Escort distribution function of work done and diagonal entropies in quenched Luttinger liquids

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(Received 24 September 2014; revised manuscript received 10 December 2014; published 22 December 2014)

We study the escort probability distribution function of work done during an interaction quantum quench of Luttinger liquids. It crosses over from the thermodynamic to the small system limit with increasing a, which is the order of the escort distribution, and depends on the universal combination $[|K_i - K_f|/(K_i + K_F)]^a$, with K_i, K_f the initial and final Luttinger liquid parameters, respectively. From its characteristic function, the diagonal Rényi entropies and the many-body inverse participation ratio (IPR) are determined to evaluate the information content of the time-evolved wave function in terms of the eigenstates of the final Hamiltonian. The hierarchy of overlaps is dominated by that of the ground states. The IPR exhibits a crossover from Gaussian to power-law decay with increasing interaction quench parameter.

DOI: 10.1103/PhysRevB.90.245132 PACS number(s): 71.10.Pm, 05.30.−d, 05.70.Ln

I. INTRODUCTION

Nonequilibrium dynamics plays an important role in many areas of contemporary physics, ranging from cosmology through condensed matter to cold atoms. Beautiful theories have been proposed and tested experimentally [1,2], focusing mostly on few-body observables. However, deeper insights into a quantum system may be gained by obtaining the full statistics of a given quantity. In particular, the full distribution function of the interference contrast of coherently split one-dimensional Bose gas was considered [3,4], and the theory of statistics of quantum work done during a time-dependent process has been worked out [5,6]. However, does the full distribution function of the given observable contain all relevant information?

Given an original probability distribution p_i, its statistical and probabilistic attributes may be scanned and revealed by studying the associated escort distribution [7], defined as $P_i = p_i^a/(\sum_n p_n^a)$, where $a > 0$ is the order of the escort distribution. For $a > 1$, the escort distribution emphasizes the more likely events and suppresses the more improbable ones. For $0 < a < 1$, the escort distribution accentuates less probable, rare events. The introduction of escort distributions turns out to be useful in many areas of science (see [7,8] and references therein), e.g., in nonextensive statistical mechanics, for the analysis of earthquakes and structural degradation of matter, the quantification of the efficiency of source coding in information theory and the entropy in black holes, the statistical analysis of financial data, the description of fractals [9], etc.

Escort distributions also facilitate the comparison of various probability distributions (PDs). In the case of slow decay at infinity (e.g., Cauchy distribution), the moments above a given one can diverge, and the usual characterization fails. However, escort distribution converges faster and can provide well-defined quantities for the moments [8].

The escort parameter is also understood as having a replicas of a system and considering only those instances when all replicas are exactly in the same state i, which occurs with probability p_i^a. In some cases [10,11], it is even more convenient to consider a replicas of a system and calculate the ath power of probabilities.

Escort distribution can reveal additional information about quantum systems as well. For example, the energy levels of electrons in a magnetic field form fractal structure, known as the Hofstadter's butterfly [12]. For a nonintegrable quantum system, the level statistics deviates from Poisson distribution and becomes more Wigner-Dysonian [13], indicating level repulsion. Such systems are expected to reveal quantum chaotic behavior [7], and might possess complicated PDs, whose hidden structures can be revealed by the escort PDs.

Recently, much attention has been focused on the PD function of work done during a quantum quench and on the closely related Loschmidt echo [6,14–18]. Therefore, we investigate the escort PD function of work done in a notoriously strongly correlated system, i.e., a Luttinger liquid (LL) after an interaction quench [19], and show that it is connected to the diagonal Rényi entropies [20,21], where the diagonal elements of the density matrix in the instantaneous basis are used. A LL is realized in many one-dimensional fermionic, bosonic, and spin systems [22,23]. Although the Luttinger model is far from being nonintegrable, it is useful to reveal the merit of focusing on the escort PD in this exactly solvable and physically relevant model, before departures from integrability are taken into account.

The escort PD of work done at zero temperature is

$$P_a(W) = \frac{1}{\sum_n p_n^a} \sum_m p_m^a \delta(W - E_m), \tag{1}$$

with $a > 0$, where W is measured with respect to the ground-state energy difference between the initial and final Hamiltonians, and

$$p_m = |\langle m|G_0\rangle|^2. \tag{2}$$

Here, $|G_0\rangle$ is the initial many-body ground-state wave function, while $|m\rangle$'s are the many-body eigenstates with energy E_m of the final Hamiltonian, obtained after a quantum quench.

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The corresponding escort characteristic function of the unnormalized escort distribution is defined as

$$G_a(t) = \sum_m p_m^a \exp(iE_m t), \tag{3}$$

and the PD from Eq. (3) becomes normalized when $G_a(t)/G_a(0)$ is Fourier transformed, and, by definition, $G_1(0) = 1$.

II. ESCORT DISTRIBUTION FUNCTION OF WORK IN LL'S

A LL is described by bosonic soundlike collective excitations, regardless of the statistics of the original system. The LL Hamiltonian is given by [22]

$$H(t) = \sum_{q\neq 0}\left[\omega(q,t)a_q^+ a_q + \frac{g(q,t)}{2}(a_q a_{-q} + a_q^+ a_{-q}^+)\right], \tag{4}$$

where $g(q,t) = g_2\Theta(t)|q|$, with g_2 the strength of the quenched interaction, and the kinetic energy changes as $\omega(q,t) = [v_0 + \delta v\Theta(t)]|q|$. Assuming K_i and K_f are the initial and final LL parameters [22], respectively, the relative LL parameter is $K = K_f/K_i$ [18], which determines the angle θ of Bogoliubov rotation from the initial to the final Hamiltonian in equilibrium as $\sinh^2(\theta) = (1-K)^2/4K$. The final-state dispersion in equilibrium is $\omega_q = v|q|$, with v the sound velocity.

The Luttinger Hamiltonian has been extensively used [22,23] to describe the equilibrium low-energy properties of one-dimensional systems. However, there is growing evidence in recent literature that the Luttinger model can successfully account for the nonequilibrium dynamics of lattice models as well, in spite of the presence of high-energy modes in the latter [18,24,25]. Based on the similarity of the Loschmidt echo and Eq. (3), at least the short- and long-time behavior of the escort characteristic function is expected to agree with the dynamics of the lattice models. In addition, our calculational method is completely different from those used in previous approaches [18,26], namely, the respective overlaps in Eq. (1) are calculated explicitly here using the known ground- and excited-state wave functions of a Luttinger liquid [18,27,28].

The excited states are constructed by populating the bosonic vacuum. Working in the basis of the final Hamiltonian [after a Bogoliubov rotation of $H(t>0)$ to render it diagonal], the eigenfunctions having a finite overlap with the initial state are of the form

$$|m\rangle = \prod_{q>0}|n_q\rangle = \prod_{q>0}\frac{1}{n_q!}(b_q^+ b_{-q}^+)^{n_q}|0\rangle, \tag{5}$$

i.e. having the same number of bosons in a given q and $-q$ state. In this basis,

$$|G_0\rangle = \prod_{q>0}|G_0^q\rangle = \prod_{q>0}\frac{1}{\cosh(\theta)}\exp[-\tanh(\theta) b_q^+ b_{-q}^+]|0\rangle. \tag{6}$$

From the specific structure of the bosonic wave function [18,27,28], we get

$$G_a(t) = \sum_m |\langle m|G_0\rangle|^{2a}\exp(iE_m t) = \prod_q \sum_{n_q} \left|\langle n_q|G_0^q\rangle\right|^{2a}\exp\left(iE_{n_q} t\right), \tag{7}$$

and $\langle n_q|G_0^q\rangle = \tanh^{n_q}(\theta)/\cosh(\theta)$. For a given mode, the overlap is calculated as

$$\sum_{n_q}\left|\langle n_q|G_0^q\rangle\right|^{2a}\exp\left(iE_{n_q}t\right) = \frac{\cosh^{-2a}(\theta)}{1-\tanh^{2a}(\theta)\exp(2i\omega_q t)}, \tag{8}$$

where $E_{n_q} = 2n_q\omega_q$, and the factor of 2 comes from the pair of entangled boson modes at a given $\pm q$ [29].The numerator comes from the normalization factor, while the denominator accounts for the overlap of multiboson wave functions. The escort characteristic function yields

$$G_a(t) = \prod_{q>0}[\cosh^{2a}(\theta) - \sinh^{2a}(\theta)\exp(2i\omega_q t)]^{-1}. \tag{9}$$

This is evaluated in closed form using

$$\begin{aligned}\prod_{q>0}&\frac{1}{1-\tanh^{2a}(\theta)\exp(2i\omega_q t)}\\ &= \exp\left\{-\sum_{q>0}\ln[1-\tanh^{2a}(\theta)\exp(2i\omega_q t)]\right\}\\ &= \exp\left[\sum_{n=1}^{\infty}\sum_{q>0}\frac{1}{n}\tanh^{2an}(\theta)\exp(2in\omega_q t)\right]\\ &= \exp\left[\frac{L}{2\pi}\sum_{n=1}^{\infty}\frac{1}{\alpha-2invt}\frac{\tanh^{2an}(\theta)}{n}\right],\end{aligned} \tag{10}$$

where an exponential cutoff, $\exp(-\alpha|q|)$, was used for the bosonic modes. This yields

$$\ln\left[\frac{G_a(t)}{G_a^\infty}\right] = \frac{L}{2\pi}\frac{\tanh^{2a}(\theta)}{\alpha-2ivt} \times {}_3F_2\left(1,1,1+\frac{i\alpha}{2tv};2,2+\frac{i\alpha}{2tv};\tanh^{2a}(\theta)\right), \tag{11}$$

where ${}_3F_2(a;b;z)$ is the generalized hypergeometric function [30], L is the system size, and

$$G_a^\infty \equiv G_a(t\to\infty) = [\cosh(\theta)]^{-La/\pi\alpha} = |\langle 0|G_0\rangle|^{2a}, \tag{12}$$

with $t \ll L/v$. This is the $2a$th power of the respective ground-state overlaps, extending the result for $a = 1$ [18]. All but the ground-state overlap dephase, although $G_a(0)$ can be arbitrarily large in the $a \to 0$ limit. This is the generalization of the many-body orthogonality catastrophe to the escort distribution case.

The normalized escort PD is obtained from the characteristic function using Eq. (11) as

$$\mathcal{G}_a(t) \equiv \frac{G_a(t)}{G_a(0)} = [1-\tanh^{2a}(\theta)]^{L/2\pi\alpha}\frac{G_a(t)}{G_a^\infty}. \tag{13}$$

Based on this, we observe that both $\mathcal{G}_a(t)$ and the escort distribution of work $P_a(W) = \int\frac{dt}{2\pi}\exp(-iWt)\mathcal{G}_a(t)$ depend on the interaction and escort parameter through $\tanh^{2a}(\theta)$ (apart from a possible velocity renormalization, representing an overall scale factor), which is translated to the universal combination $[|K_i - K_f|/(K_i + K_F)]^a$. Therefore, by varying the interaction strength and the escort parameter appropriately,

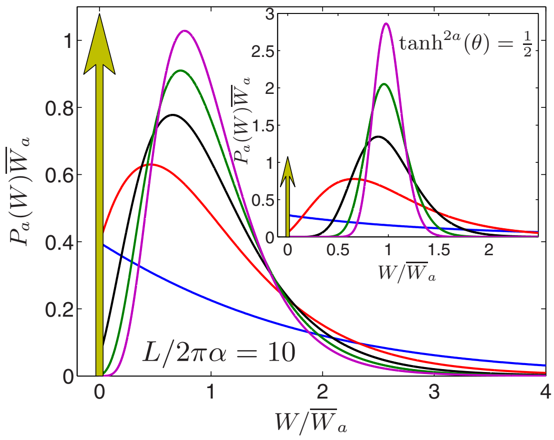

FIG. 1. (Color online) The escort PD function of work is plotted for $L/2\pi\alpha = 10$ and $\tanh^{2a}(\theta) = 0.1$ (blue), 0.3 (red), 0.5 (black), 0.7 (green), and 0.9 (magenta) with peak position from left to right. The vertical arrow at $W = 0$ denotes the Dirac delta function with spectral weight P_a^{ad}, given in Eq. (14). Inset: The evolution of the $\tanh^{2a}(\theta) = 0.5$ case for $L/2\pi\alpha = 1$ (blue), 10 (red), 40 (black), 100 (green), and 200 (magenta). For the last four, the spectral weight of the Dirac δ peak is practically zero.

the escort work distribution function remains unchanged. In addition, the escort PD remains invariant under the $(K_i,K_f) \leftrightarrow (K_f,K_i) \leftrightarrow (1/K_i,1/K_f) \leftrightarrow (1/K_f,1/K_i)$ changes of the initial and final Hamiltonian.

For $a = 1$, we immediately get the characteristic function of work [5], whose absolute value is the Loschmidt echo [18]. The Fourier transform of $\mathcal{G}_1(t)$ gives the PD to find the system in a given energy eigenstate after the quench. However, it does not reveal how many different eigenstates live on the same energy shell. The Fourier transform of $\mathcal{G}_{a\neq 1}(t)$ contains information about the number of states within a given energy shell as well, i.e., about degeneracies. The escort distribution function of work done during the quench is visualized in Fig. 1.

These results are nonperturbative in the interaction strength, and agree qualitatively with the perturbative, nonescorted distribution of work done [26]. The finite a-escorted probability to stay in the adiabatic ground state is

$$P_a^{ad} = [1 - \tanh^{2a}(\theta)]^{L/2\pi\alpha}, \tag{14}$$

signaled by the Dirac δ peak at zero energy and $P_a(W < 0) = 0$. The a-escorted expectation value and variance of work follow from expanding $\ln \mathcal{G}_a(t)$ in t as

$$\overline{W}_a = \frac{Lv}{\pi\alpha^2}\frac{\tanh^{2a}(\theta)}{1 - \tanh^{2a}(\theta)}, \tag{15a}$$

$$\sigma_W^2 = \frac{4Lv^2}{\pi\alpha^3}\frac{\tanh^{2a}(\theta)}{[1 - \tanh^{2a}(\theta)]^2}. \tag{15b}$$

We emphasize again that $\overline{W}_a$ is measured with respect to the ground-state energy difference, and it decreases/increases for $a \gtrless 1$ as the probabilities are further suppressed/enhanced, respectively. Nevertheless, $\overline{W}_a > 0$, in accordance with the second law of thermodynamics.

In the so-called small system limit [31], defined by $L\tanh^{2a}(\theta)/2\pi\alpha \ll 1$, an exponential distribution with rate parameter $2v/\alpha$ accounts for the escort distribution, though most of the spectral weight is concentrated to the $W = 0$ Dirac δ peak. This is also corroborated by $\sigma_W/\overline{W}_a \xrightarrow{\tanh^{2a}(\theta)\to 0} \infty$. In the opposite, thermodynamic limit $[L\tanh^{2a}(\theta)/2\pi\alpha \gg 1]$, achievable by increasing L or θ or decreasing a, the distribution develops a sharp and narrow peak, centered at $\overline{W}_a$ and carrying almost all the spectral weight, as expected from the central limit theorem, since $\sigma_W/\overline{W}_a \xrightarrow{L\to\infty} 0$ from Eqs. (15). Around $\overline{W}_a$, there is a large number of degenerate overlaps with small individual probabilities. In the extreme $a = 0$ limit, all probabilities in Eq. (1) become identical, and $P_0(W)$ yields the many-body density of states of a LL.

With increasing a, the large probability states are favored, and the escort distribution approaches that in the small quench limit and the peak moves towards zero energy and disappears, and decays monotonically for larger energies. This indicates that low total energy states are more similar to the initial states and appear with larger probabilities in the time-evolved wave function.

In the opposite, decreasing a region, the escort distribution enhances the role of low-probability states and the number of states around a given energy determines the distribution. Therefore, states with large total energy and large degeneracy overwhelm the smaller number of low-energy states and dominate the distribution.

III. DIAGONAL ENTROPIES

The global information content of the quenched wave function in terms of the eigenstates of the final Hamiltonian is conveniently characterized by the diagonal von Neumann or Shannon [20,21,32] and Rényi entropies which we obtain from $G_a(t)$ as well. Note that these characterize the information content of the original PD and not the escort ones, though the entropies of the escort PDs can also be evaluated similarly. Additionally, the diagonal von Neumann entropy was argued to satisfy the properties of a thermodynamic entropy [20], and exhibits different behavior in integrable and nonintegrable systems. This is defined as

$$S_1 = -\sum_m \rho_{mm}\ln(\rho_{mm}), \tag{16}$$

where the ρ_{mm}'s are the diagonal elements of the density matrix in the instantaneous basis [21]. In the present case, the probability p_m in Eq. (2) corresponds to the diagonal elements of the density matrix, ρ_{mm}. The basic thermodynamic requirements of an entropy are also satisfied by the Rényi entropies, which can also be calculated from the diagonal elements of the density matrix. From the specific structure of the escort characteristic function of work done, these follow immediately. Setting $t = 0$, and using the definitions of the Rényi entropies, we get

$$S_a = \frac{1}{1-a}\ln\left(\sum_m p_m^a\right) = \frac{1}{1-a}\ln[G_a(0)]$$
$$= \frac{L}{2\pi\alpha}\frac{1}{a-1}\ln[\cosh^{2a}(\theta) - \sinh^{2a}(\theta)]. \tag{17}$$

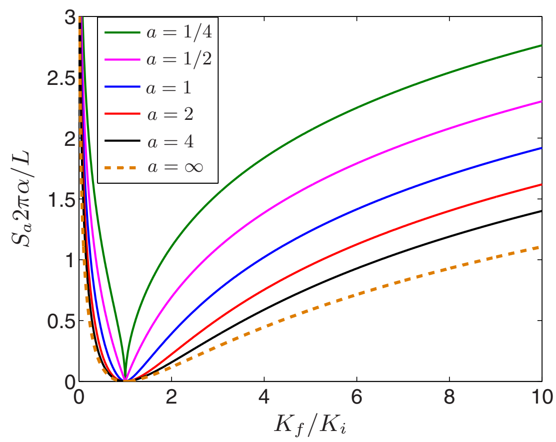

FIG. 2. (Color online) The Rényi entropies as a function of $K = K_f/K_i$ for $a = 1/4$, $1/2$, 1, 2, 4, and ∞ (from top to bottom) are plotted.

With increasing a, the overlaps with low probabilities die out. The diagonal entropies are small when the probabilities are dominated by only a few states, since $\sum_m p_m^a$ is still sizable, therefore its logarithm is vanishingly small. On the other hand, when a large number of final states are contributing to the expansion of the time-evolved wave function, the respective probabilities are small, therefore their logarithm is enhanced, leading to the growth of the entropies. Therefore, the Rényi entropies quantify the amount of entanglement between the initial and final Hamiltonian as well as the quantum fluctuations.

The Rényi entropy with $a \to \infty$ retains only the largest diagonal element of the density matrix. From this, the largest probability, $p_{\max} = \max_m p_m$, which is also the weight of the most probable configuration, is connected to the entropies as

$$p_{\max} = \exp(-S_\infty) = \left(\frac{K_i + K_f}{2\sqrt{K_i K_f}}\right)^{-L/\pi\alpha}. \tag{18}$$

Comparing this to Eq. (14), it is identified as the probability to stay in the ground state, $|\langle 0|G_0\rangle|^2$, which then dominates over the large number of low-probability excited-state overlaps.

While a direct computation of the von Neumann entropy would be rather difficult for the present case, similarly to other instances [10], it follows from the Rényi entropies as the $a \to 1$ limit as

$$S_1 = \frac{L}{2\pi\alpha}[\cosh^2(\theta)\ln\cosh^2(\theta) - \sinh^2(\theta)\ln\sinh^2(\theta)], \tag{19}$$

which plays the role of the thermodynamic entropy after the quench [20]. Various entropies as a function of the LL parameter are plotted in Fig. 2. In the small quench limit ($K \approx 1$), it becomes a nonanalytic function of K for $a < 1$ as $S_a \sim |K-1|^{\min(2,2a)}$, and becomes a nonanalytic function of K for $a < 1$.

From the escort characteristic function of work, other entropies can be calculated such as the nonextensive Tsallis entropies. It is defined as

$$S_a^{Tsallis} = \frac{1}{a-1}\left(1 - \sum_m p_m^a\right) = \frac{1 - G_a(0)}{a-1}, \tag{20}$$

being nonextensive for any $a \neq 1$, and becoming extensive for $a = 1$ when it reduces to the von Neumann entropy. Note that the Tsallis entropy could become extensive for certain values of $a \neq 1$ for certain models as well [33] (e.g., for the transverse field Ising chain).

Another useful characteristic of the difference between the initial state and the eigenstates of the final Hamiltonian is the many-body inverse participation ratio (IPR) [34,35]. It measures the inverse number of many-body eigenstates of the final Hamiltonian over which the initial state is distributed, and contains information about localization in Fock space. In disordered problems, it is a useful quantity to diagnose real-space localization as it probes the number of lattice sites over which a single-particle wave function is extended, and it scales differently with the system size for spatially localized or extended single-particle states (see, e.g., Ref. [36]).

The many-body IPR follows from the escort characteristic function of work done as well, similarly to the diagonal entropies. For the quenched LL, it reads

$$\text{IPR} = \sum_m p_m^2 = G_2(0) = \left(\frac{K_i}{2K_f} + \frac{K_f}{2K_i}\right)^{-L/2\pi\alpha}, \tag{21}$$

and its logarithm S_2 is plotted in Fig. 2. For small quenches, $K_f = K_i + \delta K$ with $|\delta K| \ll 1$, the IPR $\approx \exp[-L(\delta K)^2/K_i^2 4\pi\alpha]$ decays as a Gaussian with the quench parameter δK. For sizable quenches, $K_f \gtrless K_i$, however, it crosses over to a power-law decay, IPR $\sim (K_f/K_i)^{-\text{sign}(K_f-K_i)L/2\pi\alpha}$, with respect to the LL parameter. These are roughly consistent with recent numerics [34]. The behavior of the IPR can be understood from the nonescorted, $a = 1$ distribution of work done. From our previous analysis in Sec. II, for small quenches, the contribution of low-energy states in the expansion of the time-evolved wave function is dominant over high-energy ones, and the IPR decays slowly. With increasing quench size, however, the central limit theorem holds and most of the spectral weight comes from the large number of degenerate states located around the average energy.

The escort PD relies on the eigenenergies and overlaps in Eq. (1), containing all the information about the system at hand. However, this is somehow "too much" information, and statistical methods are required for its analysis such as the normal and escort PD as well as related quantities such as the IPR. This parallels closely with the importance of the (generalized) IPR [36], identifying de/localized states in disordered system, but still requiring the knowledge of all the eigenstates, similarly to our case.

IV. CONCLUSIONS

Our calculations can be extended for higher-dimensional and/or gapped bosonic systems as well. For example, quenching a one-dimensional gapless system to a gapped phase, the

IPR decays exponentially with the gap Δ as

$$\mathrm{IPR} = \exp\left(-cL\Delta/v\right), \tag{22}$$

with $c > 0$, and, in particular, $c = (\sqrt{2} - 1)/4$ when quenching to the semiclassical limit of the sine-Gordon model [37]. It would also be interesting to explore the behavior of the escort distribution of work done in other models, e.g., the Rabi model [38], and for local quenches such as the x-ray edge problem [39].

To conclude and answer the question raised at the beginning of the paper, the PD allows for calculating arbitrary expectation values of a given quantity, but it cannot resolve the interplay of degeneracies and individual probabilities. An escort PD, on the other hand, is capable of revealing this additional information. In addition to demonstrating this for a Luttinger liquid, the diagonal Rényi entropies and the inverse participation ratio are shown to follow also from the escort characteristic function of work done.

ACKNOWLEDGMENTS

Useful discussions with J. Pitrik and Sz. Vajna are gratefully acknowledged. This research has been supported by the Hungarian Scientific Research Funds No. K101244, No. K105149, and No. K108676, by the European Research Council Grant No. ERC-259374-Sylo, and by the Bolyai Program of the HAS.

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