire world (e.g. Chinese ghost-towns, failed European ethanol and bio-diesel factories, etc.). Therefore, in the field of actual usefulness, the Yellow Cross can have high expectations of being put to the spotlight.

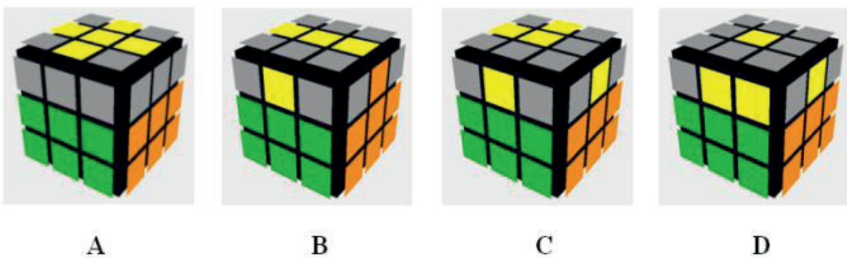


Figure 26: Possible positions of edge cubes after arranging middle row
Source: self-made

Example: The possible changing of flexible technology requirements compared to the planned order is possible in this phase, without changing the output criteria, or the points of equilibrium. A similar variable might be e.g. the inclusion of changes in tax and other financial requirements that can be handled in a flexible manner. I basically assume that a well-planned and long-term predictable economic environment may result in Output criteria that are close to the originally planned business requirements; therefore, they have no need of being rearranged into new states of equilibrium. Following the cube's logic, if the Yellow Cross is on the front side immediately after the solution of the middle row, the solution of the cube is quite simple, since the only remaining task is to rotate the corner cubes to their respective positions. This state can be assumed during project development if the Output expectations of the project form the Yellow Cross, which means the project or investment can be completed without changes (Figure 26, state “C”). If the finishing phase is like the “B” or “C” states in Figure 26, the project must be rearranged into a new state of equilibrium, for which a moderate intervention is advisable. If, however, the “D” cube state defines the state of project development, meaning not a single Output expectation is as it was assumed to be in the project planning, a major rearrangement of the state of equilibrium and serious re-planning is necessary, which is usually time-consuming (and also needs one-two additional algorithms) and can delay the project’s finishing phase.

3.2.1.5. Positioning yellow corner cubes, and arranging sustainability criteria to a finished state

In this rotation sequence, we move all four yellow corner cubes in place, making sure that the yellow top row isn’t color matched with the row beneath it.

Figure 27: Independent solution of yellow side
Source: self-made.

A multitude of various possibilities/algorithms were developed for this rotation in the last few years, and listing these would be too time-consuming, not to mention needless. For us to be able to rotate the corner cubes, it’s sufficient to define an easier combination, which can be repeated multiple times, therefore resulting in the solution of the yellow side from any given starting state.

In Figure 28, we can see a case when only two corner cubes are in the wrong place, with them having the yellow colors on the same side. The cube must always be held in a way that the two corner cubes to be rotated face rightwards. We also have to be mindful to have the side which has the yellow colors of the corner cubes we want to rotate facing upwards. As a start, let’s rotate the right column downwards, then rotate the top row (clockwise). After this, let’s rotate the left side backwards, the top row again (clockwise), then rotate the left column downwards, after which comes the top row twice (clockwise). As a finish, we rotate the left column upwards. This process must be repeated for the right side as well. In case two neighboring corner cubes have the yellow colors on opposing sides, we also use this algorithm but hold the cube in a way that the yellow side faces upwards, and the cubes we want to rotate face rightwards. In any other possible scenario, we can rotate the yellow corner cubes to their place in two steps.

We also use this rotation combination in case of three corner cubes being oriented wrongly, meaning facing outwards from the front side. We start the combination with the “wrong” corner cube which is closest to the one that’s in the correct place. As a result of this rotation, the next corner cube also gets placed in its position, or faces the front side with the yellow color. Therefore, we get a state similar to that of Figure 28, or a different one where two “wrong” corner cubes are neighboring, meaning on the same side. Using the rotation combination seen on Figure 28 from this state, we can easily do the rotations, correcting the corner cubes.

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2
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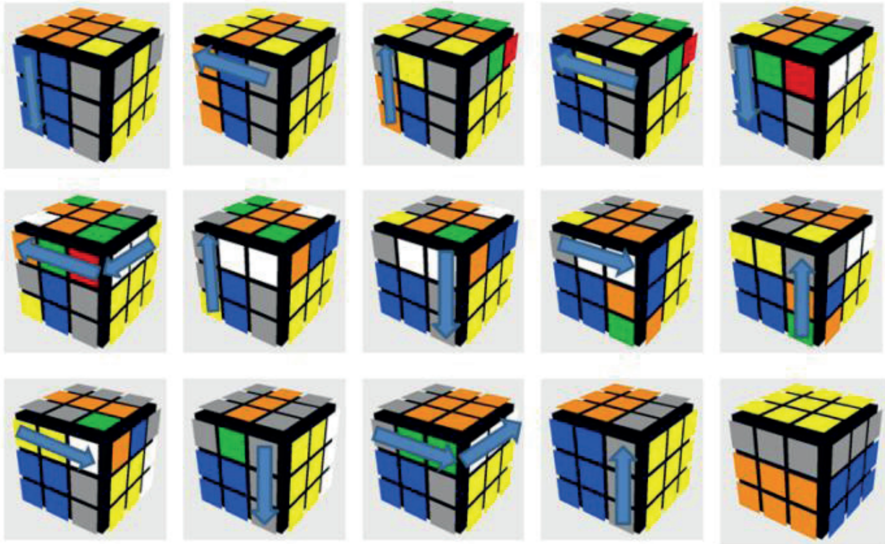


Figure 28: Positioning yellow corner cubes, providing sustainability requirements
Source: self-made.

"LOW CARBON INTERPRETATION" NO. 5:

After the bottom (yellow) side's corner cubes are in place, we can continue with arranging Output requirements. By completing the Yellow Cross, we can put the system in a state of equilibrium that means clear criteria to the "consumer" side, or affiliates, political decision makers. Finalizing the attributes of the Output side is done by arranging the corner cubes to their proper positions. I assume that one of the keys for sustainable business strategies is if the project or development abides by market conditions in a way that they're arranged by at least four strategic objective systems. This can be done easily with the help of the four yellow corner cubes. These have a total of 12 inherent attributes, which is a very big subset in terms of the cube. With the various sides of the cube, we can define a total of 54 attributes, out of which 3 are inherent in each corner cube, respectively. This means that this single rotation algorithm defines the orderliness of the system attributes by 22%. Though the multi-dimension problem solution theory for Rubik's Cube will be introduced in the next chapter, this simple correspondence shows that there are some system elements (cubes/attributes) which have a strong impact on the state of equilibrium of the entire status space with their various positions. The search for points of equilibrium using Game Theory solutions shown in the process of specialized literature can be necessary in this case as well, if the corner cubes are not in their proper positions. The search for points of equilibrium related to project development can be imagined during actualization as searching for the states of equilibrium of the corner cubes' inherent attributes (3 in total) in the status space. This can be defined as a function as follows:

Let φ_i be payoff functions optimizing three objective statuses, while the vectors $\boldsymbol{u}_1$, $\boldsymbol{u}_2$, and $\boldsymbol{u}_3$ are strategic vectors, and we can define a three-person game of infinite kind, with at least three different points of equilibrium, where the appropriate strategy vectors, $\boldsymbol{u} = (\boldsymbol{u}_i)_{i=1}^{3}$.

$$\varphi_i(\boldsymbol{u}) = \varphi_i(\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3,)$$

Example: The "possible changing of flexible technology requirements compared to the planned order is possible in this phase, without changing the Output criteria or the points of equilibrium," as mentioned in *Example No. 4,* can be expanded with the fact that neighboring attributes with a direct influence (three sides of corner cube) have finalized cooperation strategies. Implementing the technological change and the corrected financing construction which follows it can be as such. These attributes define the project's "shelf-life," meaning its sustainability in a changing economical environment. We have to know that economical points of equilibrium, meaning attributes that have an impact on business sustainability, are ever changing, with these changes taking place quickly. During the planning of investments or in making business plans, this is a factor which is hard to balance, which means that the investments related to mandatory sustainability criteria (enviro-protection, renewable energy production, climate-friendly, etc.) may quickly get into an impossible objective state. This is one thing that the use of the sustainability algorithm of project planning based on Rubik's Cube may help with.

During the rotation sequence, few connections change, which signifies that the optimization of cross-effecting correspondences needs a short time interval, and not much work, but the above mentioned intensive sorting effect makes the execution very important.

3.2.1.6. Linking top and bottom row with edge swap, strict sync of Input/Output variables

In this rotation sequence, we have to move all yellow edge cubes to their various positions. This is the state of the cube, for which everyone can see that their cube is in harmony, and only a very minor step is between them and their objective, success. The first phase of harmoniously sorting yellow and white sides can be seen on Figure 29.

Figure 29: Sorting yellow and white sides by main attributes in status space
Source: self-made.

Similarly to what's been said in regard to the White Cross, we can either position either two or all four edge cubes by rotating the yellow side during the solution. If we move two edge cubes, they can either be neighboring or opposite of each other. We use the same algorithm for both cases, but if the cubes which are to be swapped are opposite of each other, we have to do the rotation sequence twice.

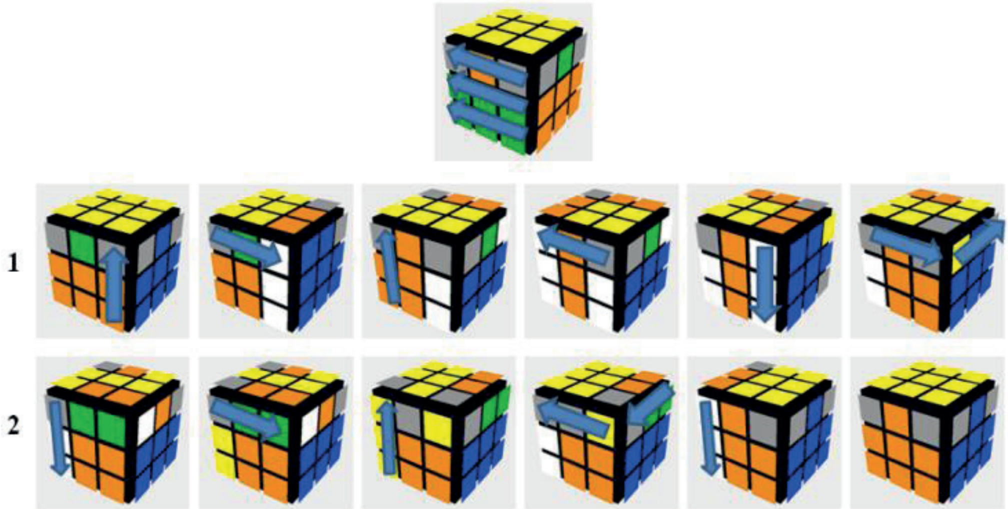


Figure 30: Positioning sealing side's yellow edge cubes
Source: self-made.

During the positioning of the edge cubes, we have to keep the two cubes which we want to swap opposite of each other, and to our left side. Now, let's rotate the right column upwards, then its top row (counter-clockwise), followed by the rotation of the left column upwards, and its top row (clockwise). After this, we have three white cubes in front of us to the right: let's rotate these to the bottom row (Figure 30, upper part). Now, let's make two rotations on the top row (counter-clockwise), then rotate the left column downwards. Rotating the top row (counter-clockwise), and the left column backwards brings the edge cube back in front of us, and the left column will have two white cubes (Figure 30 lower part), to which we can arrange the third by rotating the top row twice (clockwise). The last step is moving this finished white column back to the other white cubes by rotating them downward.

"***LOW CARBON INTERPRETATION" NO. 6:***

The goal of the rotations is linking the Input (white) and Output (yellow) sides. During the process of equilibrium search, we're talking about the strict syncing of the most important Input and Output requirements. By rotating the yellow side's edge cubes to their proper place, the strategic fixed points (meaning the four definitive middle cubes) and the input variables of the Input side form a direct, non-changeable connection with the Output variables and requirements. Practically, we finish the whole process/planning/development with this edge swap.

Example: the edge swap shows us how all the Input and Output attributes important for the planning of the project are finalized. Such a case can be if the political requirement system of the Input side is finalized in regards to the program's realization Output. During the project's evolution, we can handle changes or fixation of "corruption factors" or global variables in a similar manner.

3.2.1.7. Corner swap, defining the final state of equilibrium for system attributes

Corner swap is the final phase of the solution of the cube, and the definition of the final state of equilibrium for the system attributes (Figure 31).

Figure 31: The cube is in a state of equilibrium
Source: self-made.

The state of the cube in this phase is well-known – either three corners are in the wrong place or all four of them. Solving three corners leads us directly to the solution of the fourth, which means this doesn't need further learning. If we don't want to learn more, faster solution algorithms, it is sufficient to be familiar with a single algorithm for this phase, since using this multiple times will lead to the corner cubes being positioned in their proper place.

If we have a corner cube which is positioned properly, we begin by holding it to our left and starting the task on the right column. Let's rotate the front yellow row twice (clockwise), by which we bring a white row up, and rotate the right column backwards twice as well, making an L (Figure 32, upper part). Now, let's rotate the front row once (clockwise) and the left column downwards (Figure 32, upper part, fourth cube), finally restoring the L by rotating the front row again (counter-clockwise). Now, we can make this L into an I by rotating the right column backwards twice. Now, let's rotate the front row once (clockwise), followed by rotating the left column upwards. As a finishing touch, we only have to rotate the front row once (counter-clockwise), which puts white together with white, yellow with yellow, and continue to repeat this rotation sequence until all the corner cubes are in place. If two corner cubes weren't in place, we do it twice; if three, we do it thrice. We know multiple algorithms which can deliver the corner cubes to their "destinations" from various positions faster. Obviously, knowing and using these may shorten the time required for solution.

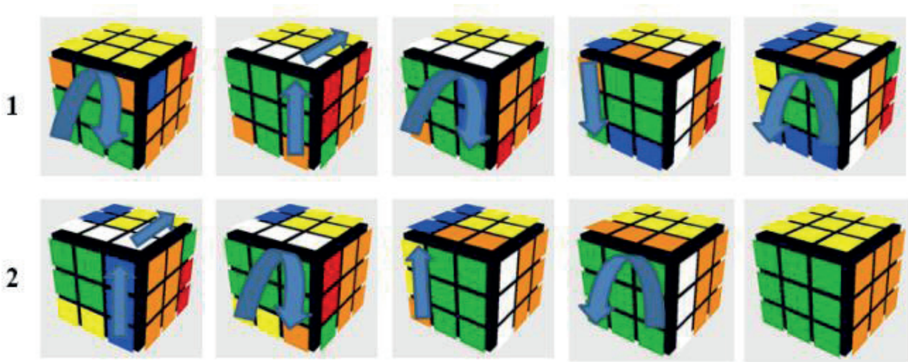


Figure 32: Swapping corner cubes
Source: self-made.

"LOW CARBON" INTERPRETATION NO. 7:

The goal of the rotation sequence is to define sustainability criteria and to set the final state of equilibrium. During the corner cube swap, the rotations have the characteristic of comparing and checking all the attributes inherent in the Input side and the cube side. The edge swap is done for at least three different sides, but usually, the swap of all four corner cubes happens with edge swaps. By modeling the little details of the project planning or development, we can say that the analysis system gets a finalized frame by these edge swaps. Via the corner cubes which have three inherent attributes, four times three, for a total of twelve relevant attributes end up in a final state of equilibrium, which is perhaps the most important rotation sequence in the entire solution process. During the project planning using Rubik's Cube, we can call this process of searching for the final state of equilibrium abiding by the sustainability criteria. As we can see in the above mentioned rotations, the point of equilibrium for the Output side (Yellow Cross, solving yellow corners) can be done during the solution process multiple times, but the 3D assortment only means abiding by the sustainability criteria if the corner cube swaps are done.

Searching for the points of equilibrium/sustainability optimum of sealing corner cubes: one of the most important values, the final harmony of the development project or strategy, is given by the rotation combination based on syncing three different attributes. Without this, there is no final coordination between the Input and Output sides, meaning the flexibility of the entire system drops significantly since it did not adapt requirements posing the "shelf-life" or capability to adapt to the various possible changes of the system attributes.

In light of the above mentioned, we can define three different strategy programs during the process of low-carbon strategy planning:

A. The existence of a technologically sufficient planning option (to avoid over-planning and obsoleteness)
B. Optimization of liquidity and financial sustainability is met (safe self-sufficiency and revenue for at least 10 years).
C. Avoiding detrimental project effects on the relevant product areas (functionally self-sufficient system).

Mathematically defining the above mentioned goals is no easy task; furthermore, writing the Game Theory payoff functions after this also requires the definition of specialized requirement systems.

Our task can e.g. be written as a three-person game, where $\boldsymbol{u}_1$, $\boldsymbol{u}_2$, $\boldsymbol{u}_3$ are the strategy vectors, and $\boldsymbol{u} = (\boldsymbol{u}_i)_{i=1}^{3}$ is the simultaneous strategic vector. This means:

$$\varphi_i(\boldsymbol{u}) = \varphi_i(\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3,) = c^T_{i1}\,\boldsymbol{u}_1 + \boldsymbol{c}^T_{i2} + \boldsymbol{u}_2 + \boldsymbol{c}^T_{i3} + \boldsymbol{u}_3 = \boldsymbol{c}^T_i + \boldsymbol{u}$$

are the objective functions and strategy vectors, therefore the

$$\boldsymbol{A}_1\,\boldsymbol{u}_1 + \boldsymbol{A}_2\,\boldsymbol{u}_2 + \boldsymbol{A}_3\,\boldsymbol{u}_3 \geq \boldsymbol{b}$$

requirement holds true for them. In this case, the coefficients will be the vectors and matrixes derived from our previous model coefficients (doing the function can be seen on page 27).

Example: Finding the final acceptable planning option (from both a financial and technological point of view) is a good example of this (using a technological solution which offers realistic return), since if this can't be realized, the development might even be detrimental to society. However, if the sustainability criteria are met, e.g. the European Union shouldn't have the (quite common) cases, where if financing is cancelled for various development environments, it makes (in the best scenario) the related activities falter (e.g. waste collection systems, waste management) or (in the worst scenario) the entire product path falls apart (e.g. entrepreneurial incubation programs or R&D programs).

3.2.2. Summarizing the evaluation of process analysis

The processes of project planning and development based on the row by row solution of the 3×3×3 Rubik's Cube show us the correspondence of the sustainable use and correspondence systems of the resources around us, which makes building our development and strategy concepts around this advisable in the future. The process regulation based on the solution process of Rubik's Cube is a swift, effective and low-cost protocol; furthermore, the demonstrated process analysis showed us that if it's not disregarded, the criteria of long-term (sustainable) operations are met, which means that we may suppose (with a high probability) that the result of the entire process won't be detrimental to society.

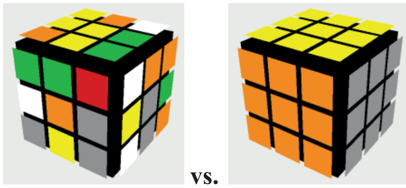


Figure 33: Cube in entropic and equilibratory states
Source: self-made.

By solving the cube, we imitated the process of project development, meaning the road from complete disorder to the state of complete order. The complete state of equilibrium for Rubik's Cube is the solved state. It's no coincidence that when someone sees a cube in disorder, the first things that comes to their mind is to solve it, since the desired state is the cube which has only single-color sides (Figure 33). Rubik's Cube has inherent harmony even in its color setting; as I've already mentioned, the choice of colors by the developer was intentional and the neighboring logic of colors is not the work of coincidence. Without the mystification of the cube, we can state that it already has an inherent and colorful harmony even in its visual appeal, which makes us assume that seamless and perfect logic supports its construction.

During the theoretical process analysis, the goal of demonstrating the various rotations was to show what kinds of cube interactions are assumed behind the advancement from state to state, meaning the effects of which cubes/attributes on each other we have to analyze during the rotation process. I didn't define the exact locations and interactions for these during the research, but the division of the process to phases did happen, and I also synced the solution phases to the mechanisms of project development. The correspondences verified that the two logical processes may support each other. During the process evaluation, we proved that sustainability criteria can be synced to some solution algorithms of the 3×3×3 Rubik's Cube, and the correspondence systems of the cube's various sides defines a 3D perception and planning strategy which shows the process of investment development from a new scientific perspective.

In Table 12, I summarized the various definition levels which mean definable intervals in the process of project development as well, and in places where I deemed it necessary, I also portrayed correspondences of the search for states of equilibrium using Game Theory methods, which can be put into a state of equilibrium with project attributes inherent in the various colors or phases – for the sake of sustainability.

Table 12: Evaluation of modeling process, and results

CUBE INTERPRETATIONS (number of rotation algorithm)	LEVEL OF MODEL DEVELOPMENT	/LOW-CARBON/ PROJECT ATTRIBUTE IN QUESTION	CORRELATION WITH GAME THEORY
NO. 1	INPUT	"White cross" – defining the starting criteria	A stage definable by an n-person zero sum game of infinite kind.
NO. 2	INPUT	"White corner" – defining the sustainable development routes, equilibrium search, non-cooperative optimum	According to functions on Nash-equilibrium, non-cooperative strategy, definable by games of finite kind.
NO. 3	MIDDLE CUBE	"Middle row" – anchoring of relation points, achieving equilibrium, arranging two-dimensional attributes, positioning fixed point	Positioning edge cubes is possible with conflict alleviation methods. Fixed point positioning is advised to be done with zero sum game.
NO. 4	MIDDLE CUBE	"Yellow cross" – indirect synchronizing of input/output sides	Definable by oligopolistic games of finite kind, or a method of equal compromise.
NO. 5	OUTPUT	"Yellow corner" – interpretation of sustainability attributes during the arrangement of outputs	Definable by three-person game of infinite kind, needs Nash-equilibrium.
NO. 6	OUTPUT	"Yellow side edge-switch" – strict synchronizing of input/output sides	Definable by zero sum game, conflict alleviation method, and cooperative strategy.
NO. 7	OUTPUT	"Corner switch" – the phase of setting the final balance, achieving equilibrium, finalizing sustainability attributes	Oligopolistic games by functions based on either cooperative equilibrium strategy or Nash-equilibrium. Cooperative strategy.

Source: self-made

3.3. Theory of "low-carbon" optimization using Rubik's Cube

There are a few good software development processes which are used mainly for industrial developments, but each of them has their various pros, cons, and limitations. Models are usually related to some corporation and organization as well, who further develop, support and promote these methods. However, a specific development model will never be applicable to every project, and if the program developers want to adopt a method for their own development cycles, they have to include the special factors of technologies, limits of resources, time needed to go on the market, and quickly changing consumer needs.

We also have to mention (as part of the research topic) a known software development method (Rubik's Cube software development methodology - RCM), which offers a solution for the problem of modeling software "shelf-life" with its specific objective system methodology. The RCM model concept is an especially effective method for the development of old software applications, and making outlet software applications desirable to consumers. This is a kind of recycling, which can help us spare our resources and time (and it's also applicable to the EU's low carbon concept). The basis of RCM is the most widely known solution method to Rubik's Cube, the *layer by layer* method, which basically follows the solution row by row and is easily interpretable during the process of reprogramming the different software. The analogy between the solution of Rubik's Cube and software development was realized by Indian software developers in 2011, which was partially introduced to practice and thus spared programmers from an excessive load of work. Making outlet software applications desirable to consumers is an exceptionally money and time consuming process, which can be decisively lowered with the RCM method, meaning the concept proved to offer a new and cost-effective method of software (Fogarassy, 2012).

My low carbon optimization theory using Rubik's Cube has an important characteristic, which is to analyze a project on various levels (Input, Output, connection) using the real interactions between various project attributes. This helps us spare a lot of time and effort by neglecting the needless analyses. The system connections assigned to the sides of the cubes (edge cube attributes, corner cube attributes) make the direct analysis of some attributes outright unnecessary, meaning not all connections have to "communicate." The "communications" between these system elements can therefore be reached by defining simple border-area connections, or through transferred system connections.

3.3.1. Sustainability correspondences of "low-carbon" developments

When we analyze the technological applicability during the process of a project development, it's important to note that we don't have to directly consider the market opportunities regarding Outputs, but the correspondence between the two exists and is included through their interactions. Another similar example is negotiation on the questions of liquidity when analyzing a given financial conformity, which isn't directly dependent on the market demand, but both have an influence on each other, which connection is included even without analyses – assuring proper applicability – by the methodology using Rubik's Cube. The previously mentioned LEDS (low-emission development strategies) of the UN is based on the above, which the UN has wanted to implement since 1992, but the economical interpretation of the program couldn't be defined in the last few decades.

Domestic objective system of the main priorities of the « low-carbon economy » (based on Fogarassy, 2013):

- We have to try and improve the effectiveness of all resources, most importantly, energy sources. We have to maintain our energy transformation systems in a much more effective manner, including the local and maximum use of the heat energy byproduct of electric energy production.

- Intentional consumption must be realized on a very high level – be it environmental protection or taking social liability: it has to appear on the production, trade and personal levels.
- Local production and consuming is preferred. No matter what kind of demand comes up, be it energy, material or service, it has to be satisfied by local supply. Energy sources have to be produced in low emission systems, using as much renewable and alternative energy as possible to reduce CO_2 emission to the lowest possible amount.
- All waste has to be minimized – recycle it, return it, reduce it, because this spares a lot of resources and energy.

In the case of the "low-carbon-economy," this complex requirement system is incredibly hard to realize or to integrate its basic theories into investment processes, where the BAU (business as usual) requirements fundamentally disregard sustainability criteria. However, these criteria or priorities can be included in various development and investment projects through the solution of Rubik's Cube, which models the objective system of the "low-carbon" developments well with its structure interpretable in multi-dimensions.

3.3.2. Optimization of sustainability criteria and the theory of 3D problem management

The management of complex risks is assisted by 1, 2, and 3 viewpoint, simultaneous problem handling methods with the use of Rubik's Cube. The base of the cube consists of six immovable small cubes which are the basis for the 1D problem handling (Figure 34). We assign the attributes of project development or investment to these small cubes, which define the core of development in the process, meaning they define unchangeable fixed points in the important areas (e.g. technology, regulations, financing, market).

Figure 34: The "skeleton" of Rubik's Cube is made up of rotatable but immovable middle cubes
Source: self-made.

The number of edge cubes is 12, which serve as the basis for 2D problem handling by allowing the optimization or movement of two attributes simultaneously (e.g. technological regulations and financing). This practically means analysis along the (x,y) axis pair (Figure 35).

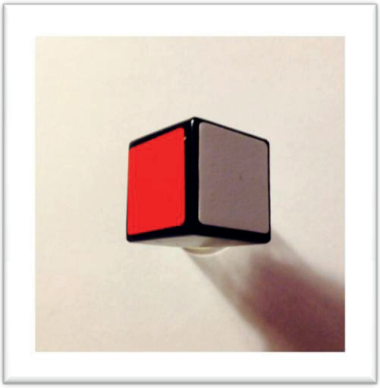


Figure 35: Edge cube of Rubik's Cube, which needs to match two colors
Source: self-made.

There are eight corner cubes, which serve as a basis for specialized 3D problem handling with its simultaneously movable or optimizable three attributes. This basically means analysis along the (x,y,z) axis triple (Figure 36).

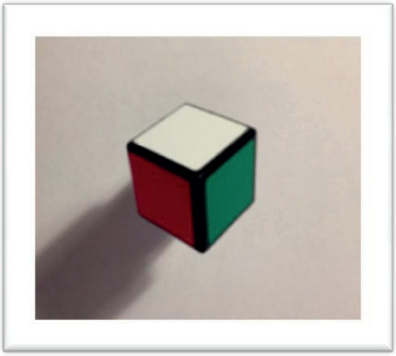


Figure 36: Corner cube, which needs to match three colors
Source: self-made.

We assume that the six-sided, 3×3×3 Rubik's Cube has a side and small cube that harmonizes with each of the elements of project development:

A. Middle cube – this is a stable/fix element and an attribute of all the cube's sides, as well as the phases of project development. In the case of the 3×3×3 cube, we define 6 middle cubes, and these stable cubes/fixed attributes also fundamentally outline and define the process of sustainable project planning.

B. Edge cube – this means a direct connection between two colors and attributes. The number of edge cubes, in the case of the 3×3×3 Rubik's Cube, is twelve. These edge cubes define a 2D inherence of attributes, by which the connected attributes also define the syncing and the system criteria together during the evolution of the project.

C. Corner cube – the most advanced element of coordinating project planning or development, since it optimizes three different attributes during the process of development. Between the matching of the three colors, meaning the three sets of attributes, it defines a very direct and complex correlation. White side (Input) corner cubes mean a stable element to project structure, capable of analyzing all viewpoints and attributes, while the yellow side (Input) corner cubes realize the selection of sustainability, harmonic energy use and exclusion/correction of detrimental developments.

According to my hypothesis, the low carbon project planning or development method is a parallel protocol using the *layer by layer* solution method for Rubik's Cube. By assigning the various project attributes to the colors of the cube, we can achieve the realization of a specialized, sustainable project development process, for which the specifics of development and process are offered by the 1, 2, and 3D nature of the various attributes or attribute sets, making them handle better during development programs. The objective systems of both low-carbon developments and solution method of Rubik's Cube follow the same guiding principle, meaning they both try to achieve the state of equilibrium by following the route of logical and low energy consumption.

3.3.3. Method of problem handling in 1D, 2D and 3D using Rubik's Cube

Project planning and development is basically a process optimization based on the collective handling of different attributes in a way that the examined segments are placed into the most harmonic constellation compared to each other. In case of a supposed "low-carbon optimization protocol," there is a need to create four different determination areas (attribute groups), which can be associated with the 3×3×3 cube's different colored sides. Two opposing sides (white and yellow) would be our project's input and output sides. The attribute groups which determine our optimization can be the following in a demonstrational project: optimization of strategic goals system (red side), analysis of market opportunities (green side), the area of actualization and technological criteria system (blue side), monetary effects (orange side), the attributes summarizing input-side goals (white side), and, last but not least, the attributes summarizing output-side goals (yellow side).

One of the most important characteristics of the low-carbon optimization concept (based on the software development experiences from India) is that the analysis of various projects on multiple levels is based on the analysis of the relevant interactions of various pieces, therefore avoiding the analysis of irrelevant interactions spares tremendous time and effort (low-carbon solution). The system connections assigned to the various sides of the cube (edge cube characteristics, and corner cube characteristics) makes the direct examination of various attributes irrelevant, meaning that not all system connections actually have to "communicate" with each other. The "communications" between these system elements can therefore be reached by simple border-area connections, or through transferred system connections.

When we analyze technological applicability during the process of project development, it is important to note that we don't have to directly consider the market opportunities regarding Outputs, but the correspondence between the two exists, and is included through their interactions. Another similar example is negotiation on the questions of liquidity when analyzing a given financial conformity, which isn't directly dependent on the market's demand, but both influence each other, which connection is included even without analyses – assuring proper applicability – by the methodology using Rubik's Cube.

It is important to note that some attributes require more interaction between each other than others. This means that some attributes (which are parallel to the cube's color positions) can only be placed in their correct position, or have finalized characteristics, if the other attributes are collectively optimized. It is also obvious from this aspect that there is a reason for using a 3D interpretable project development model.

Fundamentally, logical planning and modeling happens in 2D strategic models (e.g. logframe matrix – LFA/Logical Framework Approach), for which we imaged a case as to what the 2D interpretation of Rubik's Cube would mean. Figure 37 proves that the 2D (x,y) cube structure could perhaps handle more project attributes together, but wouldn't define the correlation between them, which can only be done by a 3D interpretation.

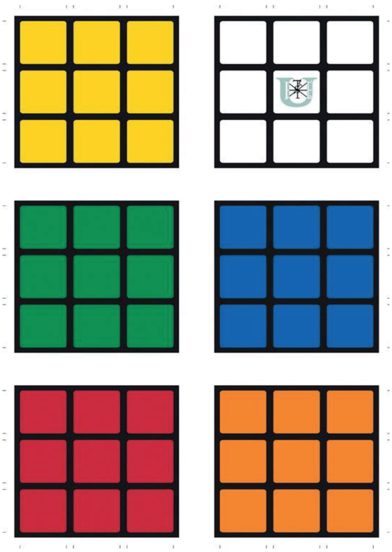


Figure 37: 2D Interpretation of Rubik's Cube structure
Source: self-made.

We can fundamentally say that if the attributes that have an impact on project development are placed correctly, the 3D interpretation zone of Rubik's Cube is the new pro of practice. Ernő Rubik, the inventor of Rubik's Cube, also had the goal of showing architect students how the elements are connected, how they move in 3D, and how their connections change with said movement, meanwhile helping them develop a sense for 3D interpretation. The novel 3D protocol helps to realize our development projects in the future by fixing incorrect developments time- and cost-efficiently. Tables 13-17 show the various attributes assigned to their respective sides/places, and the multi-dimensional interpretation. We can also say that the 3D problem management allows for the more detailed analysis of Input and Output criteria, which was properly followed through during the process of project planning, when we rearranged and solved the in detail cube earlier. (Rearranging the cube means that the edges or colors are matched via the algorithm relocating the cubes to their respective proper positions.)

In the next Table (Table 13), I summarized the characteristics of the input and output sides of an actual project. The typified goal of project development in this case is the advancement from fossilized energy resources to renewable ones or to a combined system. The four main agents which were assigned to the four colored sides were decided upon by professional evaluation, individual weighting and the process of dominance analysis in Tables 14 and 17. Of these four colors, red represents "criteria of laws and regulations for strategic program development," green represents "examination of market opportunities," blue represents "technological criteria system," and orange represents "summarization of monetary effects." Also, on the various sides (main agents), I defined cube characteristics which represent two or three individual attributes through the connection system of the cubes themselves. The characteristics, which are unimportant and unrelated to the development and actualization of the project per se, are assigned to the middle cubes, of which there is one on each side, with one defining attribute – obviously though, even if there are fixed cubes on the Input and Output sides as well, they aren't associated with main agents. The attributes which include two or three different factors were assigned to the edge and corner cubes. The defining of the attributes and their association to the cubes was done by their usefulness. The attributes which are determinable one way (point-like) were named 1D (x), those which are determinable two ways were named 2D (x,y), and those which are determinable three ways were named 3D (x,y,z) attributes.

Table 13: Meanings of input (white/W) and output (yellow/Y) sides of Rubik's Cube

SIDE COLORS	**MEANING OF COLORS** 1D – single-dimension trait (x) 2D – dual-dimension trait (x,y) 3D – tri-dimension trait (x,y,z)
WHITE (F)	**INPUT**: Defining input requirements, basic system of the product or provision defined along the matching of state regulations and market. **IMAGING OF WHITE SIDE:** MIDDLE WHITE (1D) ✓ energy rationalization (W) EDGE WHITE (2D) ✓ strategic base-connection (WR), ✓ basic technological requirement (WB), ✓ financing expectation (WO), ✓ basic market positioning (WG), CORNER WHITE (3D), ✓ basic requirement of payoff (OGW), ✓ conformity to technological criteria and funding instruments (OBW), ✓ syncing basic goals with technological threats and innovation priorities (BRW), ✓ designating strategically synced market segment at basic criteria (RGW).
YELLOW (Y)	**OUTPUT:** Pareto optimal product or provision system outlined by taking into consideration the maximum values of resource-usage opportunities. **IMAGING OF YELLOW SIDE:** MIDDLE YELLOW (1D) ✓ resource-optimized energy consumption/profitable production (Y) EDGE YELLOW (2D) ✓ strategic congruence of energy and CO2 scale (RY), ✓ optimizing technological threat minimization (BY), ✓ tax and benefit criteria in the energy-production system (OY), ✓ monetary criteria of artificial and actual advancement to the market (GY), CORNER YELLOW (3D) ✓ structure compatible with strategic goals systems, where "shelf-life" is also guaranteed (RYG), ✓ planning option sufficient in both monetary and technological terms – meaning a technological solution which guarantees a positive cost-benefit rate (OYB), ✓ production and provision conditions sustainable on the market (GYO), ✓ long-term and legit option, where the chosen technological solution supports the strategic goals to the utmost level (BYR), (endorsement of sustainability criteria through development). (Definitions: D=dimension, W=white, Y=yellow, G=green, R=red, B=blue, O=orange)

Source: self-made

Table 14: Meanings of attribute side Red (R) of Rubik's Cube

SIDE COLORS	MEANING OF COLORS 1D – single-dimension trait (x) 2D – dual-dimension trait (x,y) 3D – tri-dimension trait (x,y,z)
RED (R)	**Criteria of laws and regulations for strategic program development**: Providing the defining information, synergies, cooperations for the planned profile on a corporate, local, sectoral, regional or union economic policy level. MIDDLE RED (1D) ✓ realization of local/corporate strategy (R) EDGE RED (2D) ✓ tracking and matching of marketing strategy and economic policy priorities (RG), ✓ defining technological systems for cost-clear versions, matching the techno-parameters of financing priorities to the project (RB), ✓ strategic congruence of energy and CO2 scale (RY), ✓ strategic base-connection (RW) CORNER RED (3D) ✓ syncing basic goals with technological threats and innovation priorities (BRW), ✓ designating a strategically synced market segment at basic criteria (RGW), ✓ structure compatible with strategic goals systems, where "shelf-life" is also guaranteed (RYG), ✓ long-term and legit option, where the chosen technological solution supports the strategic goals to the utmost level (BYR) (Definitions: D=dimension, W=white, Y=yellow, G=green, R=red, B=blue, O=orange)

Source: self-made.

Table 15: Meanings of attribute side Green (G) of Rubik's Cube

SIDE COLORS	MEANING OF COLORS 1D – single-dimension trait (x) 2D – dual-dimension trait (x,y) 3D – tri-dimension trait (x,y,z)
GREEN (G)	**Examination of market opportunities**: Evaluation of market opportunities and positions in artificial and actual market segments. MIDDLE GREEN (1D) ✓ the price in the supply and demand equilibrium can be planned (G) EDGE GREEN (2D) ✓ tracking and matching of marketing strategy and economic policy priorities (RG), ✓ effects of market changes on the financing system, analysis of foreign currency risk factors, and global effects (OG), ✓ monetary criteria of artificial and actual advancement to the market (GY), ✓ basic market positioning (WG). CORNER GREEN (3D) ✓ structure compatible with strategic goals systems, where "shelf-life" is also guaranteed (RYG), ✓ production and provision conditions sustainable on the market (GYO), ✓ basic requirement of payoff (OGW), ✓ designating strategically synced market segment at basic criteria (RGW). (Definitions: D=dimension, W=white, Y=yellow, G=green, R=red, B=blue, O=orange)

Source: self-made

Table 16: Meanings of attribute side Blue (B) of Rubik's Cube

SIDE COLORS	MEANING OF COLORS 1D – single-dimension trait (x) 2D – dual-dimension trait (x,y) 3D – tri-dimension trait (x,y,z)
BLUE (B)	**Technological criteria system**: Matching of market opportunities and technological solutions. Research of the techno-risks and opportunities is advised. MIDDLE BLUE (1D) ✓ technological usage that abides by BAT technological requirements (B) EDGE BLUE (2D) ✓ defining technological systems for cost-clear versions, matching the techno-parameters of financing priorities to the project (RB), ✓ basic technological requirement (WB), ✓ optimizing technological threat minimization (BY), ✓ the most cost-efficient technological solution with both high quality and innovation-level (BO). CORNER BLUE (3D) ✓ conformity to technological criteria and funding instruments (OBW), ✓ planning option sufficient in both monetary and technological terms – meaning technological solution which guarantees a positive cost-benefit rate (OYB), ✓ syncing basic goals with technological threats and innovation priorities (BRW), ✓ long-term and legit option, where the chosen technological solution supports the strategic goals to the utmost level (BYR). (Definitions: D=dimension, W=white, Y=yellow, G=green, R=red, B=blue, O=orange)

Source: self-made

Table 17: Meanings of attribute side Orange (O) of Rubik's Cube

SIDE COLORS	MEANING OF COLORS 1D – single-dimension trait (x) 2D – dual-dimension trait (x,y) 3D – tri-dimension trait (x,y,z)
ORANGE (O)	**Summary of monetary effects**: Type of financing, relevance of government tools, tax, foreign currency risks, liquidity questions. MIDDLE ORANGE (1D) ✓ time for payoff, corporate value (O) EDGE ORANGE (2D) ✓ tax and benefit criteria in the energy-production system (OY), ✓ financing expectation (WO), ✓ the most cost-efficient technological solution with both high quality and innovation-level (BO), ✓ effects of market changes on the financing system, analysis of foreign currency risk factors, and global effects (OG) CORNER ORANGE (3D) ✓ planning option sufficient in both monetary and technological terms – meaning technological solution which guarantees a positive cost-benefit rate (OYB), ✓ basic requirement of payoff (OGW), ✓ conformity to technological criteria and funding instruments (OBW), ✓ production and provision conditions sustainable on the market (GYO). (Definitions: D=dimension, W=white, Y=yellow, G=green, R=red, B=blue, O=orange)

Source: self-made.

Even though the choice of definition of Rubik's Cube's attribute sides and its cubes was random (Tables 14., 15., 16., 17.) in our following project, it's still advised to examine their usefulness with some kind of function-like connection, because we can define the importance of the attributes and the preference comparisons. Also, in order to increase the reliability of the model, we employed a new method to weight criteria and define dominance.

3.4. Process of project development with Rubik's Cube using Game Theory interpretations

The low-carbon project planning and project development using Rubik's Cube is a specially constructed planning concept which – as of now – is a one of a kind concept that can interpret factors with an impact on processes in 3D. For "setting" the equilibrium point of the economical or resource-usage of input and output sides, and to describe the relation between them, I used Game Theory solutions which weren't used for this purpose during scientific research before.

Used before the process of modeling, the evaluation of the process of tolerance in the sense of engineering means the determination of the allowed maximum differentiation from the determined sizes, quantities or qualities. In the case of Game Theory algorithms, I researched the following: which method is the same as the solution process model of Rubik's Cube in terms of its attributes, and in what scale does it differ from it while still remaining representative. For the Game Theory algorithms I was searching for, I used the process of tolerance, meaning I was researching the admissible differences between the attributes of the cube and the parameterization of the Game Theory functions.

During the complex modeling, I analyzed the Game Theory models one by one, and through the process of modeling I assigned the relevant models to the various rotation algorithms (interpretations). I separated the attribute groups of the cube to three different aggregations, which are INPUT side attributes, MIDDLE CUBE side attributes, and OUTPUT side attributes. I used Game Theory methods to determine the points of equilibrium between the three attribute groups. The gist of this was that where the attribute elements were tagged with a "not allowed difference" by the SMART (Simple Multi Attribute Ranking Technique) analysis, I listed parameters which lead to the points of equilibrium (Nash equilibrium) through strategic models. Both the analyses and the modeling were conducted via a three-stage system; therefore I also conducted the Game Theory modeling of the entire process on three levels, meaning the matching of three different types of Game Theory models (or three different cost-functions).

The Game Theory payoff functions referring to the various modeling levels were made by analyzing the attributes of the Input side, the middle cube, and the Output side, which were tagged with a "not allowed difference" by the SMART (Simple Multi Attribute Ranking Technique) analysis in their respective attribute groups, which I took and optimized as sustainability strategies interpreted in a business environment.

3.4.1. Imaging algorithms of Input-side

The project begins in this phase. We can find the answer to the following question: what do we have to keep in mind when starting a project? The incorrect rotation of the first layer, or row of cubes, results in incorrect continuation, therefore, we can't approach the next layer.

We can easily explain this with a simple energy-transaction. If we change our initial energy-supply system in a way that the old one still has a life expectation of 20-40% of its estimated use duration, then we may end up with a considerable financial loss if we intervene. To avoid ending up in such a situation, we can use e.g. a Nash equilibrium to calculate the optimal intervention time.

GAME THEORY MODELING OF INPUT SIDE (LEVEL 1)

Enviro-orientated developments are fundamentally against the economic development priority system (e.g. the program for lowering greenhouse gases and for the use of fossilized energy sources contradict each other, since the former promotes the minimization of energy consumption, while the latter promotes the increased use of pollutants). When planning the first layer, this can be used in the process of project planning in terms of regulation policy and financing policy (Figure 38). We also have the same situation concerning the water base defense and the rising requirements of favored water-dependant energy plants. In case of various projects, we have to include the criteria of non-cooperative competitors as well for the sake of realizing clear business regulations and sustainable business strategies. In this situation, it is incredibly hard to find the Nash equilibrium, but it is imperative nevertheless since the project can't be further developed in a controversy.

Definition:

By the definition for the Nash equilibrium:

At the equilibrium point of a $J = (n, S, (\varphi_i)_{i=1}^{n})$ n- member game or strategy, we classify a point (strategic n), where

$$\varphi_i(x_1^*, \ldots, x_{i-1,}^* x_i^*, x_{i+1,}^*) \geq \varphi_i(x_1^*, \ldots, x_{i-1,}^* x_i, x_{i+1,}^*)$$

holds true not strictly for every i=1,…..,n player. Therefore, the point of equilibrium is called a Nash equilibrium.

Thesis:

Following the completion of the first layer, only the connection with a Nash equilibrium can be further developed, meaning that we can only rotate the cube further from this position. The first layer always correlates with the second layer's middle cube, and can only be the same color.

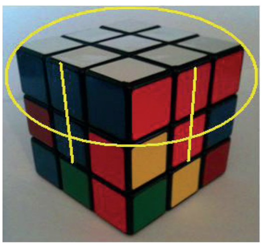


Figure 38: Equilibrium point for the first row or layer (circled), where the middle cube is always the same color (illustrated by the lines).
Source: self-made.

Proof:

Let $x^* = (x_1^* \ldots\ldots, x_n^*)$ be one point of equilibrium for the game. In this instance, in case of any given $y = (y_1 \ldots \ldots y_n) \in S$:

$$\varphi_k(x_1^*, \ldots \ldots, x_k^*, \ldots \ldots, x_n^*) \geq \varphi_k(x_1^* \ldots \ldots, y_k, \ldots \ldots x_n^*) \ \ (k = 1,2,3 \ldots, n),$$

from where, through simple addition, it is obvious that $\varphi(x^*,x^*) \geq \varphi(x^*,y)$. Based on this, a well-performing algorithm can be provided to define the points of equilibrium which have an impact on planning and to solve the fixed problems of the aggregations.

Example:

During the planning of biomass-based renewable energy production, whether the high amount of water consumed can have a detrimental effect on the project's profitability and can become the criteria for use of the most effective technology is a critical point. Therefore, the question and the criteria is viewed as strictly technological in nature, and we try to match the strategy and Game Theory optimum with the corner cube which has 3D attributes (colors are red-green-white), where white means input, red means regulation criteria, and green means technological solutions, which we handle collectively (Figure 39).

Luckily, solving water distribution problems plays a major role in Game Theory solutions, but we can usually reach the points of equilibrium that provide criteria for the outlines of an assured system usage only through defining many intricate function-correspondences, calculating mathematic correlations for which is quite difficult. Multi-purpose water usage and the interests and cost-functions of those connected to it offer different optimums, which usually suppose a game of multi-player and nonlinear nature, and yet which is somehow still a non-cooperative game based on some kind of Nash equilibrium.

To define the problem – according to the low-carbon developments using Rubik's Cube – I made a three player optimization regarding water usage for the process of strategic planning using Rubik's Cube, based on the guide by Szidarovszky and Molnár (2013).

Multi-purpose water usage as a decision-method task has been a problem for decades, and one with many solution options. In our case, we're searching for one on a non-cooperative three-player (agricultural consumer (for irrigation), industrial consumer (for cooling), and household consumer (for functional uses)) Nash equilibrium. The central element of the low-carbon strategy problem is how the agricultural (biomass producer) water usage project developer will decide whether the project has enough water out of the resources at hand.

SMART VALUE (V)	CUBE TYPE AND DOMINANCE	MAIN AGENT INHERENT ATRIBUTES
73	cornercube with more dominating attribute	basic criteria designation of matching set of strategies and technologies (WGR)
57	edge cube with less dominating attribute	strategy, regulation base connection (WG)
87	cornercure with more dominant attribute	abidement by technological criteria and funding instruments (WOG)

Figure 39: 3D attributes of "white-green-red" corner cubes (WGR), the technological solution that assures payoff (optimized for three-person water usage)
(Dimensions from left to right: SMART value, Cube type and dominance, Main agent inherent attributes)
Source: self-made.

The problem has three dimensions, where the Rubik solution is the issue of the input side. The basis of water usage can be water, underground water, and purified wastewater. Let k = 1, 2, 3 be the three players who can follow variations of decision during their decision phase as follows:

The strategy for each player can be described by a five variable vector:

$$\boldsymbol{x_k} = (\boldsymbol{f_k}, \boldsymbol{t_k}, \boldsymbol{k_k}, \boldsymbol{f_k^*}, \boldsymbol{t_k^*}).$$

where

f_k = local water	*t_k = local underground water*	*k_k = purified wastewater*
f_k^* = import water	*t_k^* = import underground water*	

The payoff function for the total amount of water used for each player is as follows en:

$$\phi_k = \boldsymbol{f_k} + \boldsymbol{t_k} + \boldsymbol{k_k} + \boldsymbol{f_k^*} + \boldsymbol{t_k^*}$$

All players have two common complicating criteria, one of which states that the amount of used water may not be less than the minimal requirement *D_k^min*, while the other states that it may not be more than the maximum requirement of technology (*D_k*), either. (These sustainability criteria are to avoid wasting water.)

$$f_k + t_k + k_k + f_k^* + t_k^* \geq D_k^{min}$$

$$f_k + t_k + k_k + f_k^* + t_k^* \leq D_k$$

In addition, the agricultural player (k=1) has to introduce two additional criteria for water usage, which have the following variables:

G = *group of plants exclusive to underground water*
ai = *rate of plants (i) by entire agricultural area*
wi = *water-dependence of plants (i) by hectare*
T = *group of plants which can be watered with purified wastewater*
$W = \sum_i a_i w_i = total\ water - dependence\ for\ all\ plants\ by\ acre$

We know that the underground water supply offers the best quality water while purified wastewater offers the worst, so we have to define the volume of plants (sensitive) in the agricultural portfolio which can't be watered with purified wastewater. The water requirement which draws solely from the underground water sources may not exceed the water-dependence of the plants which are exclusive to clean, quality underground water:

$$\frac{t_1 + t_1^*}{f_1 + t_1 + k_1 + f_1^* + t_1^*} \geq \frac{\sum_{i \in G} a_i w_i}{W}$$

the equation converted to linear form:

$$a_1 f_1 + (\alpha_1 - 1)t_1 + a_1 k_1 + \alpha_1 f_1^*(\alpha_1 - 1)t_1^* \leq 0$$

$$\text{where } \alpha_1 = \frac{\sum_{i \in G} a_i w_i}{W}$$

Similarly, the rate of use and availability of purified water can also be modeled. The water requirement for purified wastewater may not exceed the total available amount, either. This correspondence gives the volume of plants that can either only or also be watered thus (e.g. plants for energy use).

$$\frac{t_1}{f_1 + t_1 + k_1 + f_1^* + t_1^*} \geq \frac{\sum_{i \in T} a_i w_i}{W}$$

equation converted to linear form:

$$-\beta_1 f_1 - \beta_1 t_1 + (1 - \beta_1) k_1 - \beta_1 f_1^* - \beta_1 t_1^* \leq 0$$

$$\text{where } \beta_1 = \frac{\sum_{i \in T} a_i w_i}{W}$$

For the other players, we similarly have to define the correspondences of the functions defined by complications, for which the system can be found in the cited publications, before adding numeric data.

In light of the above facts, it can be stated that if we design our agricultural systems for the use of biomass as energy by allocating the complicating energy source (in this case, water) into an equilibrium state right at the beginning with Game Theory methods, then the planning process is also applicable to the sustainability criteria system. The actual result of the entire analysis can be one of the following: either we won't over-calculate water usage (over-calculate, as in the allocation won't be disproportionate), or we will discard the project entirely because it doesn't abide by the sustainability criteria, since if it's clear at this point that the amount of water at hand is insufficient to reach the Pareto optimal production state, then the shortage of water causes a water-deficit in the analyzed system.

3.4.2. Defining input and output connections with Game Theory correlations

GAME THEORY MODELING OF MIDDLE CUBE CONNECTIONS (LEVEL 2)

Keeping the middle cube in position and solving the row or layer imitates the zero sum game, since the position of the middle cube can not be changed, so it serves as a fix point for the rotation of the other cubes. Their position is fixed (meaning they can't be rotated out of their position, or correspondence systems) and their defined value elements can be considered constant (Figure 40).

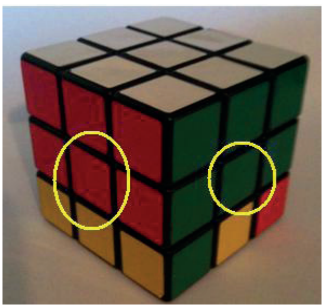


Figure 40: Zero sum games are always illustrated with the fixed middle cube (circled), which serve as criteria for the optimization of edge cubes (two colors).
Source: self-made.

Definition:

A J game with $n-$ players is called a constant sum game, if the sum of the wins and losses of the player is a constant c, regardless of strategy.

Formula:

$$\sum_{i=0}^{n} \varphi_i(x) = c \ (x \ \in S).$$

Where $c = 0$, the game is *zero sum*.

Thesis:

With the zero sum game, we do a constant sum optimization because the resource has a limited sum due to the fixed point trait; therefore, the goal is to harmonically divide the resources at hand, and we search for the point of equilibrium of the attribute group (Figure 41). During the SMART analysis, we verified that the orange middle cube of Rubik's Cube shows a "not allowed difference" attribute. Currently, the inherent attribute group of the orange side is the monetary value of the project, and the time needed for payoff. The analysis of this trait with Game Theory optimization methods shows us how the fixed resources of the low-carbon project will optimize themselves into a Nash equilibrium.

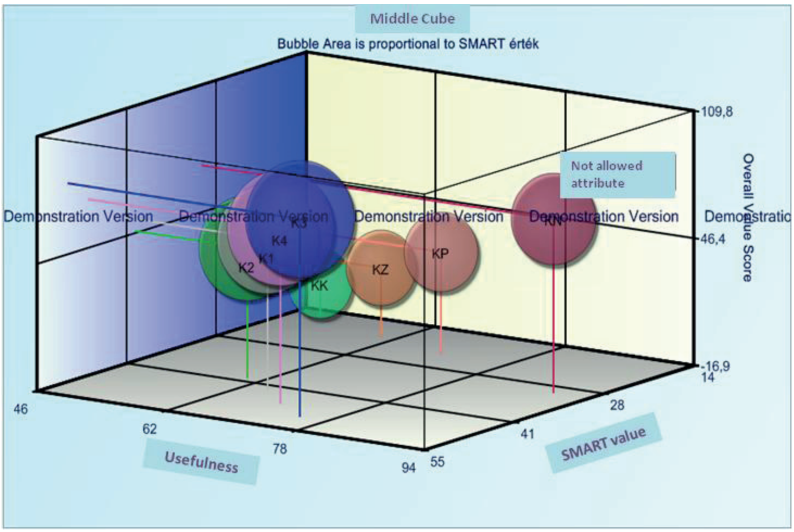


Figure 41: Prohibited attribute of the SMART analysis (time for payoff, value of project is not in equilibrium with the other attributes)
(Title: Middle Cube, Dimensions from left to right: Usefulness, SMART value, Value score).
Source: self-made

The imbalance on Figure 41 can be ascribed to the insufficiency of the stability of external factors which have an impact on the payoff of the investment. We have to analyze the circumstances of market entry of the newcomer.

It isn't easy to solve the problem if there are attributes in the group which are non-market elements (externals) but nevertheless have an impact on the time required for payoff (e.g. tax- and regulation policy, pollution control, foreign currency policy, etc.).

Proof:

I defined the points of Nash equilibrium for the middle cubes of Rubik's Cube (four different fixed attributes) by searching for the attributes which aren't part of the Pareto optimal state.

$n -$ player constant sum games can be used to demonstrate the points of equilibrium for the four different attributes.

If we take a $(x_1^*, x_2^*, x_3^*, x_4^*) \epsilon\ S$ point of equilibrium, we can define that

$$\varphi_1(x_1^*, x_2^*, x_3^*, x_4^*) \geq \varphi_1(x_1, x_2^* x_3^*, x_4^*) \text{ for every } x_1 \in S_1.$$

and

$$\varphi_2(x_1^*, x_2^*, x_3^*, x_4^*) \geq \varphi_2\left(x_1^*, x_{2,} x_3^*, x_4^*\right) \text{ for every } x_2 \in S_2.$$

and

$$\varphi_3(x_1^*, x_2^*, x_3^*, x_4^*) \geq \varphi_3(x_1^*, x_2^*, x_3, x_4^*) \text{ for every } x_3 \in S_3.$$

and

$$\varphi_4(x_1^*, x_2^*, x_3^*, x_4^*) \geq \varphi_4(x_1^*, x_2^*, x_3^*, x_4) \text{ for every } x_4 \in S_3.$$

The game is zero sum, therefore

$$\varphi_1(x_1, x_2, x_3, x_4) + \varphi_2(x_1, x_2, x_3, x_4) + \varphi_3(x_1, x_2, x_3, x_4) + \ \varphi_4(x_1, x_2, x_3, x_4) = 0\ ,$$

The second equality goes as follows

$$\varphi_1(x_1^*, x_2^*, x_3^*, x_4^*) + \varphi_2(x_1^*, x_2^*, x_3^*, x_4^*) + \varphi_3(x_1^*, x_2^*, x_3^*, x_4^*) + \ \varphi_4(x_1^*, x_2^*, x_3^*, x_4^*) = 0$$

For either attribute to get a "not allowed difference" tag, as $\varphi_1(x_1, x_2^* x_3^*, x_4^*)$

$$\varphi_1(x_1^*, x_2^*, x_3^*, x_4^*) \geq \varphi_1(x_1, x_2^* x_3^*, x_4^*)$$

Prevalent for a constant sum game's every strategy as follows:

$$\begin{aligned}\varphi_1(x_1^*, x_2^*, x_3^*, x_4^*) + \varphi_2(x_1^*, x_2^*, x_3^*, x_4^*) + \varphi_3(x_1^*, x_2^*, x_3^*, x_4^*) + \ \varphi_4(x_1^*, x_2^*, x_3^*, x_4^*) \\ \geq \varphi_1(x_1, x_2^* x_3^*, x_4^*) + \varphi_2(x_1^*, x_2^*, x_3^*, x_4^*) + \varphi_3(x_1^*, x_2^*, x_3^*, x_4^*) + \ \varphi_4(x_1^*, x_2^*, x_3^*, x_4^*)\end{aligned}$$

The point of equilibrium of the four player constant sum game ceases, if a shift in strategy happens for either of the factors:

$$(x_1^*, x_2^*, x_3^*, x_4^*) \longrightarrow (x_1, x_2^*, x_3^*, x_4^*)$$

thus the shift in strategy (the change of any element of strategies) leads to inequality,

$$\varphi_1(x_1^*, x_2^*, x_3^*, x_4^*) \geq \varphi_1(x_1, x_2^*, x_3^*, x_4^*)$$

This inequality-system states that if player one chooses a strategy different from x_i^* and thus leaves the $(x_1^*, x_2^*, x_3^*, x_4^*)$ equilibrium and the game itself, his payoff-function can only be either equal to or lower than that of the others. If the fourth player differs in a not allowed manner but the others don't change their strategies, then his payoff-function will also be equal to or lower compared to the $\varphi_{1,2,3}(x_1^*, x_2^*, x_3^*, x_4^*)$ of the others.

$$\varphi_{1,2,3}(x_1^*, x_2^*, x_3^*, x_4^*) \geq \varphi_4(x_1^*, x_{2,} x_3^*, x_4^*)$$

Since this is a zero sum game, meaning the total payment can neither get higher or lower, the payoff-function of the $\varphi_{1,2,3}(x_1^*, x_2^*, x_3^*, x_4^*)$ factors will either be equal to or greater as well.

3.4.3. Imaging algorithms of Output-side

One of the popular types of non-cooperative Game Theory solutions is conflict alleviation methods. From these, we can highlight the axiomatic solution system of Nash, which creates axiom aggregations in order to assure the solution always places on the Pareto-line. The Kálai-Smorodinsky solution defines the minimum reachable or the last available point (meaning worst acceptable) to the solution of the conflict by defining the worst possible leaving point of the conflict.

GAME THEORY MODELING OF OUTPUT SIDE (LEVEL 3)

The phase of setting the final equilibrium state by the corner switch on the leaving side, the equilibrium search, and the finalization of the sustainability criteria can usually only be done with cooperative strategy.

Definition:

Cooperative games can be defined by the following concepts. $N = \{1, \ldots\ldots, n\}$ as in *aggregation of players*, where the S subset is known as a *coalition*: $S \subseteq N$. Let S be an aggregation of the subsets, meaning the aggregation of possible coalitions. The N main aggregation is called *coalition total*.

Thesis:

In low-carbon investment concepts, the project generates energy drawn from renewable sources, but the produced electricity can only reach the consumer if the owners of both the green electricity producer (Investor/B) and the electricity system (System/H) agree with each other that the product reaches the consumer through the system. A criterion of cooperation is that the investor pays a usage/transport fee to the owner of the system, and the owner acknowledges that instead of the previous (fossilized) product, he transports a private product via the system, and in a lower volume. As compensation, the system gets the pay from the investor. This compromise, in essence, means that there has to be a valid agreement on provisioning conditions on the market. We tried to match the "green-yellow-orange" attribute cube of the previously established Rubik's Cube project planning method with the model, and to assign the proper strategy to the cooperation.

Proof:

We can introduce our conflict-alleviation method with a two-player game. In the example, let the players' strategies be represented by S_1 and S_2, and the two payoff-functions by φ_1 and φ_2. The aggregation of possible payoffs will therefore be 2D, and can be shown as follows:

$$H = \{\varphi_1(x, y), \varphi_2(x, y) \mid (x, y) \in S_1 \times S_2\}$$

In this case, as always, the payoff of both players aims at maximization, but naturally the various payoffs of one player depend on that of the other and the fact that raising one player's payoff will lower the other's stands as a rule. Therefore, the objective is to find a solution that is acceptable to both the investor and the system owner, meaning both parties simultaneously. We also have to state that in case of the agreement not being "signed," both parties get a lower payoff, or a punishment.

Standard representations:

$$\boldsymbol{f}_* = (f_{1*}, f_{2*})$$

this will be our standard payoff vector, where we assume that there is a $(f_1, f_2) \in H$ where $f_1 > f_{1*}$, and $f_2 > f_{2*}$. The problem is defined mathematically with the pair. This pair was defined in Figure 5. We also assume that aggregation H is not open, convex, or bounded, so in the case of:

$$(f_1, f_2) \in H \; and \; \bar{f_1} \leq f_1, \bar{f_2} \leq f_2$$

$(\bar{f_1}, \bar{f_2}) \in H$ and bounded in both coordinates, meaning

$$sup\,\{f_i|(f_1, f_2) \in H\} < \infty$$

in case of $i = 1{,}2$.

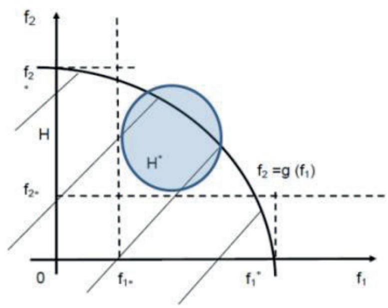


Figure 42: Figure of conflict state with the position of the payoff-function
Source: self-made

We also assume that the borderline of H is the graph of a $f_2 = g(f_x)$ function, which is strictly falling in f_1 and is concave. The graph of function $g -$ is usually called the Pareto line; therefore, the conditions of satisfying the optimum criteria of sustainability can be met here. We must also take into consideration with the game and solution criteria that no rational player will accept a compromise that means a worse payoff than the payoff without agreement.

This way, we can tighten the payoff aggregation as follows:

$$H^* = \{f_1, f_2|f_1 \geq f_{1*}, f_2 \geq f_{2*}, (f_1, f_2 \in H\}$$

Conclusion:

We concluded an unorthodox Game Theory optimum search on the different (cube) levels for the low-carbon planning of the project development process. During the Game Theory optimum search, I defined a theoretic model structure, which means fundamentally placing three different types of Game Theory solutions after each other, while keeping tabs on which Game Theory method is most efficient for featuring the various economic criteria systems:

1. Cube level one: non-cooperative three player game (for the correction of not allowed differences on Input side),
2. Cube level two: non-cooperative zero sum game (for the correction of not allowed differences of middle cube connections),
3. Cube level three: conflict alleviation method with two player game (for the correction of not allowed differences on Output side).

The three different Game Theory models can together define the states of Nash equilibrium required during project development, which help achieve sustainability during the realization of the project. The sufficient selection of Nash equilibrium is possible through the SMART value definition based on the correspondence system of the cubes. An introduction to this will be given later in this document. However, we must stress that the Game Theory row that I selected (three person cooperative game, non-cooperative zero sum game, conflict-alleviation method) is applicable mainly for typified energetic development, and a strictly defined economical environment (Hungary and Central Eastern Europe). Therefore, we can say that economic externals or development goals that differ from these can allow different Game Theory sequences to be used as well.

3.5. Weighting criteria and cube attributes; the Churchman - Ackoff process of dominance

As I already mentioned during multi-dimension problem handling, the two opposing cube sides (white and yellow) will be our project's Input and Output sides, respectively, while the attribute sets/main agents defining the low-carbon optimization in our demonstrational project will be as follows:

- ✓ optimization of strategic objective system (red side)
- ✓ analysis of market opportunities (green side)
- ✓ actualization, technological requirements (blue side)
- ✓ matching financial effects (orange side)

We might ask – what's the reason behind defining these main agents as the important attribute sets? Naturally, the designation of main agents took place after a careful systematic pre-selection, the process and methodology of which can be summarized as follows:

During the selection, we basically need to weight those attributes that have an impact on the development process for us to be able to select the most important attributes, which give the definitive criteria of the planning of the project or the actualization of investments. The six sides of the 3×3×3 Rubik's Cube were linked to the attribute sets but, as we've already mentioned, the opposing sides of the cube (white and yellow) give the Input and Output sides of the project process, respectively. In this correlation, the definition of the main agents is narrowed to four dominant attributes, which are later assigned to the red, blue, green and orange sides. In the methodology section, we already described the specialized steps that gave basis to this selection.

During the evaluation process and the visualization of the various attributes of the main agents, I used the SMART (Simple Multi Attribute Ranking Technique) software, so I also applied the process of dominance and the data it uses to the software's data input criteria as well. The implemented method uses three different coefficients during the estimation, namely:

1.) Summarizing results, applying them to the analysis
2.) Queries between professional groups
3.) Direct interventions from professional groups via online correction

The operative steps of the guided process are as follows:

Step 1: Arranging layers or attributes (with professional estimation). Interval: 1-100.

Step 2: Defining subgroups from groups defined by layers, selection of control attribute, which will be attributes definable by 50. All control attributes are sorted to their various subgroups. We weight the attributes using professional estimation.

Step 3: Determining importance, using professional estimation. We can define three categories of the subgroups by the control attribute, using the professional estimation:

- more important than control attribute (+)
- less important than control attribute (-)
- as important as control attribute (0)

At the end of the process, the algorithm of the method generates finalized results to the attributes according to the given values and answers. Using this, it sorts the attributes into order, and sets the sensitivity, meaning the point for which any given attributes show the not allowed difference.

The fossilized-renewable energy changing systems I analyzed can only support steps 1 and 2 if we don't define subgroups but assign the control attributes to all four sides. It is advisable to include a professional team for step 3 if the basic criteria so suggest. During the process of dominance, I provided five different attribute groups, out of which four were selected. I assigned these to the various sides of the cube and defined the level of dominance of each main agent. In the case of fossilized-renewable energy changing systems, the assumed protocol for weighting attributes and cube sides is as follows:

Step 1:

- ✓ Summarization of financial effects
- ✓ Technological requirement system
- ✓ Strategic program positioning
- ✓ Examination of market opportunities
- ✓ Adaptation of law and regulations

Step 2:

Group 1: weighting of attributes

Summarization of financial effects:	90 --------▶	orange
Adaptation of law and regulations:	70 -----------▶	green
Examination of market opportunities:	60 --------------▶	blue
Technological requirement system:	50 -----------------▶	red
Strategic program positioning:	30 -----------------▶	not present

Control attribute: Technological requirement system (50 %)

Step 3:

Summarization of financial effects:	+
Adaptation of law and regulations:	+
Examination of market opportunities:	+
Technological requirement system:	0
Strategic program positioning:	-

Results of the Churchman-Ackoff process of dominance:

When summarizing the results of the process of dominance seen above, we can say that defining the preference of the four main agents, which have an impact on the project by transforming Churchman and Ackoff's method, is realizable quite easily. The main agent and the side/color of the cube match quite well and the correctness of the alternative (x_j) is as follows (Table 18):

$$x_j = \sum_{i=0}^{n} w_i a_i \quad j = 1,2,3,4$$

$$w_i \ = i - th\ attribute's\ weight\ w_i > 0;\ a_i = i - th\ value\ by\ attribute\ a_i > 0$$

Table 18: Weighted attributes, importance, and guide to process of dominance

Attribute group	Weighted attributes (w_i)	Importance analysis	Notes
1. Summarization of financial effects	90	+	Orange (O) attribute, primary preference, starting side
2. Legal and regulatory adaptation	70	+	Green (G) attribute, since the cube usually sorts itself clockwise, we put opposing attributes on the opposing side.
3. Examination of market opportunities	60	0	Blue (B) attribute, has lowest weight (in this case, this doesn't hold true).
4. Technological requirement system	50	0	Red (R) attribute, position of control attribute, backside of the cube, point of equilibrium.
5. Strategic program positioning	30	-	Not present attribute. Its pairing with other dominant attributes should be examined. as per the professional decision, it is integrated to attribute 2, with a 2.7 to 9 ratio.

Source: self-made.

The sorting logic, which can be clearly seen in Table 18, is that the most dominant attribute goes to the top side (in our case, this is orange – O), and the weakest attribute opposite to it (which is red – R). The reason for this is that the description of their correlation profile (including contradictions and errors) can be described best if it's done through two other attributes (from the left, and the right). In our dominance list, the attributes sorted by their weakening should be assigned counter-clockwise by their "rate of weakening." The gist of the order is that attribute groups which have a stronger dominance are assumed to be in a stronger order, while the attribute groups that show weaker dominance might fall further from the point of equilibrium. The cube's top side will be taken by the attribute group showing the strongest relevance, while the one that shows the weakest will be positioned opposite to it, on the cube's bottom side. Rotating the cube into order usually happens clockwise in different algorithms, meaning they are optimized via the fastest route after the various steps. This is why we define the other two sets counter-clockwise (according to their dominance values), which is why the stronger cube attribute group falls right to the orange side (which is green), and the weaker, third most dominant to its left side (which is blue).

3.6. Application of usefulness functions to link attributes of main sides

One of the most important steps of project development using Rubik's Cube is how we choose the attribute elements of the analysis system from the attribute group (of one main agent) which was defined by process of dominance beforehand. The attribute groups, meaning the main agents assigned to the cube's sides, have part-attributes defined by the single-colored middle cubes, the dual-colored edge cubes, and the tri-colored corner cubes. The selection of small cubes is dependent on the basic outline of Rubik's Cube, meaning that we considered the white side handled as Input, and the yellow side considered Output, while also taking into consideration the surfaces which represent attribute groups opposite of, or next to each other (the other four sides), which are part of the optimum assortment of our project's environmental effects during the process of optimization. I used multi-variable usefulness functions to define the inner part-attributes of the main agents assigned to the cube sides, in other words, the small cubes.

Using multi-variable usefulness functions

Decision makers must take the prevention of environmental problems and those other economic problems into consideration that can surface due to the cessation of various products and processes. Without including the benefits and setbacks of these consequences, there can be no decision. The multi-attribute usefulness theory handles problems where the effect of the decision is defined by two or more variables. We generally assume that all attributes have either discrete or continuous values. For the sake of simplicity, let's assume that discrete attributes were defined in a way that higher usefulness values are matched with higher attribute values, if everything else is unchanged:

Let $x = x_1, x_2, \ldots x_{nb}$ be the attributes,

and x = $\langle x_1, x_2, \ldots, x_n \rangle$ be the values of the attribute vectors, to define the

$U(x_1, \ldots, x_n)$ usefulness function.

System of preferences, interpretation of multi-variable usefulness functions

Multi-attribute usefulness theory assumes that usefulness functions have a well-defined structure. The accepted theoretic approach says that we identify regularities in the preferences of behavior and by using the so-called representation theses we can show that the attribute which has a preference system can be defined by a usefulness function as follows:

$$U(x_1, \ldots, x_n) = f\ [f_1(x_1), \ldots, f_n(x_n)]$$

where f is hopefully a simple function, e.g. an addition. It's obvious that this correspondence is similar to how we used the probability webs to break the summarized probability distribution function. This is important because we also demonstrate the probability distribution of the various attribute groups of Rubik's Cube in a network-like manner.

Preferences without insecurity

The basic regularity found in the deterministic preference structure is called *preferential independence.* Two attributes, X_1 and X_2 are preferentially independent of X_3, if the preference between $\langle x_1, x_2, x_3 \rangle$ and $\langle x_1', x_2', x_3' \rangle$ isn't dependent on X_3 's exact value.

This definition – *preferential independence* – is important to the Rubik's Cube methodology's defining of dominance, because this "sorting" is based on quick optimization which only includes important attributes, which is why e.g. Indian software developers also chose this method (RCM) to renew old software applications.

If X_1, ..., X_n attributes are both preferentially independent, the behavior preference of the agent can be defined by maximizing the function below:

$$V(x_1,\ldots x_n) = \sum_i V_i\,(x_i)$$

where each V_i is a value function, dependent only on the X_i attribute.

This type of value function is called an *additive value function.* Additive value functions offer a perfectly seamless method of describing value functions of important attributes and are applicable to many "real life" situations. Even if the cost preference function doesn't hold true completely, this value function can still offer a proper option for the preferences of main agents. This is true even for a case when the cost preference function is only damaged in intervals, which compile scenarios rarely happening in real life.

We can see in the demonstration above that an investment decision can be based on the important definitive agents, main attributes via classic decision-making mechanisms, even while disregarding main attributes like

- risks of market and regulation circumstances,
- effects on environment, climate change, etc.

helping us in making our investment decisions. In the case of the methodology based on the solution of Rubik's Cube, that for the inner attributes of the cube sides (main agents), we can't base a decision on preference independence, but when we have to make decisions on the comparison of main agents' connections, this is outright mandatory.

One of the most valuable characteristics of the sustainability evaluation using Rubik's Cube, is that it can handle the various usefulnesses assigned to either main agents (cube sides) or definitive attributes, together with the correspondence systems and the other main agents (assigned to the cube's other sides). Besides 2D (x,y) it can also define 3D (x,y,z) connections as well, and can identify the various attributes with effects via the rotations. The reason for this is that the dual-colored cubes can have 2D, while the three-colored cubes can have 3D attributes assigned to them, meaning it's applicable to handling of attributes which are linked to two or three main agents simultaneously.

The attributes inherent in the cubes can be interpreted in various ways. (Since the white and yellow sides mean the Input and Output sides of the process, respectively, their functional interpretation differs from that of the other four sides during the solution). In order to make the connection clear, I'll demonstrate the correspondence on the next illustration (Figure 42). The 2D marking is for cubes and attributes which are dual-colored (e.g. the blue-orange edge cube on Figure 42), while the 3D marking is for cubes or connection attribute characteristics which have three colors. On illustration 43, the blue-red-yellow corner cube can be called a typified 3D attribute.

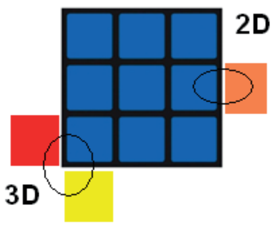


Figure 43: 2D and 3D interpretation of cube side, or main agent
Source: self-made.

2D cube attributes mean that the edge cubes have attributes assigned to them which are influenced by the other main agent (orange). 3D cube attributes mean that (in our case, the) blue corner cube has attributes which are influenced by two other main agents (red and yellow), and vice versa. Marking the elements of the main agents/cube sides with an attribute therefore has to depend on its position on this specific side as well. This means that marking the small cubes with attributes and defining their usefullness and dominance in the main agents is possible in regards to this. I did the usefulness analysis with the SMART (Simple Multi Attribute Ranking Technique) attribute evaluation software, which was based on the process evaluation and Game Theory optimization levels shown in Table 12. Table 19 shows how the levels of the project planning model were synced with the attribute analyses of the SMART software application.

Table 19: Syncing SMART evaluation levels to modeling levels

SMART EVALUATION LEVEL	LEVEL OF MODEL DEVELOPMENT	/LOW-CARBON/ PROJECT ATTRIBUTE IN QUESTION (1D/2D/3D - # of inherent traits)
Level 1	INPUT	"White cross" – defining the starting criteria (1D, 2D)
	INPUT	"White corner" – defining the sustainable development routes, equilibrium-search, non-cooperative optimum (3D)
Level 2	MIDDLE CUBE	"Second row" – anchoring of relation points, achieving equilibrium, arranging two-dimensional attributes, positioning fixed point (1D, 2D)
	MIDDLE CUBE	"Yellow cross" – indirect synchronizing of input/output sides (1D, 2D)
Level 3	OUTPUT	"Yellow corner" – interpretation of sustainability attributes during the arrangement of outputs (3D)
	OUTPUT	"Yellow side edge-switch" – strict synchronizing of input/output sides (2D)
	OUTPUT	"Corner switch" – the phase of setting the final balance, achieving equilibrium, finalizing sustainability attributes (3D)

Source: self-made.

3.6.1. Interpretation of three-level logical analysis

The three-way layout of the search for optimum using Game Theory models (Input, middle cube, Output) already showed us that though the planning levels of the project development based on the solution of Rubik's Cube follow the solution logic, it is advisable to brake the process of searching for equilibrium into greater units, meaning some phases (NO_{1-7}) should be merged.

During the preparation of the three-level analyses, the first two solution phases (NO_1, NO_2) were brought to the Input level. The next two solution phases (NO_3, NO_4) were assigned to the middle cube, and the last three (NO_5, NO_6, NO_7) to the Output. Therefore, I merged the seven phases connected to the *layer by layer* (meaning row to row) solution method of the 3×3×3 Rubik's Cube, and defined three analysis levels, which are as follows:

1. Cube level 1: Correction of not allowed differences on the Input side (Figure 43.),
2. Cube level 2: Correction of not allowed differences on the middle cube level (Figure 44.),
3. Cube level 3: Correction of not allowed differences on the Output side (Figure 45.).
   (Note: I defined "not allowed difference" as a not sustainable attribute in the attribute set)

On the first cube level (or "layer"), I marked 21 attributes by defining 9 cubes. These, interpreted together with the neighboring cubes, which are edge cubes (4×2) and corner cubes (4×3), and also a single middle cube, are also defined in the other (yellow, blue, green, and red) main agent dimensions (Figure 44).

Figure 44: Interpretation of first cube level (Input side)
Source: self-made.

The next level can be specified by defining the middle interpretation zone (1×1×3) of the cube. In this case, we put 12 inner attributes into the interpretation dimension. This means four edge cubes (4×2) and four middle cubes (4×1), meaning we have to calculate with eight cubes as system elements (Figure 45).

Figure 45: Interpretation of second cube level, or middle cube connections
Source: self-made.

In the last step of defining the interpretation dimensions (levels), similarly to the first step, I marked 21 attributes by defining 9 cubes. These, interpreted together with the neighboring cubes, which are edge cubes (4×2) and corner cubes (4×3), and also a single middle cube, are also defined in the other main agent dimensions (Figure 46).

Figure 46: Interpretation of third cube level (Output side)
Source: self-made.

We therefore made attribute sets for the SMART evaluation by grouping the middle, edge and corner cubes of the first, second and third rows by solution level. We can say that in the case of cube attributes definable by values, we have to realize that we can define our entire system with 9+8+9= 26 complex attributes, which have another 54 independent attributes, apart from the main agents (6x9). We also have to stress that because of the complex attribute handling (1D, 2D, 3D), this analysis can also clearly define the correspondence systems of the various attributes.

3.7. SMART (Simple Multi Attribute Ranking Technique) analysis

To interpret the attributes of the cubes and to define the attributes associable to the small cubes, I chose the SMART (Simple Multi Attribute Ranking System) method, which can handle and illustrate 2D and 3D attributes at the same time. I chose the analysis method as defined in the methodology segment, which method counts as a one of a kind software application in terms of visually illustrating different attributes.

The process of the SMART analysis was as follows:

1. Evaluating the results of process of dominance conducted on main agents, input of data,
2. Defining the small cube attributes of examination levels, and the estimated usefulness values,
3. Creation of SMART Tables and illustration in 3D.

Using the results of the Churchman-Ackoff process of dominance analysis, the beginning data of the SMART evaluation is as follows (where I defined the color/attribute matches according to the results of said process of dominance analysis):

Group 1: weighting of attributes

Summarization of monetary effects:	90 --------▶ orange
Adaptation of law and regulations:	70 -----------▶ green
Examination of market opportunities:	60 --------------▶ blue
Technological criteria system:	50 -----------------▶ red
Strategic program positioning:	30 -----------------▶ not present

In the previous chapter, we answered the question of how we can match the solution algorithms with the different levels for the process of project planning based on Rubik's Cube, and, with the Churchman-Ackoff method, we get the four most important attributes from the list of attributes which have an impact on it, namely those we can match to the cube's sides. If white (W) is the Input side, then the most dominant attribute group is matched with the orange (O) side, which gives us our WO base side pair, from where we continue clockwise around the white side and the following side pairs of white-blue (WB), white-red (WR), and white-green (WG) will define the relevant attributes (agents) in the planning process.

The sorting criteria is that the most dominant attribute gets placed on the top (in our case, this is orange – O), and the least dominant attribute goes opposite to this side, meaning the bottom (in our case, this is red – R). The reason for this is that the description of their connection profiles (including the contradictions and errors) can be defined best if it happens via a transaction through two other attributes (left and right sides). In our dominance list, the weakening attributes located in the middle are arranged by their "order of weakening," namely counter-clockwise. The gist of the ranking is that the attribute groups that show a stronger dominance are supposedly in better order, while the attribute group of weaker dominance is supposedly further from the point of equilibrium. The attribute group with the strongest relevance will be placed on the top, the weakest relevance attribute group will be opposite to this side, and finally, we define the remaining two groups by their "order of dominance," namely, counter-clockwise. Since the solution of the cube usually happens clockwise in the various algorithms, the parts are optimized towards the point of equilibrium through the steps of process following the shortest route to solution, which explains why we position the most dominant attribute groups to the green side on the right of our starting orange side, and the third strongest dominant attribute group to the blue side on the left of our starting orange side.

Definition of usefulness functions

Compared to our analysis method, the SMART software offers a general function-definition method. The data and criteria introduced in the next structure can be simply added to the database of the program and be evaluated with the help of the plug-in algorithms.

SMALL CUBE NUMBER	USEFULNESS (1-100) (assumed)	CONNECTION, DIMENSION VALUE	SMART VALUE (V)	CUBE TYPE AND DOMINANCE	MAIN AGENT INHERENT ATTRIBUTES

When compiling the tables, the conversion of usefulness functions into usefulness values took place by applying the following steps:

1. the maximum value is assigned a score of 100,
2. the minimum value is assigned a score of 0,
3. after defining the two border values, we can also define the middle usefulness value (meaning half-useful compared to the maximum), which is assigned a score of 50,
4. after defining the maximum border value and the middle value, we can also define the usefulness value in-between them, which is assigned a score of 75,
5. after defining the minimum border value and the middle value, we can also define the usefulness value in-between them, which is assigned a score of 25.

Using a similar method, we can get inner function values and we can mathematically assign an interpolation to the points that we get this way. The sufficiency of the alternatives is defined by the weighted mean of the usefulness value. I indicated the connection dimension values (which is also clearly visible in Table 10) as follows: 3/3 for the three-level connections (corner cubes), 2/3 for three-level connections (edge cubes), and 1/3 for the fixed middle cubes. In regards to this, the usefulness-function for main graph or main attribute is as follows:

$$V(x_1, \ldots . x_n) = \sum_i V_{i=\frac{1}{3};\frac{2}{3};\frac{3}{3}} \left(x_i = \frac{\sum_{i=0}^{n} w_i a_i}{\sum_{i=0}^{n} w_i} \right) \qquad i = 1, \ldots . . n;\ n = 9$$

$$w_i = i - number\ attribute's\ weight, w_i > 0;$$

$$a_i = i - attribute\ based\ value\ \ a_i > 0$$

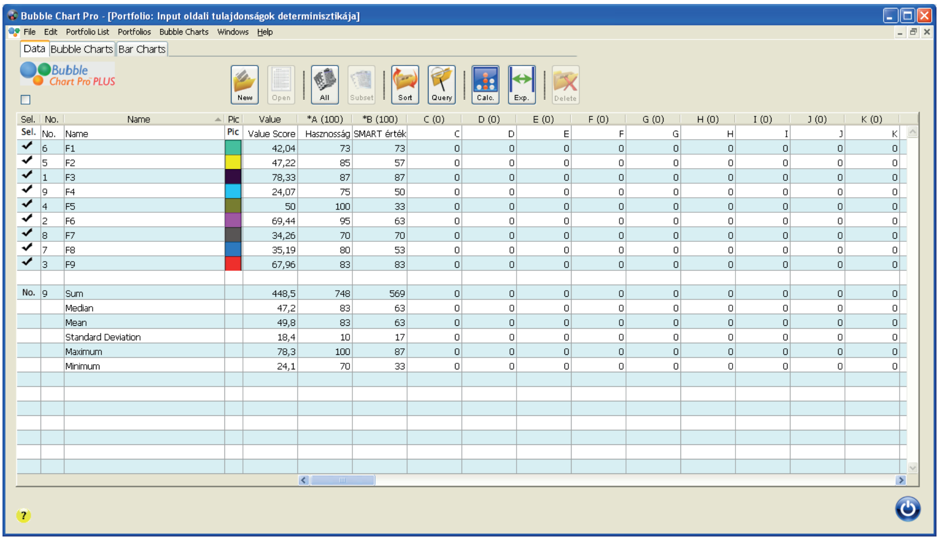
According to the equation above, by portraying the SMART values for the various levels, we obtain clear knowledge on the attributes which have an impact on the different dimensions of usefulness for the main agent. We can view the inherent attributes of the main agents (O,R,G,B), their relations to each other, and the usefulness attributes of the Input side in Table 20.

Table 20: Generating SMART Input values for data insertion ('►X' = usefulness)

MIDDLE WHITE (1D) – energy rationalization Dominance: 100	(100) max value
OUTER WHITE (2D) Dominance: Orange/90; Green/70; Blue/60; Red/50	
strategy, regulation base connection (WG),	100/70 ► 85
basic technological requirement (WR),	100/50 ► 75
financing expectations (WO),	100/90 ► 95
basic market positioning (WB)	100/60 ► 80
CORNER WHITE (3D) –	
basic criteria of market payoff (WOB),	100/90/60 ► 83,3
conformity to monetary tools and regulation criteria (WOG),	100/90/70 ► 86,6
syncing basic goals with technological threats and market priorities (WBR),	100/60/50 ► 70
basic criteria designation of matching set of strategies and technologies (WGR),	100/70/50 ► 73,3

Source: self-researched

After the input of data generated in Table 20, the SMART Bubble Chart Pro (demo version) creates the attribute illustrations via the "Value Score" point rating system, which is useful because the attributes compared to each other can be differentiated visually as well, regardless of that happening by their correctness or their strategic usefulness. Figure 47 shows the input data table of the SMART program.



Sel.	No.	Name	Pic	Value	*A (100)	*B (100)	C (0)	D (0)	E (0)	F (0)	G (0)	H (0)	I (0)	J (0)	K (0)
Sel.	No.	Name	Pic	Value Score	Hasznosság	SMART érték	C	D	E	F	G	H	I	J	K
✓	6	F1		42,04	73	73	0	0	0	0	0	0	0	0	0
✓	5	F2		47,22	85	57	0	0	0	0	0	0	0	0	0
✓	1	F3		78,33	87	87	0	0	0	0	0	0	0	0	0
✓	9	F4		24,07	75	50	0	0	0	0	0	0	0	0	0
✓	4	F5		50	100	33	0	0	0	0	0	0	0	0	0
✓	2	F6		69,44	95	63	0	0	0	0	0	0	0	0	0
✓	8	F7		34,26	70	70	0	0	0	0	0	0	0	0	0
✓	7	F8		35,19	80	53	0	0	0	0	0	0	0	0	0
✓	3	F9		67,96	83	83	0	0	0	0	0	0	0	0	0
No.	9	Sum		448,5	748	569	0	0	0	0	0	0	0	0	0
		Median		47,2	83	63	0	0	0	0	0	0	0	0	0
		Mean		49,8	83	63	0	0	0	0	0	0	0	0	0
		Standard Deviation		18,4	10	17	0	0	0	0	0	0	0	0	0
		Maximum		78,3	100	87	0	0	0	0	0	0	0	0	0
		Minimum		24,1	70	33	0	0	0	0	0	0	0	0	0

Figure 47: 2D figure of the results table of the SMART program
Source: self-made based on SMART program

The equilibrium state of the Input side is unstable, as evidenced by the F1 attribute, which is the basic attribute designation of matching set of strategies and technologies (WGR/white-green-red). Figure 7 shows the attributes and their positions in the attribute group. If we click on the sphere, we get the coordinates (x,y,z) for it, which translate to special usefulness functions. Because of the 3D depiction, both the correspondence of attributes and the depth of said correspondences can be easily interpreted in Figure 48.

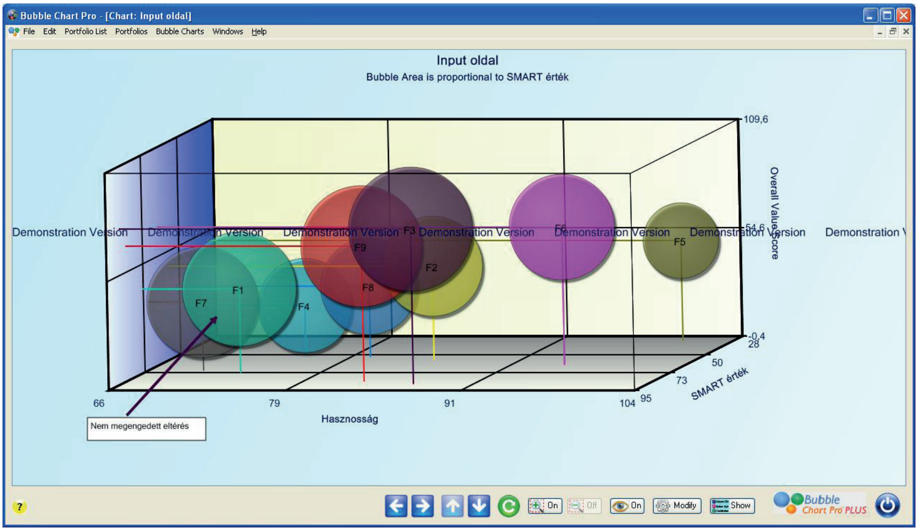


Figure 48: Depiction of results and non-equilibrium attributes of SMART program output
Source: self-made based on SMART program

With the aid of the SMART program, I evaluated the Input side (as seen on Figures 6 and 7), the middle cube side-attributes, and the Output side. The tables and summarizing illustrations related to the evaluation can be found in Appendices 2, 3, and 4.

To summarize the analysis, we can say that I was successful in assigning the project development factors to the side colors of Rubik's Cube going by the dominance of the attribute groups. The process of dominance analysis conducted with the SMART pilot program and the summary of its results is as follows:

- ✓ The definition of input and output sides of project attributes by picturing them to the white and yellow cube sides was completed,
- ✓ In the case of the main attributes, orange was the most dominant with red the least dominant; therefore, the dominance values are the highest for orange and the lowest for red. If the orange side is in the front, then we find the red side opposite to it, where green goes to the right, and blue goes to the left of orange,
- ✓ Going by the dominance, defined in the methodology part, the strongest attribute was assigned to orange, the second strongest assigned to green, the third strongest assigned to blue, while the weakest assigned to red, following the strength of dominance,
- ✓ I separated the analysis method (including the various attribute groups) into three different parts – 1) Input ► 2) Middle cube ►3) Output – for the sake of applicability of the Game Theory methods, and the SMART analysis,
- ✓ We can set the Game Theory optimization for the "selection of technology for base criteria" attribute of the Input side, because this is where the SMART program showed a not allowed difference,
- ✓ We can set the Game Theory optimization for the "monetary value of the project, and the time needed for payoff" attribute of the Middle cube side, because this is where the SMART program showed a not allowed difference,

- ✓ We can set the Game Theory optimization for the "market criteria system, balancing of market instruments and provisions" attribute of the Output side, because this is where the SMART program showed a not allowed difference,
- ✓ Therefore, the results of the analyses on project evaluation of project planning processes for projects which aim at advancement from fossilized energy sources to renewable ones is that the attributes with the most impact are as follows: selection of technology for base criteria (Input side), which has the biggest impact on reaching equilibrium, monetary value of the project, and the time needed for payoff (Middle side, or correspondence attribute group) which, if interpreted in a manner more suited to sustainability, will bring us closer to the sustainable economical value, and market criteria system, balancing of market instruments and provisions (Output side), which needs proper and balanced planning, for the imbalances it causes may lead to a failed project.

3.8. New and novel scientific results

During the analysis of the sources and the completed methodical analyses, I will introduce the results in two groups: C_n (new scientific conclusions) for the scientific results, which can later be introduced as theses after further research or proper expansion of the analyzed data, and R_n (new scientific results) for the ones that can be defined as scientific theses in the current research.

3.8.1. New scientific conclusions

C_1: The correspondence systems of project specifics which have an impact on the "shelf-life" and sustainability of enviro-orientated investments or investments that have a positive impact on climate change can be defined through Rubik's Cube's "Layer by layer" solution method and the Game Theory models assigned to the various phases of the method.

C_2: The actual use – including the designation of correct points of equilibrium for – of Game Theory models for sustainable modeling of economical events often becomes harder, due to the many criteria which come into play. During the search for sustainable points of equilibrium with Game Theory models, the simple function-like definition of the correspondence systems of compared attributes and the level-by-level handling of said correspondence systems based on simple planning phases makes realization easier and more sufficient.

C_3: I used multi-variable trial functions for the selection of attribute groups which can be defined as "having a negative impact on the equilibrium of sustainable project planning and realization." In the various phases of project development – using process of dominance and usefulness-functions – leads the project's successful realization towards the proper process and helps correct this.

C_4: According to my hypotheses, low-carbon project development or planning processes can be called parallel with the layer by layer solution method of Rubik's Cube. By assigning the various sides and colors to project attributes, we can realize a special and sustainable project development process in which the specifics of the process and development is achieved through the fact that the various attributes or attribute groups can be regarded as either one, two or three-dimensional system elements in the development programs. The same guiding theory is followed by low-carbon development and the solution system of Rubik's Cube, which strives to reach the point of equilibrium through logic and low energy input.

3.8.2. *New scientific results*

R_1: My comparison assessments verified that the various sustainability logics can be synchronized with the 3×3×3 Rubik's Cube's solution algorithms, where the relations of the cube's sides define a planning strategy that provides a new scientific approach for investment planning. The first two solution algorithms out of the seven correlate with the Input factors of the investment planning process, the third and fourth describe the correspondence system of the starting and the finishing phases, while the remaining three can be associated with the attributes of the Output side.

R_2: My analyses verified that the state of equilibrium for the Input side of the project development process, which also guarantees sustainability, can also be defined using a simple constant sum game or non-cooperative game of finite kind. The correspondence systems and the relative points of equilibrium of the Input and Output sides can also be described with the conflict alleviation method, zero sum game, or oligopolistic game of finite kind. The point of equilibrium for the Output side of the development process can also be described with a cooperative oligopolistic game or cooperative strategies for Nash equilibrium.

R_3: I used different calculations to verify that the corner cubes of the 3×3×3 Rubik's Cube have a key role in the search process for the point of equilibrium or sustainability optimum. Their rotation combination, based on synchronizing three attributes, may offer investment programs a perfect Nash equilibrium. Without setting this, there's no final balance between the Input and Output sides, and the flexibility of the system is greatly lowered since it didn't adapt the criteria which mean the ability of adapting to the possible changes in system attributes, and the relative elongation of "shelf-life."

R_4: One of the most valuable characteristics of the sustainability evaluation using the 3×3×3 Rubik's Cube is that it can handle the correspondence system and various usefulness cases assigned to the attributes defining the evaluation simultaneously with all the other defining attributes (which are assigned to the remaining sides of the cube). It can define both the two-dimensional (x,y) and three-dimensional (x,y,z) connections and can identify the attributes which have an impact on the various factors using the order of rotation. We can associate dual-colored edge cubes with 2D attributes and tri-colored corner cubes with 3D interpretations, meaning it can also handle attributes which simultaneously belong to two or three main agents.

R_5: I also showed that the process of sustainable business planning can be aptly defined with Game Theory models which model a sustainable project development process for the layer by layer solution method algorithms of Rubik's Cube. My examinations verified that a project development process can be considered sustainable if the following criteria are met:

A. The existence of a technologically sufficient planning option (to avoid over-planning and obsoleteness)
B. Optimization of liquidity and financial sustainability is met (safe self-sufficiency and revenue for at least 10 years)
C. Avoiding detrimental project effects on the relevant product areas (functionally self-sufficient system).

4. CONCLUSIONS AND SUGGESTIONS FOR THE INTERPRETATION OF SUSTAINABLITY

As it became obvious from the process of cited literature, we can find very different approaches on the interpretation of sustainability in economics. It is therefore a basic dilemma when defining sustainability systems/strategies whether we should employ the theoretical correspondences of either weak or strong sustainability for the various interpretations. The difference between weak and strong sustainability is basically defined by the relations between environmental and artificial funds. The theory of weak sustainability states that environmental and artificial funds can replace each other. However, the relatively new index of total economic value (TEV) – which is an up-to-date economic adaptation that also includes environmental value – still can not properly handle the resource transformation questions regarding the time-value of money. This question was sufficiently answered by sustainable economic value (abbreviated as SEV), which is an estimation method that can – with the inclusion and goal-oriented use of local information – portray the changes in both environmental, social and technological fund-elements in an integrated manner, which is only partially realized by total economic value. Said total economic value (abbreviated as TEV) supports one of the newer economic branches, the so-called system of "low-carbon economy."

The gist of the low-carbon economy concept is that it prefers structures which operate with low energy- and matter input on the level of the local economy or market, therefore considering the criteria for long-term proper handling of resources as assured. Therefore, we can rightfully rely on the priority system of the low carbon economy during the interpretation of criteria connected to sustainability, which is a concept aiming for equilibrium defined with what we can call the currently most advanced approach. Obviously, the sustainability interpretations search for solutions on the usage of resources, realizable concepts and actual models, the use of which will result in equilibrium. We may therefore think that to model sustainability in the future, the only sufficient scientific approaches are the ones that can offer well-defined and clear-cut solutions.

Main conclusions of reviewing cited literature:

- The economic interpretation of sustainability still has many inherent difficulties, as a result of which its imaging is far from the level of operative realization. Today, the sustainability criteria and economic criteria can most definitely be caught in the professional guides on low-carbon economy, which are considered acceptable enviro- or climate strategy guides by both political decision-makers and market participants.
- The question of the handling and sustainable use of resources opened new possibilities in the search for economic equilibrium in the last few years, one of which was the highly interesting and amusing solution of using Game Theory methods. Despite the many disappointments and losses of trust that Game Theory strategy models suffered in terms of economical decisions, we can say that the new theoretical approaches, new ways of finding the proper Nash equilibrium, and the use of simplified models is a reassuring occurrence coming from the field of mathematic modeling.
- During the process of cited literature, I already mentioned the new low-carbon software development approach, which can mean a new recycling method for old and out-of-date software. This solution can be used to satisfy new consumer needs through software applications, while using much smaller inputs of materials and effort, which means re-entering the market with our "dead" product. The secret of this software-regeneration is using the layer by layer, meaning row by row, solution method of Rubik's Cube as a base, which imitates the solution of the 3×3×3 Rubik's Cube, thereby "knocking" the development process into the proper state of equilibrium through a multitude of short interactions, which also correlates with new consumer needs.

Main conclusions of my personal research:

- The sustainability criteria and the classic "layer by layer" solution method of Rubik's Cube can be synchronized with each other. The analysis of connections between the basic structure, the sustainability interpretation, and the solution algorithms of Rubik's Cube verify that the process leading to the cube's configurational balance is a process of searching for equilibrium, which in our case is also applicable to model the equilibrium search of enviro-defense processes.
- My analyses on Game Theory strategy models show that in today's practice, we can find a multitude of economic strategy models that don't really work as intended. The reason for this is basically the over-complication of the models and the inclusion of the multitude of factors and criteria. In order to save the process of modeling and the actual mechanisms of the models from falling into the category of "too complicated, no thanks," we require a simplified and yet correctly working model that is easy to interpret, can be properly loaded with different data, and easy to correct. My three-level "unorthodox Game Theory optimum search" model, which is compatible with Game Theory models, offers a solution for this challenge.
- During the analysis of the Game Theory models and the strategic optimum search systems, I came to the conclusion that it is more beneficial to use smaller, individual and unique Game Theory solutions which have different reactions in a business environment to describe the process of equilibrium search instead of using complex multi-factor model-structures to describe the entire process in the form of functions. In case of development processes for renewable energy production, in other words, advancement from fossilized to renewable energy sources, by dividing the development program to three levels, then using *non-cooperative Game Theory method* for the first, *constant or zero sum game* for the second, and finally, to define output criteria, *cooperative Nash-equilibrium search with multi-player oligopolistic game* for the third, offers a more beneficial result.
- The unorthodox Game Theory method I described suggests – during the phase of actual use – that we use function characteristics which are flexible time-wise for the various levels (input, middle cube/correspondence, and output); therefore, it may prove applicable to model more complicated processes if we form an optimum search process through the consecutive use of many simple Game Theory models. These methods/games can also be changed and flexibly adapted to different economic criteria systems, according to the changes in business environment.

5. EXPLANATIONS AND COMMENTS TO THE RUBIK'S CUBE LOGIC

The topic of the dissertation is the novel approach of the characteristics related to sustainability by applying game theory models. The primary goal of the research is to support the planning process – despite the economic and political instability – of sustainable investments aiming at environment protection or greatly influencing climate change.

Game theory solutions are used widely nowadays to carry out the tasks of strategy-creation or to solve other technical and resource consumption related problems in business life. They plan an important role in solving situations in which the opposing interests of more decision makers must be taken into consideration at the same time and the circumstances of the situation depend highly on individual decisions of different decision makers and on the effects of decision strategies on each other.

Game theory solutions may find the sufficient balance point to develop decisions in many cases, however, we usually face situations in which, due to the great number and difficulty of factors influencing the circumstances, game theory solutions show more balance points. This makes the selection of the right decision more difficult, or in a worse case, they cannot find solutions (which are difficult to write down with the use of mathematical relations) among the circumstances. Therefore, during processing the relevant literature, I emphasized the introduction of the classic and the new approach of the relations of economic value and sustainability, because the interpretation of sustainability still carries many difficulties which cause the practical implementation to be hardly achieved.

Many think that the key of interpreting sustainability is to be able to conceptualize the criteria-system and requirements of sustainability through function-like relations, as well. The mathematical interpretation of sustainability factors, the introduction of sustainable economic balance or company strategies through a game theory approach, and interpreting the search for classic and sustainable economic balance points are challenges for which many theories, scientific papers, generative formulas and a great number of scientific attempts have been connected for decades now, but none of them has been fully successful.

The project-development method shown in the dissertation, based on the functionally structured solving process of the Rubik's Cube, which uses a layer by layer method to search for the optimal solution of sustainable project-development by using game theory solutions, uses a conceptionally different approach compared to earlier ones.

The essence of the shown Rubik logical approach is that the factors influence the actors of the economy or the circumstances around the actors are handled individually. The balance points related to them are organized into levels by game theory solutions that can primarily be organized linearly, then the solutions possessing balance points are flexibly adapted. The specialty of the method lies in the fact that through the method of solving the Rubik's Cube, the development processes can be carried out in a way in which primarily the criteria of sustainability are considered, again and again.

The above conventionalized "pure" game theory solutions, in other words, the ones which give certain balance points, are guaranteed by the selection method introduced during the low-carbon project-development. The important characteristics are then assigned to the corresponding sides of the Rubik's Cube and to the small tiles during the Churchman - Ackoff method and the SMART (Simple Multi Attribute Ranking Technique) utility analysis.

This way, the unorthodox game theory method - based on the solving of the Rubik's Cube – is able to map the external criteria system of market mistakes (which is outside the market criteria system, but influences it greatly) burdening the economic environment, and it is the primary criteria of existence of carrying out sustainable environment-protection investments.

It is a social requirement that through exactly defining, typifying, and summarizing the external effects of developments supporting sustainable economic structures, patterns (being able to written down by mathematic functions) should be defined that can conceptualize the currently relevant criteria-system of sustainability for market actors and political decision makers.

Following the previous logic, "the low-carbon Rubik's Cube project-development method" presented in the dissertation tries to find a solution for the materialization of sustainability, technological, and other, primarily "cleantech" economic development processes so that they can happen quickly and in the cheapest way possible but feasibly in the long run.

The resource environment related to renewable energy source investments was considered as a primary research field during the development of "the low-carbon Rubik's Cube project development method." These contain much opposition; however, it has improved very quickly throughout the past few years. The Rubik's Cube method can, hopefully, create the basis of a project-development practice which can eliminate the implementation of non-sustainable investments in the future, and development processes can be avoided which direct the indicators of welfare to a negative trend.

During the analysis of the set of values or the economic interpretation of sustainability, it became obvious that the traditional value measuring systems are not able to create a criteria-system that provides a basis of a system for sustainable economic evaluation and for the design of economic structures feasible on the long run. The project development uses Rubik's logic and is based on game-theory to give a new approach in this process.

Proven hypotheses:

- H1: The correspondence system of project characteristics that influence the feasibility and sustainability of investments aimed at environmental protection or which influence climate-change positively can be described by models.
- H2: During the search for balance, the function-like description of the connection between the factors to be compared can be performed by using game-theory methods.
- H3: The multivariable test functions are capable of selecting the characteristic groups that predominantly influence the successful implementation of a project.

Partly proven hypotheses:

- H4: By the individual solving algorithms of the 3x3x3 Rubik's Cube, the sustainability principles can be synchronized and the correspondence system of the sides of the cube writes down a space-approach and planning strategy, which provides a new scientific approach during the planning process of investments.

As a summary, it was proven in this dissertation – in a credible way – that the scientific theory, according to which the solving algorithm of the Rubik's Cube, the "*Layer by layer"* method, is suitable for creating models for project-development processes. However, the correspondence system of certain project characteristics can be represented with the corresponding game-theory models in a way that different environment and climate-friendly investments can be planned easily from the aspect of human resources and of preserving and improving environmental factors as well, in a low-risk economic environment.

Understanding the solving method and the connection system of Rubik's Cube, its character tiles (middle, corner and edge tiles) can ensure – through game-theory methods – that the criteria-system conceptualized by sustainable economic value concepts can be modeled through the shortest time, with the consideration of using a relatively small amount of resources.

APPENDIX

A1: BIBLIOGRAPHY

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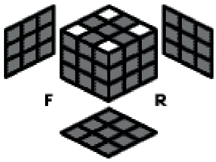
A2: CORNER-FIRST METHOD

Description and formulae of Corners-First solution method for Rubik's Cube

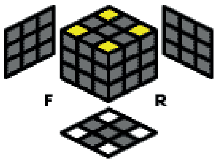
Based on the manual by Victor Ortega and Josef Jelinek

Stage I: Solve the corners

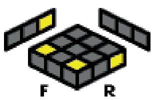
1. Orient U corners



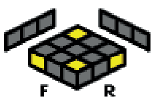
2. Orient D corners



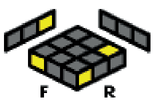
Rotate the whole cube so that D becomes U. Orient the corners depending on which of the seven patterns below you see:



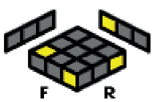
letter T pattern:
R U R' U' F' U' F



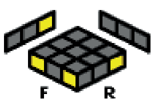
letter L pattern:
F R' F' U' R' U R



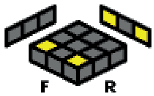
sune pattern #1:
R U2 R' U' R U' R'



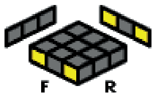
sune pattern #2:
R U R' U R U2 R'
(inverse of #1 in both respects)



letter pi pattern:
R U R2 F' R2 U R'

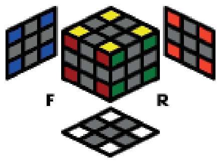


letter U pattern:
R' F' U' F U R

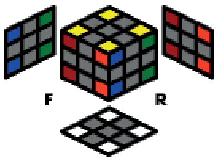


letter H pattern:
R2 U2 R' U2 R2

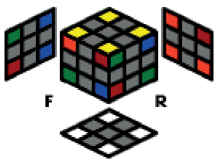
3. Permute all corners, by method of number of solved "edges"



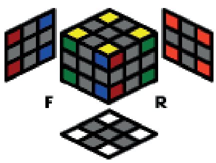
Proceed with one of the following sequences depending on how many solved edges you have:



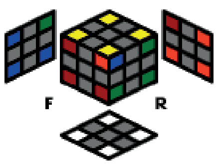
0:
F2 R2 F2



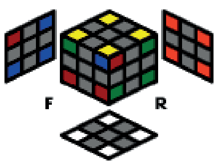
1 (DB solved):
R U' F U2 F' U R'
((UB solved): R' U R' B2 R U' R)



2 (DB and UB solved):
R2 U F2 U2 R2 U R2



4 (D solved):
F2 U' R U' R' U F2 U R U R'

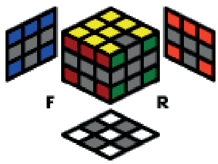


5 (UF not solved):
R U' R F2 R' U R F2 R2

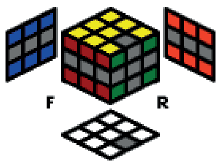
Stage II: Solve the edges

At this point, align corners and position centers. The cube is now fully symmetrical, except for the edges.

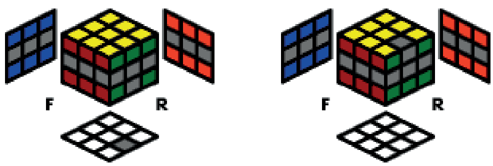
4. Solve three U edges



5. Solve three D edges



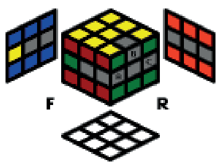
6. Solve one more U or D edge, depending on which is easier

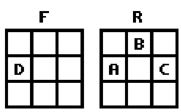


7. Solve last U edge, orient middle layer

a) U edge in the middle layer

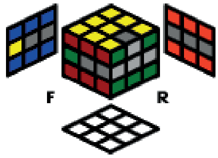
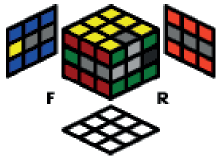
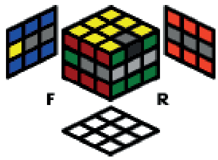
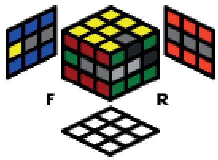
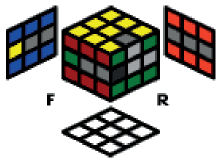
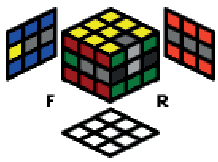
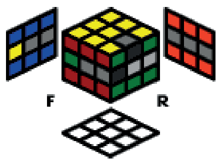
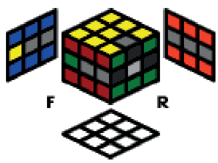
In the diagram below, edges A, B, or C are oriented correctly (C) if that facelet's color matches the adjacent center or the opposite center. Otherwise it is incorrectly oriented (I).





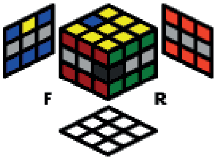
(facelet D belongs in facelet position B)
("adjacent center" between A and C)

Edge A	Edge B	Edge C	Pattern	Sequence

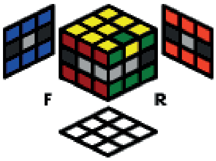
C	C	C		⇨R ⇨R' ⇦R' ⇨R
C	C	I		⇨R' ⇦R' ⇨R' ⇦R'
C	I	C		⇦R ⇨R2 ⇨R
C	I	I		R' ⇦R ⇨R' ⇨R
I	C	C		R2 ⇨R ⇦R ⇨R ⇦R'
I	C	I		R' ⇨R' ⇦R' ⇦R ⇨R ⇦R
I	I	C		R' ⇦R' ⇨R ⇨R
I	I	I		R' ⇨R' ⇨R' ⇦R2 ⇦R

b) U edge in position but twisted

There will be 1 or 3 twisted edges in the middle layer:



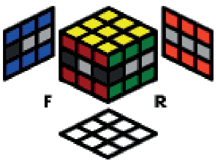
FR twisted:
R U2 R' ⇨⇨R2 ⇦R' U2 R'



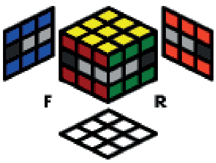
FR not twisted:
R' ⇦R' ⇦R' ⇦R'

c) U edge solved

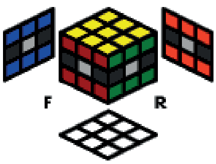
There will be 0, 2, or 4 edges twisted in the middle layer:



2 adjacent (FL and FR):
R2 F ⇩F' R2 F ⇧F'

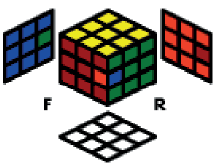


2 opposite (FL and BR):
F ⇩F' R2 F ⇧F' R2

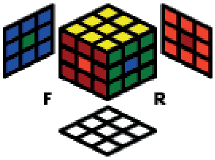


all four:
R F2 R2 ⇨R ⇨⇨R F2 R2 ⇨R
all four, when all edges are positioned correctly:
F' U' F (⇨ R)4 F' U F

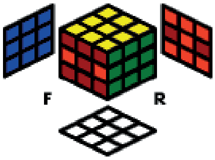
8. Position middle layer



send FR to BL, BL to BR, and BR to FR:
R2 ⇦R2 (⇨)



exchange centers with opposites:
⇩⇩⇦⇩⇩(⇨)



exchange FR with BR, FL with BL:
R2 ⇨⇨R2 (⇨⇨)

►READY

A3: SMART CHART AND RESULTS ILLUSTRATIONS FOR INPUT ATTRIBUTES

Deterministics of Input (white side) attributes:

Generating SMART Input values for data insertion ('►X' = usefulness)

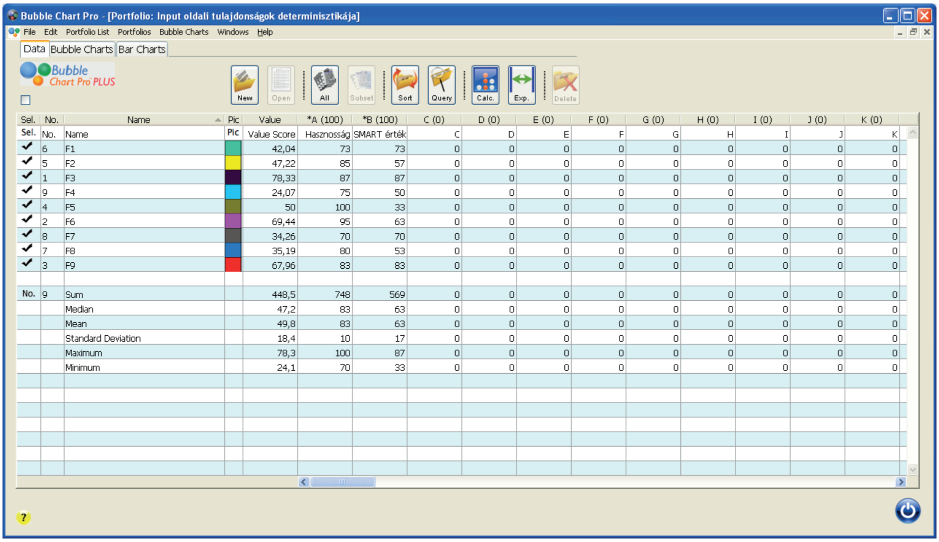
MIDDLE WHITE (1D) – energy rationalization	Dominance: 100	(100) max value

EDGE WHITE (2D) Dominance: Orange/90; Green/70; Blue/60; Red/50	
strategy, regulation base connection (WG),	100/70 ► 85
basic technological requirement (WR),	100/50 ► 75
financing expectations (WO),	100/90 ► 95
basic market positioning (WB)	100/60 ► 80
CORNER WHITE (3D)	
basic criteria of market payoff (WOB),	100/90/60 ► 83,3
conformity to monetary tools and regulation criteria (WOG),	100/90/70 ► 86,6
syncing basic goals with technological threats and market priorities (WBR),	100/60/50 ► 70
basic criteria designation of matching set of strategies and technologies (WGR),	100/70/50 ► 73,3

SMALL CUBE NUMBER	USEFULNESS (1-100) (rounded)	CONNECTION, DIMENSION VALUE	SMART VALUE (V)	CUBE TYPE AND DOMINANCE	MAIN AGENT INHERENT ATTRIBUTES
F1	**73**	3/3	**73**	corner cube with more dominating attribute	basic criteria designation of matching set of strategies and technologies (WGR)
F2	**85**	2/3	**57**	edge cube with less dominating attribute	strategy, regulation base connection (WG)
F3	**87**	3/3	**87**	corner cube with more dominant attribute	adherence to technological criteria and funding instruments (WOG)
F4	**75**	2/3	**50**	edge cube with less dominant attribute	basic technological requirement (WR)
F5	**100**	1/3	**33**	MIDDLE CUBE, dominance comparison	energy rationalization (W)
F6	**95**	2/3	**63**	edge cube with more dominating attribute	effects of market changes on the financing system, analysis of foreign currency risk factors, and global effects (WO)
F7	**70**	3/3	**70**	corner cube with average attribute	syncing basic goals with technological threats and market priorities (WBR)
F8	**80**	2/3	**53**	edge cube with less dominant attribute	basic market positioning (WB)
F9	**83**	3/3	**83**	corner cube with average or more dominant attribute	financially stable market payoff (technological solution which provides real return) (WOB)

Source: self-researched.

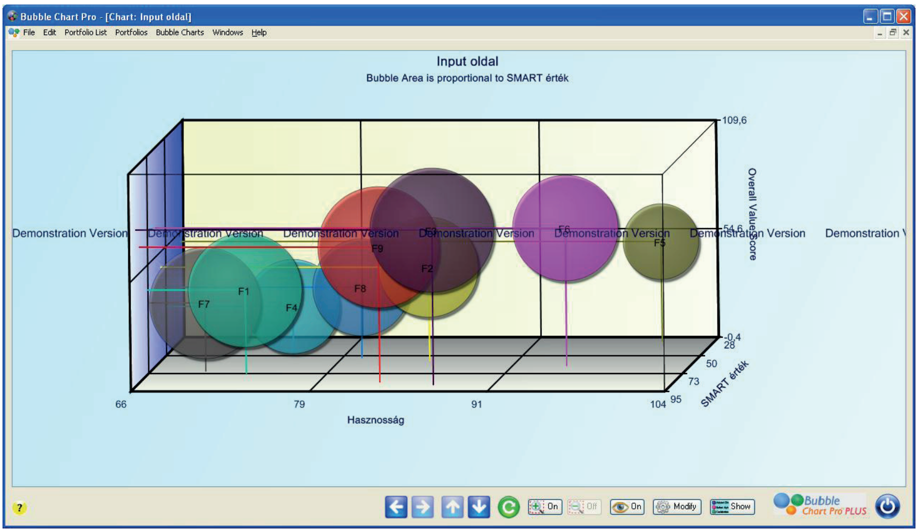
2D Figure of the results table of the SMART program


Bubble Chart Pro - [Portfolio: Input oldali tulajdonságok determinisztikája]

Sel.	No.	Name	Pic	Value Score	*A (100) Hasznosság	*B (100) SMART érték	C (0)	D (0)	E (0)	F (0)	G (0)	H (0)	I (0)	J (0)	K (0)
✓	6	F1		42,04	73	73	0	0	0	0	0	0	0	0	0
✓	5	F2		47,22	85	57	0	0	0	0	0	0	0	0	0
✓	1	F3		78,33	87	87	0	0	0	0	0	0	0	0	0
✓	9	F4		24,07	75	50	0	0	0	0	0	0	0	0	0
✓	4	F5		50	100	33	0	0	0	0	0	0	0	0	0
✓	2	F6		69,44	95	63	0	0	0	0	0	0	0	0	0
✓	8	F7		34,26	70	70	0	0	0	0	0	0	0	0	0
✓	7	F8		35,19	80	53	0	0	0	0	0	0	0	0	0
✓	3	F9		67,96	83	83	0	0	0	0	0	0	0	0	0
No.	9	Sum		448,5	748	569	0	0	0	0	0	0	0	0	0
		Median		47,2	83	63	0	0	0	0	0	0	0	0	0
		Mean		49,8	83	63	0	0	0	0	0	0	0	0	0
		Standard Deviation		18,4	10	17	0	0	0	0	0	0	0	0	0
		Maximum		78,3	100	87	0	0	0	0	0	0	0	0	0
		Minimum		24,1	70	33	0	0	0	0	0	0	0	0	0

Notes: Hasznosság = Utility; SMART érték = SMART value

Depiction of results and non-equilibrium attributes of SMART program output:



Notes: Input oldal = Input side; Hasznosság = Utility; SMART érték = SMART value

The 3D illustration shows not allowed differences compared to the state of equilibrium for F7 and F1 attributes. According to the analyses, the balancing of technological criteria systems (e.g. water usage) should be done using the optimum search with the Game Theory method.

A4: SMART CHART AND RESULTS ILLUSTRATIONS FOR OUTPUT ATTRIBUTES

Deterministics of Output (yellow side) attributes:

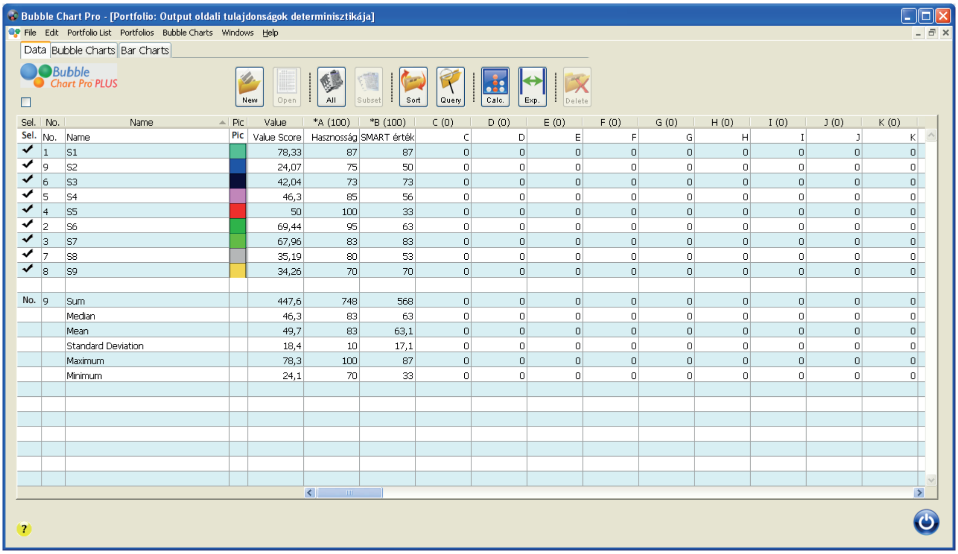
Generating SMART Output values for data insertion (’►X’ = usefulness)

MIDDLE YELLOW (1D) – energy-efficient resource use/profitable production (Y)	
EDGE YELLOW (2D)	(100)
minimizing technological risk (RY),	100/50 ► 75
strategic conformity of the energy-CO2 scale (GY),	100/70 ► 85
tax and discount opportunities in energy production system (OY)	100/90 ► 95
financial criteria of artificial and actual advancement to market (BY)	100/60 ► 80
CORNER YELLOW (3D)	
technology and regulation matching strategic goal systems (YGR),	100/70/50 ► 73,3
planning option acceptable financially and by market (YOB),	100/90/60 ► 83,3
presence of technological requirements sustainable in market environment (YBR),	100/60/50 ► 70
technological solution is real financial option long-term (YOG),	100/90/70 ► 86,6

SMALL CUBE NUMBER	USEFULNESS (1-100) (rounded)	CONNECTION, DIMENSION VALUE	SMART VALUE (V)	CUBE TYPE AND DOMINANCE	MAIN AGENT INHERENT ATTRIBUTES
S1	**87**	3/3	**87**	corner cube with more dominant attribute	technological solution is real financial option long-term (YOG)
S2	**75**	2/3	**50**	edge cube with more dominant attribute	minimizing technological risk (RY)
S3	**73**	3/3	**73**	corner cube with average attribute	technology and regulation matching strategic goal systems (YGR)
S4	**85**	2/3	**56**	edge cube with average attribute	strategic conformity of the energy-CO2 scale (GY)
S5	**100**	1/3	**33,3**	MIDDLE CUBE, dominance comparison	energy-efficient resource use (Y)
S6	**95**	2/3	**63**	edge cube with more dominant attribute	tax and discount opportunities in energy production system (OY)
S7	**83**	3/3	**83**	corner cube with less dominant attribute	planning option acceptable financially and by market (YOB)
S8	**80**	2/3	**53**	edge cube with average attribute	financial criteria of artificial and actual advancement to market (BY)
S9	**70**	3/3	**70**	corner cube with average attribute	presence of technological requirements sustainable in market environment (YBR)

Source: self-researched.

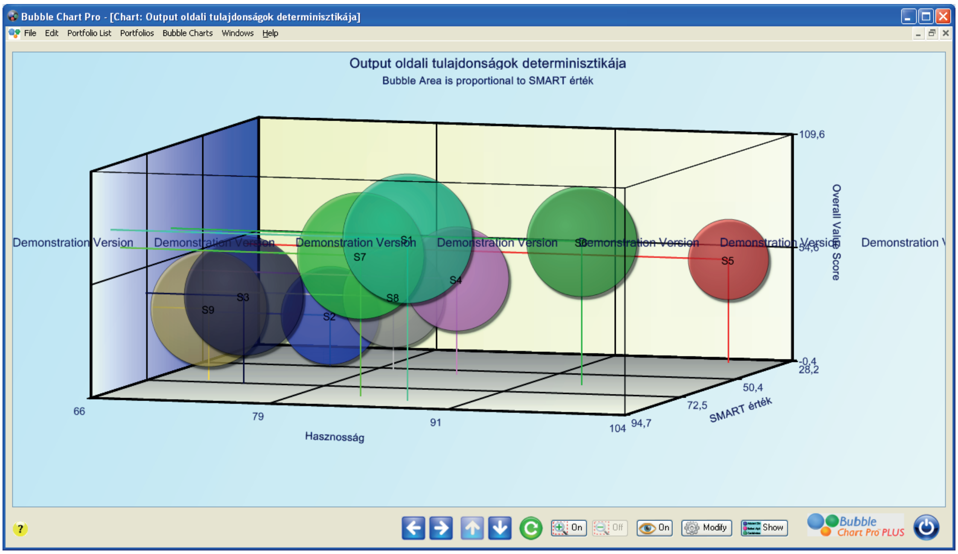
Character table of Output side in SMART program:



Sel.	No.	Name	Pic	Value Score	*A (100) Hasznosság	*B (100) SMART érték	C (0)	D (0)	E (0)	F (0)	G (0)	H (0)	I (0)	J (0)	K (0)
✓	1	S1		78,33	87	87	0	0	0	0	0	0	0	0	0
✓	9	S2		24,07	75	50	0	0	0	0	0	0	0	0	0
✓	6	S3		42,04	73	73	0	0	0	0	0	0	0	0	0
✓	5	S4		46,3	85	56	0	0	0	0	0	0	0	0	0
✓	4	S5		50	100	33	0	0	0	0	0	0	0	0	0
✓	2	S6		69,44	95	63	0	0	0	0	0	0	0	0	0
✓	3	S7		67,96	83	83	0	0	0	0	0	0	0	0	0
✓	7	S8		35,19	80	53	0	0	0	0	0	0	0	0	0
✓	8	S9		34,26	70	70	0	0	0	0	0	0	0	0	0
No.	9	Sum		447,6	748	568	0	0	0	0	0	0	0	0	0
		Median		46,3	83	63	0	0	0	0	0	0	0	0	0
		Mean		49,7	83	63,1	0	0	0	0	0	0	0	0	0
		Standard Deviation		18,4	10	17,1	0	0	0	0	0	0	0	0	0
		Maximum		78,3	100	87	0	0	0	0	0	0	0	0	0
		Minimum		24,1	70	33	0	0	0	0	0	0	0	0	0

Notes: Hasznosság = Utility; SMART érték = SMART value

3D illustration of Output side attributes in SMART program:



Notes: Output oldali tulajdonságok determinisztikája = Output side attributes; Hasznosság = Utility; SMART érték = SMART value

The 3D illustration shows not allowed differences compared to the state of equilibrium for S9 and S3 attributes. According to the analyses, the balancing of the market criteria system (e.g. proper market platform) and services should be done using the optimum search with the Game Theory method.

M5: SMART CHART AND RESULTS ILLUSTRATIONS FOR MIDDLE CUBE ATTRIBUTES

Deterministics of middle cube attributes:

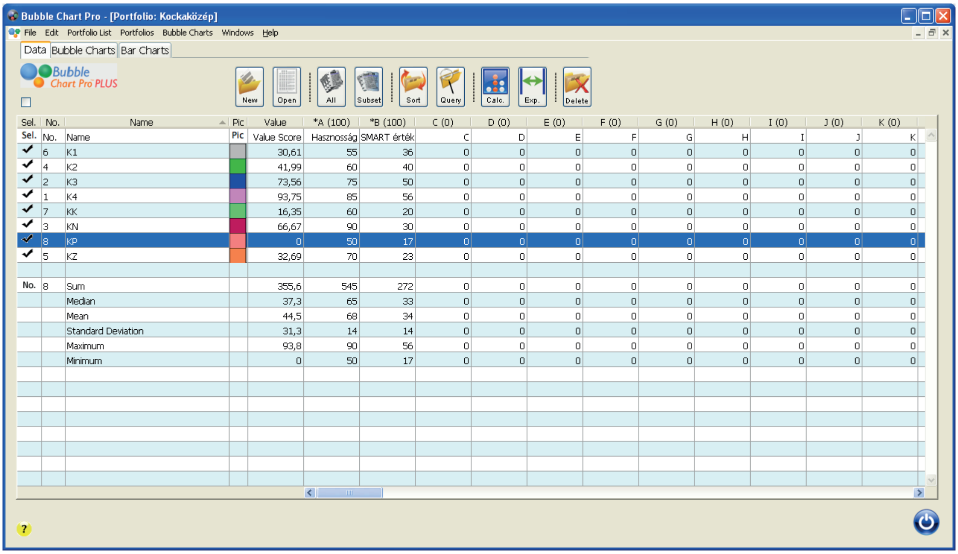
Generating SMART output values for data insertion ('►X' = usefulness)

MIDDLE CUBES (1D)	
KN – Orange middle – return time, value of firm	► (90)
KZ – Green middle – realization of local/corporate strategy	► (85)
KK – Blue middle – price plannable in supply/demand equilibrium	► (95)
KP – Red middle – proper technological use	► (80)
EDGE CUBES (2D)	
K1 – tracking market strategy and syncing it with technological priorities (BR)	60/50 ► 55
K2 – matching parameters of economic tech systems to regulations (GR)	70/50 ► 60
K3 – analysis of market changes, foreign currency risk factors, and global effects (OB)	90/60 ► 75
K4 – designating most economic strategic goal with long-term profits (OG)	90/70 ► 85

SMALL CUBE NUMBER	USEFULNESS (1-100) (rounded)	CONNECTION, DIMENSION VALUE	SMART VALUE (V)	CUBE TYPE AND DOMINANCE	MAIN AGENT INHERENT ATTRIBUTES
KN	**90**	1/3	**30**	MIDDLE CUBE (1D) – return time, value of firm (O)	SUMMARIZING FINANCIAL EFFECTS
KZ	**70**	1/3	**23**	MIDDLE CUBE (1D) – realization of local/corporate strategy (G)	STRATEGIC PROGRAM PLACEMENT/ LAW AND REGULATION CRITERIA
KK	**60**	1/3	**20**	MIDDLE CUBE (1D) – price plannable in supply/demand equilibrium (B)	ANALYSIS OF MARKET OPPORTUNITIES
KP	**50**	1/3	**17**	MIDDLE CUBE (1D) – proper technological use (R)	TECHNOLOGICAL CRITERIA SYSTEM
K1	**55**	2/3	**36**	EDGE CUBE tracking market strategy, and syncing it with technological priorities (BR)	ANALYSIS OF MARKET OPPORTUNITIES / TECHNOLOGICAL CRITERIA SYSTEM
K2	**60**	2/3	**40**	EDGE CUBE matching parameters of economic tech systems to regulations (GR)	STRATEGIC, LAW AND REGULATION CRITERIA / TECHNOLOGICAL CRITERIA SYSTEM
K3	**75**	2/3	**50**	analysis of market changes, foreign currency risk factors, and global effects (OB)	SUMMARIZING FINANCIAL EFFECTS / ANALYSIS OF MARKET OPPORTUNITIES
K4	**85**	2/3	**56**	EDGE CUBE designating most economic strategic goal with long-term profits (OG)	SUMMARIZING FINANCIAL EFFECTS / STRATEGIC, LAW AND REGULATION CRITERIA

Source: self-researched.

Character table of middle cube side-attributes in SMART program:

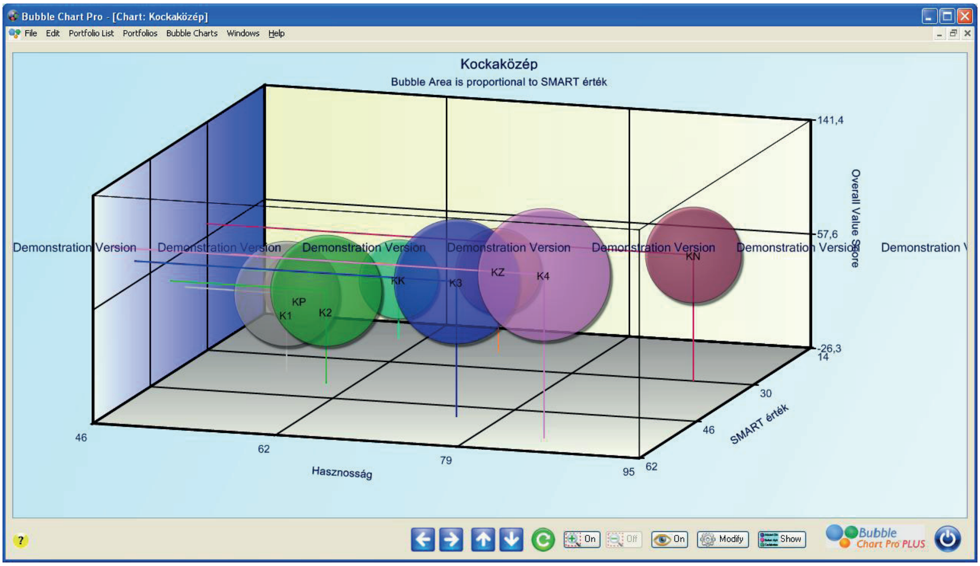


Bubble Chart Pro - [Portfolio: Kockaközép]

Sel.	No.	Name	Pic	Value Score	Hasznosság	SMART érték	C	D	E	F	G	H	I	J	K
✓	6	K1		30,61	55	36	0	0	0	0	0	0	0	0	0
✓	4	K2		41,99	60	40	0	0	0	0	0	0	0	0	0
✓	2	K3		73,56	75	50	0	0	0	0	0	0	0	0	0
✓	1	K4		93,75	85	56	0	0	0	0	0	0	0	0	0
✓	7	KK		16,35	60	20	0	0	0	0	0	0	0	0	0
✓	3	KN		66,67	90	30	0	0	0	0	0	0	0	0	0
✓	8	KP		0	50	17	0	0	0	0	0	0	0	0	0
✓	5	KZ		32,69	70	23	0	0	0	0	0	0	0	0	0
No.	8	Sum		355,6	545	272	0	0	0	0	0	0	0	0	0
		Median		37,3	65	33	0	0	0	0	0	0	0	0	0
		Mean		44,5	68	34	0	0	0	0	0	0	0	0	0
		Standard Deviation		31,3	14	14	0	0	0	0	0	0	0	0	0
		Maximum		93,8	90	56	0	0	0	0	0	0	0	0	0
		Minimum		0	50	17	0	0	0	0	0	0	0	0	0

Notes: Hasznosság = Utility; SMART érték = SMART value

3D illustration of middle cube attributes in SMART program:



Notes: Kockaközép = Middle Cube Area; Hasznosság = Utility; SMART érték = SMART value

The 3D illustration shows not allowed differences compared to the state of equilibrium for KN and K4 attributes. According to the analyses, the balancing of the orange middle cube main attribute set's "Project's return value and time" should be done using the optimum search with the Game Theory method.

A6: APPENDIX OF ILLUSTRATIONS

A7: APPENDIX OF TABLES

ACKNOWLEDGEMENTS

I would like to thank for the professors of Szent Istvan University PhD School of Business Management their extensive support and generous help.