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Effect of Pavement Stiffness on the Shape of Deflection Bowl

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Abstract – The paper introduces a new method for calculating the elastic moduli of pavement layers. The method requires only two input parameters: the thickness of the upper „bound" layer and the Falling Weight Deflectometer (FWD) or Improved Benkelman Beam Apparatus (IBBA) measurement data. The authors developed a continuously differentiable regression function, which can be applied to describe the shape of the deflection bowl. Additional parameters of the deflection bowl (e.g. radius of curvature, position of inflexion point) can be calculated based on the regression function. FWD measurements were simulated running the BISAR (Bitumen Stress Analysis in Roads) software on different pavement variations. Outputs of the simulations were further processed with self-developed software. As a result, a series of diagrams were elaborated, by which the elastic moduli of the pavement layers can be determined.

Stiffness / pavement layers / elastic moduli / deflection bowl / BISAR

Kivonat – A pályaszerkezet merevségének hatása a behajlási teknő alakjára. Útpályaszerkezetek esetében a megfelelő rehabilitációs eljárás kiválasztása igen nagy gazdasági jelentőséggel bír. Ezért a szerkezetek állapotának megfelelő ismerete nélküli döntéshozatal igen költséges lehet. Emiatt különösen fontos, hogy az FWD (Falling Weight Deflectometer) vagy IBBA (Improved Benkelman Beam Apparatus) eszközzel mért elmozdulások elemzésével olyan többletinformációhoz jussunk, ami a döntést megkönnyíti a gyakorló mérnök számára. Az FWD vagy IBBA eszközzel mért deformációs vonalra illesztett függvényből levezetett görbületi sugár (R_0) és a burkolatvastagság (h) ismeretében a kötött rétegek alján jelentkező megnyúlásokat jól lehet becsülni. A BISAR (Bitumen Stress Analysis in Roads) programmal végzett számítások statisztikai elemzése pedig azt mutatja, hogy a D_0 (központi behajlás) és R_0 (görbületi sugár) paraméterek ismeretében lehetőség nyílik a kötött és szemcsés rétegek modulusának visszaszámolására.

Merevség / pályaszerkezet rétegek / rugalmassági modulus / behajlási teknő / BISAR

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INTRODUCTION

Forest roads with asphalt pavement represent the basis of the forest road networks in Hungary. Properly maintained asphalt pavements offer a high level of service. While traffic load of forest road networks have grown, expenses for their maintenance remained lower than required in the last three decades. As a result, these roads are in poor condition, generally. Renovation projects demand the knowledge of the roads' bearing capacity. The term "bearing capacity", although widely used at pavement management projects, is hard to define. In fact, direct measurement of bearing capacity is impossible. Instead, one can measure the deflection caused by a known load, and calculate the bearing capacity afterwards.

The traffic transfers its loads to road pavements through the tyres of the vehicles. Due to this, shearing stresses originate from vertical loads (pressing, beating, shaking, bending etc.) and horizontal stresses (braking, accelerating, wearing) (Kosztka 1978, 1986). These stresses affect each pavement layer differently, such as the elastic and plastic (permanent) deformation, the break and the structural realignment (Boromisza 1976). All these structural changes appear on the surface of the pavement as deformations, and the so-called deflection bowl or deformation surface forms.

To measure the evolving deformations several methods have been elaborated. Currently the measuring procedures based on absorbed oscillation are widely used. These are called Falling Weight Deflectometers, FWD. These deflectometers, operating with impulses, often drop a given weight from a given height onto a disc with anti-shock – using the potential energy – then they record the evolving displacements (Kosztka et. al. 2008). Researchers of the Institute of Geomatics and Civil Engineering at the University of West Hungary developed a new instrument to measure the full deflection bowl with the Benkelman beam (Markó et. al. 2013). The development was based on the Benkelman beam, extending its properties with automated data logging and the ability to measure multiple points of the deflection bowl. The Improved Benkelman Beam Apparatus (IBBA) continuously measures the vertical displacement of one point on the surface of the pavement, together with the horizontal position of the truck (*Figure 1*).

Figure 1. Falling Weight Deflectometer (left) and Improved Benkelman Beam Apparatus (right) in action.

The deflection bowl recorded during the test provides much more information about the current state of the pavement structure than the central deflection in itself. Therefore we can define its bearing capacity, remaining lifetime, and the thickness of the needed strengthening layer more precisely. Choosing the applicable rehabilitation procedure in the case of a given pavement structure has a really great economic significance. Without appropriately knowing the condition of the pavement,decisions could become very expensive. This is why it is so important to gain additional information by analysing the deflections, which makes it easier for the practising engineers to make decisions. We started our work with this approach, and summarized our results in this paper.

1 MATERIAL AND METHOD

1.1 Estimating the deflection bowl with functions

When deflection is measured to define the bearing capacity, displacements are measured and recorded only in certain distances from the load. This makes it necessary to fit functions onto the discrete measurement points to get the complete plot of all the evolved deflections. It is practical to apply functions describing the deflection bowl, because this way the geometrical attributes that are important regarding the stressed pavement can be defined with comparatively easy calculations.

Because of the surface sinking caused by mining (e.g. tunnel building), functions were already elaborated long time ago. Most authors (Aversin, Martos, Beyer, Bals etc.) suggested functions similar to the Gaussian bell curve (Fazekas 1978). Suggestions can be found to describe the deformation curve of pavements in Hothan and Schäfer's (2004) summary work.

Hossain (1991) used exponential function to estimate the deformation caused by external load:

$$D(x) = ae^{bx} \tag{1}$$

where x : distance from the centre of the load [m] and a, b : parameters.

On the basis of their examinations the effect of the upper and stiffer layers can be experienced in the decrease of the "a" parameter, while the effect of the lower high-solidity layers causes increasing "b" parameter value. The "a" and "b" parameters depend on the strength of materials characteristic of the pavement. The exponential function is able to estimate the FWD or IBBA measurements with high correlation, though it cannot reproduce the natural shape of the deflection bowl. Therefore it is not suggested to use it despite the high correlation coefficient.

Jendia (1995) tries to describe the whole deformation curve by substituting the exponential function in the central range of the deformation line $0 \le x \le r$ with a hexic polynomial:

$$D(x) = \begin{cases} c_3x^6 + c_2x^4 + & \\ +c_1x^2 + c_0 & 0 \le x \le r \\ ae^{bx} & x \ge r \end{cases} \tag{2}$$

Jendia first defines "a" and "b" values of the unknown parameters. He specifies the second derivative's equality at the joint of the functions, that is, the continuity of the curve. Therefore, there are three constraints for the c_3, c_2, c_1 and c_0 parameter.

He creates the last independent variant iteratively by minimizing the difference between the values measured on the second and third sensor of the FWD device and the calculated deflections. The method of Jendia can reach only low equality with the data points in spite of its high demand of calculation (*Figure 2*).

Grätz (2001) makes it possible to describe the deflection bowl with a single function:

$$D(x) = \frac{w_a + w_b x^2}{1 + w_c x^2}. \tag{3}$$

With the help of the suggested rational fractional function[1] the three factors that describe the whole bowl can be defined (*Figure 2*. Grätz (1)). The correspondence with the measured results can be further increased if a fourth degree of the polynomial is used:

[1] The rational fractional function is a mapping of the set of scalars, where we give the association with the quotient of two polynomials.

$$D(x) = \frac{w_a + w_b x^2 + w_c x^4}{1 + w_d x^2 + w_e x^4}\,. \tag{4}$$

Using the altered function higher correlation can be achieved, though the layer parameters cannot be concluded from the equation's factors as the coefficients depend on all the layers differently (*Figure 2.* Grätz (2)). To describe the deformation curve in practice, it is suggested to apply functions with which one bowl parameter can be deduced, which describes a special layer of the pavement (e.g. radius of curvature).

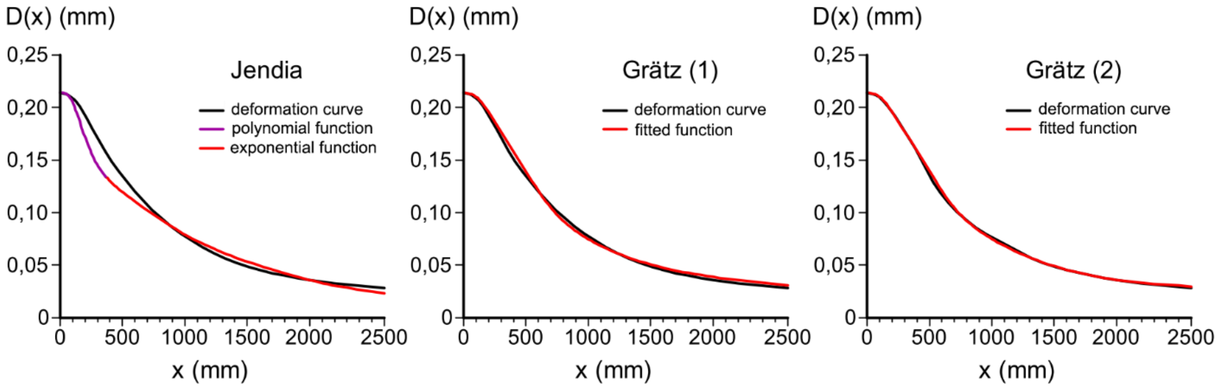

Figure 2. Comparing different deflection bowl functions (Hothan and Schäfer 2004)

In his study, Daehnert (2005) introduces two function types from the French literature (Ph. Leger and P. Autret) which shows good correspondence with the theoretical deformation curve:

$$D(x) = D_0 e^{-\left(x^2 b\right)} \tag{5}$$

and

$$D(x) = D_0 \frac{a}{x^2 + a} \tag{6}$$

where D_0 the maximum deflection in the axis of the load [m].

The function (6) was originally developed to process the Lacroix deflectograph data. In its structure, it is similar to the Agnesi[2] Witch Curve (Scharnitzky 1989). Cser (1961) uses function (6) to model the evolving deformations directly under wheel load with the substitution of $a = 3r^2$

$$D(x) = D_0 \frac{3r^2}{x^2 + 3r^2} \tag{7}$$

where r the radius of the loaded surface considered to be evenly dissolved and circle shaped [m]. The curve has an inflexion point at the edge of the wheel load ($x = r$). The function can follow the evolving deformations only in a restricted extent, as the inflexion point is fixed.

[2] Maria Gaetana Agnesi (Milan, May 16th, 1718 – Milan, January 9th, 1799), Italian linguist, mathematician and philosopher, the honorary member of the University of Bologna.

1.2 Estimating the deformation curve on the basis of mechanics functions

Starting with the Boussinesq stress formulas, the value of D_0 deflection under the centre of the $d = 2r$ diameter flexible circle plate can be deduced (Papagiannakis and Masad 2008):

$$D_0 = \frac{2pr}{E_e}\left(1-\mu^2\right) \tag{8}$$

where
D_0 : vertical deflection measured in the load axis [mm],
E_e : the modulus of the flexible halfspace [MPa],
p : surface distributed load [MPa],
r : radius of the loading plate [mm],
m : the Poisson factor [–].

Beside the central deflection, Odemark calculated the deformation curve of the flexible halfspace with. E_e modulus, loaded in the usual way using the $y = f(p,r, E_e)$ function. The second differential at its $x = 0$ point estimates the curve's value very well. The R_0 radius of curvature, in the case of one-layer half-space, can be calculated with the following formula (Nemesdy, 1985):

$$R_0 = \frac{E_e r}{p\left(1- m^2\right)}. \tag{9}$$

Both functions give the same result in the case of homogeneous, infinite halfspace, so it is obvious that there is functional relation between the central deflection and the radius of curvature. Take the quotient of the equivalent modulus provided by the two equations:

$$c = \frac{2r^2}{R_0 D_0} \tag{10}$$

where "c" factor is the quotient of the two moduli, which is $c = 1$ in the case of homogeneous infinite half space.

It requires very long calculations to define deformations on the surface of the homogeneous half-space using Boussinesq's theory. Therefore, to simplify this, it is practical to take an estimating function. When determining the estimation function it is necessary to start from the geometric restrictions that are the boundary conditions. On the base of the above functions the following conditions can be stated: $x = 0$, where $D(x) = D_0$, and the second derivative $D''(x) » 1/R_0$ of the requested $D(x)$ function at $x = 0$. Additionally, the defined D_0 and R_0 values require mechanics conditions (10), too. Searching for the function that satisfies the conditions we start with the function type suggested by Cser (1961):

$$D(x) = D_0 \frac{d^2}{c \times x^2 + d^2} = D_0 \frac{1}{c\left(\frac{x}{d}\right)^2 + 1} \tag{11}$$

In the suggested function "c" is the so-called *shape factor*, which influences the shape of the deformation curve (Primusz – Tóth 2009; Primusz – Markó 2010). It can easily be seen that the estimation function really satisfies the $x = 0$ and $D(x) = D_0$ conditions. After defining the shape of the deflection bowl we can define the radius of curvature (*Figure 3*).

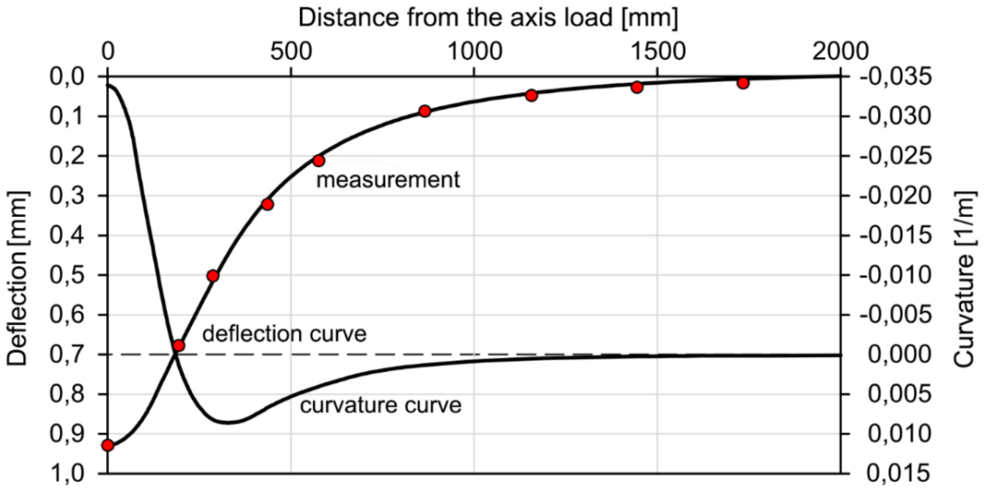

Figure 3. Curve of deflection bowl and curvature under load

We estimated the radius of curvature of the oscular circle belonging to an optional point of the $D = D(x)$ function with the $k(x) \approx D''(x)$ function:

$$k(x) \approx \frac{\vartheta^2}{\vartheta x^2}\left(\frac{D_0 4r^2}{cx^2+4r^2}\right) = 8\,\frac{D_0\, r^2 c\,(3cx^2-4r^2)}{(cx^2+4r^2)}. \tag{12}$$

The negative sign of the curve means that in the case of positive bending moment, the centre (0 point) of the oscular circle described with the curve radius is located on the $-D$ driven side of the axis. The curve alteration is presented in *Figure 3*. The minimal radius of curvature at $x = 0$ is

$$R_0 = \frac{2r^2}{D_0 c}. \tag{13}$$

It can be seen that the mechanics condition (10) is also satisfied, so the function is a good estimation of the mechanically defined deformation curve.

1.3 Estimation of the strain rising at the bottom of the bound layer

Knowing the radius of curvature derivated from the fitted function on the measured deformation points, and the thickness of the overlay, the strains rising at the bottom of the bound layers can be estimated with the following formula:

$$\varepsilon = \frac{h}{2R_0} = c \cdot D_0 \frac{h}{4r^2} \tag{14}$$

where

ε : strain in the load axis,
h : thickness of the bound layer,
R_0 : radius of curvature in the load axis.

The conditions defined on the bound layer are satisfied if Hooke's law is present and the elastic modulus is equal for compression and for tension (Primusz – Tóth 2009).

1.4 Computer simulation with the BISAR software

The simulation is basically an examination where the expected and real behaviour of the system is being studied through the physical or computer model of a process.

Applying the simulation model we are able to provide appropriate inputs for the model of the system, operate it and observe the outputs.

Through the simulation of the pavements, we can observe what deformations evolve at the places of the sensors that record the deflections under external load, typical of the FWD devices, and how much stress evolves in each structural layer. Using the simulation, the deflection curve recorded on the pavements can be provided with more information, therefore, more exact pavement diagnostics are possible (Huang 2003).

1.4.1 Setting the simulation model

Nowadays, the most popular and most accepted method of defining the stresses evolving in pavements is the application of computer software. One of the oldest and most referred software is the BISAR (Bitumen Stress Analysis in Roads), developed by the SHELL Research Center. The software can calculate stress, strain, and deflection in an elastic multilayer system loaded with vertical load. The layers are characterized by their layer thickness, elastic modulus, Poisson factor and the adhesion defined at the boundaries of the layers. The whole system is supported by an infinite elastic half-space.

We used the DOS version of the BISAR software to carry out the simulation, because this way – after generating the starting data files – we could run batched calculations. The data files contained the structure of the pavements to be calculated in one procedure (number, thickness, modulus of layers etc.), the rate and place of load, and the coordinates of points where the calculation of stresses and strains are needed. The BISAR simulation ran in the case of two- and three-layer systems.

1.4.2 Pavement models used in simulation

The layers of the pavements can basically be divided into three groups: subgrade (together with frost protecting and/or improving layers), base layer, and overlay. Each group can be divided into further layers, so an average real pavement can be built of 3–5 layers (*Figure 4. a*). As to their material, the layers can be bound with bitumen (sometimes hydraulic) or can be unbound. As most pavement models are able to consider the material characteristics with the help of the elastic modulus and the Poisson factor, it is suggested to close up the unbound and bound layers and handle them as a whole instead of increasing the number of layers (Yoder – Witczak 1975).

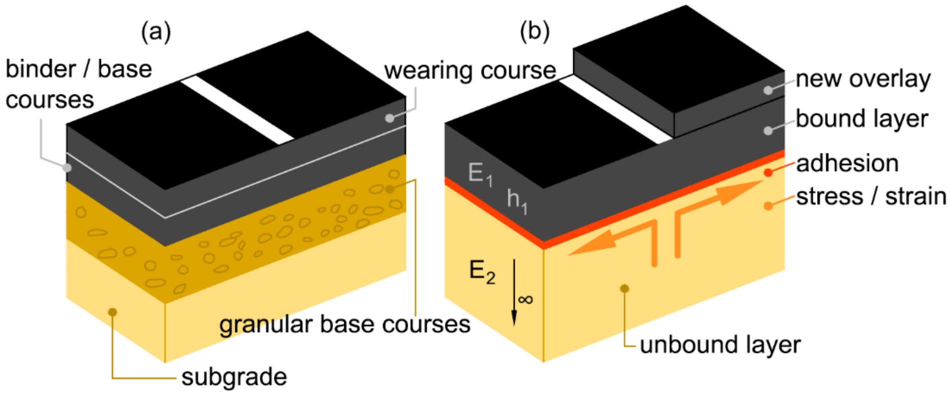

Figure 4. The structure of the pavement models in the simulations

Accordingly, bound and unbound pavement layers can be distinguished (*Figure 4. b*). The pavement behaviour models are able to consider the collaboration defined between the layers. Full slip should be assumed between the unbound granular layers and the bound overlays, while in the case of reinforcement – between the old and new overlays – full adhesion should be expected, even if it causes smaller stress to the old asphalt layers. During simulation we examined the behaviour of the existing pavements with two-layer systems, while we used three-layer systems in the case of reinforced pavements. The two-layer systems are the idealized models of the existing pavements, in which the bottom layer refers to the unbound granular layers and subgrade, while the top bound layer refers to the overlays. There is probable frictional adhesion between the bound and unbound layers. In the case of the two-layer pavement models, the modulus of the top bound layer is between 1000 and 8000 MPa, the modulus of the bottom unbound half-space varied between 20 and 500 MPa. The thickness of the top bound layer varied between 50 and 500 mm, and we divided the examined range logarithmically examining 7 different values. Thus, we examined the two-layer systems in 12×12×7 = 1008 combinations (*Table 1*).

Table 1. The parameters of the pavement models examined during simulation

Model	Nr.	Modulus	Var.	Poisson	Thickness	Var.	Adhesion	Sum
Two-layered	1	20 – 500	12	0,5	Infinite	–	1	1 008
	2	1000 – 8000	12	0,5	50–500	7		
Three-layered	1	20 – 500	12	0,5	Infinite	–	1	15 552
	2	1000 – 8000	12	0,5	50–300	6		
	3	5000 – 15000	3	0,5	20–120	6	0	

Notes: Layer ordinal (Nr.) from bottom to top, Layer-modulus (Modulus [MPa]), Variation (Var.), Poisson-factor [–], Thickness [mm], Adhesion [0: full adhesion, 1: full slip], All variations (Sum)

The three-layer models evolve from the two-layer systems with the addition of a reinforcement layer; this reinforcement layer can help in the examination of the bearing capacity of pavements. During calculations the rigidity modulus of the new asphalt layer used for reinforcement was 5000, 10000 and 15000 MPa. We then increased the thickness of the reinforcement layer by 2 cm up to 12 cm, so altogether 15552 variations evolved (Primusz – Markó 2010). Beside the layer modulus and thickness, the Poisson factor gets different values in the case of different materials; however, its practical definition is rather difficult as its value depends on tension and temperature (Pethő 2008; Szentpéteri – Tóth 2014). In the case of general road construction materials its value is usually between 0.2 and 0.5. The effect of the cross contraction factor's changes was examined in detail by De Jong, Peutz and Korswagen (1973), and Tam (1987). The researchers stated that changing the Poisson factor had little effect on the primary design parameters (strain, stress, deflection). That is, the effect on deflections caused by changing the Poisson factor is rather small compared to the layer thickness or the layer modulus. If the Poisson factor is increased from 0.2 to 0.5, the deflections will decrease by only few percent (Van Gurp 1995). Based on the above research in the BISAR simulation, we took each layer with the value $\mu = 0.5$. In this case, the analytical functions will largely be simplified; therefore, it is – in several respects – practical to choose this value.

1.4.3 Calculating stresses

The BISAR software is able to handle several loads and calculate their superposition. The loads and the examined points are placed in one frame of reference and can be arbitrarily defined by the *x*, *y*, *z* coordinate triplet. During the simulation we took size $F = 50$ kN single wheel load, which affects the top layer vertically, and scatters evenly a radius $r = 0.15$ m elastic circle plate (p = 0.707 MPa). The distance of the examined points – measured from the load axis – was equal to the usual sensor set-up of the FWD device. In respect to the asphalt overlays' lifetime, the most important stress is the strain of the bottom edge line caused by the vertical deflection load. Therefore, in the fixed positions we examined not only the vertical deflections, but the strains defined on the bottom plane of the bound layers. After running the BISAR software, we evaluated the result text files with a self-developed program. We considered the vertical deflections calculated by the BISAR as a result of an FWD measurement during our further analysis.

2 RESULTS AND DISCUSSION

During the first part of the evaluation we examined how much effect each layer had on the evolving deformations in the case of a given pavement. This question is described in detail in Van Gurp's study (1995).

Figure 5 shows how much the layers of a three-layer structure effect the surface deflections (Van Gurp 1995). Naturally, the distribution changes with the modifying of the layer thickness or stiffness. According to the study, the thicker and stiffer the upper layers are, the more important the darkened areas of *Figure 5* become. The figure demonstrates well that the bearing capacity of subgrade has the greatest influence on the peak value of the surface deflections, and 900 mm from the load axis, the measured deflection represents the deflection of the subgrade in 100%. It can also be observed that if we consider the subgrade and base layer as one, then the effect of the top bound (asphalt) layer expands only 300 mm from the load axis, so it mostly affects the central deflections. This theory can be examined with the BISAR simulation.

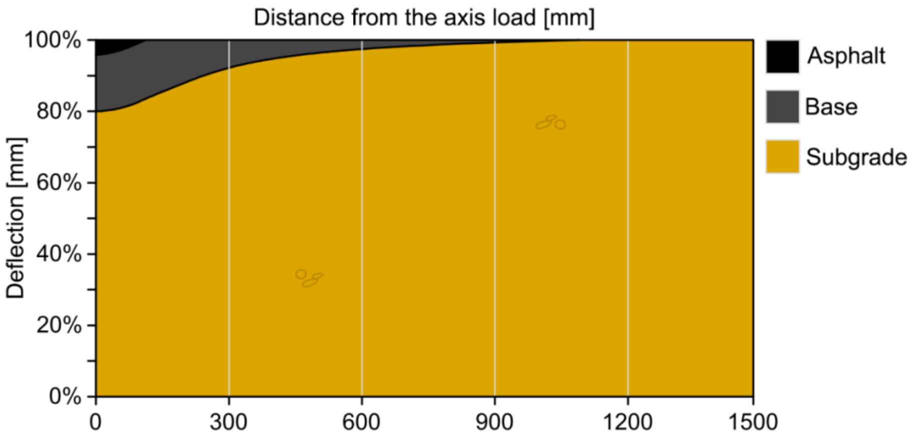

Figure 5. The effect of each layer on the surface deflection (Van Gurp 1995)

In the case of the examined pavement models, we calculated the maximum deflections on the surface, and also the partial deflections evolving at the bottom of the bound layer with the BISAR software. Thereby we had both deformation curves by structure. We fitted the function (11) onto these deflections, which resulted in two "c" shape factor values. By comparing the two values it can be stated that between the deformation curves evolving on the surface and at the bottom of the bound layer, the difference depends on the (*h*) layer thickness. This relation is graphically presented in *Figure 6*; the surface deformation curve is marked as c_t, while the one being at the bottom of the bound layer is marked as c_b (*Figure 6. a*). The thicker the bound layer is (*h*), the bigger the extent of the difference is (*Figure 6. b*). The differences of shape factors can be originated from the differences of deflections. Comparing the deflections of the 1008 two-layer systems, only the central deflections showed measurable differences in the range of 0–1 mm (70% of the D_0 values fell in between).

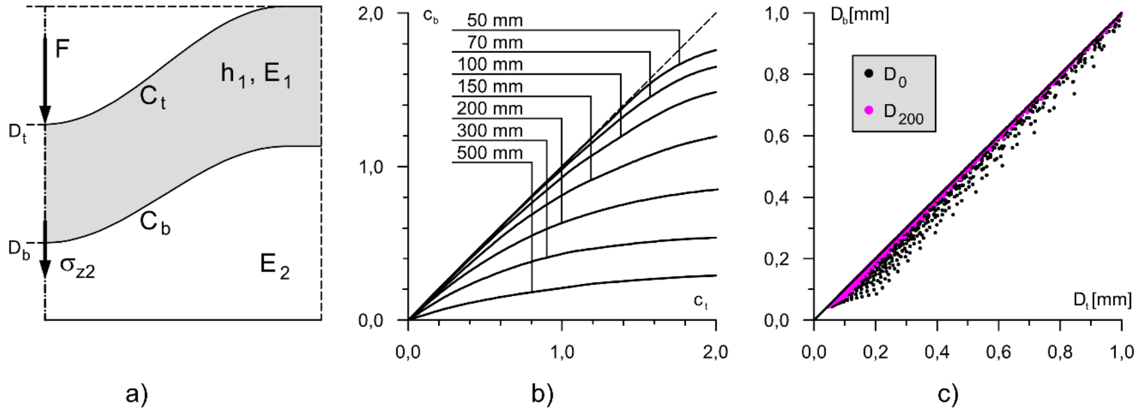

Figure 6. The changes of the "c" parameter of the function fitted onto the deformations evolving on the surface and at the bottom of the bound layer

Going farther from the load axis (200–300 mm) the deflections calculated on the surface and at the bottom of the bound layer were entirely the same (this result also confirms our statements at the beginning of this chapter). That is, the differences of shape factors can mainly be explained with the changes of the central deflection, as the thickness of the bound layer influences the compression of the layer itself (*Figure 6. c*).

Now, if we examine the results from the practical aspect and accept the assumption that the deflections measured on the pavement surface are nearly the same as the ones evolving at the bottom of the bound layer, that is $D_t(x) \approx D_b(x)$, then the shape factor (c_b) that characterizes the bottom of the bound layer can well be estimated on the base of the surface measurement: $c_t \approx c_b$. As the practical measurements always have the possibility of mistakes, and there are several factors (e.g. temperature) that modelling cannot count with, hereafter we will not make any difference between the two shape factors.

2.1 Analysis of the two-layer system

Using the results of the BISAR software we looked for relationship between the parameters deduced from the shape of the deflection bowl (Primusz and Tóth 2009), and the layer parameters of the two-layer system.

The examination revealed that the "c" shape factor, the quotients of the layer moduli (*K*) and the thickness of the bound layer (*h*) have very close correspondence. The graphical evaluation of the results is shown in *Figure 7*.

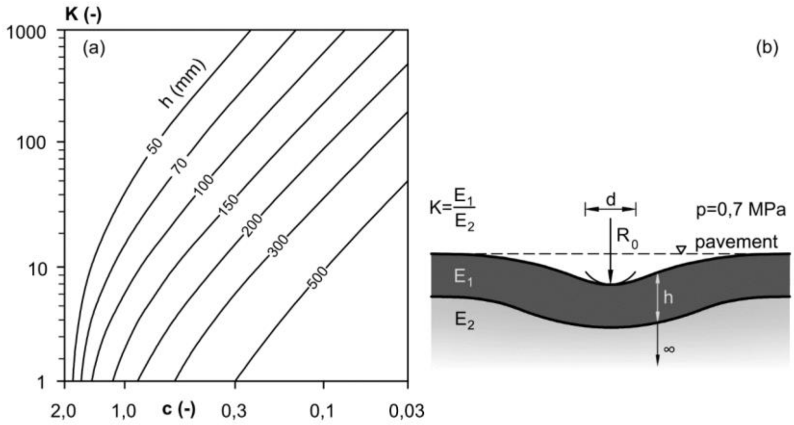

Figure 7. Function between the shape factor (c) and the rate of the layer moduli (K)

With the help of the graph the idealized two-layer model of a given pavement – knowing the bound layer thickness – can be induced from the FWD or IBBA measurements. We made the statistic model of the $K = f(c,h)$ function connection in two steps. The examinations showed that the rate of the K moduli and the σ_{z2} compressive stress evolving on top of the bottom layer are related similarly to the one in *Figure 7*. As the K and the σ_{z2} values do not depend on the accuracy of the function fitted on the deflections – that is the "c" shape factor – it is practical to first describe this relationship. We received the best result using the following model:

$$Y = \frac{a}{b \cdot X^c + 1} \tag{15}$$

with the next substitution: $Y = \sigma_{z2}$ and $X = h^d \sqrt{K}$. We defined the model's *a, b, c* and *d* parameters with the STATISTICA program:

$$\sigma_{z2} = \frac{0,8}{20,816 \cdot \left(h^{1,4}\sqrt{K}\right)^{1,393} + 1} = \frac{0,8}{20,816 \cdot h^{1,95} K^{0,70} + 1} \tag{16}$$

The accuracy of fitting is clearly shown by the very high $R^2 = 0,9977$ value. Then we looked for a relation between the "c" shape factor and the σ_{z2} value. There was clear polynomial function with $R^2 = 1$:

$$\sigma_{z2} = 0,0392c^6 - 0,2749c^5 + 0,6907c^4 - 0,8332c^3 + 0,5424c^2 + 0,2588c \tag{17}$$

After plotting the value pairs, the hexic polynomial could be estimated with a line without the significant decrease of the fitting's rate ($R^2 = 0,9954$):

$$\sigma_{z2} \approx 0,4205 \cdot c. \tag{18}$$

Substituting function (18) in function (16), we get the wanted relationship:

$$c \approx \frac{1,9}{20,816 \cdot h^{1,95} K^{0,70} + 1} \tag{19}$$

or reordering to *K*:

$$K \approx 0,0131 \left[h^{-1,95} \left(\frac{1,9}{c} - 1 \right) \right]^{1,428} \tag{20}$$

the *K* factor here shows the stiffness of the layers correlated with each other.

2.2 Estimating the modulus of the granular layers

According to the study of Hoffmann (1988), if the pavement and the subgrade are considered as a two-layer system, knowing the radius of curvature and the central deflection, the E-modulus of the subgrade can be directly calculated. This statement can be checked knowing the results of the BISAR simulations. Using the deformation curves of the 1008 two-layer systems, the radius of curvature of the systems can be defined with the function (13). The related D_0, R_0 and E_2 data rows are graphically presented in *Figure 8.*

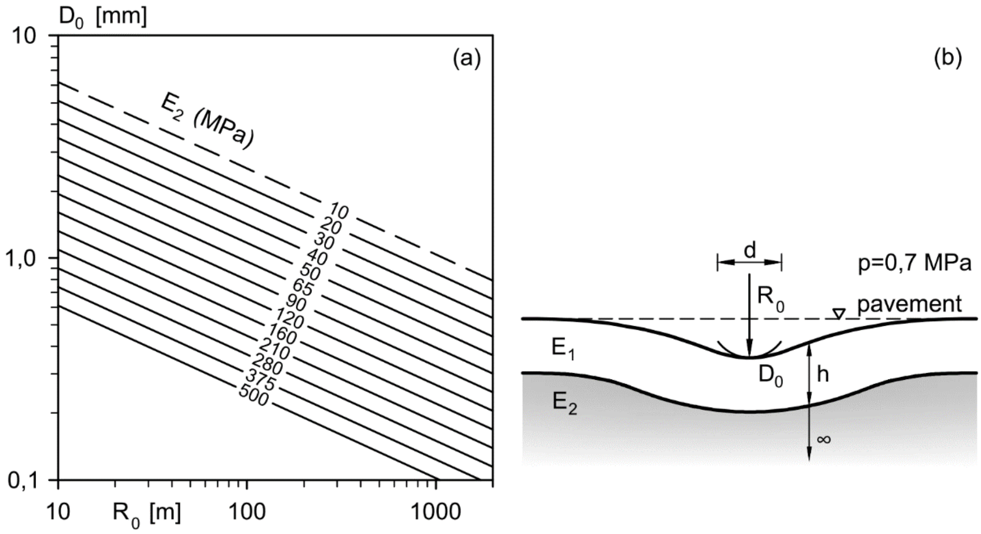

Figure 8. The E-modulus of the bottom layer can be defined by the centre deflection and the radius of curvature

Figure 8 shows that the data rows define a plane in dual logarithmic frame of reference. So the E-modulus of the bottom layer can be defined by the following function relation:

$$E_2 = a \cdot D_0^b \cdot R_0^c \tag{21}$$

The linear figure of the function:

$$\log(E_2) = \log(a) + b\log(D_0) + c\log(R_0) \tag{22}$$

The following general equation can be derived with the $Y = \log(E_2)$, $X_1 = \log(D_0)$, $X_2 = \log(R_0)$, $b_0 = \log(a)$, $b_1 = b$ and $b_2 = c$ substitution:

$$Y = b_0 + b_1 X_1 + b_2 X_2 \tag{23}$$

Here b_0 represents the intersection, while b_1 and b_2 show the partial slopes. The regression factors can be defined with the least square method as before (Orbay 1990). The results of the calculations made with the STATISTICA program are shown in *Table 2*.

Table 2. The statistic characteristics of the function constants fitted onto the two-layer system

h (cm)	N = 1008	β	Deviation (β)	B	Deviation (B)	t(1005)	p
	Intersection	–	–	3.08794	0.000767	4024.03	0.00
5–50	Direction tangent (1)	–1.56581	0.000598	–1.62284	0.000620	–2617.99	0.00
	Direction tangent (2)	–1.05669	0.000598	–0.62894	0.000356	–1766.77	0.00

R^2 = 0.9998, F(2.1005) = 3515439, p < 0,0000, α = 0,05 and the residual deviation: 0.00525

According to the examination, there is very strong correspondence between the E-modulus of the bottom layer, the vertical deflection interpreted at the load axis, and the radius of curvature. The high R^2 also shows this. So the E_2 modulus can be estimated with the following function from the evolving deflections:

$$E_2 = 1224{,}45 \cdot D_0^{-1{,}623} R_0^{-0{,}629}\,. \tag{24}$$

To avoid big numbers we gave the D_0 in millimetres, the R_0 radius of curvature in metres, while the joint modulus of the granular layers E_2 in MPa in the equation. According to the function (13) R_0 depends on the "c" shape factor, therefore, the (24) can be changed as follows:

$$E_2 = 111,73 \cdot D_0^{-0,994} \cdot c^{0,629} \tag{25}$$

According to the statistical model, it is not necessary to know the bound layer thickness to estimate the joint modulus of the granular layers, so it can be defined from the FWD or IBBA measurements without any destruction. The defined condition parameter may be useful especially for the Pavement Management Systems (PMS).

2.3 Estimating the modulus of the bound layers

The definition of the modulus of the bound layers is done using the following simple formula:

$$E_1 = K \cdot E_2 \tag{26}$$

where
E_1 : modulus of the bound layer [MPa],
E_2 : modulus of the unbound granular layer [MPa],
K : rate of the layers compared to each other [–].

In the function (26), K is the rate of the bound and unbound layers compared to each other, which is calculated with the formula (20). To estimate the E_2 modulus we use the formula (24). Therefore we proved that in the case of two-layer pavement models, the moduli of the layers can unequivocally be calculated back from the deformation curve, so it is not necessary to use the iterative backcalculation methods.

2.4 Analysing the three-layer system

With the help of the BISAR software we modelled 15552 three-layer pavement variants. We calculated the evolving stresses and strains at the bottom of the reinforcement layers 'built onto' the original pavements and the whole bound layer thickness. The calculation was based on Ambrus's (2001) former results. He demonstrated that at the bottom of the reinforcement layer of pavements having the same deflection curve but different structure, the same strains evolve in every case. That is, if the pavement deflection curve (its radius of curvature) is known, then the rate of the necessary reinforcement can be directly estimated.

We could not find regression relationship between the R_0 radius of curvature of the deflection curves calculated with the BISAR software and the strains evolving directly at the bottom of the reinforcement layer. The reason for this is that we assumed full adhesion between the two layers, so it actually behaved as one layer. Therefore, we later only dealt with the strains evolved at the bottom of the whole bound layer thickness. We managed to draw the following statistic model:

$$\log(\varepsilon_b) = -0,522 \cdot \log(R_0) - 0,533 \cdot \log(\Delta h) - 0,189 \cdot \log(E_{AC}) + 5,088 \tag{27}$$

or

$$\varepsilon_b = 122463 \cdot R_0^{-0,522} \cdot \Delta h^{-0,533} \cdot E_{AC}^{-0,1888} \tag{28}$$

where
ε_b : strain evolving at the bottom of the bound layer after the reinforcement [με],
R_0 : the radius of curvature of the pavement before the reinforcement [m],
Δh : the thickness of the reinforcement layer between 20 and 120 mm,
E_{AC} : the modulus of the reinforcement layer between 5000 and 15000 MPa.

The results of the calculations carried out with the STATISTICA software are shown in *Table 3*. Knowing ε_b, it is possible to define the necessary asphalt reinforcement layer. To do this, the asphalt fatigue functions used in asphalt mechanics have to be applied. The principle of the method is that the material is able to tolerate a certain strain during limited load repetitionswithout failure. That is why the evolving ε strain is equivalent with a repetition number, such as a unit axis crossing number (Ambrus 2001). The fatigue function of the material should be defined with laboratory examinations, though today, several estimating functions can be used (Bocz 2009).

Table 3. The statistic characteristics of the function constants fitted onto the three-layer system

h (cm)	N = 15 552	β	Deviation (β)	B	Deviation (B)	t(15548)	p
2–12	Intersection	–	–	5.08800	0.01863	273.09	0.00
	Direction tangent (R)	–0.83663	0.002617	–0.52207	0.00163	–319.69	0.00
	Direction tangent (H)	–0.42522	0.002617	–0.53302	0.00328	–162.49	0.00
	Direction tangent (E)	–0.11296	0.002617	–0.18882	0.00437	–43.16	0.00

$R^2 = 0.8935$, $F(3.15548) = 43491$, $p < 0.0000$, $\alpha = 0.05$ and the residual deviation : 0.10747

Currently in Hungarian road maintenance practice, the critical strain is defined directly at the bottom of the reinforcement layer. One reason for this is that the old asphalt layer becomes cracked, so we cannot count on its long-term load-bearing ability. This approach sometimes results in exaggeration, as it expects only the new layer to resist the external loads, while the old asphalt layers are still able to participate in the force-game. Counting with the existing asphalt layers is also hampered by the fact that only the fatigue ability of loose asphalt mixtures could be examined with 2- or 4-point bending test. It is very circuitous to make a test piece out of the core samples drilled out of existing pavement for these examinations. Today, the cracking-drawing test (Indirect Tensile Test, ITT) makes it possible to use samples directly drilled out of the pavement and define its fatigue characteristics (Pethő – Tóth 2012). The old asphalt material's fatigue criterion should be defined with the least squares method from the results of the laboratory experiment:

$$N_f = k \cdot \left(\frac{1}{\varepsilon_0} \right)^n \tag{29}$$

where

N_f : the entire load repetition number,

k, n : material constants,

ε_0 : horizontal strain in $\mu\varepsilon$ in the centre of the test piece.

Based on the function (29) a statement can be made in connection with the strain of the old asphalt layer. The importance of the function (28) is that the strain can be estimated at the bottom of the existing asphalt layers after the reinforcement. Comparing the two functions the base function of a design procedure can be deduced, which will design upon the fatigue characteristics of the old rather than the new asphalt.

3 SUMMARY

The function suggested by us can be fitted not only onto deflection curves calculated with the FWD or IBBA, but also the ones calculated with the BISAR software. We showed that by knowing the deflection curve and the thickness of the bound layer, without using further iteration procedures (backcalculation), we could define the modulus of the examined pavement's layers. The modulus calculated this way can certainly not be matched with the result of any laboratory tests. The practical benefit of the procedure is thatwith the defined moduli, we can create a pavement model whose behaviour – shape alterations under wheel load – well approximates the real pavement.

Knowing the radius of curvature we can calculate the strain of the bottom of the bound layer; knowing the strain, we can calculate the existing pavement's lifetime. The analysis of the three-layer models made it possible to estimate the strains evolving at the bottom of the existing asphalt layer after building the reinforcement layer, and so we can establish the theoretical possibility of a harmonic and economic reinforcement design method. The elaborated modelling procedure on the network level has the capability to be the base of a pavement management system. On the project level, the appropriately parameterized two-layer pavement model can help plan more professional reinforcement layers.

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