

Electing the Pope

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Abstract

Few elections attract so much attention as the Papal Conclave that elects the religious leader of over a billion Catholics worldwide. The Conclave is an interesting case of qualified majority voting with many participants and no formal voting blocks. Each cardinal is a well-known public figure with publicly available personal data and well-known positions on public matters. This provides excellent grounds for a study of spatial voting: In this brief note we study voting in the Papal Conclave after the resignation of Benedict XVI. We describe the method of the election and based on a simple estimation of certain factors that seem to influence the electors' preferences we calculate the power of each cardinal in the conclave as the Shapley-Shubik index of the corresponding voting game over a convex geometry.

Keywords and phrases: Papal Conclave, voting games, game over convex geometry, Shapley-Shubik index, minimal winning coalitions.

JEL codes: C71, D72

1 Introduction

The election of Pope Francis has attracted much attention among Catholics and non-Catholics alike. As the religious leader of some 1.2 billion Catholics worldwide, the Pope has an enormous influence on world politics – far more than what being the head of the smallest ministate would imply. The media hype is enhanced by the complex voting procedure entwined with old traditions and secretive elements. We are, however interested in the Papal Conclave – the electing body – for another reason. The voting power literature studies examples that are either instances of qualified weighted majority voting, such as that in the European Union (Kóczy, 2012) where each of the voters have a given weight or examples of voting in a parliament or a similar voting body. In the latter case the voters are all individuals with equal weights, but they form parties or other voting blocks. Party discipline

tends to be high, so that party members almost always vote the same. If this is always true we might as well consider a weighted voting where the party leader chooses a position and all party members follow. In practice there may be absent, or rebel voters making the analysis somewhat more complex (Kóczy, 2011). In the Papal Conclave we only have a set of cardinals with certain policy preferences, possibly with suspected voting blocks, but no formal parties. Especially not with parties that could employ sanctions on rebel voters. In sum: each cardinal chooses his position independently of others.

The new pope is elected by the Papal Conclave consisting of the cardinals under 80 using a two-thirds majority. In the election of Pope Francis this meant 117 cardinals – 115 participating. The voting rules are very elaborate where, until the required majority is reached, several voting rounds might take place. With this process the positions of the participating cardinals might and do change. In our simple analysis, however we ignore the changes of preferences, the strategic games that may be played. We seek to determine the more influential cardinals in the voting process based on their positions. Our assumption is that the elected Pope will then be similar to these powerful players, they themselves are likely candidates, although some would likely use their influence to elect someone else.

Since Shapley and Shubik (1954) adopted the Shapley value to measure a priori voting power, we use simple games to model voting situations. In these games there are no actual payments, *power* itself is the payoff of the game. A coalition is either *winning* and has a payoff of 1, or *losing* getting 0. We are interested in swing voters who can turn a losing coalition into a winning one. Due to the symmetry of the voting situation, the swing is always the 77th supporting cardinal; in the theory of the Banzhaf power index (Banzhaf, 1965), where we look for *critical* players, the same player is critical for the coalition.

If we assume that any combination of 77 cardinals is equally likely, we are very remote from reality. There are very strong interest groups based on location (Italy, America, Europe) and progressiveness. We assume that a candidate that is acceptable to both a liberal and a conservative cardinal will also be acceptable to a moderate cardinal. These, so-called games on convex geometries (Edelman, 1954) presume that only convex coalitions may form. Both the Shapley-Shubik index (Bilbao, 2000) and the Banzhaf index (Bilbao, Jiménez, and López, 1998) has been extended to such games.

In the following we introduce our notation, explain the geometry of the game and present our *papabili*.

2 Power indices

2.1 Simple games

First we introduce the usual terminology and notation. Let $N = \{1, \dots, n\}$ be the set of the players. $v : 2^N \rightarrow \mathbb{R}$, where $v(\emptyset) = 0$, is a transferable utility (TU) cooperative game (henceforth *game*) with player set N . For any player i and any coalition S : $v'_i(S) = v(S \cup \{i\}) - v(S)$, that is $v'_i(S)$ is player i 's marginal contribution to coalition S in game v . Let v'_i stand for player i 's marginal contribution function in game v . Player i is a null-player in game v if $v'_i = 0$. Finally, $|A|$ is for the cardinality of set A .

A voting situation or voting game is a pair (N, \mathcal{W}) , where the players are the voters and \mathcal{W} denotes the set of *winning coalitions*. We consider *simple voting games* where

1. $\emptyset \notin \mathcal{W}$ and $N \in \mathcal{W}$,
2. if $C \subseteq D$ and $C \in \mathcal{W}$, then $D \in \mathcal{W}$,
3. if $S \in \mathcal{W}$ and $T \in \mathcal{W}$, then $S \cap T \neq \emptyset$.

Condition 3 requires the game to be *proper*, that is, a motion and its opposite cannot be approved simultaneously.

Let $\bar{\Gamma}_N$ denote the set of *proper simple voting games over the player set N* . We can also write a simple voting game in the form of a transferable utility game v , where $v(S) = 1$ if $S \in \mathcal{W}$ and 0 otherwise. The term "simple" comes from having coalitions with payoffs 0 or 1 only.

We study the players' ability to change decisions. If, by joining a losing coalition, a player can turn it winning, we call the player *swing*. *Voting power* then refers to this ability to change decisions.

Given a game v of $\bar{\Gamma}_N$, an *a priori measure of voting power* or *power measure* $\kappa : \bar{\Gamma}_N \rightarrow \mathbb{R}_+^N$ assigns to each player i a non-negative real number $\kappa_i(v)$, its *power* in game v ; if for any game v of $\bar{\Gamma}_N$: $\sum_{i \in N} \kappa_i(v) = 1$, then it is also a *power index*.

In the following we explain some of the well-known indices. The *Shapley-Shubik index* ϕ (Shapley and Shubik, 1954) applies the Shapley value (Shapley, 1953) to simple games: Voters arrive in a random order; if and when a coalition turns winning the full credit is given to the last arriving, the *pivotal* player. A player's power is given as the proportion of orderings where he is pivotal, formally for any simple voting game v player i 's Shapley-Shubik index in game v is as follows

$$\phi_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{s!(n-s-1)!}{n!} v'_i(S) ,$$

where $s = |S|$.

The *Banzhaf measure* ψ (Penrose, 1946; Banzhaf, 1965) is the vector of probabilities that a party is *critical* for a coalition, that is, the probabilities that it can turn winning coalitions into losing ones. Formally, for any simple voting game v player i 's Banzhaf-measure in game v is as follows

$$\psi_i(v) = \frac{\eta_i(\mathcal{W})}{2^{n-1}},$$

where $\eta_i(\mathcal{W})$ is the number of coalitions in \mathcal{W} in which i is critical. When normalized to 1, we get the *Banzhaf index* β (Coleman, 1971). Formally, for any simple voting game v player i 's Banzhaf index in game v is as follows

$$\beta_i(v) = \frac{\eta_i(\mathcal{W})}{\sum_{j \in N} \eta_j(\mathcal{W})}.$$

2.2 Games over convex geometries

The set $\mathcal{L} \subseteq 2^N$ of sets of coalitions is a convex geometry (Edelman, 1985) if two properties are satisfied:

1. $\emptyset \in \mathcal{L}$ and \mathcal{L} is closed under intersection.
2. if $S \in \mathcal{L}$ and $S \neq N$ then there exists $i \in N \setminus S$ such that $S \cup i \in \mathcal{L}$.

The set \mathcal{L} collects convex coalitions. We call $i \in S$ for $S \in \mathcal{L}$ an extremal point if $S \setminus \{i\} \notin \mathcal{L}$. We denote the extremal points of S by $\delta(S)$. For games over convex geometries we can also define winning and losing coalitions.

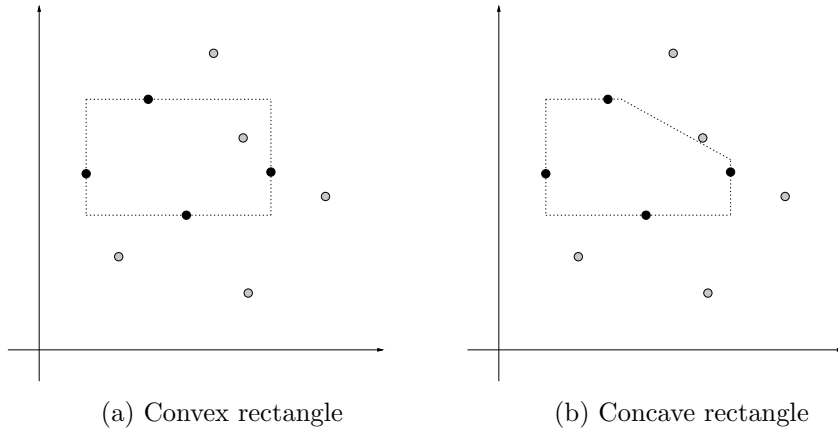


Figure 1: Examples of coalitions

We say that i is a convex swing in coalition S if $S \in \mathcal{L}$, $i \in \delta(S)$, $v(S) = 1$, but $v(S \setminus \{i\}) = 0$. Note that if $i \in \delta(S)$, $S \setminus \{i\} \in \mathcal{L}$ this property only tells about the transition between feasible coalitions. If these extremal players continue to find the coalition acceptable, a situation where internal players

would like to leave the coalition is not conceivable. Therefore no power can be derived from turning a convex coalition into a non-feasible one.

3 The Papal Conclave

3.1 A brief history

Despite the importance of the bishop of Rome in the Catholic Church, relatively little is known about the first popes. The Bible itself provides no rules “how bishops were selected, but the process of choosing the seven deacons detailed in Acts (6:1–6) influenced the way they were.” (Baumgartner, 2003) Peter probably appointed Linus as his successor, while the first account of an election is that of Fabian in 236, although records for the time before 400 are considered unreliable. Over the centuries the election of the new Pope has changed a lot. Until the late renaissance times Papal elections have often been bitter, even violent conflicts between strong candidates coming from strong Roman families, between parties supporting Rome versus the emperor (first the Byzantine then the Holy Roman emperor) or candidates loyal to the king of France or to Italy. Initially the voting game was not even proper: multiple popes could be elected simultaneously creating a temporary schism in the Church. To eliminate this awful situation the Third Lateran Council (1179) prescribed a two-third majority of all voting cardinals (Baumgartner, 2003). The idea was that no faction can elect its “own” Pope, who is, on the other hand resented by the opposition. The two third majority implied that no ‘strong’ candidates could be elected. In practice, however, making it to the election was difficult and often only a very small portion of cardinals were present. In 1191 only 8 cardinals voted of the 31 cardinals then alive. Such elections undermined the legitimacy of the pope and cardinals lived increasingly in Rome to be available for elections.

In 1241 the cardinals have been locked up in a building and the word conclave, coming from *cum clave* (“with keys”) can since then be used for the electing body. While the number of cardinals have been kept low, for a long time it was limited to 24, reaching a compromise was not always trivial. According to the records during the 1268-72 interregnum decision was reached after 34 months of discussion (Colomer, 1998). To avoid similar situations the elected Gregory X issued the bull *Ubi Peculiarum* that specified some rules of the conclave in great detail. In 1294 the hermite Pietro de Murrone was elected to become Celestine V. While becoming the pope has been a dream for many, he considered it a burden and wanted to abdicate. Before the actual abdication he announced to apply the conclave’s rules “however a vacancy might occur in the papacy”. (Baumgartner, 2003). The vote after his abdication is the first instance when the actual voting records have survived. The voting has not been anonymous and a two-third majority plus one vote was required “to ensure that a cardinal’s vote for himself did

not provide the margin of victory” (Baumgartner, 2003). For a long time yet it was common that cardinals name more than one candidate on their ballots, so that the voting was actually approval voting with a qualified majority.

Since then the rules have changed little. John Paul II (1996) introduced the clause that after 33 unsuccessful rounds simple majority suffices to elect the pope. This rule has been criticised that it does not force an agreement upon the cardinals making a simple majority enough to elect a pope if the electors hold out for long enough (BBCNews, 2007). BenedictusXVI (2007) essentially reverted to the original requirement of a two-third majority.

3.2 The cardinals

While the election is at the influence of the Holy Spirit, those are ultimately the cardinals who cast their votes and in the past a number of objective factors have clearly been good predictors of the final outcome. Magister (2013) lists four characteristics of the cardinals – besides their name: nationality, age, who made them cardinal and whether they belong (or have belonged) to the Curia. It would be beyond the scope of this paper to review the discussions why these might play a role, but a few points may be noted. The bishop of Rome has been more often than not Italian; whether a pope is Italian or not, is a central question. Currently Latin America is the continent with the largest number of Catholics and despite having only 19 cardinals together with cardinals of North America and some European Latin countries have a strong voting block. Ultimately the question is whether the pope is European or non-European. After the long reign of John Paul II many favoured an older pope; now the feelings are at least ambiguous. Cardinals at the curia are well known and well connected in the Church but lack pastoral experience and may be more affected by the so-called Vatileaks scandals. This trade-off might play an important role, too. We consider two characteristics: physical distance from Rome and a measure of conservatism. The first is a one-dimensional proxy for nationality and the associated “games”, the second has become important since the Church faces serious challenges from the rapidly changing society.

We measure distance by the distance of the birthplace of the cardinal (obtained from the cardinals’ Wikipedia pages) from Rome. For conservatism we use a very simple Google-metric: of the number of hits for the searches *Is “X” conservative?* and *Is “X” liberal?* we take the percentage of the first. For comparability reasons we convert this percentage value to ‘basis points’ by multiplying it by 10000 and taking its upper integer part. These two scores uniquely determine the location of a cardinal on the policy space. Both measures are admittedly imperfect. There are a few cardinals, who were born far from Rome, but who for various reasons spent their religious life in the Roman Curia. On the other hand our Google-metric might

be biased by many factors (e.g. the cardinal's popularity or nationality). In spite of that when compared with comments by Vatican insiders, cardinals known for their conservatism tend to score high on our chart, while known liberals usually score low. Note that a liberal cardinal may still be seen very conservative by the society.

3.3 The majority

Given the distribution of cardinals over this two-dimensional space we look for coalitions that (i) are convex and (ii) have the required majority. Observe that coalitions with supermajority, that is with more members than the required quota are not interesting for us, since here none of the cardinals are critical, no-one can push his own policy. In coalitions with the (exact) majority each cardinal on the borderline of the coalition is swing, that is, his presence is essential for the coalition. What are the convex coalitions in this setting? A cardinal c with position (c_x, c_y) belongs to a coalition S if and only if there exist four (not necessarily distinct) cardinals e, f, g, h in S , such that $e_x \leq c_x \leq g_x$ and $f_y \leq c_y \leq h_y$.

In order to find the influence of the individual cardinals we must first identify all such coalitions. These coalitions are really rectangles spanned by 2 or more positions in the policy space. For each rectangle we must identify the cardinals on the borderline – each of these cardinals get a point for being swing/critical. The value of each cardinal is given as the total points collected, normalized to 1. Note that due to the fact that cardinals only differ in their positions, the game is symmetric. In particular, the probability of forming any of the minimal winning coalitions is the same. In such a game the Shapley-Shubik index coincides with the Banzhaf index we calculated.

3.4 The algorithm for identifying the minimal winning coalitions

The cardinals, denoted by c^1, c^2, \dots, c^{115} are treated as points in the plane where the horizontal axis (x) represents the distance from Rome and the vertical axis (y) represents the conservatism of the cardinals.

In the following we calculate the influence of each cardinal by finding all rectangles corresponding to minimal winning coalitions and checking how frequently a particular cardinal is placed on the outline of such a rectangles. First we provide the general outline of our method and then provide a more efficient algorithm

1. Select the left, right, bottom, and top side of our rectangle: Chose an arbitrary not necessarily different quadruple of cardinals $S = \{l, r, b, t\} \subset$

N , such that

$$\begin{aligned} l_x &= \min_{k \in S} k_x \\ r_x &= \max_{k \in S} k_x \\ b_y &= \min_{k \in S} k_y \\ t_y &= \max_{k \in S} k_y \end{aligned}$$

2. Check the number of cardinals with positions in the closed rectangle spanned by S . If it is exactly 77, S spans a minimal winning coalition.
3. Check that the rectangle has not been considered before.
4. Give points to cardinals on the borderline.
5. Rank according to the points given.

Since the points need not be disjoint, there are more than $\binom{|N|}{4}$ or in our case about 7 million rectangles to look at.

A more systematic search is more economical. The detailed algorithm we have used can be found in Appendix A.

4 Results

In Table 1 we present a list of the cardinals together with their coordinates and the number of minimal winning coalitions where they are critical. Distance is given in kilometers, while conservatism in basis points.

Table 1: Cardinals in the Conclave of 2013 ranked by the number of minimal winning coalitions where they are critical

rank	cardinal	c_x (km)	c_y (bp)	score
1	George Pell	15902	4477	599
2	Francisco Javier Errázuriz Ossa	11921	4617	572
3	Jorge Bergoglio	11162	4507	546
4	Leonardo Sandri	11162	4967	536
5	Théodore-Adrien Sarr	4181	185	517
6	Juan Luis Cipriani Thorne	10873	4247	497
7	Telesphore Placidus Toppo	6039	7264	481
8	Agostino Vallini	34	4992	461
8	Jean-Pierre Ricard	603	6396	461
10	James Michael Harvey	7686	734	459
11	Francisco Robles Ortega	10563	3134	455
12	Dominik Duka	961	6394	444

Table 1: Cardinals in the Conclave of 2013 ranked by the number of minimal winning coalitions where they are critical

rank	cardinal	c_x (km)	c_y (bp)	score
13	Antonio María Rouco Varela	1653	817	440
14	Baselios Cleemis	7183	6391	426
15	Odilo Scherer	10408	4399	424
16	Luis Antonio Tagle	10403	2568	420
17	Béchara Boutros Raï	2214	877	419
17	Norberto Rivera Carrera	10384	2973	419
19	Rubén Salazar Gómez	9392	113	410
20	André Vingt-Trois	1107	925	397
21	Juan Sandoval Íñiguez	10376	1560	391
22	John Tong Hon	9279	6064	387
23	Antonio Maria Vegliò	213	4649	385
23	Jorge Urosa	8365	1491	385
25	Lluís Martínez Sistach	859	1190	383
26	Velasio de Paolis	83	5183	379
26	Crescenzo Sepe	176	3364	379
28	Roger Mahony	10201	5863	377
29	Francesco Monterisi	323	5013	375
30	Raffaele Farina	220	4609	366
31	Giuseppe Betori	119	2871	365
32	John Onaiyekan	3843	5956	362
33	Angelo Amato	351	2524	355
34	Raúl Eduardo Vela Chiriboga	10234	5526	352
35	Josip Bozanić	414	2616	347
36	Jean-Louis Tauran	1105	5643	341
37	Justin Francis Rigali	10201	5295	337
38	Carlo Caffarra	390	3734	332
38	João Braz de Aviz	9911	3119	332
40	George Alencherry	6857	5589	325
41	Seán Patrick O'Malley	8984	5300	311
42	Julio Terrazas Sandoval	10325	216	308
43	Raymundo Damasceno Assis	9062	2676	298
44	Carlos Amigo Vallejo	1449	2273	297
45	Mauro Piacenza	401	4179	292
46	Manuel Monteiro de Castro	1755	2327	279
47	William Levada	10224	4244	276
48	Rainer Woelki	1092	5499	271
49	Domenico Calcagno	432	3643	267
50	Nicolás de Jesús López Rodríguez	10836	6034	266
51	Ennio Antonelli	99	5542	262
52	José Policarpo	1834	2447	261

Table 1: Cardinals in the Conclave of 2013 ranked by the number of minimal winning coalitions where they are critical

rank	cardinal	c_x (km)	c_y (bp)	score
53	Angelo Bagnasco	423	4488	258
54	Stanisław Dziwisz	1029	5378	253
55	Cláudio Hummes	10317	7841	245
56	Severino Poletto	424	3931	243
57	Angelo Comastri	108	1206	239
58	Ivan Dias	6183	3299	237
59	Paolo Sardi	453	4891	235
60	Timothy M. Dolan	8148	5217	230
61	Giovanni Lajolo	503	5010	228
62	Óscar Andrés Rodríguez Maradiaga	9758	4327	205
63	Fernando Filoni	463	4759	200
63	John Njue	9509	4911	200
65	Dionigi Tettamanzi	498	4979	195
66	Edwin Frederick O'Brien	6879	5247	191
67	Pham Minh Man	9557	4600	188
68	Francesco Coccopalmerio	466	3855	184
69	Giovanni Battista Re	487	4631	168
70	Vinko Puljić	516	3387	163
71	Gianfranco Ravasi	490	3961	153
72	Antonios Naguib	2258	5087	143
73	Seán Brady	1978	5054	129
74	Franc Rodé	503	4388	126
74	Giuseppe Versaldi	503	4657	126
76	Geraldo Majella Agnelo	9119	3995	124
77	Antonio Cañizares Llovera	1182	3195	116
78	Attilio Nicora	527	3602	113
79	Oswald Gracias	6183	5309	104
80	Angelo Scola	506	4424	91
81	Philippe Barbarin	1093	5004	90
82	Jaime Lucas Ortega y Alamino	8653	4770	88
83	Polycarp Pengo	5937	5000	78
84	Santos Abril y Castelló	1144	3401	73
85	Paolo Romeo	526	4277	69
86	Reinhard Marx	1126	3447	64
86	Wilfrid Napier	8215	4204	64
88	Daniel DiNardo	7382	4941	55
89	Giuseppe Bertello	532	3851	51
90	Kazimierz Nycz	1140	3629	46
91	Marc Ouellet	7911	4605	45
92	Kurt Koch	665	3779	42

Table 1: Cardinals in the Conclave of 2013 ranked by the number of minimal winning coalitions where they are critical

rank	cardinal	c_x (km)	c_y (bp)	score
92	Donald Wuerl	7334	4927	42
92	Raymond Leo Burke	7822	4813	42
95	Tarcisio Bertone	538	4696	39
96	Stanisław Rylko	1033	3675	31
97	Francis George	7749	4579	28
98	Karl Lehmann	736	3836	24
99	Paul Josef Cordes	1077	4852	15
99	Malcolm Ranjith	7643	4509	15
101	Thomas Christopher Collins	7153	3790	14
102	Walter Kasper	778	4599	13
103	Wim Eijk	1293	4842	10
104	Péter Erdő	811	4008	9
104	Audrys Bačkis	1671	3826	9
106	Christoph Schönborn	971	4139	1
107	Joachim Meisner	1083	4698	0
107	Godfried Danneels	1231	4043	0
107	Zenon Grocholewski	1335	3877	0
107	Anthony Olubunmi Okogie	4047	4727	0
107	Gabriel Zubeir Wako	4098	4474	0
107	Robert Sarah	4165	4077	0
107	Peter Turkson	4319	3887	0
107	Laurent Monsengwo Pasinya	5004	3936	0
107	Jean-Claude Turcotte	6588	4632	0
Total				28192

5 Discussion

The obtained ranking contains some interesting findings. Firstly note Pope Francis at rank 3. With this our ranking method proved to be more successful to identify the next Pope than many ‘well-informed’ analysts. Wakin (2013) reports on rumors that Bergoglio was a runner-up candidate in the 2005 election, however, he was not mentioned among the *papabili*, the people who are likely to become the next pope, in 2013 (Matthews, 2013). On the other hand the *papabili* ended in much lower positions, Peter Turkson, a strong African candidate turned out to be a null player, that is, a player who is never critical.

A closer inspection of the results also reveals that Pope Francis was right when he mentioned that the conclave had to go to the other end of the world to find the suitable person to become the new pope: all our top

candidates are from far away places, in fact our top candidates are the cardinals with the birthplaces most distant from Rome. This is hardly a surprising result. The Median Voter Theorem (Black, 1948) stating that the society will choose the median voter's choice assumes that the voters' positions are distributed unidimensionally and that decisions are taken with simple majority. As soon as a qualified majority is required, the median voter becomes a null player, while voters with more extreme positions gain influence. The results do not generalise to preferences in a multi-dimensional space (for a discussion see (Caplin, Andrew and Nalebuff, Barry, 1991)), but the intuition is similar: Very central cardinals will rarely be critical (if at all), while cardinals with relatively extreme location or level of conservatism are likely to do well. While the list is topped by cardinals with very distant birthplaces, they are soon followed by cardinals who have extreme positions along the liberal/conservative axis. The first Italian, Augustino Vallini is also the cardinal born closest to Rome. To test if the results are due more to the majority voting with a qualified majority or due to the two-dimensional location of the cardinals, we repeated the calculations with simple majority, but the positions of the top three papabili did not change.

While it would be difficult to repeat the same exercise for the election of Pope Benedict XVI, one must note that being a German cardinal Joseph Ratzinger had a near-median distance, but was considered rather a conservative. With such a position he would have likely scored high in our ranking.

A Algorithm to find minimal winning coalitions

Let ξ be the inverse of a (not necessarily unique) permutation of indices that arranges the cardinals in a non-decreasing order according to their x coordinates. Hence $c^{\xi(1)}$ is the closest to Rome and $c^{\xi(115)}$ is the farthest from it. Similarly let η be the inverse of a permutation that arranges the cardinals in a non-decreasing order according to their y coordinate, where $c^{\eta(1)}$ indicates the most liberal among them.

1. Let \mathcal{R} collect feasible rectangles and set $\mathcal{R} = \emptyset$ and let $\mathbf{s} \in \mathbb{Z}^{115}$ denote a score function with $\mathbf{s} = \mathbf{0}$.
2. Starting from $c^{\xi(1)}$ take the leftmost cardinal as l and allow all cardinals to be considered for r, t, b .
3. Starting from $c^{\eta(1)}$ take the cardinal b with the lowest c_y satisfying $b_x \geq l_x$ and $b_y \leq l_y$ and allow all cardinals to be considered for r, t .
4. Starting from $c^{\xi(115)}$ take the rightmost cardinal as r if it satisfies $r_x \geq b_x$ and $r_y \geq b_y$ and allow all cardinals to be considered for t .

5. Starting from $c^{\eta(115)}$ the cardinal t with the highest c_y satisfying $t_y \geq l_y$, $t_y \geq r_y$, $t_x \geq l_x$ and $t_x \leq r_x$.
6. If the rectangle has exactly 77 cardinals, check if it is in the list of rectangles \mathcal{R} . If it is not
 - (a) Store it in the list \mathcal{R} .
 - (b) Give points to each cardinal on the borderline.
7. Take the next t while $t_y \geq c_y^{\eta(77)}$.
8. Take the next r while $r_x \geq c_x^{\eta(77)}$.
9. Take the next b while $b_y \leq c_y^{\eta(39)}$.
10. Take the next l while $l_x \leq c_x^{\eta(39)}$.

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