Calculation of Imaging Properties of Metamaterials

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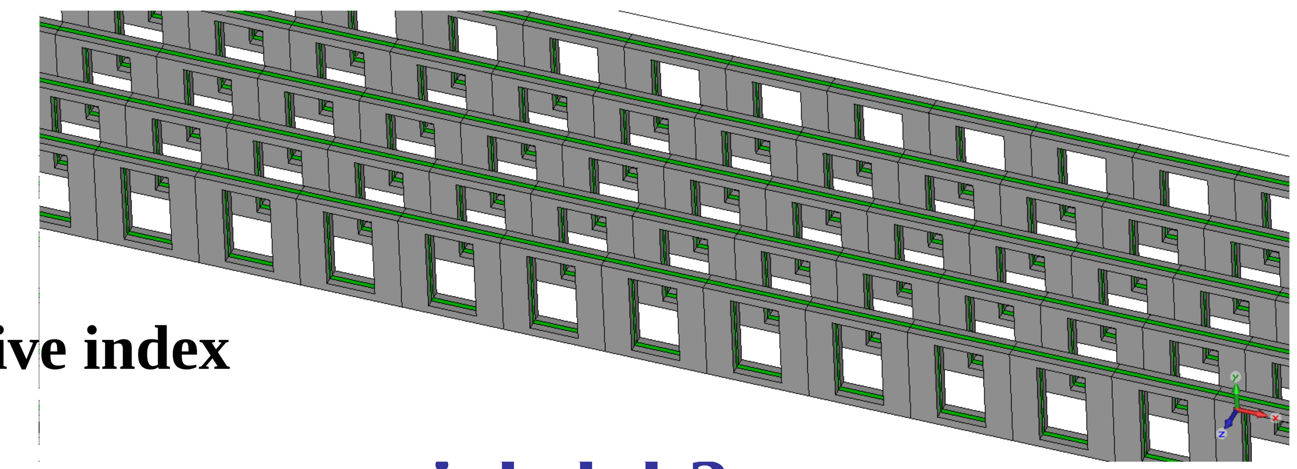
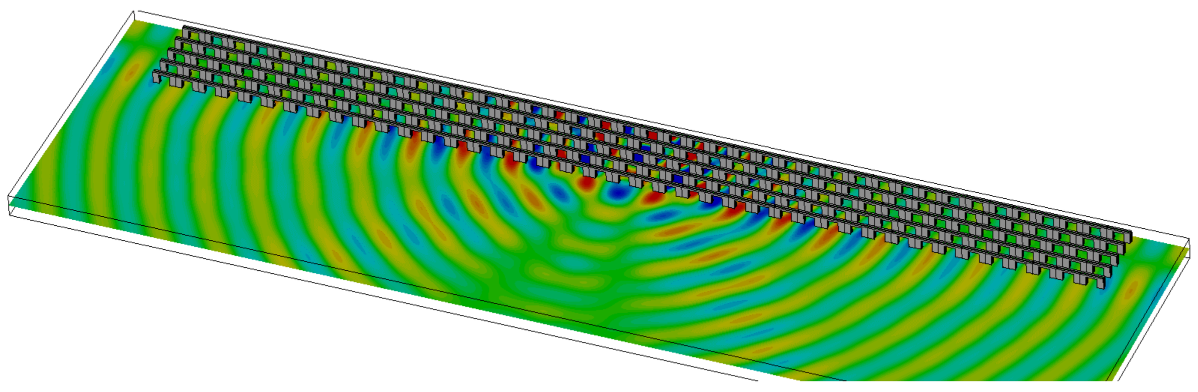
Abstract: **A method is proposed that combines full wave simulations and the transfer matrix method to efficiently simulate imaging systems containing metamaterial slabs in order to provide the information, what an observer would see applying the system. The dispersion relations of periodic metamaterial slabs are retrieved from the S parameters calculated with full wave simulation of the unit cell. The dispersion relation is then utilized in the transfer matrix method to calculate the imaging properties of the metamaterial slab. The developed procedure is applied to obtain the virtual distance of a point source placed behind a Fishnet metamaterial slab with respect to the observation angle. If the image distance is known for all observation angles, the image of an extended object can be simply calculated.**

What are metamaterials?



- Periodic or aperiodic structures with **subwavelength feature sizes**
- They can be handled **as homogeneous materials**
- Can exhibit extraordinary electromagnetic properties like **negative or zero refractive index**

Problem: What would an observer see, when looking through a metamaterial slab?

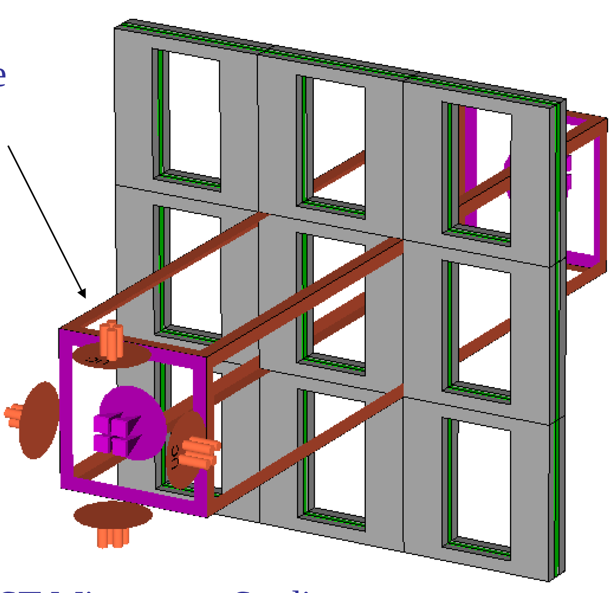


- **Full wave simulation** of an **extended area of the fine structure** is required
  → **high computational cost**
- A **blurred spot** is found **due to abberation** and **internal reflections**
  → **no exact image position**
- **Virtual image** (image behind the surface) is **not found**

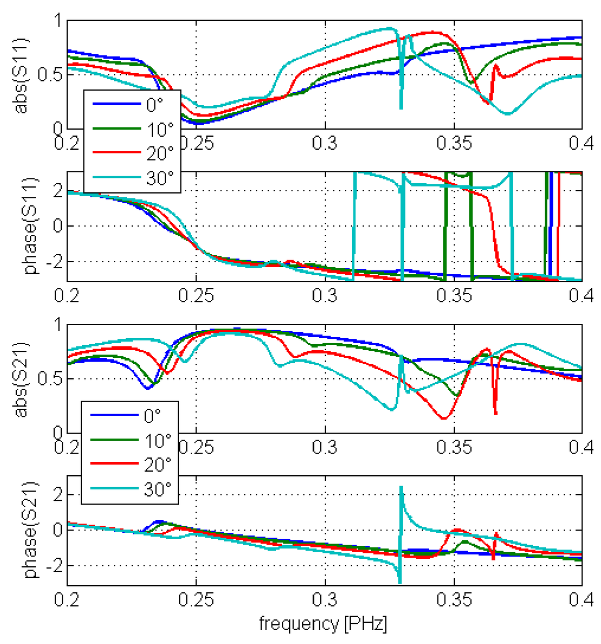
1. Retrieving the Dispersion Relation of Metamaterial Slabs

- Full wave simulation of **one unit cell** to determine the **S-parameters for all incident angles**
- The **dispersion relation** $k_z(k_x, \omega)$ can be extracted **by inverted expressions** of S_{11}, S_{21}

Bloch type boundary conditions



CST Microwave Studio

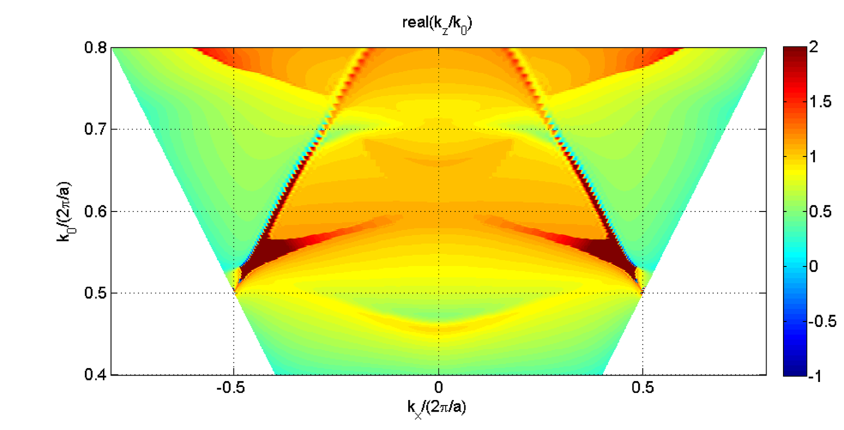


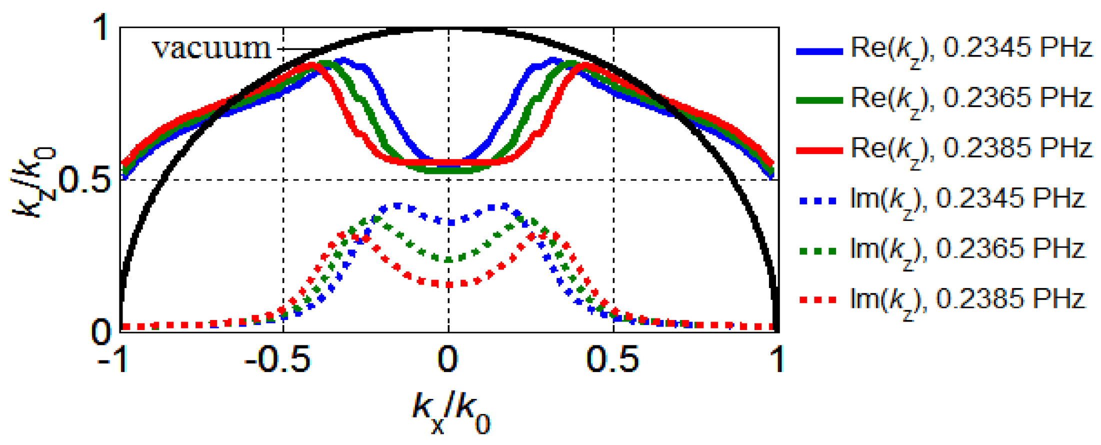
$$\Gamma = (\xi - 1)/(\xi + 1)$$

$$k_z = \frac{\mathrm{Im}\left\{\ln\left(\frac{S_{21}}{1 - S_{11}\Gamma}\right)\right\} + 2m\pi}{d} - i\,\frac{\mathrm{Re}\left\{\ln\left(\frac{S_{21}}{1 - S_{11}\Gamma}\right)\right\}}{d}$$

$$\xi = \pm\sqrt{\frac{(1 + S_{11})^2 - S_{21}^2}{(1 - S_{11})^2 - S_{21}^2}}$$

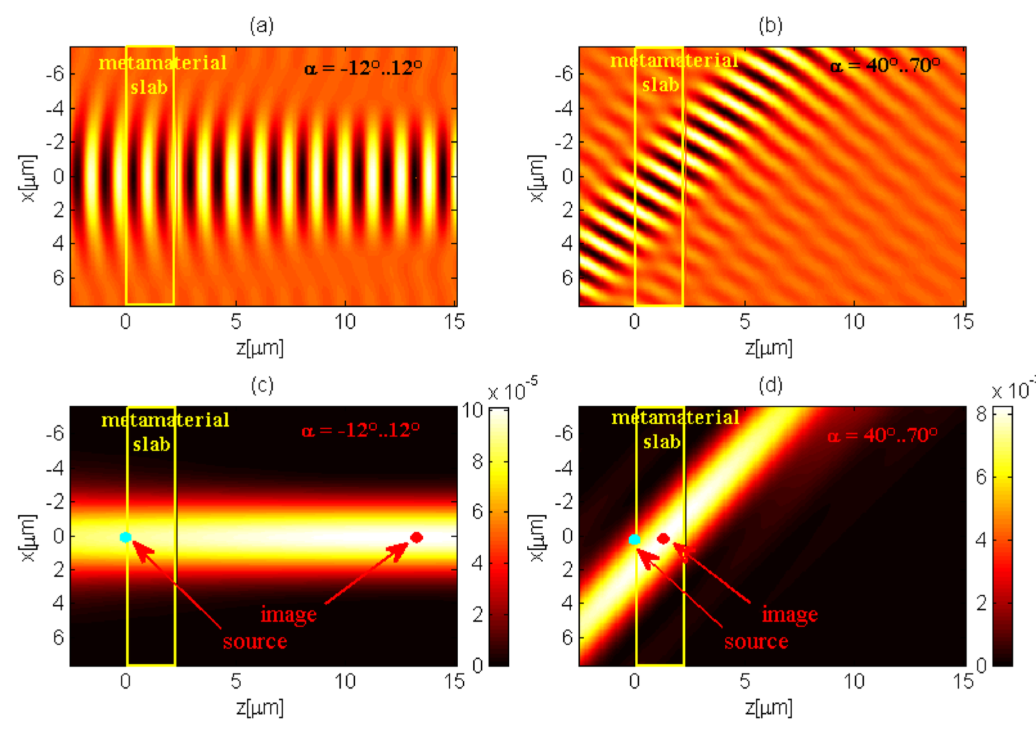
$$k_x = k_0 \sin\alpha$$



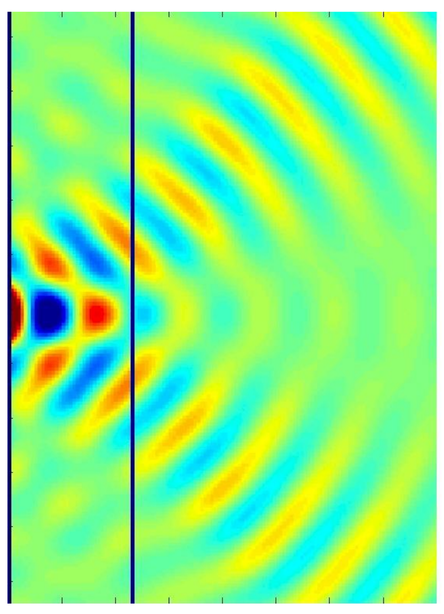


2. Transfer Matrix Method to Determine the Image Position

- **Decompose** the field of a **point source** into **plane waves** by Fourier transform
- Calculate the **transfer function** $T(k_x)$ for each plane wave, where k_z is **determined from the dispersion relation** $k_z(k_x, \omega)$
- Summation of the plane waves in the image plane and inverse Fourier transform

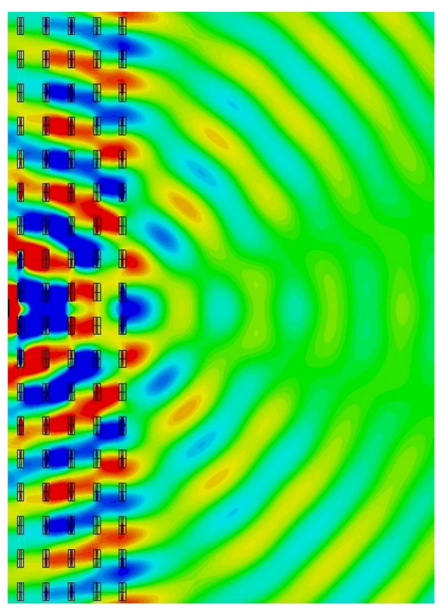


2D



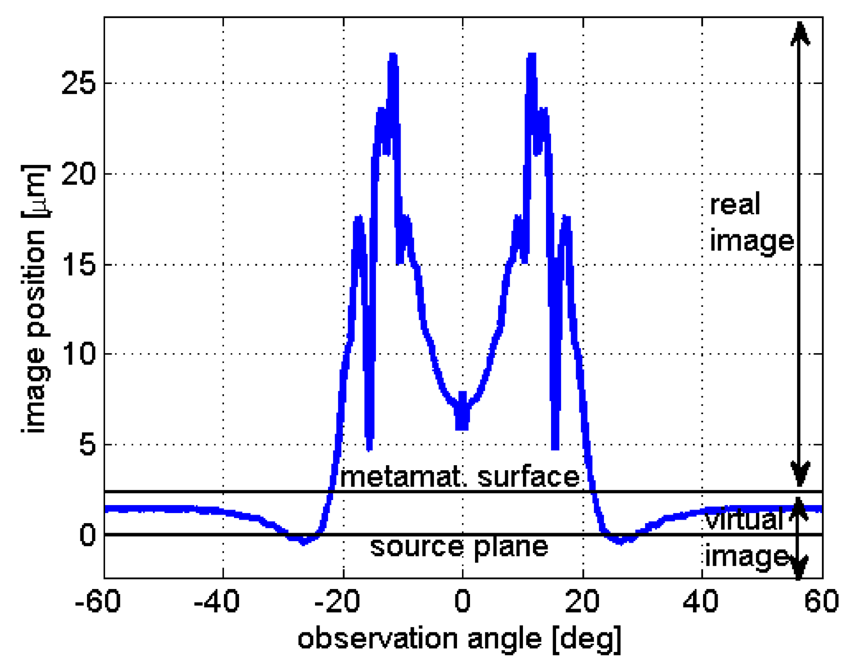
Reduced 2D simulation with Transfer Matrix method and dispersion relation

3D



3D full wave simulation with CST Microwave Studio

- The effect of **internal reflection** can be **eliminated**
  $T_{slab}(k_x) = (1 - \Gamma)\exp(ik_z d)\,(1 + \Gamma)$, instead of
  $$T_{slab}(k_x) = \frac{4}{(\xi+1)\left(\frac{1}{\xi}+1\right)\exp(-i\,k_z d) + (\xi-1)\left(\frac{1}{\xi}-1\right)\exp(i\,k_z d)}$$
- The effect of **aberration** can be **eliminated by summing** plane waves only **in a narrow angle range** instead of summing all plane waves
  → **an angle dependent image position** can be determined
- The **virtual image** can be found by **backward calculations**



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