Lyapunov-Krasovskii functionals for evaluating $\mathcal{H}_\infty$ performance of platoons of communicating vehicles

Gabriella Varga, Gábor Rödönyi, Péter Gáspár and József Bokor

***Abstract*—Robust performance of finite length homogeneous vehicular platoons is analyzed, where disturbances and constant delays in the inter-vehicle communication are present. The goal of the paper is to compare and evaluate the accuracy and computational cost of several Lyapunov-Krasovskii functional (LKF) based methods. The potential of the approach in analyzing platoon performance is that it can be extended to cope with time-varying delays, large scale systems, vehicle model uncertainties and platoon heterogeneity. Simple delay-independent, delay-dependent and discretized complete LKF based performance criteria are presented.**

I. INTRODUCTION

The challenge in designing and analyzing longitudinal control systems for a platoon of vehicles is due to wide range of problems one should deal with simultaneously. The vehicle is a nonlinear system with unknown components and time-varying parameters, the control action may be saturated and delayed by the engine and brake systems, the sensor signals are affected by noise, the communication is imperfect (packet loss, transmission delay, discrete event property) and the platoon may consists of vehicles with very different dynamics. Furthermore, it is not enough to check stability and performance of a finite length platoon, the notion of string stability - the ability of the vehicle string to attenuate disturbances as it is propagated along the platoon - must be examined in order to guarantee the scalability of the control algorithms [1]–[4].

A wide range of tools are available which analyze some components of the above set of problems. For example string (or mesh) stability of interconnected systems is analyzed by means of vector Lyapunov functions [5]. Heterogeneity in platoon dynamics is discussed in [6], the effect of random packet loss is examined in [7] and vehicle model uncertainty in [8].

The effects of communication and actuation delays are analyzed by many authors [9]–[14]. In each of these papers constant delay is assumed and a frequency domain criteria is proposed for the $\mathcal{H}_\infty$-norm of the error propagation transfer function being less than one. There are several drawbacks of the frequency-domain approaches. (1) They cannot be generalized to cope with time-varying delays and time-varying parametric uncertainties. (2) There are platoon control algorithms where the error propagation transfer function cannot be computed. For example when predecessor and leader following controllers with constant spacing policy receive both spacing and acceleration demand information. (3) For inputs and outputs not belonging to signal spaces inducing $\mathcal{H}_\infty$-norm on the system cannot be analyzed in this way. For example a peak-to-peak analysis to determine the maximal spacing errors require ℓ_1-norm computation [8].

These obstacles in the elaboration of a unified framework for the analysis of vehicular platoons motivate the consideration of Lyapunov-Krasovskii approaches applied in many papers in the area of networked control systems. In one of the most related papers, nonlinear interconnected fuzzy models with delayed control inputs are analyzed and a decentralized controller is designed by using LKF and explicit model transformation for a radar gimbal stabilization system [15].

The goal in this paper is to compare and evaluate the accuracy and computational cost of several LKF based methods including a simple delay-independent, two simple delay-dependent methods with implicit and explicit model transformations, respectively, and a discretized complete LKF based method. In order to be able to evaluate these methods, the constant communication delay case is considered where the performance level can be computed accurately.

The paper is organized as follows. In Section II the platoon model is presented. The LKF methods to be tested are presented in Section III and the results are summarized in Section IV.

A. Notations

Let $\mathcal{L}^n$ denote the set of all mappings $x : [0, \infty) \mapsto \mathbb{R}^n$ which are Lebesgue-measurable. Let $\mathcal{L}_2^n$ denote the square integrable signals with norm defined by $\|x\|_2^2 = \int_0^\infty \|x(t)\|^2 dt$, where $\|x(t)\|$ denotes the Euclidean norm, i.e. $\mathcal{L}_2^n = \{x \in \mathcal{L}^n : \|x\|_2 < \infty\}$. The induced $\mathcal{L}_2$-norm of linear time-invariant (LTI) systems that are bounded and analytic in the open right-half plane is computed as $\|G\|_\infty = \sup_{\omega\in\mathbb{R}} \bar{\sigma}(G(j\omega))$. The ith element of vector x is denoted by x_i. Matrix inequality $M > 0$ denotes that M is symmetric and positive definite. The transpose of a matrix M is denoted by M^T.

II. PLATOON MODEL

Let the ith vehicle be described by the following third-order continuous-time state-space model

$$\dot{p}_i(t) = v_i(t), \tag{1a}$$

$$\dot{v}_i(t) = q_i(t) + d_i(t), \tag{1b}$$

$$\dot{q}_i(t) = -\frac{1}{\tau_i} q_i(t) + \frac{g_i}{\tau_i} u_i(t), \tag{1c}$$

where p_i, v_i denote position and velocity, d_i is a disturbance representing both outer effects and modeling error, q_i is an

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internal state such that the acceleration of the vehicle is $a_i(t) = q_i(t) + d_i(t)$. The leader vehicle is driven by a human driver, u_0 denotes the acceleration demand computed from the pedal signals. u_i, $i = 1, 2, ..., n$ for the follower vehicles denote the acceleration demand generated by the controllers.

Let the distributed platoon controller be given and designed for a constant spacing policy introduced in [16]. It utilizes information from local radars (relative position e_i and relative speed δ_i) and receives acceleration a_i through a V2V communication network from both the preceding and lead vehicle. The effect of the network is modeled by a constant time-delay, denoted by h. Taking inter-vehicle communication delays into account, the controller can be described by the following equations

$$u_1(t) = -k_1\delta_1(t) - k_2e_1(t) + a_0(t-h) \tag{2a}$$

$$\begin{aligned} u_i(t) = &- k_{1\beta}\delta_i(t) - k_{2\beta}e_i(t) \\ &+ k_{a0}a_0(t-h) + k_{a1}a_{i-1}(t-h) \\ &- k_{1\alpha}(v_i(t-h) - v_0(t-h)) \\ &- k_{2\alpha}(p_i(t-h) - p_0(t-h)), \quad i = 1, ..., n \end{aligned} \tag{2b}$$

where the k_* are constant parameters, $\delta_i := v_i - v_{i-1}$ and $e_i := p_i - p_{i-1} + L$ with prescribed spacing L set to zero in the analysis without loss of generality.

If (2) is inserted into (1c), the closed-loop system depends on both the actual disturbance inputs and their delayed values. In order to simplify the analysis additional dynamics can be introduced for the disturbances so the system contains only single state delays. It is common practice to normalize the disturbances as

$$d_i(t) = W_{d,i}(s)d_{0i}(t), \qquad \|d_{0i}\|_2 \leq 1, \qquad i = 0, .., n \tag{3}$$

where $W_{d,i}(s)$ is a strictly proper transfer function with state-space realization

$$\dot{x}_{di}(t) = A_d x_{di}(t) + B_d d_{0i}(t) \tag{4a}$$

$$d_i(t) = C_d x_{di}(t) \tag{4b}$$

Putting (1), (2) and (4) together and applying state transformation $p_i \to p_i - p_{i-1}$ and $v_i \to v_i - v_{i-1}$, the closed-loop platoon model reveals the form

$$\dot{x}(t) = Ax(t) + A_h x(t-h) + Bw(t) \tag{5a}$$

$$z(t) = Cx(t) + Dw(t) \tag{5b}$$

where $x = [x_{d0}^T, q_0, x_{d1}^T, e_1, \delta_1, q_1, \cdots, x_{dn}^T, e_n, \delta_n, q_n]^T$, $w = [u_0, d_1, \cdots, d_n]^T$, $z = [e_1, \cdots, e_n]^T$ and $D = 0$.

III. Analysis of $\mathcal{H}_\infty$ performance of systems with single state delay

In this section a set of LMI conditions are provided which differ in complexity, i.e. the number of variables and the size of the LMIs. The accuracy of each methods when applying for vehicular platoons with constant communication delays are evaluated in the next section.

Consider the linear time invariant system (5) with initial condition

$$x(t) = \phi(t), \qquad t \in [-h, 0] \tag{6}$$

where $\phi : [-h, 0] \to \mathbb{R}$ is a given continuous function. Let $x_t(\xi)$ denote $x(t + \xi)$ for $\xi \in [-h, 0]$. All of the following criteria for

$$\|C(sI - A - e^{sh}A_h)B + D\|_\infty < \gamma \tag{7}$$

i.e. the H_∞ norm of system (5) being less than $\gamma > 0$ can be derived based on [17, Proposition 8.3].

A. Simple delay-independent criterion (DI-LKF)

Let the LKF candidate be

$$V(x_t) = x^T(t)Px(t) + \int_{t-h}^{t} x^T(\theta)Qx(\theta)d\theta. \tag{8}$$

In [18] LMI conditions are derived for the stability of system (5) with $w(t) = 0$.

Theorem 1: Consider the system (5) with initial condition $\phi \equiv 0$. If there exist real matrices $P = P^T > 0$, $Q = Q^T$ such that

$$\begin{bmatrix} A^TP + PA + Q & PA_h & PB & C^T \\ A_h^TP & -Q & 0 & 0 \\ B^TP & 0 & -\gamma I & D^T \\ C & 0 & D & -\gamma I \end{bmatrix} < 0 \tag{9}$$

then system (5) is asymptotically stable and has H_∞ norm less than γ.

B. Simple delay-dependent criterion based on explicit model transformation (EM-LKF)

By using an explicit model transformation

$$x(t-h) = x(t) - \int_{-h}^{0} \dot{x}(t+\theta)d\theta$$

and LKF candidate

$$\begin{aligned} V(x_t) \;\; = \;\; & x(t)^TPx(t) + \int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}(\eta)Z\dot{x}(\eta)d\eta d\theta \\ & + \int_{t-h}^{t} x^T(\eta)Qx(\eta)d\eta. \end{aligned} \tag{10}$$

as in [19], the following theorem can be derived.

Theorem 2: Consider the system (5) with initial condition $\phi \equiv 0$. If there exist real matrices $P = P^T > 0$, $Q = Q^T$, $Z = Z^T > 0$, and Y, W such that

$$\begin{bmatrix} \Lambda_{11} & * & * & * \\ \Lambda_{21} & \Lambda_{22} & * & * \\ -hY^T & -hW^T & -hZ & * \\ \Lambda_{41} & hB^TZA_h & 0 & \Lambda_{44} \end{bmatrix} < 0 \tag{11}$$

where $*$ denotes symmetric elements and

$$\begin{aligned} \Lambda_{11} &= PA + A^TP + Y + Y^T + hA^TZA + Q + C^TC \\ \Lambda_{21} &= A_h^TP - Y^T + W + hA_h^TZA \\ \Lambda_{41} &= B^TP + hB^TZA + D^TC \\ \Lambda_{22} &= -Q - W - W^T + hA_h^TZA_h \\ \Lambda_{44} &= hB^TZB + D^TD - \gamma^2 I \end{aligned}$$

Then system (5) is asymptotically stable and has H_∞ norm less than γ.

C. Simple delay-dependent criterion based on implicit model transformation (IM-LKF)

The LKF candidate is the same as for the explicit model transformation (10), but in this case the so called implicit model transformation is applied [17, Section 5.5.2]. The conditions for stability is presented in [17, Prop. 5.16].

Theorem 3: Consider the system (5) with initial condition $\phi \equiv 0$. If there exist real matrices $P = P^T > 0$, $S = S^T$, $X^T = X$ and Y such that

$$\begin{bmatrix} \Lambda & * & * & * \\ A_h^T P - Y^T & -S & * & * \\ B^T P + D^T C & 0 & D^T D - \gamma^2 I & * \\ -YA_0 & -YA_1 & -YB & -\frac{1}{h}X \end{bmatrix} < 0 \tag{12}$$

where $\Lambda = PA + AP + S + hX + Y + Y^T + C^T C$, then system (5) is asymptotically stable and has H_∞ norm less than γ.

D. Discretized complete LKF (DF-LKF)

Consider the following complete quadratic LKF

$$\begin{aligned} V(x_t) &= x^T(t)Px(t) + 2x^T(t)\int_{-r}^{0} Q(\xi)x(t+\xi)d\xi \\ &\quad + \int_{-r}^{0}\int_{-r}^{0} x^T(t+\xi)R(\xi,\eta)x(t+\eta)d\eta d\xi \\ &\quad + \int_{-r}^{0} x^T(t+\xi)S(\xi)x(t+\xi)d\xi \end{aligned} \tag{13}$$

The matrix functions Q, S and R can be chosen to be piecewise linear continuous functions [20]. For this, the interval $[-h, 0]$ (or $[-h, 0] \times [-h, 0]$) is divided into N (or N by N) segments of length $l = \frac{h}{N}$. Each segment indexed by p or (p, q) can be described with the help of matrix parameters Q_p, S_p, $R_{pq} = R_{qp}^T$, $p, q = 0, 1, 2, \ldots, N$ so that for $0 \le \alpha \le 1$ and $0 \le \beta \le 1$

$$\begin{aligned} Q(-pl + \alpha l) &= (1-\alpha)Q_p + \alpha Q_{p-1} \\ S(-pl + \alpha l) &= (1-\alpha)S_p + \alpha S_{p-1} \end{aligned}$$

and

$$R(-pl + \alpha l, -ql + \beta l) = \begin{cases} (1-\alpha)R_{pq} + \beta R_{p-1,q-1} + (\alpha-\beta)R_{p-1,q} & \alpha \ge \beta \\ (1-\beta)R_{pq} + \alpha R_{p-1,q-1} + (\beta-\alpha)R_{p,q-1} & \alpha < \beta \end{cases}$$

Before presenting Theorem 4 some notations must be introduced.

$$\begin{aligned} \bar{Q} &= \begin{bmatrix} Q_0 & Q_1 & \ldots & Q_N \end{bmatrix} \\ \bar{S} &= \frac{1}{l}\,\mathrm{diag}(S_0\ S_1\ \ldots\ S_N) \\ \bar{R} &= \begin{bmatrix} R_{00} & R_{10}^T & \ldots & R_{N0}^T \\ R_{10} & R_{11} & \ldots & R_{N1}^T \\ \vdots & \vdots & \ddots & \vdots \\ R_{N0} & R_{N1} & \ldots & R_{NN} \end{bmatrix} \\ \Delta &= \begin{bmatrix} \Delta_{11} & * & * \\ Q_N^T - A_h^T P & S_N & * \\ -B^T P - D^T C & 0 & -D^T D + \gamma^2 I \end{bmatrix} \\ \Delta_{11} &= -PA - A^T P - Q_0 - Q_0^T - S_0 - C^T C \\ S_d &= \mathrm{diag}\{S_0 - S_1, S_1 - S_2, \ldots, S_{N-1} - S_N\} \end{aligned}$$

$$\begin{aligned} R_d &= \begin{bmatrix} R_{d11} & R_{d12} & \ldots & R_{d1N} \\ R_{d21} & R_{d22} & \ldots & R_{d2N} \\ \vdots & \vdots & \ddots & \vdots \\ R_{dN1} & R_{dN2} & \ldots & R_{dNN} \end{bmatrix} \\ R_{dpq} &= l(R_{p-1,q-1} - R_{pq}) \\ D^s &= \begin{bmatrix} D_{01}^s & D_{02}^s & \ldots & D_{0N}^s \\ D_{11}^s & D_{12}^s & \ldots & D_{1N}^s \\ D_{w1}^s & D_{w2}^s & \ldots & D_{wN}^s \end{bmatrix} \\ D_{0p}^s &= \frac{l}{2}A^T(Q_{p-1} + Q_p) + \frac{l}{2}(R_{0,p-1} + R_{0p}) \\ &\quad -(Q_{p-1} - Q_p) \\ D_{1p}^s &= \frac{l}{2}A_h^T(Q_{p-1} + Q_p) - \frac{l}{2}(R_{N,p-1} + R_{Np}) \\ D_{wp}^s &= \frac{l}{2}B^T(Q_{p-1} + Q_p) \\ D^a &= \begin{bmatrix} D_{01}^a & D_{02}^a & \ldots & D_{0N}^a \\ D_{11}^a & D_{12}^a & \ldots & D_{1N}^a \\ D_{w1}^a & D_{w2}^a & \ldots & D_{wN}^a \end{bmatrix} \\ D_{0p}^a &= -\frac{l}{2}A^T(Q_{p-1} - Q_p) - \frac{l}{2}(R_{0,p-1} - R_{0p}) \\ D_{1p}^a &= -\frac{l}{2}A_h^T(Q_{p-1} - Q_p) + \frac{l}{2}(R_{N,p-1} - R_{Np}) \\ D_{wp}^a &= -\frac{l}{2}B^T(Q_{p-1} - Q_p) \end{aligned}$$

Theorem 4: Consider system (5) with initial condition $\phi \equiv 0$. If there exists real matrices $P = P^T$, Q_p, $S_p = S_p^T$, $R_{pq} = R_{qp}^T$, $p, q = 0, 1, \ldots, N$ such that

$$\begin{bmatrix} P & \bar{Q} \\ \bar{Q}^T & \bar{R} + \bar{S} \end{bmatrix} > 0 \tag{14a}$$

$$\begin{bmatrix} \Delta & * & * \\ -D^{sT} & R_d + S_d & * \\ -D^{aT} & 0 & 3S_d \end{bmatrix} > 0 \tag{14b}$$

satisfied, then the system is asymptotically stable and has H_∞ norm less than γ.

IV. Results

The homogeneous vehicle platoon of length $n+1$ is tested with the following parameter setting: $\tau_i = 0.7$, $g_i = 1$, $k_1 = 0.7$, $k_2 = 0.1127$, $k_{1\alpha} = 0.4642$, $k_{2\alpha} = 0.0564$, $k_{1\beta} = 0.2358$, $k_{2\beta} = 0.0564$, $k_{a1} = 0.0449$, $k_{a0} = 0.9551$, $A_d = -5$, $B_d = 5$ and $C_d = 1$. The tests are carried out by computing the $\mathcal{H}_\infty$ norms of SISO systems by different computation methods.

A good approximation of the true performance levels can be computed by zero-order hold transformation of system (5) to discrete-time with a sufficiently small sampling time $T_s = 0.01$s. It is chosen to be a divisor of the tested delay values, i.e. $h(k) = kT_s$. Then the delays can be interpreted as a chain of shift operators. $\mathcal{H}_\infty$ performance levels $\gamma(w_j \to z_i)$ from input w_j to output z_i of this system serve as reference values and are denoted by $\gamma_R(w_j \to z_i)$.

A. Effect of network sampling time

First this discrete-time reference model is compared with a multi-rate discrete-time model proposed in [8], where measurements available locally are sampled with T_s and those received from the network are sampled by T_n, an integer multiple of T_s. The goal of the comparison is to evaluate the effect of rare network sampling times. In the range of $h \in [0, 0.1]$ it can be concluded that

- The network sampling time has significant effect on the channel $u_0 \to e_1$. The performance level increases about 8.5% at $T_n = 0.2$s with respect to the case $T_n = T_s$.
- For other input-output pairs the effect of network sampling time is not significant.
- On channel $u_0 \to e_1$ the relative difference between the two models is less than 10%.

B. Evaluation of LKF methods

In Figures 1 and 2 the accurate performance levels computed based on the discrete-time reference model are shown from inputs u_0 and d_0, respectively, to the spacing errors e_i. It can be seen that the spacing error is decreasing along the platoon (with i increasing), i.e. the platoon is string stable.

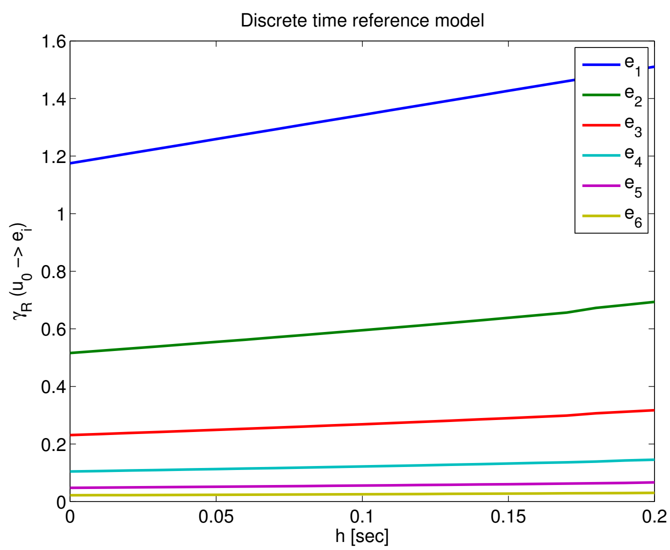

Fig. 1. Accurate performance values as function of network transmission delay $\gamma_R(h)$ from input u_0

It can also be seen that the worst-case gain is increasing as the delay increases. It is not surprising therefore, that the delay-independent conditions (DI-LKF) are infeasible. Due to the feed-forward terms in the controller a sufficiently long constant delay can completely deteriorate the performance.

Results of other LKF methods are comparable with the true values plotted in Figures 1 and 2, therefore the relative errors of the computed performances

$$\frac{\gamma_* - \gamma_R}{\gamma_R} 100 \qquad [\%]$$

are shown in Figures 3-5. Here $*$ denotes one of the LKF methods. For the channel $u_0 \to e_1$ all methods arrives to the accurate results γ_R. From Figures 3-5 it can be concluded that the discretized complete LKF method achieves the accurate values even with single linear matrix functions $Q()$ and $R(,)$.

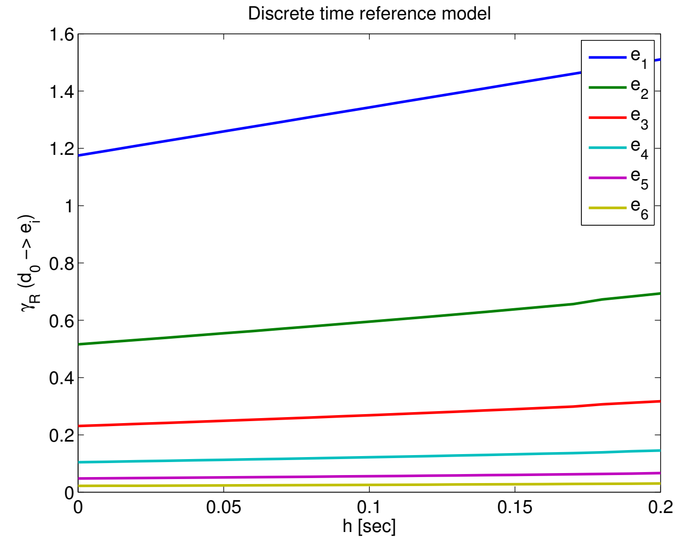

Fig. 2. Accurate performance values as function of network transmission delay $\gamma_R(h)$ from input d_0

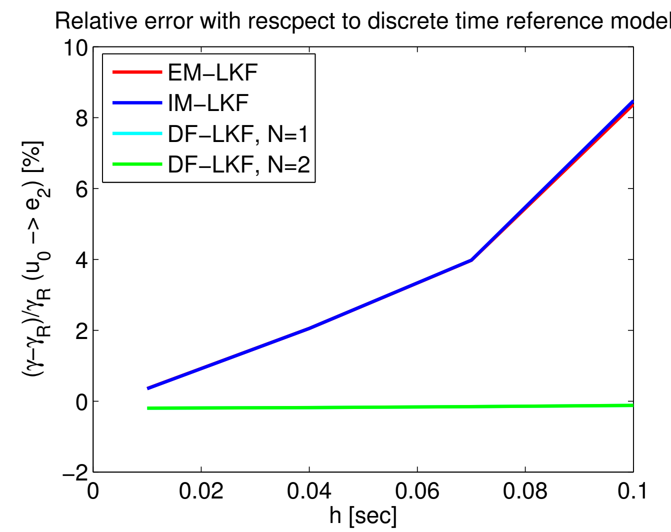

Fig. 3. Relative performance values for LKF methods with respect to the reference values, as functions of network transmission delay from input u_0 to output e_2. The discretized complete LKF methods (DF-LKF) approximately coincide with the values of the reference model

In Table I the performance and computational cost of the methods for $n + 1 = 5$ vehicles and for a small and a larger delay are shown. It can be observed that as the number of variables increases so the computation time and the accuracy increases.

V. Conclusions

Lyapunov-Krasovskii functionals are used to analyze the effects of constant network transfer delays on performance of vehicular platoons. It is shown that the discretized complete LKF approach with very few segments ($N = 1$) results in accurate performance values. Its computational cost is tolerable so the approach is potentially applicable to extend the analysis toward handling time-varying delays, uncertainties and distributed Lyapunov function approaches.

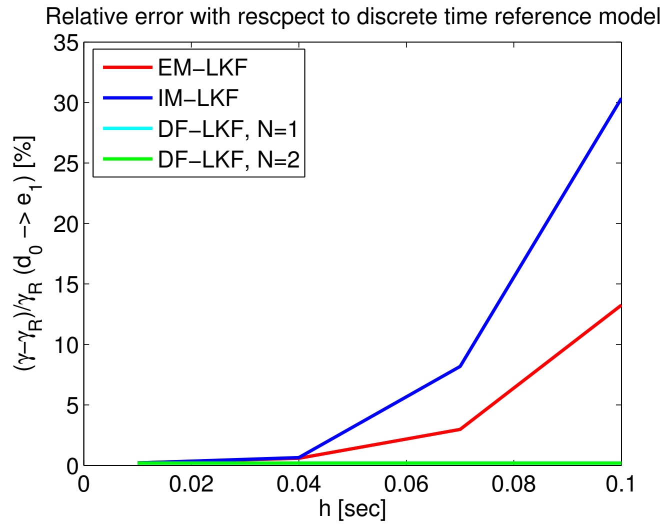

Fig. 4. Relative performance values of for LKF methods with respect to the reference values, as functions of network transmission delay from input d_0 to output e_1. The discretized complete LKF methods (DF-LKF) approximately coincide with the values of the reference model

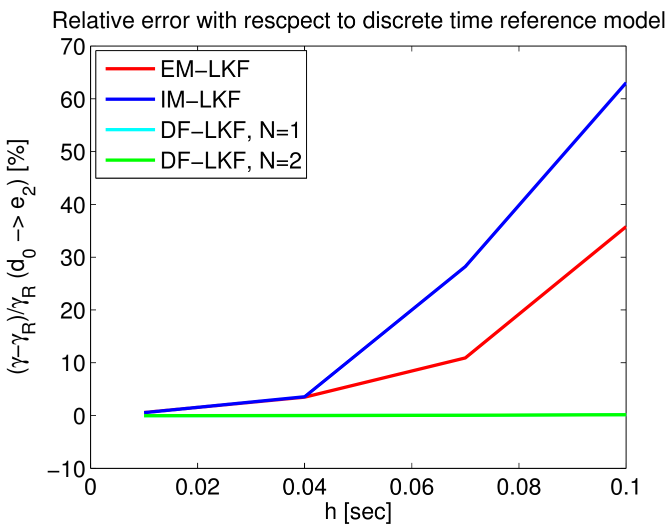

Fig. 5. Relative performance values of for LKF methods with respect to the reference values, as functions of network transmission delay from input d_0 to output e_2. The discretized complete LKF methods (DF-LKF) approximately coincide with the values of the reference model

TABLE I. PERFORMANCE AND COMPUTATIONAL COST OF THE METHODS FOR $n+1=5$ VEHICLES. THE ACCURATE VALUES ARE $\gamma_R(u_0 \to e_4, h=1) = 0.1038$ AND $\gamma_R(u_0 \to e_4, h=1) = 0.1188$

		EM-LKF	IM-LKF	DF-LKF,N=1	N=2
h=1	$\gamma, u_0 \to e_4$	0.1048	0.1048	0.1035	0.1036
	computation time	12.6	8.6	29.1	94.7
h=10	$\gamma, u_0 \to e_4$	0.1345	0.1351	0.1184	0.1185
	computation time [s]	12.6	10.2	46.9	95.9
	number of variables	366	266	576	986

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