NONLINEAR REGRESSION MODEL FOR PERMEABILITY ESTIMATION BASED ON ACOUSTIC WELL-LOGGING MEASUREMENTS

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1. Abstract

Full waveform sonic logging is routinely applied in the oil industry practice. The primary and secondary waves inform about the porosity, elastic parameters and the orientation of in-situ stresses acting around the borehole. The separation of Stoneley waves propagating in the drillhole enables to determine the permeability of hydrocarbon reservoirs. Permeability is generally estimated from the inversion processing of Stoneley-interval transit-time data. Statistical methods can also be found in the literature, which offer quicker and simpler solution for revealing the empirical relationships between the characteristic data of Stoneley-waves and permeability. These methods do not require the prior knowledge of porosity. Case studies assume a linear connection between the relative decrease in Stoneley-wave's velocity of porous formations and the natural logarithm of permeability, which usually gives only a rough estimate. In this study, the regression model is improved by the determination of a more accurate nonlinear relationship between Stoneley-wave slowness and permeability. The new statistical algorithm is tested over a wide domains of petrophysical parameters using an exactly known permeability model and synthetic well logs. The exponential model is also applied to the statistical processing of real well-logging data, where the estimated permeability log is compared to laboratory data measured on core samples. The quality checks of synthetic and field results show that the application of the nonlinear model is highly recommended to get a more accurate and reliable estimation of hydrocarbon reserves.

2. Introduction

Sonic logging is a widely used borehole geophysical method in hydrocarbon exploration. Primary porosity can be determined from refracted P-wave transit times when the sonic travel times of the matrix and fluids filling the pore space are known (RAYMER et al. 1980). When different types of fluids are present in the rock, also their saturations must be quantified. The porosity calculated by acoustic logs is essential for estimating the secondary porosity of fractured formations, because the porosity obtained from acoustic method does not contain the secondary porosity part of the effective porosity. This porosity value must be compared to that of another method such as neutron-porosity or density measurement that includes both the primary and secondary components of the effective porosity (ELLIS and SINGER 2007).

The use of full waveform data in open holes – beyond determination of porosity – gives further possibilities. Using the shear and surface wave transit-times, it is possible to reveal the presence and orientation of fractures induced in the formation by natural or artificial processes, the anisotropy and magnitude of the stress-field acting in the environment of the borehole and the elastic parameters and permeability of the formation. The above characteristics are of fundamental importance for planning the hydraulic layer-fracturing, for the determination of fluid inflow to the well, with which it is possible to optimize the exploitation procedure (BADRI et al. 2000). The physical background of sonic wave

propagation with several hydrocarbon industry applications were summerized by HALDORSEN et al. (2006). The direct determination (through drilling) of formation permeability is possible using nuclear magnetic resonance (NMR) logging, but this method is very expensive. This is why in our domestic practice it is not widely used today (COATES et al. 1999). The use of NMR log is promising for the future, because beyond the determination of effective porosity it is possible to determine the distribution of pore-sizes and the viscosity of the fluid filling the pore space. The latter quantity is in relation with fluid mobility (permeability/viscosity ratio) which is one of the basic parameters used in modeling sonic wave propagation in porous-permeable formations. The indirect (in-situ) methods for the calculation of permeability are based on the determination of formation porosity and bound-water saturation. If we evaluate the current methods used for data processing, we see that all methods have their weaknesses, because the results are often in contradiction with those determined from core samples or the estimation error of permeability may reach one (one and half) order of magnitude. Thus, the introduction of a new (independent) method could be of utmost importance. The common application of the new method and the existing ones can improve the accuracy and reliability of permeability estimation. The Stoneley-waves are surface waves propagating on the boundary of the mud column and the rock formation. Their velocity slightly varies with frequency. They can be generated with a monopole source, which emits wave-energy with equal intensity in all directions. Detection is done with a so called "array" type sonde, which may contain even several monopole sources and a group of receivers. In comparison with the classical borehole compensated (BHC) tools the monopole receivers have smaller spacing. Thus, higher vertical resolution can be reached. Owing to the construction of the sonde, because of the bigger source-receiver distance it is easier to separate the components of the wave train and, with the common processing of channels a higher signal-to-noise ratio can be reached. The low frequency component of the detected Stoneley-spectrum is the most sensitive one to the permeability of primary porosity reservoirs.

There have been many attempts to use the Stoneley-waves for reservoir characterization. BIOT (1955) has worked out the motion equation valid in porous media. With the aid of this equation the Stoneley-wave phenomenon may be related to the porosity and permeability of homogeneous and isotropic media, in case of elastic behaviour of the rock matrix and viscous porous fluid. The relation between the attenuation of the Stoneley-wave and permeability of the formation was predicted theoretically by ROSENBAUM (1974). The low frequency case of the dispersive wave was examined by WHITE (1983) and MATHIEU and TOKSÖZ (1984). WILLIAMS et al. (1984) have found a quantitative relation between permeability and attenuation of the Stoneley wave. The simplified models based on the Biot-theory were developed by SCHMITT et al. (1988) and CHANG et al. (1988). These were also confirmed by laboratory measurements (WINKLER et al. 1989). Based on the Biot-Rosenbaum model, TANG and CHENG (1994) developed an inversion method for processing Stoneley interval transit-time data which is still considered to be standard for the determination of sonic permeability. Within the frame of this method at several frequencies (between 0.2 and 0.8 kHz) fluid mobility is estimated with the joint inversion of Stoneley-slowness and attenuation data.

Besides the "complicated" inversion procedures there exist also some approximate procedures using empirical relations based on a multitude of field measurements, which estimate permeability from the measured and theoretical Stoneley-slowness ratios (i.e. Stoneley-index). BUFFIN (1996) published a method based on a crossplot technique, which

showed a good conformity between the estimated permeability values and the same data measured on core samples in Australian hydrocarbon exploration wells. BALA (2010) applied successfully the same statistical method for the investigation of miocen gas reservoirs in Poland. The advantage of the method is that it does not require the precalculation of porosity. Its disadvantage is that the linear or sequentially linear aproximation used by the two authors does not show high correlation between the Stoneley-index and permeability. Until now the method has been tested only on field data, therefore we found it necessary to test it on syntetic data, too.

In this study, we start from a known permeability model, then, after processing the syntetic well-logging data calculated on the model, we analyze how the exact permeability values are reconstructed. We describe that the correlation coefficient measured between the Stoneley-index and permeability can be improved if we consider the relation between the two variables non-linear. We test the statistical method on the wide domains of relevant petrophysical parameters applying homogenous and inhomogenous models. Using the non-linear regression model resulted from the syntetic tests we present also the evaluation of a field data set, which demonstrates the feasibility of the improved method.

3. Stoneley-waves in porous media

The Darcy-equation describes the flow of fluids in porous-permeable rocks

$$\frac{\partial \mathbf{u}}{\partial t} = -\frac{K}{\Phi \eta} \nabla p \,, \tag{1}$$

where K [m^2] denotes permeability, Φ [v/v] is porosity, η [Pa·s] is viscosity, $\mathbf{u}$ [m] is the relative displacement vector of the fluid and p [Pa] is the pore pressure. If the rock-matrix is considered to be incompresible the volumetric deformation of the (ideal) fluid filling the pore-space fullfils the Hook-equation

$$-\nabla \mathbf{u} = \frac{\Phi}{B_f} p \,, \tag{2}$$

where B_f [Pa] is the bulk modulus of the pore fluid (the ratio of the applied pressure and volumetric deformation). Eliminating $\mathbf{u}$ from equations (1) and (2) we get the following diffusion relation

$$\frac{\partial p}{\partial t} = C_0 \nabla^2 p \,, \tag{3}$$

where $C_0 = KB_f / \eta \Phi$ is the diffusion coefficient. In the case of elastic porous medium BIOT (1962) has found the relation between stress and strain. Based on this, calculation of the diffusion coefficient in equation (3) needs to be modified

$$C = \frac{C_0}{1 + \dfrac{B_f}{\Phi B}\left[(1-\alpha)(\alpha - \Phi) + \dfrac{\alpha^2}{1 + (4\mu/3B)}\right]} \ , \tag{4}$$

where B is the bulk modulus of the dry rock (matrix) and μ [Pa] is the shear modulus. (The porosity of the dry and fluid saturated formation is considered to be equal). The elasticity parameter α is defined by the Biot–Gassmann-equation

$$B_c = B + \alpha^2 M = B + \left(1 - \frac{B}{B_s}\right)^2 \left(\frac{\Phi}{B_f} + \frac{\alpha - \Phi}{B_s}\right)^{-1} \ , \tag{5}$$

where B_c is the bulk modulus of the porous (saturated with water or hydrocarbon) formation and B_s is the same for the case of rock matrix.

The slightly dispersive Stoneley-wave is the slowest component of the sonic wave-train exhibiting the highest amplitude. The studied frequency range of this Stoneley-wave is generally between 0-10 kHz (the centre frequency is around 0.5 kHz) and it propagates, at low frequency as a tubewave in the borehole. The velocity of the tubewave in an impermeable formation ($v_{St}^{(imp)}$) is

$$v_{St}^{(imp)} = \frac{v_m}{\sqrt{1 + \dfrac{B_m}{\mu}}} \ , \tag{6}$$

where v_m [m/s] is the propagation velocity of the sonic wave in the mud and B_m is the bulk modulus of the mud. In porous-permeable formations the tubewave displaces the fluid particles relative to the matrix. This burns up some part of the wave energy. This phenomenon is accompanied by the attenuation and slowing down of the sonic waves. At low frequencies wave velocity becomes a complex quantity $v = \left[B^* / \rho_m\right]$, where B^* represents the complex bulk modulus of the mud and ρ_m is its density.

Because of pressure both the borehole of radius a and the pore fluid exhibit radial displacement. Based on the diffusion relation (3), considering mud pressure to be a harmonic function of time, the velocity ($v_{St}^{(p)}$) of tubewave in a permeable formation is given by the following equation (NORRIS 1989)

$$v_{St}^{(p)} = v_{St}^{(imp)}\left[1 - \Phi\frac{\mu}{\mu + B_m}\frac{B_m}{B_f}\sqrt{\frac{C_0}{C}}\sqrt{\frac{iC_0}{\alpha^2\omega}}\right] \ , \tag{7}$$

where ω [Hz] is the angular frequency of the compression wave (*i* is the imaginary unit). Since the diffusion coefficient depends on permeability and porosity, the propagation velocity in a permeable formation according to equation (7) decreases from $v_{St}^{(imp)}$ to $v_{St}^{(p)}$. The imaginary part of equation (7) describes the attenuation of the Stoneley-wave. This quantity increases with porosity and permeability, but decreases with the diameter of borehole. The above theoretical relation is confirmed by laboratory measurements, too. WINKLER et al. (1989) conducted observations on nearly homogeneous Berea sandstone and also on other samples produced artificially, in the 0–100 kHz frequency range. The dispersive properties of the Stoneley-waves were studied using model calculations by BRIE et al. (2000). They found that with growing fluid mobility the phase velocity and attenuation grow, also, significantly as a function of frequancy. They demonstrated that in the low frequency range propagation velocity reacts more significantly to the variation of permeability while in the high frequency range attenuation is the most sensitive to that. Based on these observations, it is possible to designate an optimal frequency around which both measurements can be applied. The above theory represents the base of those data processing methods which determine permeability based on Stoneley interval-times.

## 4. Non-linear permeability model

BUFFIN (1996) plotted the square of the measured Stoneley interval-times $\left(\Delta t_{St}^{(m)}\right)$ at low frequencies and the square of shear-wave interval-times $\left(\Delta t_{s}^{(m)}\right)$ normalized to the measured bulk density $\left(\rho_{b}^{(m)}\right)$ in the same co-ordinate system. On this crossplot the impermeable formations are represented along a line which is called shale-baseline. This line gives the calculated (theoretical) value of the Stoneley-wave slowness $\left(\Delta t_{St}^{(c)}\right)$ which does not include the effects of hole-diameter and other environmental factors (i.e. drilling mud and mudcake)

$$\Delta t_{St}^{(c)} = \sqrt{\frac{\rho_{mf}}{\rho_b} \Delta t_s^2 + \Delta t_{mf}^2} \ , \tag{8}$$

where ρ_{mf} and Δt_{mf} are the bulk density of the mudfiltrate and its sonic interval-time. Based on equation (8) the slope of the shale line is ρ_{mf} and the intersection with ordinate axis is Δt^2_{mf} in the given co-ordinate system. Relative to the shale base-line the measurement points which appear towards the bigger Stoneley interval-time (slowness) values indicate permeable formations on the crossplot. It was determined from field observations that permeability of primary porosity rocks is directly proportional to the magnitude of deviation from the shale base-line. The decrease of Stoneley-wave velocity relative to the impermeable formation is expressed by the Stoneley- index which is defined as the ratio of the measured and calculated Stoneley travel times

$$I_{St} = \frac{\Delta t_{St}^{(m)}}{\Delta t_{St}^{(c)}} \ . \tag{9}$$

The statistical procedures given in literature assume a linear relationship between permeability and Stoneley-index, where the regression coefficients depend on litology, porosity and pore content. Both the field results of other authors and our own calculations show that the above relation is not typically linear and this is mostly significant in the range of small permeabilities.

Figure 1

Statistical algorithm for permeability prediction
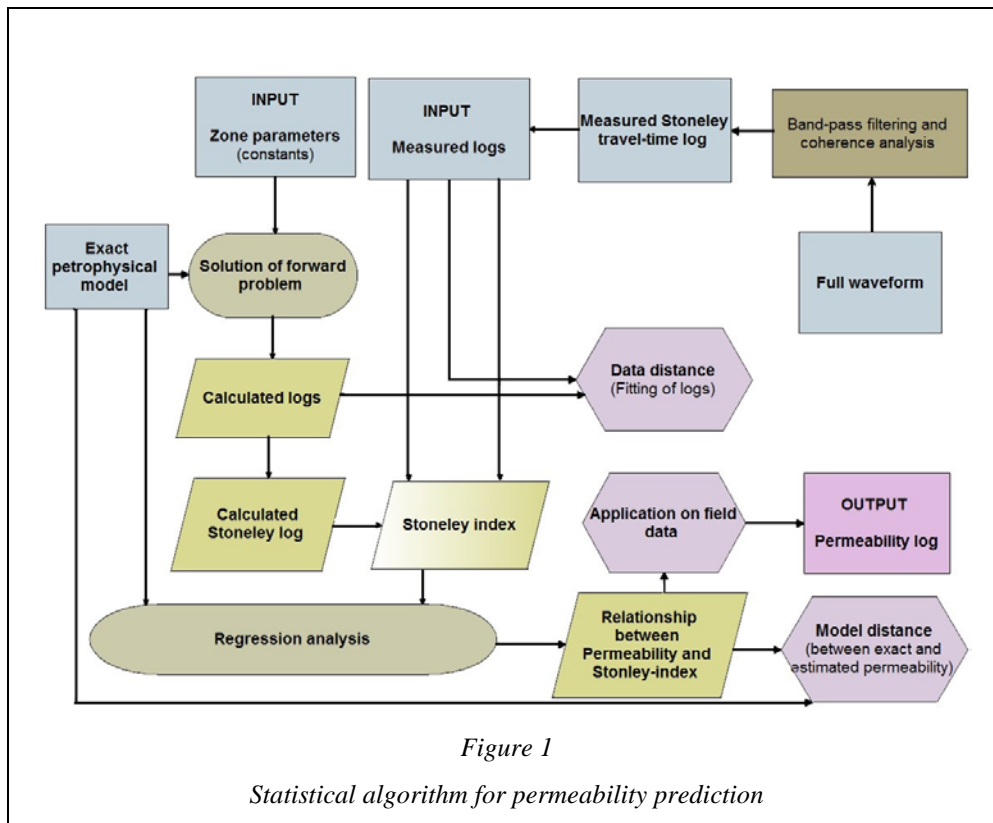
In Figure 1 the basic steps of our suggested modeling procedure are shown. Using the workflow, it is possible to determine the relationship between the Stoneley-index and permeability. The input data of the statistical procedure is given by the measured borehole logs such as natural gamma-ray intensity (GR [API]), density measured by gamma-gamma probe (ρ_b [g/cm^3]), shear-wave interval-time (Δt_s [µs/m]) and Stoneley interval-time (Δt_{St} [µs/m]). Well-logging data can be related to the parameters of the petrophysical model. A part of the model parameters changes rapidly with depth, others can be regarded constant within a longer zone. The first group of parameters should be determined point-by-point during the evaluation of the reservoirs. The porosity (Φ [v/v]), water saturation in the flushed zone (S_{xo} [v/v]), shale volume (V_{sh} [v/v]), sand volume (V_{sd} [v/v]) and permeability (K [mD]) are grouped among those reservoir parameters which can be determined individualy or by using an inversion method. The other group including zone parameters are set from laboratory measurments, literature or preliminary knowledge of the

measurement area. The forward problem can be solved by using constant values of longitudinal wave slowness of mud filtrate (Δt_{mf}), density of mudfiltrate (ρ_{mf}), longitudinal wave slowness of the shale (Δt_{sh}), shale density (ρ_{sh}), longitudinal wave slowness of the sand (Δt_{sd}), sand density (ρ_{sd}), hydrocarbon P-wave slowness (Δt_{hc}), hydrocarbon density (ρ_{hc}), natural gamma-ray intensity of the sand (GR_{sd}) and shale (GR_{sh}).

The functions describing the relation between well-logging data and model parameters are called theoretical probe response functions. For solving the forward problem we used the following response equations

$$\rho_b^{(c)} = \Phi\left[\rho_{mf}S_{xo} + \left(1 - S_{xo}\right)\rho_{hc}\right] + V_{sh}\rho_{sh} + V_{sd}\rho_{sd} \,, \tag{10}$$

$$GR^{(c)} = GR_{sd} + \frac{1}{\rho_b^{(c)}}\left(V_{sh}GR_{sh}\rho_{sh} + V_{sd}GR_{sd}\rho_{sd}\right), \tag{11}$$

$$\Delta t_S^{(c)} = \sqrt{3}\left\{\Phi\left[\Delta t_{mf}S_{xo} + (1 - S_{xo})\Delta t_{hc}\right] + V_{sh}\Delta t_{sh} + V_{sd}\Delta t_{sd}\right\}, \tag{12}$$

$$\Phi + V_{sh} + V_{sd} = 1 \,. \tag{13}$$

The theoretical values of Stoneley slownesses can be calculated by equation (8). Equation (13) is the material balance equation valid in unit bulk volume. Based on the measured and calculated Stoneley slownesses, it is possible to formulate the Stoneley index which can be related to permeability values originating from other (independent) sources. According to our experience the regression function is

$$\ln(K) = a - be^{-cI_{St}^{\kappa}} \,, \tag{14}$$

where *a, b, c* are regression coefficients and κ is a suitably scaled factor. The value of permeability ($K^{(St)}$) estimated by regression analysis can be compared, in case of field measurements with permeability data determined from other methods or, in case of syntetic tests, with exact permeability ($K^{(e)}$) values. The fitting of the different results can be measured by a model distance

$$D_m = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(K_i^{(St)} - K_i^{(e)}\right)^2} \cdot 100[\%] \,, \tag{15}$$

where *N* is the number of depth-points along the borehole. The distance between the measured and calculated data is called data distance. The strenght of linear dependence between the Stoneley-index and permeability is traditionally characterized by the Pearson's correlation coefficient (*r*). But, for the characterization of the reliability of model (14) the rank correlation coefficient is a more suitable quantity. In case of non-linear relationships it

gives a more realistic evaluation and it is less influenced by outlying data (SPEARMAN 1904). For the calculation of rank correlation we used the following approximate formula

$$R = 1 - \frac{6 \sum_{n=1}^{N} \delta_n^2}{N(N^2 - 1)} \; , \tag{16}$$

where δ_n is the difference between the ranks of the *n*-th data of the two variables. When the value of the above correlation coefficient is nearly 1, it shows a close (non-linear) relationship.

5. Model calculations using syntetic data

Consider that the parameters of the petrophysical model (porosity, water saturation, permeability etc.) are exactly known. By means of equation (8) and response equations (10)–(13), we can generate syntetic well logs. The values of zone-parameters of the response equations are given in table 1.

Table 1

Zone parameters of theoretical probe response equations

ρ_{mf} [g/cm^3]	ρ_{sd} [g/cm^3]	ρ_{sh} [g/cm^3]	ρ_{hc} [g/cm^3]	GR$_{sd}$ [API]	GR$_{sh}$ [API]	Δt_{mf} [μs/m]	Δt_{sd} [μs/m]	Δt_{sh} [μs/m]	Δt_{hc} [μs/m]
1.0	2.65	2.4	0.15	10	160	630	184	330	1050

By contaminating the synthetic data with random noise of different levels, it is possible to calculate quasi measured borehole logs to test how exactly the statistical procedure reconstructs the parameters of the exact model. This experiment evaluates the performance of the method, i.e. it characterizes its accuracy, stability and shows its noise sensitivity. During the tests, we compare the Stoneley-index to exact permeability values, which were taken from the generally accepted formula suggested by TIMUR (1968)

$$K = 0{,}136 \left(\Phi^{4.4} / S_{w,irr}^2 \right) , \tag{17}$$

where $S_{w,\,irr}$ [v/v] represents the bound-water saturation. The sand content is calculated from equation (13) deterministically. The syntetic tests are performed both on homogenous and inhomogenous models using well logs with different uncertainties, respectively.

6. Test on homogeneous model

The model parameters given in Table 2 represent the petrophysical model of a sedimentary structure (Model-I). The sequence of beds is composed of five shaly sandy layers. At the top a shaly-sand layer can be found, which is followed by a water reservoir containing smaller amount of shale. The third layer is shale, which separates the above water-bearing layer from the gas reservoir that is placed under it. The lithologic column ends with a shale bed downmost. The value of permeability covers five orders of magnitude. In the reservoirs

the proportion of bound-water is small due to a small amount of shale, about 20%, while in the impermeable layers it is 80–100%. The Stoneley-slownesses were calculated in the layers and were increased proportional to permeability. It resulted in quasi measured Stoneley interval times. The increment was chosen from the literature (0–15%). In the given case, we did not add noise to the synthetic well log data.

Table 2

Petrophysical parameters of layer-wise homogeneous model

Depth [m]	Φ [v/v]	S_{xo} [v/v]	V_{sh} [v/v]	$S_{w,irr}$ [v/v]	Δt_{St} [μs/m]	K [mD]
0–4.9	0.15	1.0	0.5	0.6	691	5.65
5–9.9	0.23	1.0	0.3	0.2	738	333
10–12.9	0.07	1.0	0.8	1.0	674	0.07
13–19.9	0.25	0.7	0.1	0.2	793	481
20–24.9	0.10	1.0	0.7	0.8	678	0.53

The crossplot of quasi measured Stoneley- and shear-wave slownesses can be seen in Figure 2.

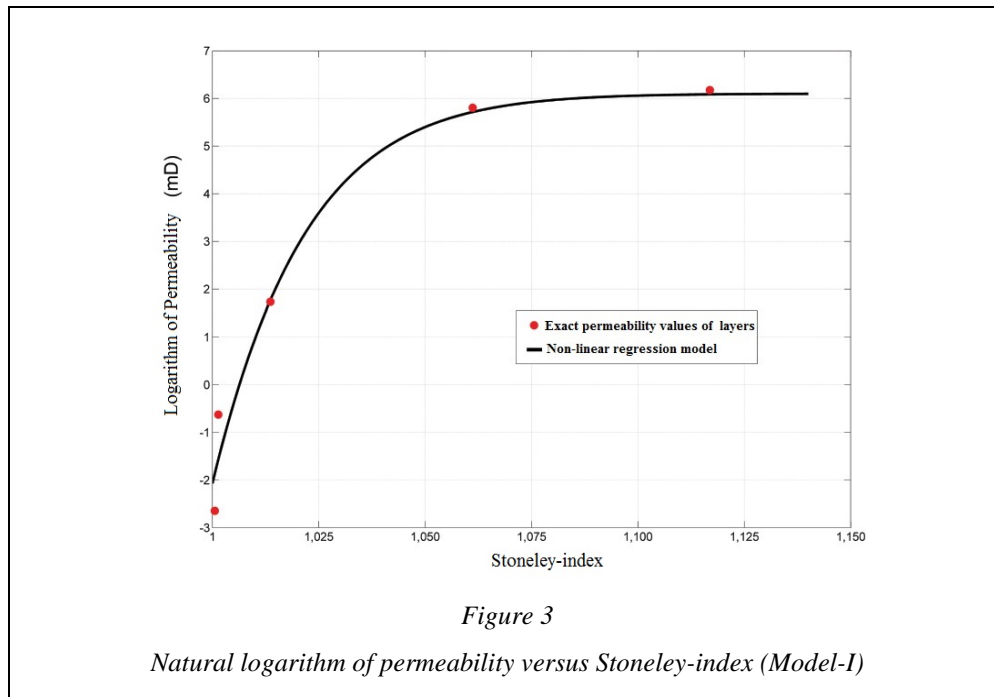*Figure 2*

Stoneley- versus shear wave slowness (Model-I)
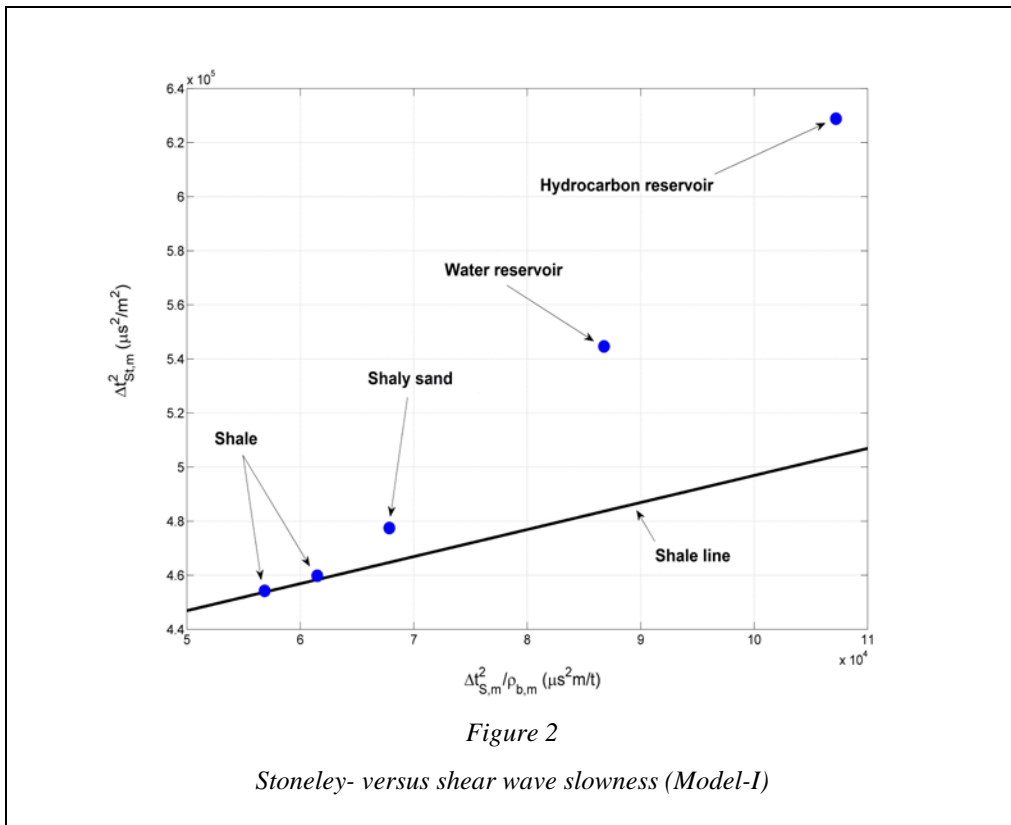
The baseline shows the impermeable layers, of which equation is

$$\Delta t_{St}^2 = \frac{\Delta t_s^2}{\rho_b} + 396900 \ .$$
(18)

The water-bearing and hydrocarbon reservoirs appear in the domain of increasing sonic interval times. The higher the deviation of the data from the shale line the higher the permeability of the sedimentary formation. The measurement of this effect is applied to permeability prediction using the acoustic wave train signals.

In noiseless situation, the Stoneley-index with permeability leads to regression relationship (14), where $a = 6$, $b = 7.28$, $c = 11.4$ (Figure 3). Compared to permeability the Stoneley-index changes several orders of magnitude smaller degree, then we chose $\kappa = 4$ in equation (14).

Figure 3

Natural logarithm of permeability versus Stoneley-index (Model-I)

In Figure 4 the applied borehole logs are shown (GR – natural gamma-ray intensity, RHOB – bulk density, DTS – shear wave slowness, DTSTC – calculated Stoneley-slowness, DTSTM – measured Stoneley-slowness, DDTST – difference of measured and calculated Stoneley-slownesses), the bulk volumes (POR – porosity, VSH – shale content, VSD – sand volume), the pore-content (SHCIRR = 1–SX0 – irreducible hydrocarbon saturation, SWIRR – bound water saturation) and the Stoneley-index (IST) as a function of relative depth. On the last track we present the Timur-permeability (PERMT) and the estimated permeability (PERMS) due to regression analysis using function (14). The

permeability values show strong similarity, the model distance calculated by equation (15) is 3.2%. Permeability stands in strong correlation with the Stoneley-index, since the value of rank correlation using equation (16) is 0.99. The regression model approximates well the known permeability values.

Figure 4

*Synthetic well logs and result of permeabililty estimation (Model-I)*
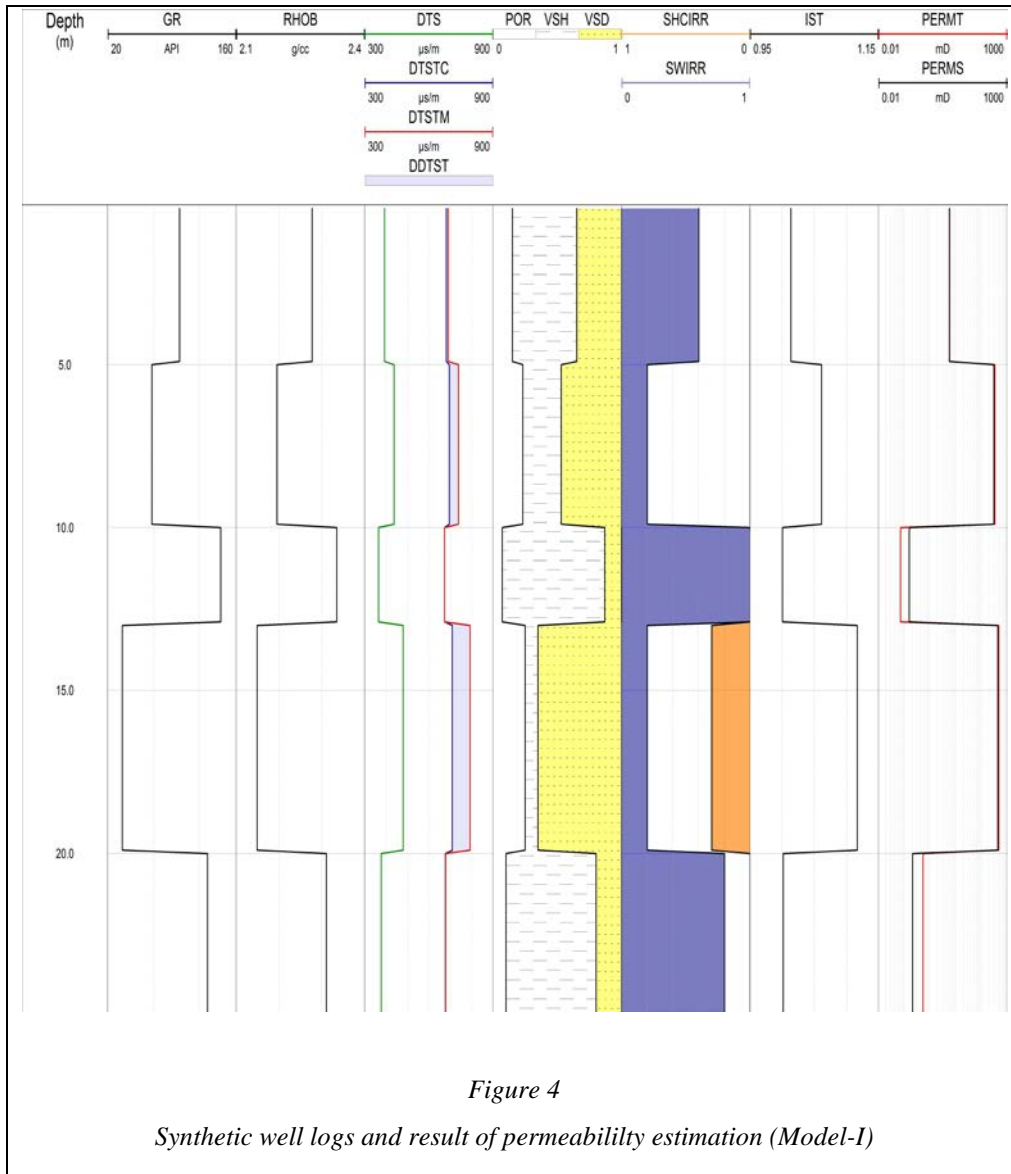
7. Test on inhomogeneous model

In real formations the petrophysical parameters can vary within the layers, hence the depth dependence of petrophysical parameters were approximated with eighteenth to twenty-second degree polynomials (Model-II). Synthetic data were calculated on the inhomogeneous model to which various degrees of Gaussian noise were added. In the case of 2% noise the model equation (14) has proved to be valid (Figure 5), where $a = 8.78$, $b = 48.22$, $c = 1.63$.

Figure 5

Natural logarithm of permeability versus Stoneley-index (Model-II)
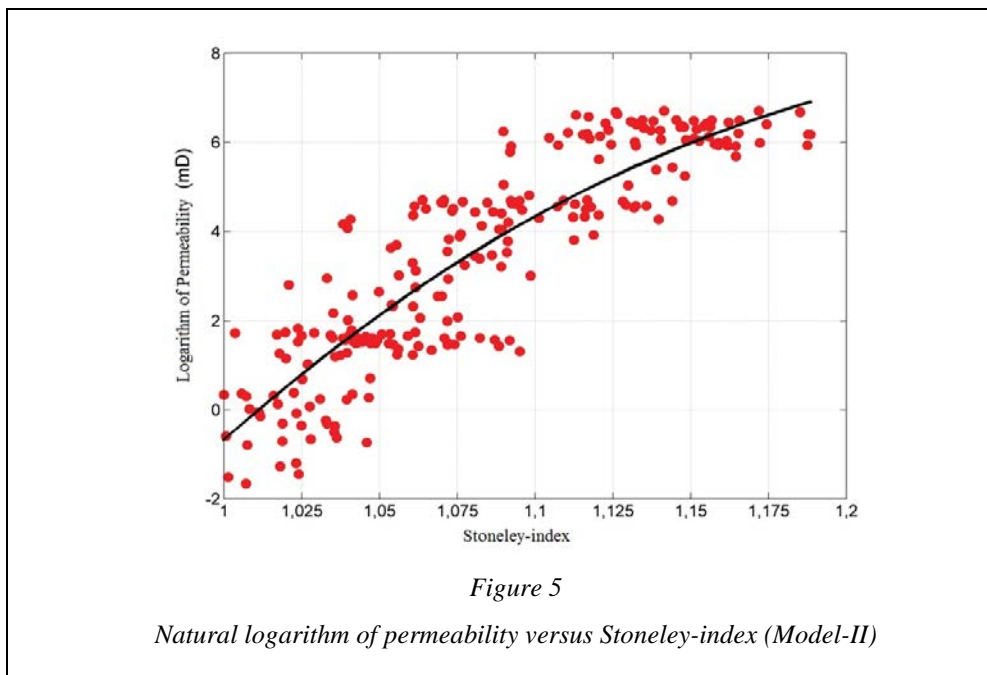
The result of permeability estimation can be seen in Figure 6. The permeability log can be estimated with an error that is proportional to the data noise. If the noise is 2%, the distance between the exact and measured permeability logs is 6.9%, although the rank correlation coefficient indicates a close relationship ($R = 0.98$).

It can be seen that in inhomogenous petrophysical model even a slight error may cause a significant estimation error. Therefore, noise sensitivity of the model was further examined. The noiseless (synthetic) data set of the inhomogenous model was loaded with different amount of Gaussian-noise (1–5%). Besides the variation of noise, the variace of three grading parameters (model distance, rank correlation, linear correlation) were observed.

Figure 6

Synthetic well logs and result of permeabililty estimation (Model-II)
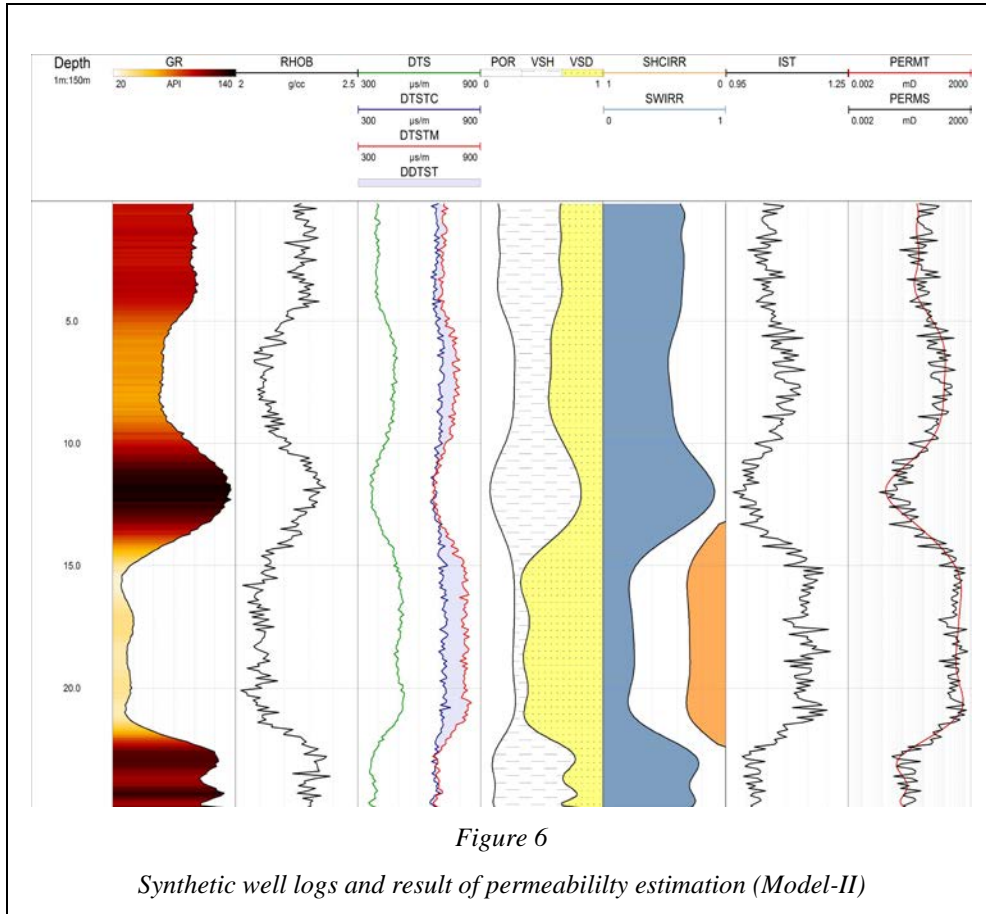
In Table 3 the result of data processing is shown, which includes the values of the regression coefficients related to formula (14). The rank correlation coefficient shows a strong relationship even at the highest noise level, although the degree of linear correlation decreases rapidly. The model distance at 5% noise level is four times higher than its initial value. The order and the sign of coefficients do not change significantly. It can be stated that the exponential model is valid in case of noisy data, however noise significantly affects the reliability of estimation. This phenomenon can be observed in the case of linear approximation, too. That is the reason, why we made an attempt for the improvement of this method with the suggestion of the non-linear model. The noise sensitivity is inherent for other well log methods, too. It may have a significant influence on the accuracy of permeability estimation.

Table 3

Regression coefficients estimated for noisy data sets and inhomogeneous model
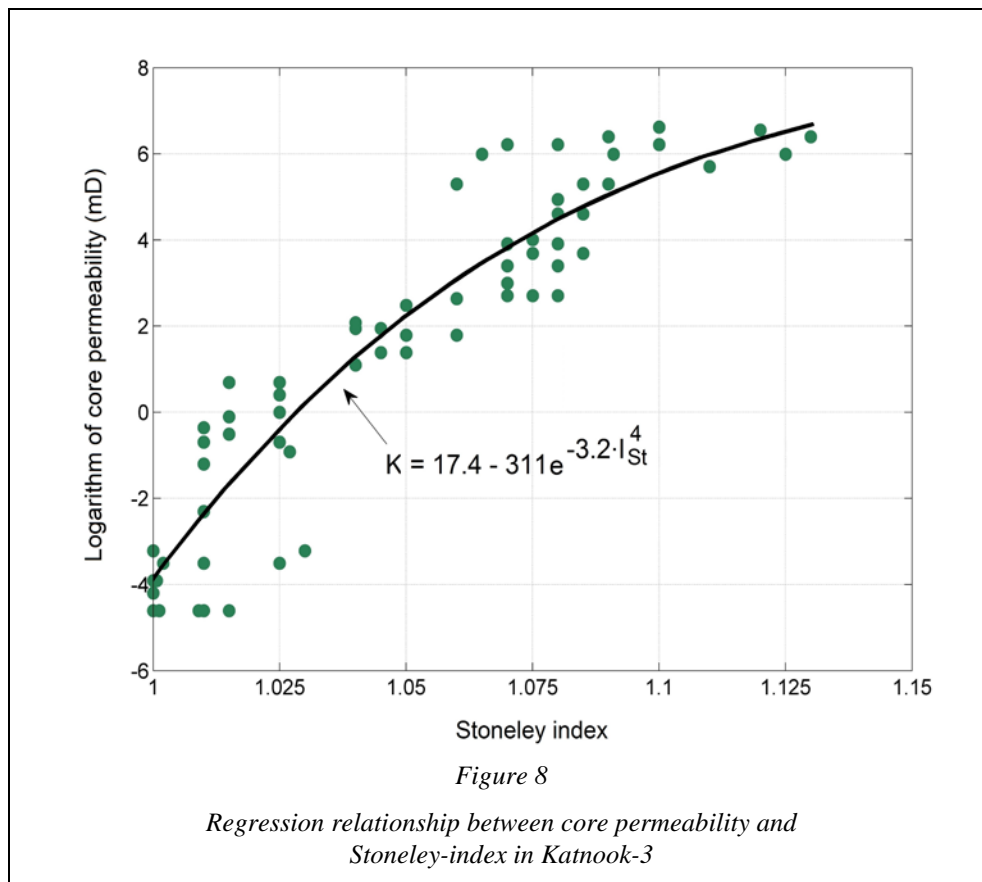
| Noise level [%]	D_m [%]	R	r	a	b	c
1	3.67	0.99	0.97	9.08	61.51	1.79
2	6.86	0.98	0.90	8.77	48.08	1.63
3	9.83	0.98	0.78	9.17	25.28	1.08
4	11.86	0.98	0.66	7.85	17.15	1.00
5	12.09	0.96	0.65	7.90	16.00	0.94

8. Case study

The Stoneley-index vs. permeability relationship can be applied to field data. For this purpose, the data set in Buffin's article (1996) was used, which allowed the validation of the estimated permeability log. The investigated borehole (Katnook-3) was drilled in a hydrocarbon-field of South-Australia (Figure 7). The Otway-basin, including the studied well-site was formed as a result of rifting between the Antarctic and Australian plates. It is approximately 500 km long, west to northwest trending. This area occurs along the southern coast of Australia, and comprises a series of Jurassic and Cretaceous basins with a

Figure 7

Location map of the South-Australian hydrocarbon field (Boult and Donley 2001)
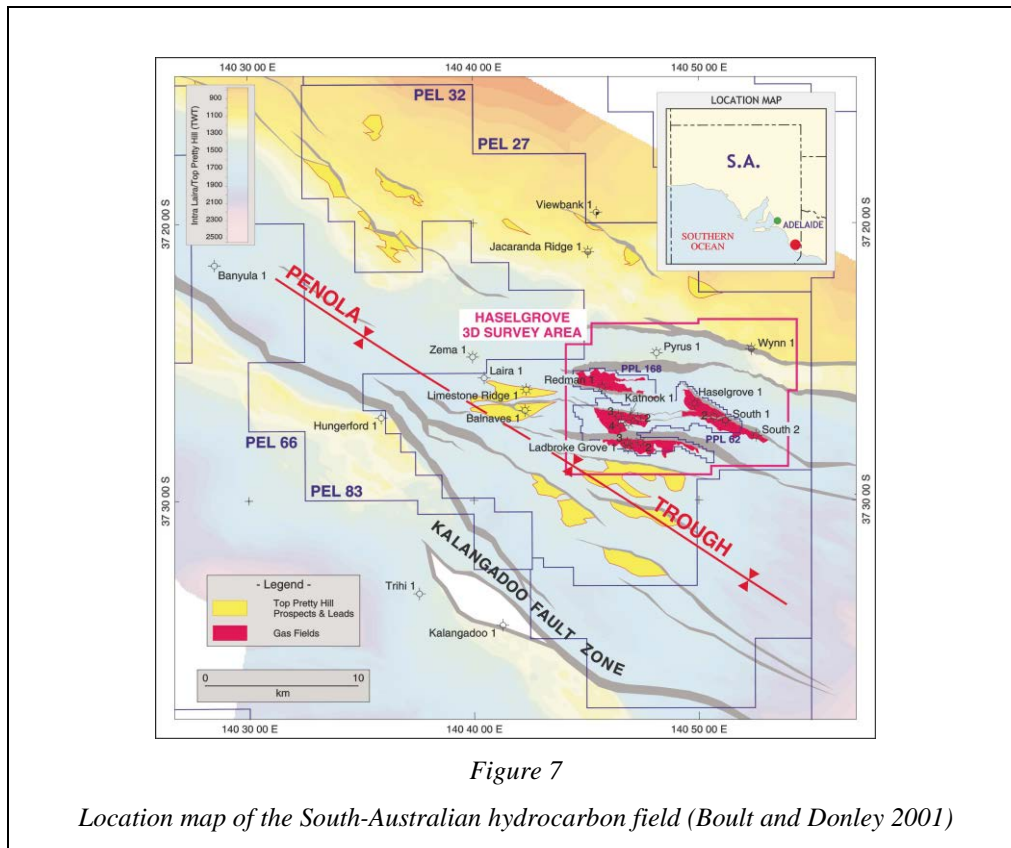
Tertiary cover. Several hydrocarbon reservoir structures can be found in Otway-basin, which exhibit high porosity and permeability in general. The early Cretaceous Pretty Hill Formation is characterized by frequent facies changes, which consist of fine- to coarse-grained sandstone and other mineral components. The main producing gas reservoir in the Otway-basin is the Pretty Hill Formation. Its porosity is often over 25%, permeabilities in excess of 1000 mD, and has flowed over 451,000 m^3 gas per day on test in the Katnook field.

The regression function on the figure is:
$$K = 17.4 - 311e^{-3.2 \cdot I_{St}^4}$$

Figure 8

Regression relationship between core permeability and
Stoneley-index in Katnook-3

The components of acoustic waveform were available in the form of well logs (DTS, DTSTC, DTSTM). Then the Stoneley-index (IST) was calculated using equations (8) and (9). In order to determine the regression function the Stoneley-index was compared to permeability values measured on core samples (Figure 8). As a result, the parameters of equation (14) regression function are $a = 17.4$, $b = 311$, $c = 3.2$. The rank correlation value is 0.98, which shows a very close connection between the Stoneley-index and core permeability (the value of linear correlation is $r = 0.78$).

Figure 9

Input well logs and result of permeability estimation in Katnook-3
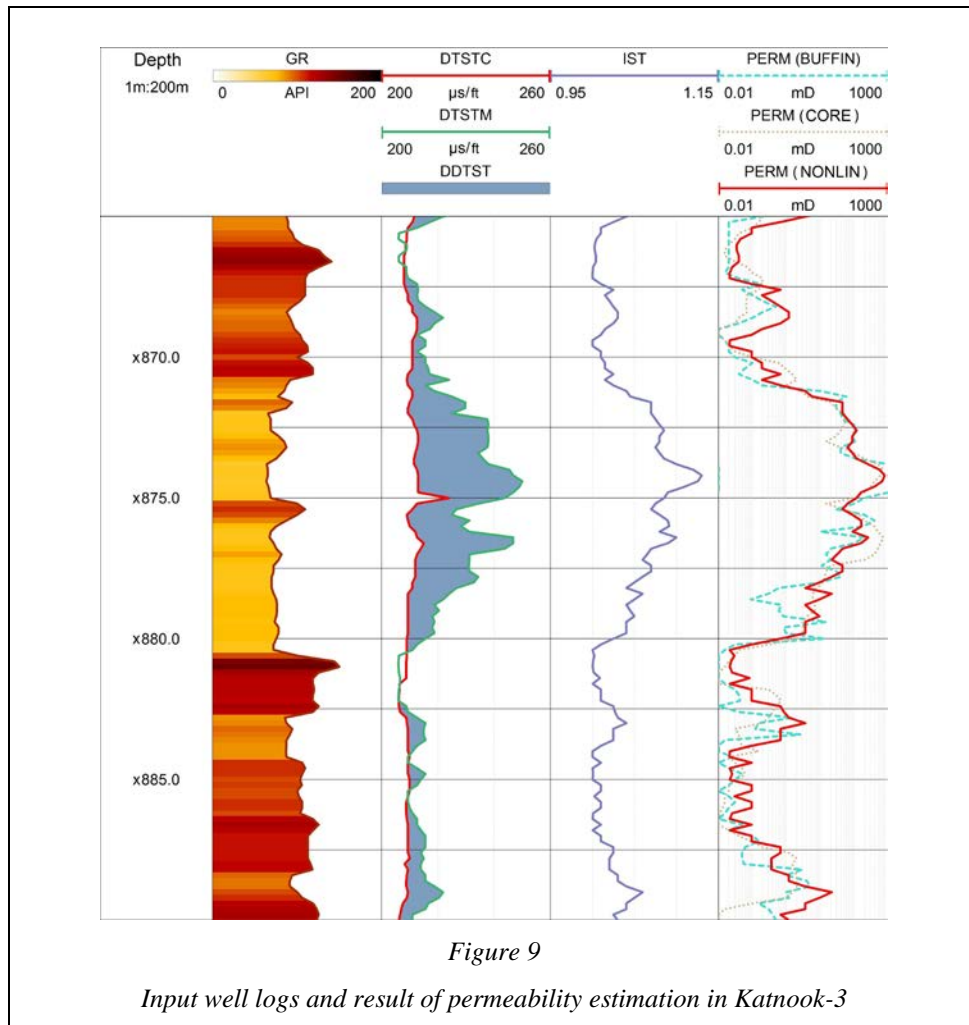
The processed borehole logs and the permeability logs estimated from Stoneley-index are shown in Figure 9, where PERM (CORE) represents the core permeability, PERM (BUFFIN) represents the permeability resulted by the (linear) model of BUFFIN (1996), and PERM (NONLIN) represents permeability due to our non-linear approximation. The core data were quite frequently available (sometimes 0.2 m). The model distance between the values estimated with the non-linear method and the laboratory measured permeability values was 7%. The model distance between the linear result of Buffin and the measured values on the core is 10.1%. The non-linear approximation provides ~30% relative improvement compared to the core data. The misfit in some intervals can be caused by rapid changes of borehole caliper, which is not taken into account by the model. Comparing this field case to the inhomogeneous synthetic test it can be seen that the dispersion of data around the regression model is smaller than in the previous case. The synthetic modeling

experiments show consistent results with those of the case study, which confirms that the suggested statistical method can be used advantageously to evaluate permeability in hydrocarbon formations.

9. Conclusions

In this study the possibility of permeability estimation from nuclear and acoustic logs is shown. Both the synthetic model analysis and the field application confirm that permeability can be derived based on Stoneley-wave arrival times with the statistical (approximate) method. The advantage of this method contrary to core measurements is its quickness and it is a direct method providing continuous (in-situ) information along the whole logging interval. The vertical resolution of this method depends on the length of the applied sonde that is usually 2 ft (0.6 m), in case of normal acoustic tools. The suggested method is a further development of a statistical method known from the literature, which assumes nonlinear relationship between Stoneley-index and permeability. In this case, the correlation coefficient value is bigger, than in case of conventional linear relationship. The field results confirm this close relation, too. The non-linear relationship is especially true in the range of small permeabilities. Therefore, with this method, the reservoirs with poor permeability can be characterized more accurately. The disadvantage of the suggested method is its noise sensitivity. Hence, the appropriate preparation of well logs and data processing is essential. To correct noise sensitivity we offer robust and resistant regression methods that are independent of the probability distribution of data and provide real estimate even in the case of having outliers in the data set. To solve this problem the iterative most frequent value method of STEINER (1997) can be applied effectively. Moreover, this method can be the base for calculation of permeability with inverse modeling. The nonlinear regression equation can be regarded as a theoretical probe response function, which forms a connection between permability and the measured Stoneley-slowness in the permeable layer. In that case, when the regression coefficients are known, the Stoneley-slowness data can be calculated with this equation in the permeable layer. During the iterative inversion procedure, permeability may be varied until the deviation between the measured and calculated Stoneley-curves becomes minimal. Before trying this method, the validity of non-linear response equation has to be certified in several well-sites and the sensitivity to permeability of the model equation have to be studied. These types of analyses will show whethe permeability be used as an inversion unknown or may reveal the problem of ambiguity.

10. Acknowledgement

11. List of symbols

Symbol	Description	Unit
a, b, c	Regression coefficients	–
B	Bulk modulus of dry rock	Pa
B^*	Complex bulk modulus of drilling mud	Pa
B_c	Bulk modulus of saturated formation	Pa
B_f	Bulk modulus of pore fluid	Pa
B_m	Bulk modulus of drilling mud	Pa
B_s	Bulk modulus of rock matrix	Pa
C_0, C	Diffusion coefficient	$m^2 s^{-1}$
D_m	Model distance (RMSE)	%
GR	Natural gamma-ray intensity	API
GR_{sd}	Natural gamma-ray intensity of sand	API
GR_{sh}	Natural gamma-ray intensity of shale	API
I_{St}, IST	Stoneley-index	–
K	Permeability in Darcy's equation	m^2
N	Number of depth points along the borehole	–
p	Pore pressure	Pa
r	Pearson's correlation coefficient	–
R	Spearman's (rank) correlation coefficient	–
PERM	Permeability estimated from well logs	mD
S_{x0}	Water saturation in flushed zone	v/v
$S_{w,irr}$, SWIRR	Irreducible water saturation	v/v
SHCIRR	Irreducible hydrocarbon saturation	v/v
t	Time	s
u	Relative displacement vector	m
v_m	Sonic wave velocity in drilling mud	ms^{-1}
v_{St}	Stoneley-wave velocity	ms^{-1}
V_{sh}, VSH	Shale volume	v/v
V_{sh}, VSD	Sand volume	v/v
α	Elasticity	–
δ	Difference between ranks of data pairs	–
Δt_{hc}	Sonic interval transit time of hydrocarbon	μsm^{-1}
Δt_{mf}	Sonic interval transit time of mudfiltrate	μsm^{-1}
Δt_s, DTS	Shear-wave interval transit time	μsm^{-1}
Δt_{sd}	Sonic interval transit time of sand	μsm^{-1}
Δt_{sh}	Sonic interval transit time of shale	μsm^{-1}
Δt_{St}, DTST	Stoneley interval transit time	μsm^{-1}
η	Viscosity	Pa·s
κ	Scaling parameter	–
μ	Shear modulus	Pa
ρ_b, RHOB	Bulk density of formation	gcm^{-3}
ρ_{hc}	Density of hydrocarbon	gcm^{-3}
ρ_m	Density of drilling mud	gcm^{-3}
ρ_{mf}	Density of mudfiltrate	gcm^{-3}
ρ_{sd}	Density of sand	gcm^{-3}

ρ_{sh}	Density of shale	gcm^{-3}
Φ, POR	Porosity	v/v
ω	Angular frequency of compression wave	Hz

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