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COMPARISON OF DEA MODELS APPLIED FOR EVALUATION OF THE RESULTS OF A PRODUCTION SIMULATION GAME

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Summary: Data Envelopment Analysis (DEA) is a method for comparing the efficiency of decision making units when the output of these units is evaluated based on the amount of inputs used. A special application area of DEA is the evaluation of student groups participating in a production simulation game. This paper shows how DEA is used to compare the performance of student groups in the simulation game, and how their results can be evaluated using the efficiency scores.
Different DEA models are used to capture the different characteristics of the operation. Basic models with radial efficiency measures are used to analyze the effect of input and output weights, and to separate the proportional decrease of inputs from the independent input reduction possibilities. Slack based measure models are applied to study the joint effect of proportional and independent input/output changes. Dynamic models are used to study the change of efficiency over time.
The paper compares the results of the applied models and analyses the differences. The results show that the application of the assurance regain models is strongly recommended. The presence of negative outputs requires the application of models which can be adapted to negative data.

Keywords: Data Envelopment Analysis, linear programming, performance evaluation, simulation game

1. Introduction

Data Envelopment Analysis (DEA) is a mathematical programming approach that is used for comparing the efficiency of decision making units (DMU) such as production and/or service systems. In contrast to other methods (e.g. ratio methods) that is used for performance evaluation, DEA is capable of handling multiple inputs and multiple outputs as well. DEA was first introduced by Charnes, Cooper and Rhodes for evaluating nonprofit organizations. In the last few decades DEA has been extensively used and investigated and it became an important research area. Several applications of DEA are reported in the literature both in service and in production sector as well.There is no any single DEA method which is always the best. Different application environments have generated different evaluation problems thus several variants of DEA models have been developed.
In this paper, we apply DEA in a higher education context to compare the performance of student groups in a production simulation game. We propose different DEA models to capture the different characteristics of the operation. In the following part of this paper first different DEA models are introduced. Next, the application environment is presented and the important differences between the suggested DEA models for the production simulation application are discussed. Finally, conclusions are drawn and the areas of future research are summarised.

2. Variation of DEA models

The objective of DEA models is to determine the efficiency of decision making units relative to each other by using the ratio of weighted output and weighted input. When we calculate this ratio we make a difference between *input oriented* and *output oriented* methods, which depends on the purpose of evaluation. In the case of input oriented models the objective is to minimize inputs while satisfying the given output levels. In the case of output oriented models the objective is to maximize outputs without requiring more of any of the observed inputs. The highest value of efficiency is equal to 1 and the lowest value is equal to 0.
The models may have a different approach to the marginal change of output. When we assume a *constant return to scale (CRS)* – or CCR – relationship between the input and output values, the size of the input does not influence the marginal change of output. When the effect of the unit change of input is not constant then a *variable return to scale (VRS)* – or BCC – relationship is assumed. (Cooper et. al, 2007)
In Data Envelopment Analysis the two most important group of efficiency measures are the radial measures and the non-radial measures. Radial measures are applied, for example, in the cases of CCR and BCC models, whereas non-radial measures are applied in the case of *slacks-based measure (SBM)* models (Tone 1999). The radial models provide information about the proportional change of all inputs (all outputs). It is assumed that all inputs (all outputs) must be decreased (increased) by the same proportion. Note, that independent input/output changes can also be explored in the second phase of this approaches. In contrast, the non-radial models search for the maximum input decrease and/or maximum output increase with the help of an objective function using the slack variables.
When CCR or BCC models are solved, generally there are several weights with zero value. From management point of view this is not acceptable, because management want to consider each output/input at the evaluation. Furthermore, large differences in weights may cause misleading evaluation. The application of the *assurance region (AR) method* helps to overcome these shortcomings of these models, by imposing constraints on the relative magnitude of the weights.
Traditionally DEA models have required the assumption that all the input and output values are semi-positive. In many applications, however, negative inputs or outputs may appear, such as loss when profit is an output variable. Many models are developed to handle negative data (for example Portela et al 2004). Sharp et al. (2006) introduced a *modified slacks based measure (MSBM)* in which both negative outputs and negative inputs may occur.
Similar to MSBM models, the *Semi-Oriented Radial Measure (SORM)* model proposed by Emrouznejad (2010), can deal with negative data. The key idea that makes SORM model different from MSBM is that it replaces an input/output variable which can take positive values for some and negative values for other DMUs by two non negative variables. One variable is for used for the positive data of the original variable, and the other variable is for the negative data of the original variable.
There exist input or output oriented and constant or variable return to scale version of the aforesaid models. In addition, further DEA models have been developed to solve new problems and to overcome the drawbacks of the earlier models.

3. Application environment

We analysed a production simulation game, which is developed by Ecosim to support education and training in the production management area (www.ecosim.hu). The objective of the game is to simulate production management decision making in a car engine manufacturing factory. The factory produces three different car engines for five different markets in 7 periods. Each market has its own demand characteristics. The car engines are assembled from parts on assembly lines operated by workers.

For the next production period (year) each student group must make sales and marketing, production, investment and financial decisions. After submitting the decisions, the simulation program generates the results of the actual production period. The results are summarized in a production report and in a financial report. Using the results and experiences of the earlier periods the student groups try to increase operational performance of the next periods.

We used different input – oriented DEA models for evaluating the performance of student groups at the end of the seventh period of the simulation game. In all cases we applied a constant return to scale model, because there is not size difference between the DMUs, thus a variable return to scale approach is not relevant.

Two outputs and four inputs were considered in the analysis. In our previous papers we presented the evaluation of the performance of student groups using different outputs (Koltai, Uzonyi 2012). In this paper the results of several DEA models addressing various modeling problems are presented. One of the outputs is *cumulated production quantity* which reflects the effect of production management decisions related to machine and worker capacity, to material requirement planning and to inventory management. The other output is *net profit* which integrates the effect of marketing, production and financial decisions. The four inputs − *cumulated number of workers, cumulated number of machine hours, cumulated sum of money spent on raw materials* and *cumulated value of credits* − represent the resources used in the production process. Consequently, the performance of the production system based on these decisions reflects student's knowledge in the related areas.

4. Results and comparison of DEA models

The performance of 18 student groups is compared using input oriented CRS, CRS-AR, SBM, MSBM and SORM model. The results are summarized in Table 1.

Table 1: Efficiency results of DEA models

Team	Output 1 Net profit	Output 2 Production Quantity	CRS	CRS-AR	SBM	MSBM	SORM
1	0,650	2,701	**1,0000**	0,9281	**1,00000**	**1,0000**	**1,0000**
2	0,097	2,714	**1,0000**	0,8109	**1,00000**	**1,0000**	**1,0000**
3	1,874	2,911	**1,0000**	**1,0000**	**1,00000**	**1,0000**	**1,0000**
4	0,186	2,448	0,9732	0,8750	0,22036	0,7033	0,9732
5	-0,269	2,327	0,9579	0,7583	**1,00000**	0,5192	0,9579
6	0,046	2,573	0,9823	0,8583	0,07051	0,6846	0,9823
7	1,656	2,778	**1,0000**	**1,0000**	**1,00000**	**1,0000**	**1,0000**
8	1,007	2,553	0,9917	0,9152	0,62730	0,6043	0,9917
9	1,714	2,977	**1,0000**	0,9999	**1,00000**	**1,0000**	**1,0000**
10	1,051	2,836	0,9982	0,9351	0,88190	0,8757	0,9982
11	0,987	2,440	0,9982	0,9473	0,75962	0,7461	0,9982
12	0,183	2,466	0,9798	0,8647	0,19680	0,6468	0,9798
13	0,675	2,368	0,9322	0,8020	0,47361	0,5573	0,9322
14	1,729	2,650	**1,0000**	0,9859	**1,00000**	**1,0000**	**1,0000**
15	0,879	2,665	**1,0000**	**1,0000**	**1,00000**	**1,0000**	**1,0000**
16	0,197	2,487	0,9508	0,8305	0,18356	0,5641	0,9508
17	0,667	2,964	0,9053	0,8250	0,42232	0,0676	0,9053
18	0,799	2,553	0,9867	0,8731	0,69966	0,7184	0,9867

Source: the authors own table

Column 2 and 3 shows the values of the two outputs applied in the evaluation. These data are properly scaled to avoid numerical problems. Column 4-8 shows the efficiency scores of the different models. In those models, which can not handle negative data, negative values were substituted by zero.
Using the basic input oriented CRS model, 7 student groups have the highest possible efficiency score. The results show that the operation of almost half of the DMUs is efficient. Furthermore the value of the efficiency score of inefficient groups is close to 1, which indicate a low discrimination power of the model. In this case, a large number of input and output weights are zero, consequently, for example, the profit has insignificant effect on the obtained efficiency scores.
Applying weight restrictions (CCR-AR), it can be observed that all DMU obtained lower scores. The number of the efficient groups is also reduced, only groups 3, 7 and 15 remained at the status of full efficiency. We applied 0.1 for the pairwise relative lower limit of the inputs, and 0.25 for the lower limit of the ratio of outputs.
It is proved, that the efficiency score of the SBM models is not greater than the CRS efficiency values (Tone, 1999). In addition, a DMU is CCR efficient if it is SBM efficient. Consequently, CCR efficient student groups remained at the efficient status under SBM evaluation. The values of SBM score of most of the inefficient groups are lower than the CRS scores. Group 5 has higher efficiency score with SBM than with CRS evaluation. This contradiction indicates that the SBM model can not be applied in this case. Note, that group 5 has negative net profit, consequently output are not semi-positive, and the efficiency scores are theoretically erroneous.
MSBM and SORM models can be used to handle negative data. According to Table 1. MSBM and SORM selected the same DMUs as efficient, but different target values are recommended. The target values recommended by the CRS, and the MSBM models for a selected student group (Team 10) is presented in Table 2. It can be seen, that the MSBM target values indicate a slightly smaller input reduction, than that of the CRS values, but with a higher production quantity.

Table 2: Target values of Team 10

	Production quantity	***Net profit***	***No. workers***	***Machine hours***	***Raw materials***	***Debt***
Original	2 836 320	1 050 699	13 662	3 284 436	5 608 796	1 632 000
CRS	2 836 320	1 050 699	12 489	3 278 483	5 410 413	1 123 659
MSBM	2 8418 60	1 050 699	12 514	3 284 436	5 423 519	1 124 375

Source: the authors own table

Note, that the SORM efficiency scores are identical with the CRS efficiency scores in Table 1. This can be explained by the fact, that Team 5 is the only team with negative output value. In this special case, the constraint belonging to this unfavourable output does not influence the production possibility set, and consequently the efficiency scores.

5. Conclusion

This article compared the results of different DEA models when the performance of student groups in a production simulation game is evaluated. Basic models with radial efficiency measures are used to analyse the effect of input and output weights, and to separate the proportional decrease of inputs from the independent input reduction possibilities. Slack based measure models are applied to study the joint effect of proportional and independent input/output changes. The results show the advantage of the application of the assurance

regain model. The presence of negative outputs requires the application of models which can be adapted to negative data.
We note, that evaluation of the teams in the presented cases is based on aggregated input and output values. The inputs and outputs in the 7 production periods are simply accumulated. The dynamic behaviour of the teams is not reflected in this aggregate approach. Applying *Dynamic DEA models* can capture the progress of teams during the decision-making process and may providing a more detailed picture about the learning process. (Koltai, Uzonyi 2013)

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