Preisach Type Hysteresis Models with Everett Function in Closed Form

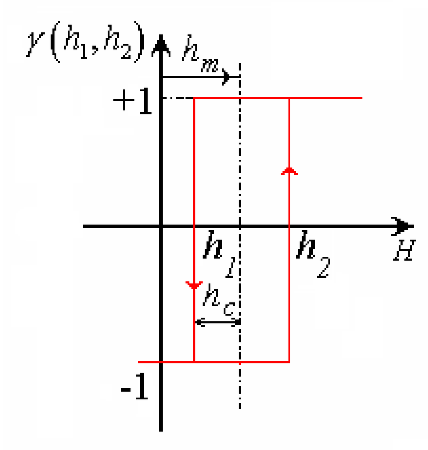
Zsolt Szabó[1] and János Füzi[2,3]

[1]Department of Broadband Infocommunication and Electromagnetic Theory, BME, Hungary
[2]Neutron Spectroscopy Department, Wigner Research Centre for Physics, Budapest, Hungary
[3]Faculty of Engineering and Information Technology, University of Pécs, Hungary

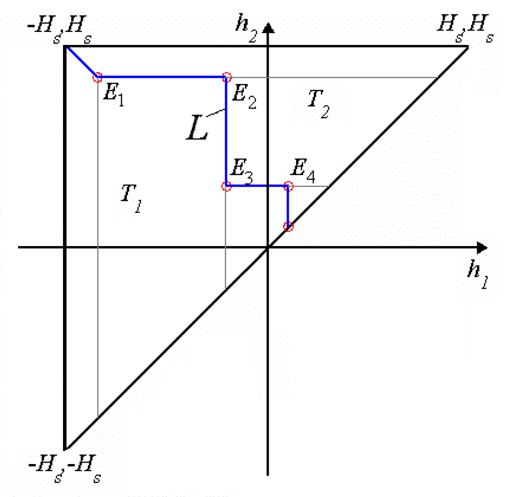
email: szabo@evt.bme.hu

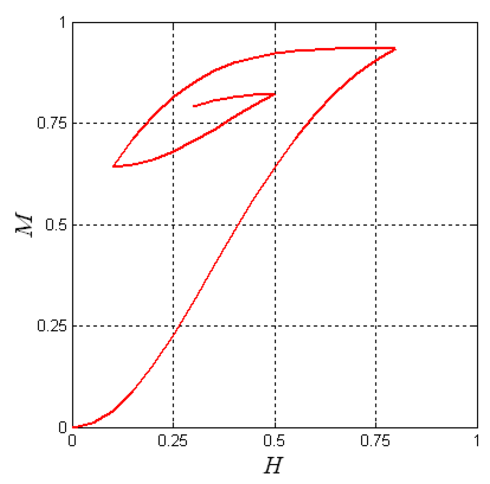
Abstract: The Preisach function is considered as a product of two special one dimensional functions, which allows the closed form evaluation of the Everett integral. The deduced closed form expression is included in static and rate dependent hysteresis models. The applicability and accuracy of the models are discussed and demonstrated by fitting measured data. The developed hysteresis models, which are freely available for research and educational purpose, proved to be fast enough to be incorporated in electromagnetic software.

1. Introduction



$$h_2 \ge h_1 \qquad h_1 \ge -H_s \qquad h_2 \le H_s$$





$$B(t)=\iint_T \mu(h_1,h_2)\gamma(h_1,h_2,H(t))\,dh_1 dh_2$$

$$E(x,y)=\int_x^y\int_x^{h_2}\mu(h_1,h_2)\,dh_1 dh_2$$

$$B(t)=E_1-2E_2+2E_3-2E_4$$

This term must be calculated

Previously calculated

$$B_{rev}(t)=k_1 H(t)+k_2\tanh\left(\frac{H(t)}{k_3}\right)$$

2. A Closed Form Expression of the Everett Function

$$\mu(h_1,h_2)=\sum_{i=1}^{n}\varphi_i(h_1)\varphi_i(-h_2)$$

$$\varphi_i(x)=\frac{a_i\, e^{-\frac{x-b_i}{c_i}}}{\left(1+e^{-\frac{x-b_i}{c_i}}\right)^2}=\frac{a_i/2}{1+\cosh\left(\frac{x-b_i}{c_i}\right)}=\frac{\alpha_i\, e^{-\beta_i x}}{\left(1+\gamma_i\, e^{-\beta_i x}\right)^2}$$

$$\beta_i=1/c_i \qquad \gamma_i=e^{b_i/c_i} \qquad \alpha_i=a_i\gamma_i$$

$$E(x,y)=\sum_{i=1}^{n}\frac{\alpha_i^2}{\beta_i^2}\,\frac{\left(e^{\beta_i x}-e^{\beta_i y}\right)+\dfrac{\left(\gamma_i+e^{\beta_i y}\right)\left(1+\gamma_i e^{\beta_i x}\right)}{1-\gamma_i^2}\log\dfrac{\left(1+\gamma_i e^{\beta_i y}\right)\left(\gamma_i+e^{\beta_i x}\right)}{\left(1+\gamma_i e^{\beta_i x}\right)\left(\gamma_i+e^{\beta_i y}\right)}}{\left(1-\gamma_i^2\right)\left(\gamma_i+e^{\beta_i y}\right)\left(1+\gamma_i e^{\beta_i x}\right)}$$

- *closed form expression, which includes only basic arithmetic operations, exponentiation to a real exponent and logarithms.*
- *can be calculated without complex mathematical libraries, and is fast enough to be incorporated in engineering software.*

$b=0\rightarrow\gamma=1 \Rightarrow$ *singular*

$$\lim_{\gamma\to 1}E(x,y)=\frac{1}{2\beta^2}\frac{\left(e^{\beta x}-e^{\beta y}\right)^2}{\left(1+e^{\beta x}\right)^2\left(1+e^{\beta y}\right)^2}$$

$c=+\infty \Rightarrow \varphi_i(x)=\alpha_i$

$$E(x,y)=\int_x^y\int_x^{h_2}\left(\sum_{i=1}^{n}\alpha_i^2\right)dh_1 dh_2=\frac{\alpha}{2}(y-x)^2$$

Rayleigh model of parabolas (1887)

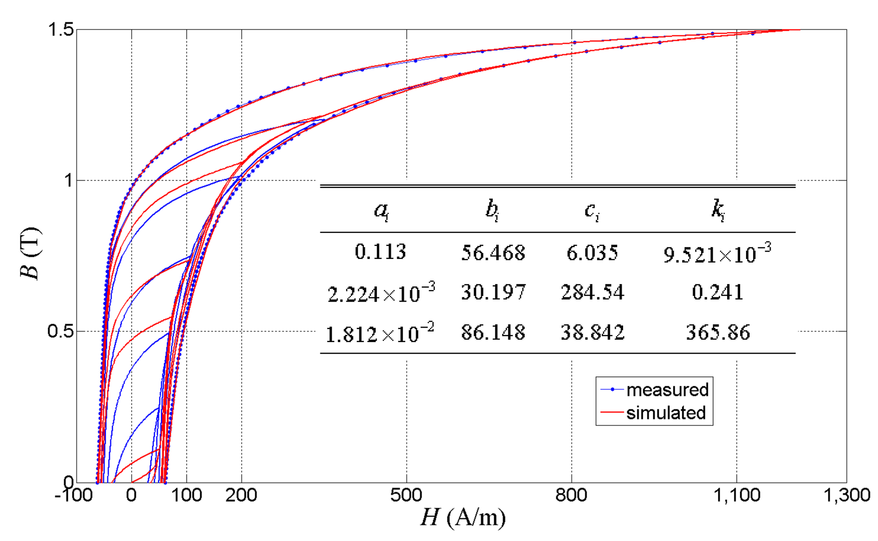
3. Static Model

$$B_l=\sum_{i=1}^{n}\frac{\alpha_i^2}{\beta_i^2}\,\frac{\left(1-e^{-2\beta_i H_l}\right)\left(\gamma_i^2-1\right)-\left(1+\gamma_i e^{-\beta_i H_l}\right)^2\log\dfrac{\left(1+\gamma_i e^{\beta_i H_l}\right)\left(\gamma_i+e^{-\beta_i H_l}\right)}{\left(1+\gamma_i e^{-\beta_i H_l}\right)\left(\gamma_i+e^{\beta_i H_l}\right)}}{\left(\gamma_i^2-1\right)^2\left(1+\gamma_i e^{-\beta_i H_l}\right)^2}$$

- the lower branch of the concentric hysteresis loop

$$B=-B_l+2\sum_{i=1}^{n}\frac{\alpha_i^2}{\beta_i^2}\,\frac{\left(e^{\beta_i H}-e^{-\beta_i H_l}\right)-\dfrac{\left(\gamma_i+e^{\beta_i H}\right)\left(1+\gamma_i e^{-\beta_i H_l}\right)}{\gamma_i^2-1}\log\dfrac{\left(1+\gamma_i e^{\beta_i H}\right)\left(\gamma_i+e^{-\beta_i H_l}\right)}{\left(1+\gamma_i e^{-\beta_i H_l}\right)\left(\gamma_i+e^{\beta_i H}\right)}}{\left(\gamma_i^2-1\right)\left(\gamma_i+e^{\beta_i H}\right)\left(1+\gamma_i e^{-\beta_i H_l}\right)}$$

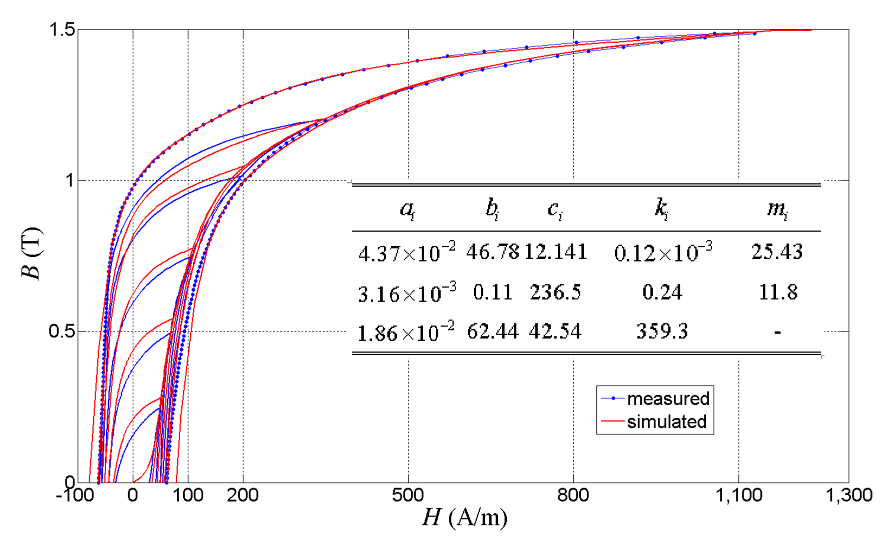
Compumag Team Problem 32



a_i	b_i	c_i	k_i
0.113	56.468	6.035	9.521×10^{-3}
2.224×10^{-3}	30.197	284.54	0.241
1.812×10^{-2}	86.148	38.842	365.86

4. Moving Model

$$H_m=H+m_1 B+m_2 B^3$$



a_i	b_i	c_i	k_i	m_i
4.37×10^{-2}	46.78	12.141	0.12×10^{-3}	25.43
3.16×10^{-3}	0.11	236.5	0.24	11.8
1.86×10^{-2}	62.44	42.54	359.3	-

5. Rate Dependent Hysteresis Model

$$\frac{dH_m}{dt}=a_m\left(H-H_m\right)-b_m\frac{dB}{dt}+c_m\frac{dH}{dt}$$

$$\frac{dB}{dt}=\mu_0\left(\frac{dH_m}{dt}+\frac{dM}{dt}\right)=\mu_0\left(1+\frac{dM}{dH_m}\right)\frac{dH_m}{dt}$$

- increasing effective magnetic field intensity

$$\frac{dM}{dH_m}=\sum_{i=1}^{n}\frac{2\alpha_i^2}{\beta_i\gamma_i}\frac{e^{-\beta_i H_m}}{\left(1+e^{-\beta_i H_m}\right)^2}\left[\frac{1}{1+\gamma_i e^{\beta_i H_m^r}}-\frac{1}{1+\gamma_i e^{\beta_i H_m}}\right]$$

- discretization

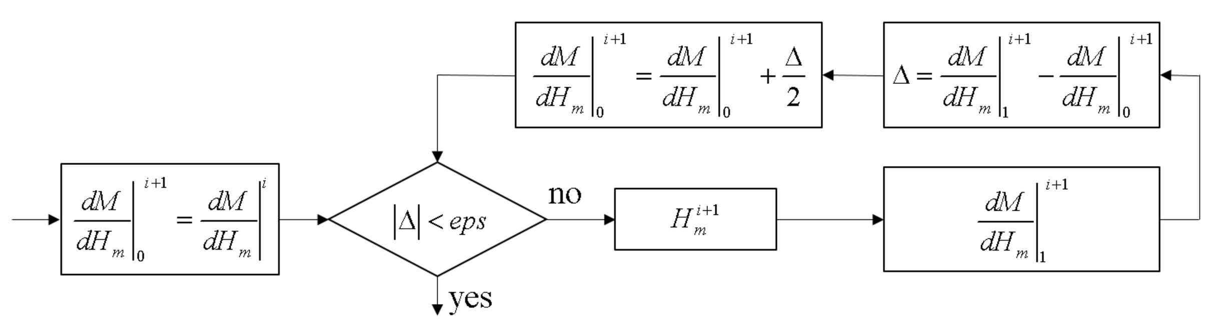
$$\left.\frac{dH_m}{dt}\right|^{i+\frac{1}{2}}=\frac{H_m^{i+1}-H_m^{i}}{\Delta t} \qquad \left.\frac{dM}{dH_m}\right|^{i+\frac{1}{2}}=\frac{\left.\dfrac{dM}{dH_m}\right|^{i+1}+\left.\dfrac{dM}{dH_m}\right|^{i}}{2}$$

$$H_m^{i+\frac{1}{2}}=\frac{H_m^{i+1}+H_m^{i}}{2} \qquad H^{i+\frac{1}{2}}=\frac{H^{i+1}+H^{i}}{2}$$

Unknown, additional iterations at each time step

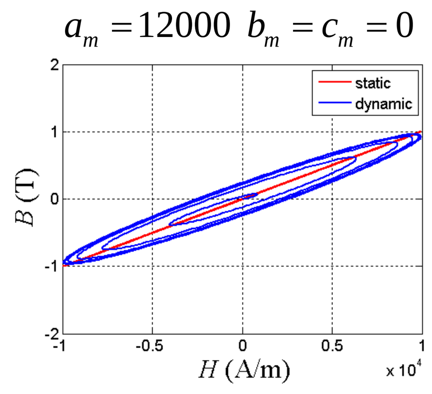
$$H_m^{i+1}=\frac{\left[1-\kappa_1+\kappa_2\left(\left.\dfrac{dM}{dH_m}\right|^{i+1}+\left.\dfrac{dM}{dH_m}\right|^{i}\right)\right]H_m^i+\kappa_3 H^{i+1}+\kappa_4 H^i}{1+\kappa_1+\kappa_2\left(\left.\dfrac{dM}{dH_m}\right|^{i+1}+\left.\dfrac{dM}{dH_m}\right|^{i}\right)}$$

$$\kappa_1=\frac{a_m\Delta t}{2} \qquad \kappa_2=\frac{b_m\mu_0}{2} \qquad \kappa_3=\kappa_1-2\kappa_2+c_m \qquad \kappa_4=\kappa_1+2\kappa_2-c_m$$

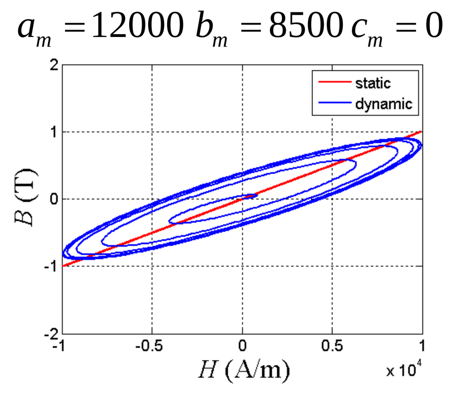


- eddy current induced hysteresis curves $B=500H_m$

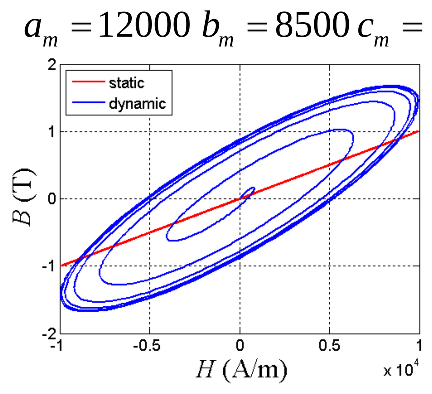
$a_m=12000\ b_m=c_m=0$



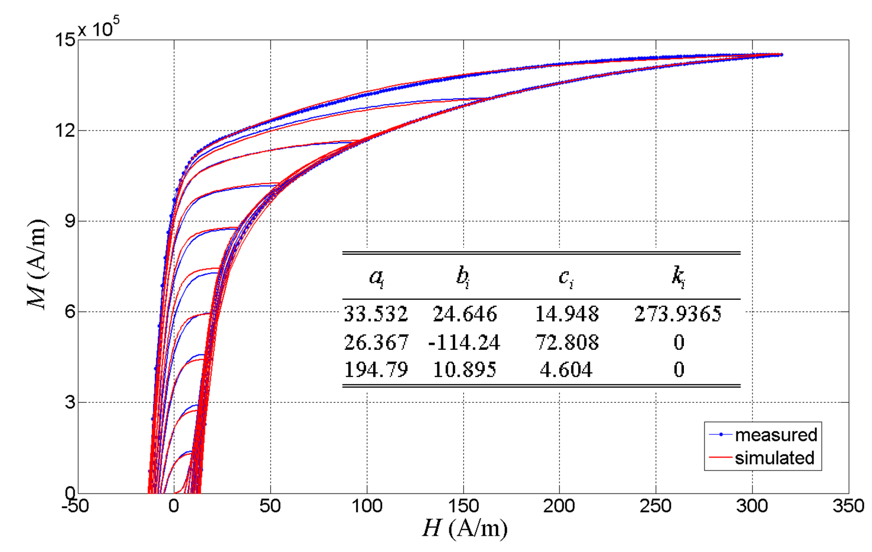
$a_m=12000\ b_m=8500\ c_m=0$



$a_m=12000\ b_m=8500\ c_m=6$

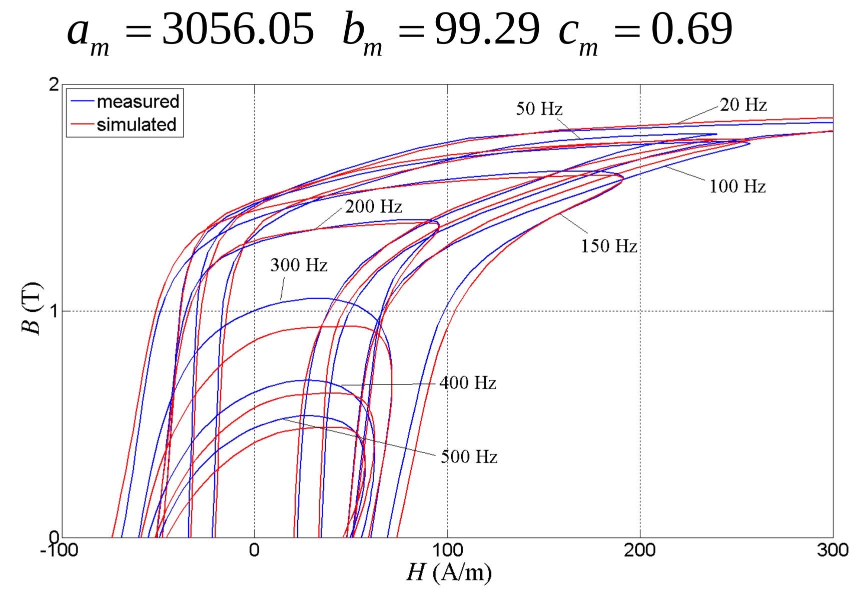


ORSI-111



a_i	b_i	c_i	k_i
33.532	24.646	14.948	273.9365
26.367	-114.24	72.808	0
194.79	10.895	4.604	0

$$a_m=3056.05\quad b_m=99.29\quad c_m=0.69$$



Acknowledgement:

This work has been supported by the Bolyai János Fellowship of Hungarian Academy of Sciences, the PIAC_13-1-2013-0186, the KMR-12-1-2012-0008 and EUREKA project MetaFer. **Work dedicated to the 650th anniversary of the foundation of the University of Pécs, Hungary.**