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QUASI-POLYNOMIAL CONTROL OF A SYNCHRONOUS GENERATOR

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A simple dynamic model of permanent magnet synchronous generator, that is used for electrical energy generation is investigated in this work using a nonlinear technique based on the quasi-polynomial representation of the dynamical model. It is well known that dynamical systems with smooth nonlinearities can be embedded in a quasi-polynomial model. Quasi-polynomial systems are good candidates for a general nonlinear system representation since their global stability analysis is equivalent to the feasibility of a LMI. Moreover, the stabilizing quasi-polynomial state feedback controller design problem is equivalent to the feasibility of a bilinear matrix inequality. The classical stabilizing state feedback problem for quasi-polynomial systems has been extended in this work with the ability of tracking time-dependent reference signals. It is shown, that the stabilizing quasi-polynomial servo controller design is equivalent to a bilinear matrix inequality. The results are applied to the model of a synchronous generator.

Keywords: quasi-polynomial systems, Lotka-Volterra systems, stability analysis, state feedback control, synchronous generator, wind turbine

Introduction

Electrical power systems should operate in an economic way with minimum possible operating cost under normal operating conditions. To ensure this, a preventive controller for power systems has been presented in [1]. It encompasses many types of control actions, including generation rescheduling, load curtailment and network switching reactive compensation.

From the viewpoint of the power grid, the electric power generation can be characterized by the operation of the electrical generators, the subject of our study. These power plants should not only be able to follow the time-varying active power demand of the consumers and the central dispatch center, but also keep the quality indicators (frequency, waveform, total harmonic distortion) of the grid on the expected level. This can be achieved by applying proper control methods based on dynamic models of plant (see e.g. [2, 3]) and the involved generators.

Because of the specialties and great practical importance of the synchronous generators in power plants, their modeling for control purposes is also well investigated in the literature. Besides of the basic textbooks (see e.g. [4]) that describe the modeling, specialized papers are also available that use the developed models for the design of various controllers [5].

A wind turbine driving permanent magnet synchronous generator is proposed in [6] with current controlled voltage source inverter, which is the best choice when the output power is small. The current control of the voltage source inverter has bidirectional active and reactive power control ability, avoiding the intricacy of the controller designing.

The class of quasi-polynomial (QP) systems plays an important role in the theory of nonlinear dynamical systems because nonlinear systems with smooth nonlinearities can be transformed into a QP form [7]. This means, that any applicable method for QP systems can be regarded as a general technique for nonlinear systems.

Previous work in the field of QP systems include [8], which proves that the global stability analysis of QP systems is equivalent to the feasibility of a linear matrix inequality (LMI). It has been shown in [9] that the globally stabilizing state feedback design for QP systems is equivalent to a bilinear matrix inequality (BMI). Although the solution of a BMI is an NP-hard problem, an iterative LMI algorithm could be used. A summary of linear and bilinear matrix inequalities and the available software tools for solving them can be found in [10]. Another control synthesis algorithm for polynomial systems is presented in [11].

The goal of this paper is to formulate the servo controller design problem for QP systems based on the results presented in [9] and to design a servo controller for a synchronous generator model using the QP controller synthesis methodology that keeps the active power at the desired level.

Basic notions

In what follows, the basic modeling assumptions and definitions to be used in the sequel are summarized briefly.

Nonlinear model of a synchronous generator

The modeling procedure of the synchronous generator is mainly based on [12] and [13], therefore, only the resulting model is presented here.

The model is based on the following simplification assumptions:

- a symmetrical tri-phase stator winding system is assumed,
- one field winding is considered to be in the machine,
- all of the windings are magnetically coupled,
- the flux linkage of the winding is a function of rotor position,
- the copper loss and the slots in the machine can be neglected,
- the spatial distribution of the stator fluxes and apertures wave are considered to be sinusoidal, and
- stator and rotor permeability are assumed to be infinite.

It is also assumed that all the losses due to wiring, saturation, and slots can be neglected.

The four windings (three stators and one rotor) are magnetically coupled. Since the magnetic coupling between the windings is a function of the rotor position, the flux linkage of the windings is also a function of the rotor position. The actual terminal voltage v of the windings can be written in the form

$$v = \pm \sum_{j=1}^{J} (r_j\, i_j) \pm \sum_{j=1}^{J} \left(\frac{\mathrm{d}\varphi_j}{\mathrm{d}t} \right), \tag{1}$$

where i_j are the currents, r_j are the winding resistances, and φ_j are the flux linkages. The positive directions of the stator currents point out of the synchronous generator terminals.

Thereafter, the two stator electromagnetic fields, both traveling at rotor speed, can be identified by decomposing each stator phase current under steady state into two components, one in phase with the electromagnetic field and another phase shifted by $90°$. With the above, one can construct an air-gap field with its maximal aligned to the rotor poles (d axis), while the other is aligned to the q axis (between poles). This method is called the Park's transformation.

As a result of the derivation in [12] the vector voltage equation is as follows

$$\boldsymbol{v}_{dFq} = -\boldsymbol{R}\,\boldsymbol{i}_{dFq} - \boldsymbol{L}\,\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_{dFq}, \tag{2}$$

with

$$\begin{aligned} \boldsymbol{v}_{dFq} &= \begin{bmatrix} v_d & -v_F & v_q \end{bmatrix}^T \\ \boldsymbol{i}_{dFq} &= \begin{bmatrix} i_d & i_F & i_q \end{bmatrix}^T, \end{aligned} \tag{3}$$

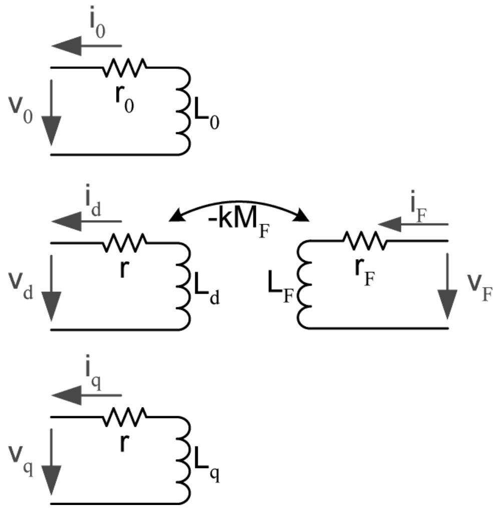

Figure 1: The equivalent circuit of the synchronous generator.

where v_d, v_q and i_d, i_q are the direct and the quadrature components of the stator voltage and current of the synchronous generator, while v_F and i_F are the exciter voltage and current of the synchronous generator. Furthermore, $\boldsymbol{R}$ and $\boldsymbol{L}$ are the following matrices (see *Fig.*1)

$$\begin{aligned} \boldsymbol{R_{RS\omega}} &= \begin{bmatrix} r & 0 & \omega\, L_q \\ 0 & r_F & 0 \\ -\omega\, L_d & -\omega\, k\, M_F & r \end{bmatrix} \\ \boldsymbol{L} &= \begin{bmatrix} L_d & k\, M_F & 0 \\ k\, M_F & L_F & 0 \\ 0 & 0 & L_q \end{bmatrix}, \end{aligned} \tag{4}$$

where r is the stator resistance, r_F is the exciter resistance of the Synchronous Generator, L_d, and L_q are the direct and the quadrature part of the stator and rotor inductance, ω is the angular velocity, and M_F is linkage inductances. The state-space model for the currents is obtained by expressing $\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_{dFq}$ from *Eq. (2)*, i.e.

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_{dFq} = -\boldsymbol{L}^{-1}\,\boldsymbol{R}_{RS\omega}\,\boldsymbol{i}_{dFq} - \boldsymbol{L}^{-1}\,\boldsymbol{v}_{dFq}. \tag{5}$$

The purely electrical model *Eq. (5)* has to be extended with the equation of rotational motion (*Eq. (6)*) that gives the mechanical sub-dynamics, that is

$$\begin{aligned} \frac{\mathrm{d}\omega}{\mathrm{d}t} &= -\frac{L_d\, i_q}{3\tau_j} i_d + \frac{-k\, M_F\, i_q}{3\tau_j} i_F + \\ &+ \frac{L_q\, i_d}{3\tau_j} i_q + \frac{-D}{\tau_j}\omega + \frac{T_{\mathrm{mech}}}{\tau_j}. \end{aligned} \tag{6}$$

$$\begin{bmatrix} \dot{i}_d \\ \dot{i}_F \\ \dot{i}_q \\ \dot{\omega} \end{bmatrix} =$$

$$\begin{bmatrix} -\frac{L_F}{H} & \frac{k M_F}{H} & 0 & 0 \\ \frac{k M_F}{H} & -\frac{L_d}{H} & 0 & 0 \\ 0 & 0 & \frac{1}{L_q} & 0 \\ 0 & 0 & 0 & \frac{1}{\tau_j} \end{bmatrix} \begin{bmatrix} -v_d \\ v_F \\ -v_q \\ T_{mech} \end{bmatrix} +$$

$$\begin{bmatrix} -\frac{rL_F}{H} & \frac{kM_F r_F}{H} & -\frac{\omega L_F L_q}{H} & 0 \\ \frac{rkM_F}{H} & -\frac{r_F L_d}{H} & \frac{\omega k M_F L_q}{H} & 0 \\ -\frac{\omega L_d}{L_q} & \frac{\omega k M_F}{L_q} & \frac{r}{L_q} & 0 \\ -\frac{L_d\, i_q}{3\tau_j} & \frac{-k\, M_F\, i_q}{3\tau_j} & \frac{L_q\, i_d}{3\tau_j} & \frac{-D}{\tau_j} \end{bmatrix} \begin{bmatrix} i_d \\ i_F \\ i_q \\ \omega \end{bmatrix} \tag{7}$$

where

$$H = k^2 M_F^2 - L_d L_F$$

$$\begin{bmatrix} \dot{i}_d \\ \dot{i}_F \\ \dot{i}_q \\ \dot{\omega} \end{bmatrix} =$$

$$\begin{bmatrix} -1.7100 & 0.5893 & 0 & 0 \\ 0.5893 & -6.6918 & 0 & 0 \\ 0 & 0 & -1.7090 & 0 \\ 0 & 0 & 0 & 0.0006 \end{bmatrix} \begin{bmatrix} -v_d \\ v_F \\ -v_q \\ T_{mech} \end{bmatrix} +$$

$$\begin{bmatrix} -0.0019 & 0.0004 & -3.4883\,\omega & 0 \\ 0.0006 & -0.0042 & 1.2022\,\omega & 0 \\ 3.5888\,\omega & 2.6489\,\omega & -0.0019 & 0 \\ -0.0004\, i_q & -0.0003\, i_q & 0.0004\, i_d & -0.0011 \end{bmatrix} \begin{bmatrix} i_d \\ i_F \\ i_q \\ \omega \end{bmatrix} \tag{8}$$

Altogether, the state equations *Eqs.* (5) and (6) have four state variables: i_d, i_F, i_q, and ω.

The manipulated input vector of the generator is $\boldsymbol{u} = \begin{bmatrix} v_F & T_{mech} \end{bmatrix}^T$, while the disturbance input vector is $\boldsymbol{d} = \begin{bmatrix} v_d & d_q \end{bmatrix}^T$. Realize that the state equations *Eqs.* (5) and (6) are bilinear in the state variables because matrix $\boldsymbol{R_{RS\omega}}$ depends linearly on ω. The obtained model is nonlinear and it has four state variables: i_d , i_F, i_q and ω [14].

Output equations of the model: The output active power equation can be written in the following form:

$$p_{out} = v_d i_d + v_q i_q \tag{9}$$

and the reactive power is

$$q_{out} = v_d i_q - v_q i_d. \tag{10}$$

Eqs. (9) and (10) are the output equations of the generator's state space model. It is important to note, that these equations are bi-linear in the state and input variables. Note, that although only the active power is to be controlled in this case, as a possible future extension of the work, reactive power can also be controlled in order to follow an external reference signal.

Quasi-polynomial representation of nonlinear systems

Let us denote the element of an arbitrary matrix $\boldsymbol{W}$ with row index i and column index j by W_{ij}. Quasi-polynomial models are systems of ordinary differential equations (ODE) of the following form

$$\dot{y}_i = y_i \left(L_i + \sum_{j=1}^{m} A_{ij} \prod_{k=1}^{n} y_k^{B_{jk}} \right), \quad i = 1, \ldots, n. \tag{11}$$

where $y \in \text{int}(\mathbb{R}^n_+)$, $A \in \mathbb{R}^{n\times m}$, $B \in \mathbb{R}^{m\times n}$, $L_i \in \mathbb{R}$, $i = 1, \ldots, n$. Furthermore, $\boldsymbol{L} = [L_1 \ \ldots \ L_n]^T$. Let us denote the equilibrium point of interest of *Eq. (11)* as $\boldsymbol{y^*} = [y_1^* \ y_2^* \ \ldots \ y_n^*]^T$. Without the loss of generality we can assume that $\text{rank}(\boldsymbol{B}) = n$ and $m \geq n$ (see [15]).

Lotka-Volterra models

The above family of models is split into classes of equivalence [16] according to the values of the products $\boldsymbol{M} = \boldsymbol{B}\,\boldsymbol{A}$ and $\boldsymbol{N} = \boldsymbol{B}\,\boldsymbol{L}$. The LOTKA-VOLTERRA form gives the representative elements of these classes of equivalence. If $\text{rank}(\boldsymbol{B}) = n$, then the set of ODEs in *Eq. (11)* can be embedded into the following *m*-dimensional set of equations, the so called LOTKA-VOLTERRA model:

$$\dot{z}_j = z_j \left(N_j + \sum_{i=1}^{m} M_{ji} z_i \right), \quad j = 1, \ldots, m \tag{12}$$

where

$$\boldsymbol{M} = \boldsymbol{B}\,\boldsymbol{A}, \quad \boldsymbol{N} = \boldsymbol{B}\,\boldsymbol{L},$$

and each z_j represents a so called quasi-monomial:

$$z_j = \prod_{k=1}^{n} y_k^{B_{jk}}, \qquad j = 1, \ldots, m. \tag{13}$$

Stability analysis using linear matrix inequalities

Henceforth it is assumed that $\boldsymbol{y}^*$ is a positive equilibrium point, i.e. $\boldsymbol{y}^* \in \text{int}(\mathbb{R}^n_+)$ in the QP case and similarly $\boldsymbol{z}^* \in \text{int}(\mathbb{R}^m_+)$ is a positive equilibrium point in the LOTKA-VOLTERRA case. For LV systems there is a well known candidate LYAPUNOV function family ([8, 17]), which is in the form:

$$V(\boldsymbol{z}) = \sum_{i=1}^{m} c_i \left(z_i - z_i^* - z_i^* \ln \frac{z_i}{z_i^*} \right), \tag{14}$$

$$c_i > 0, \quad i = 1 \ldots m,$$

where $\boldsymbol{z}^* = \begin{bmatrix} z_1^* & \ldots & z_m^* \end{bmatrix}^T$ is the equilibrium point corresponding to the equilibrium $\boldsymbol{y}^*$ of the original QP system (*Eq. (11)*). The time derivative of the LYAPUNOV function *Eq. (14)* is:

$$\dot{V}(\boldsymbol{z}) = \frac{1}{2}(\boldsymbol{z} - \boldsymbol{z}^*)(\boldsymbol{C}\,\boldsymbol{M} + \boldsymbol{M}^T\boldsymbol{C})(\boldsymbol{z} - \boldsymbol{z}^*), \tag{15}$$

where $\boldsymbol{C} = \text{diag}(c_1, \ldots, c_m)$ and $\boldsymbol{M}$ is the invariant characterizing the LOTKA-VOLTERRA form (*Eq. (12)*). Therefore, the non-increasing nature of the LYAPUNOV function is equivalent to a feasibility problem over the following set of LMI constraints (see [18] or [19]):

$$\begin{aligned} \boldsymbol{C}\,\boldsymbol{M} + \boldsymbol{M}^T\boldsymbol{C} &\leq \boldsymbol{0} \\ \boldsymbol{C} &> \boldsymbol{0} \end{aligned} \tag{16}$$

where the unknown matrix is $\boldsymbol{C}$, which is diagonal and contains the coefficients of *Eq. (14)*.

The derivation of global stability analysis for nonautonomous QP systems from the autonomous case is straightforward. The LYAPUNOV function (*Eq. (15)*) also depends on the equilibrium value of the input ($\boldsymbol{u}^*$) and has the form

$$\dot{\tilde{V}}(\boldsymbol{z}) = \frac{1}{2}(\boldsymbol{z}-\boldsymbol{z}^*)(\boldsymbol{C}\,\tilde{\boldsymbol{M}} + \tilde{\boldsymbol{M}}^T\boldsymbol{C})(\boldsymbol{z}-\boldsymbol{z}^*), \quad (17)$$

where $\tilde{\boldsymbol{M}}$ depends on the coefficient matrices of the input-affine LOTKA-VOLTERRA model (*Eq. (21)*):

$$\tilde{\boldsymbol{M}} = \boldsymbol{M_0} + \sum_{l=1}^{p} \boldsymbol{M_l}\, u_l^*.$$

The corresponding LMI feasibility problem to be solved in order to check global asymptotic stability is

$$\begin{array}{rcl} \boldsymbol{C}\,\tilde{\boldsymbol{M}} + \tilde{\boldsymbol{M}}^T\boldsymbol{C} & \leq & \boldsymbol{0} \\ \boldsymbol{C} & > & \boldsymbol{0}. \end{array} \quad (18)$$

Input-affine QP system models

An input-affine nonlinear system model with state vector $\boldsymbol{y}$, input vector u and output vector $\boldsymbol{\eta}$

$$\begin{array}{rcl} \dot{\boldsymbol{y}} & = & \boldsymbol{f(y)} + \sum_{i=1}^{p} \boldsymbol{g_i(y)}u_i \\ \boldsymbol{\eta} & = & \boldsymbol{h(y)} \end{array} \quad (19)$$

is in QP-form if all of the functions $\boldsymbol{f}$, $\boldsymbol{g_i}$ and $\boldsymbol{h}$ are in QP-form. Then the general form of the state equation of an input-affine QP system model with p-inputs is:

$$\dot{y}_i = y_i \left(L_{0_i} + \sum_{j=1}^{m} A_{0_{ij}} \prod_{k=1}^{n} y_k^{B_{jk}} \right) + \quad (20)$$
$$+ \sum_{l=1}^{p} y_i \left(L_{l_i} + \sum_{j=1}^{m} A_{l_{ij}} \prod_{k=1}^{n} y_k^{B_{jk}} \right) u_l$$

where

$$i = 1,\dots,n, \quad \boldsymbol{A_0}, \boldsymbol{A_l} \in \mathbb{R}^{n\times m}, \quad \boldsymbol{B} \in \mathbb{R}^{m\times n},$$
$$\boldsymbol{L_0}, \boldsymbol{L_l} \in \mathbb{R}^n, \quad l = 1,\dots,p.$$

The corresponding input-affine LOTKA-VOLTERRA model is in the form

$$\dot{z}_j = z_j \left(N_{0_j} + \sum_{k=1}^{m} M_{0_{jk}} z_k \right) + \quad (21)$$
$$+ \sum_{l=1}^{p} z_j \left(N_{l_j} + \sum_{k=1}^{m} M_{l_{jk}} z_k \right) u_l$$

where

$$j = 1,\dots,m, \quad \boldsymbol{M_0}, \boldsymbol{M_l} \in \mathbb{R}^{m\times m},$$
$$\boldsymbol{N_0}, \boldsymbol{N_l} \in \mathbb{R}^m, \quad l = 1,\dots,p,$$

and the parameters can be obtained from the input-affine QP system's ones in the following way

$$\begin{array}{rcl} \boldsymbol{M_0} & = & \boldsymbol{B}\,\boldsymbol{A_0} \\ \boldsymbol{N_0} & = & \boldsymbol{B}\,\boldsymbol{L_0} \\ \boldsymbol{M_l} & = & \boldsymbol{B}\,\boldsymbol{A_l} \\ \boldsymbol{N_l} & = & \boldsymbol{B}\,\boldsymbol{L_l} \end{array} \qquad l = 1,\dots,p. \quad (22)$$

The controller design problem for QP systems

Globally stabilizing QP state feedback design problem for QP systems can be formulated as follows (for a more detailed desription, see [9]). Consider arbitrary QP inputs in the form:

$$u_l = \sum_{i=1}^{r} k_{il}\hat{q}_i, \quad l = 1\dots,p, \quad (23)$$

where $\hat{q}_i = \hat{q}_i(y_1,\dots,y_n)$, $i = 1,...,r$ are arbitrary quasi-monomial functions of the state variables of *Eq. (20)* and k_{il} is the constant gain of the quasi-monomial function $\hat{q}_i$ in the l-th input u_l. The closed loop system will also be a QP system with matrices

$$\hat{\boldsymbol{A}} = \boldsymbol{A_0} + \sum_{l=1}^{p}\sum_{i=1}^{r} k_{il}\boldsymbol{A_{il}}, \quad \hat{\boldsymbol{B}},$$
$$\hat{\boldsymbol{L}} = \boldsymbol{L_0} + \sum_{l=1}^{p}\sum_{i=1}^{r} k_{il}\boldsymbol{L_{il}}. \quad (24)$$

Note that the number of quasi-monomials in the closed-loop system (i.e. the dimension of the matrices) together with the matrix $\hat{\boldsymbol{B}}$ may significantly change depending on the choice of the feedback structure, i.e. on the quasi-monomial functions $\hat{q}_i$.

Furthermore, the closed loop LV coefficient matrix $\hat{\boldsymbol{M}}$ can also be expressed in the form

$$\hat{\boldsymbol{M}} = \hat{\boldsymbol{B}}\,\hat{\boldsymbol{A}} = \boldsymbol{M_0} + \sum_{l=1}^{p}\sum_{i=1}^{r} k_{il}\boldsymbol{M_{il}}. \quad (25)$$

Then the global stability analysis of the closed loop system with unknown feedback gains k_{il} leads to the following BMI

$$\hat{\boldsymbol{M}}^T\boldsymbol{C} + \boldsymbol{C}\hat{\boldsymbol{M}} = \boldsymbol{M_0}^T\boldsymbol{C} + \boldsymbol{C}\boldsymbol{M_0} + \quad (26)$$
$$\sum_{l=1}^{p}\sum_{i=1}^{r} k_{il}\left(\boldsymbol{M_{il}}^T\boldsymbol{C} + \boldsymbol{C}\boldsymbol{M_{il}}\right) \leq 0.$$

The variables of the BMI are the $p \times r$ k_{il} feedback gain parameters and the c_j, $j = 1,..,m$ parameters of the LYAPUNOV function. If the BMI above is feasible, there exists a globally stabilizing feedback with the selected structure.

Note that (marginal) infeasibility of the BMI (*Eq. (26)*) means only that the closed loop system is not proven to be globally asymptotically stable. However, the solution k_{il} may still guarantee local stability, which is enough in several cases.

Controller design using bilinear matrix inequalities

A BMI is a diagonal block composed of q matrix inequalities of the following form

$$G_0^i + \sum_{k=1}^{p} x_k G_k^i + \sum_{k=1}^{p}\sum_{j=1}^{p} x_k x_j K_{kj}^i \leq 0, \quad i = 1,\dots,q \quad (27)$$

where $x \in \mathbb{R}^p$ is the decision variable to be determined and G_k^i, $k = 0, \ldots, p$, $i = 1, \ldots, q$ and K_{kj}^i, $k, j = 1, \ldots, p$, $i = 1, \ldots, q$ are symmetric, quadratic matrices.

The main properties of BMIs are that they are non-convex in x (which makes their solution numerically much more complicated than that of linear matrix inequalities), and their solution is NP-hard [10], so the size of the tractable problems is limited. However, there exist practically applicable and effective algorithms for BMI solution [20], [21], or [22]. In Matlab environment the TomLab/PENBMI solver [23] can be used effectively to solve BMIs.

Quasi-polynomial servo control

As in the linear case, the problem statement of the servo, or reference tracking control is as follows. Consider a nonlinear system in the form of *Eq. (20)* and an external reference signal r which is to be followed by the system output η.

It is possible to define the tracking error signal (*Eq. (28)*) whose time derivative gives the tracking error dynamics which should be stabilized together with the system (*Eq. (20)*).

$$z(t) = \int_{t_0}^{t} r(\tau) - \eta(\tau)\mathrm{d}\tau. \tag{28}$$

It is easy to see that the differential equation of the tracking error has the form (*Eq. (29)*).

$$\dot{z}(t) = r(t) - \eta(t), \tag{29}$$

$$u(t) = -K_R \int_0^t r(\tau) - \eta(\tau) d\tau - \boldsymbol{K}\, \boldsymbol{y}(t).$$

If the output equation is also in QP form, then the extended closed loop QP system can be written up in LOTKA-VOLTERRA form similar to (*Eq. (25)*) and the BMI for the globally stabilizing controller design can be formulated.

Quasi-polynomial control of the synchronous generator

Quasi-polynomial form of the synchronous generator

The bilinear nature of the state equations (*Eq. (8)*) enables us to directly apply the QP mechanism without QP embedding [16]. The system has the following set of quasi-monomials:

$$\left\{ \frac{1}{i_d}, \frac{i_F}{i_d}, \frac{i_q\,\omega}{i_d}, \frac{1}{i_F}, \frac{i_d}{i_F}, \frac{i_q\,\omega}{i_F}, \frac{1}{i_q}, \frac{i_d\,\omega}{i_q}, \frac{i_F\,\omega}{i_q}, \frac{1}{\omega}, \frac{i_F\,i_q}{\omega} \right\}$$

The QP coefficient matrices of the input-affine system (*Eq. (20)*) are

$$\boldsymbol{A_0} = \begin{bmatrix} -2.323 & 0 & 0 & 0 \\ 0.0004 & 0 & 0 & 0 \\ 0.5893 & 0 & 0 & 0 \\ -3.4883 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0.0006 & 0 & 0 \\ 0 & -6.6918 & 0 & 0 \\ 0 & 1.2022 & 0 & 0 \\ 0 & 0 & -0.7861 & 0 \\ 0 & 0 & 3.5888 & 0 \\ 0 & 0 & 2.6489 & 0 \end{bmatrix}^T$$

$$\boldsymbol{A_1} = \begin{bmatrix} 0.5893 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -6.6918 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$\boldsymbol{A_2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0006 \end{bmatrix}^T$$

Feedback structure

There is a degree of freedom in the selection of the stabilizing feedback structure. A wise choice of the feedback structure does not increase the number of monomials of the closed loop system. This way the size of the BMI to be solved remains tractable. In our case, a linear full state feedback is applied, i.e. the feedback law is in the form

$$\begin{aligned} v_F &= k_{r1} \textstyle\int_0^t r(\tau) - p_{out}(\tau) d\tau + \\ &\quad + k_1\, i_d + k_2\, i_F + k_3\, i_q + k_4\, \omega \\ T_{mech} &= k_{r2} \textstyle\int_0^t r(\tau) - p_{out}(\tau) d\tau + \\ &\quad + k_5\, i_d + k_6\, i_F + k_7\, i_q + k_8\, \omega. \end{aligned} \tag{30}$$

Controller design and verification via simulation

Using the feedback law (*Eq. (30)*), the closed loop system is also in QP form with 18 quasi-monomials:

$$\left\{ \frac{1}{i_d}, \frac{i_F}{i_d}, \frac{i_q}{i_d}, \frac{\omega}{i_d}, \frac{i_q\omega}{i_d}, \frac{1}{i_F}, \frac{i_d}{i_F}, \frac{i_q}{i_F}, \frac{\omega}{i_F}, \frac{i_q\omega}{i_F}, \frac{1}{i_q}, \frac{i_d\omega}{i_q}, \right.$$

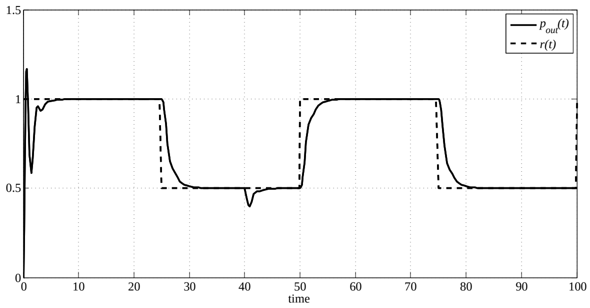

Figure 2: The controlled output p_{out} of the system (solid line) together with the reference input r (dashed line).

$$\frac{i_F\omega}{i_q}, \frac{i_d}{\omega}, \frac{i_F}{\omega}, \frac{i_q}{\omega}, \frac{i_d i_q}{\omega}, \frac{i_F i_q}{\omega} \Big\}.$$

Due to the lack of space, the 19×19 (the extra dimension comes from the tracking error dynamics) matrices of the BMI and the closed loop LOTKA-VOLTERRA system are not listed here.

The BMI (*Eq. (26)*) for the globally stabilizing feedback design case suffers from rank deficiency and the available BMI solvers stop with marginal infeasibility, which means that global stability cannot be proven using the feedback law (*Eq. (30)*). However, simulation results indicate, that the system is locally stable with the controller gain parameters yielded by the globally stabilizing BMI (*Eq. (26)*) formulated for the closed loop generator model extended with the tracking error dynamics.

$$\begin{aligned} k_{r1} &= 1.0000 & k_{r2} &= -0.0002 \\ k_1 &= -0.2932 & k_2 &= -1.6880 \\ k_3 &= -1.9609 & k_4 &= 0.2259 \\ k_5 &= 0.0001 & k_6 &= 0.0000 \\ k_7 &= -0.0001 & k_8 &= 0.1450 \end{aligned}$$

The simulated behavior of the controlled generator can be seen in *Fig.* 2 where the controlled output of the system (i.e. p_{out}) is shown together with the reference input (r). Both of them are dimensionless. The tracking properties of the system are acceptable, moreover, its disturbance rejection is also good (at time 40, a step-like change has been applied to the disturbance input v_d).

The simulation has been performed in Matlab/Simulink environment [24].

Conclusions

A novel servo control design technique based on the QP representation has been formulated in this work. As an example, an active power tracking controller has been designed for a synchronous generator model. The tracking properties of the closed loop system are satisfactory.

Further work includes the extension of the method for vector reference signals and use the degree of freedom lying in the BMI problem for formulating a robust/optimal controller design problem. Another direction of future research is to apply graph theoretical techniques for controller structure selection that applies the underlying connections between QP systems and chemical reaction networks [25].

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