THE TEMPERATURE-DEPENDENT YANG-MILLS TRACE ANOMALY AS A FUNCTION OF THE MASS GAP

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The trace anomaly or, equivalently, the interaction measure is the important thermodynamic quantity/observable, since it is very sensitive to the non-perturbative effects in the gluon plasma. It has been calculated and its analytic and asymptotic properties have been investigated with the combined force of analytic and lattice approaches to $SU(3)$ Yang-Mills (YM) quantum gauge theory at finite temperature. The first one is based on the effective potential approach for composite operators properly generalized to finite temperature. This makes it possible to introduce into this formalism a dependence on the mass gap Δ^2, which is responsible for the large-scale dynamical structure of the QCD ground state. The gluon plasma pressure as a function of the mass gap adjusted by this approach to the corresponding lattice data is shown to be a continuously growing function of temperature T in the whole temperature range $[0, \infty)$ with correct Stefan-Boltzmann limit at very hight temperature. The corresponding trace anomaly has finite jump discontinuity at some characteristic temperature $T_c = 266.5\ MeV$ with latent heat $\epsilon_{LH} = 1.41$. This is a firm evidence of the first-order phase transition in $SU(3)$ pure gluon plasma. It is exponentially suppressed below T_c and has a complicated and rather different dependence on the mass gap and temperature across T_c. In the very high temperature limit its non-perturbative part has a power-type fall off.

1. Introduction

From the very beginning and up to present days, lattice QCD remains the only practical method to investigate QCD at finite temperature and density from first principles. Recently it underwent a rapid progress[1] (and references therein). However, lattice QCD is primarily aimed at obtaining well-defined calculation schemes in order to get realistic numbers for physical quantities. One may therefore get numbers and curves for various thermodynamic quantities/observables, but without understanding what is the physics behind them. Such an understanding can only come from an analytic description of the corresponding lattice data in the whole temperature range and desirably on a general dynamical basis. So the merger between lattice and analytical approaches to QCD at finite temperature and density is unavoidable, i.e., they do not exclude each other: on the contrary, they should be complementary. In other words, numbers and curves come from thermal lattice QCD, while the analytic description of the physics behind them comes from the dynamical theory, which is continuous QCD.

The effective potential approach for composite operators[2,3] turned out to be a very effective analytical and perspective dynamical tool for the generalization of QCD to non-zero temperature[4] (and references therein). In the absence of external sources it is nothing but the vacuum energy density (VED), i.e., the pressure apart from the sign. This approach is nonperturbative (NP) from the very beginning, since it deals with the expansion of the corresponding skeleton vacuum loop diagrams in powers of the Plank constant, and thus allows one to calculate the VED from first principles. The key element in this program is the extension of our paper[3] to nonzero temperature.[4] This makes it possible to introduce the temperature-dependent gluon pressure as a function of the Jaffe-Witten (JW) mass gap.[5,6] It is this which is responsible for the large-scale structure of the QCD ground state (in what follows we will call it as mass gap, for simplicity). The confining dynamics in the gluon matter (GM) is therefore nontrivially taken into account directly through the mass gap and via the temperature-dependent gluon pressure itself, but other NP effects are also present. Being NP, the effective approach for composite operators, nevertheless, makes it possible to incorporate the thermal perturbation theory (PT) expansion in a self-consistent way. In our auxiliary work[7] we have formulated and developed the analytic thermal PT which allows one to calculate the PT contributions in terms of the convergent series in integer powers of a small α_s. In this way, we have explicitly derived and numerically calculated the first PT correction of the α_s-order to the NP part of the gluon pressure investigated and calculated previously in.[4]

2. The gluon pressure $P_g(T)$

Gathering all our results, obtained previously[4,7] and summarized in,[6] the gluon pressure $P_g(T)$ can be written down as follows:

$$P_g(T) = P_{NP}(T) + P^s_{PT}(T) = \Delta^2 T^2 - \frac{6}{\pi^2}\Delta^2 P'_1(T) + \frac{16}{\pi^2} T M(T) + P^s_{PT}(T), \tag{1}$$

where the integrals $P'_1(T)$ and $P^s_{PT}(T)$ are

$$P'_1(T) = \int_0^{\omega_{eff}} d\omega \frac{\omega}{e^{\beta\omega} - 1}, \tag{2}$$

and

$$P^s_{PT}(T) \equiv P^s_{PT}(\Delta^2; T) = \alpha_s \times \frac{9}{2\pi^2}\Delta^2 \int_{\Lambda_{YM}}^{\infty} d\omega\ \omega^2\ \frac{1}{\bar{\omega}} \frac{1}{e^{\beta\bar{\omega}} - 1}, \tag{3}$$

respectively, while the composition $M(T) = [P_2(T) + P_3(T) - P_4(T)]$ is defined via the following integrals

$$\begin{aligned}
P_2(T) &= \int_{\omega_{eff}}^{\infty} d\omega\ \omega^2 \ln\left(1 - e^{-\beta\omega}\right), \\
P_3(T) &= \int_0^{\omega_{eff}} d\omega\ \omega^2 \ln\left(1 - e^{-\beta\omega'}\right), \\
P_4(T) &= \int_0^{\infty} d\omega\ \omega^2 \ln\left(1 - e^{-\beta\bar{\omega}}\right),
\end{aligned} \tag{4}$$

where $\omega_{eff} = 1\ GeV$ and the mass gap $\Delta^2 = 0.4564\ GeV^2$ for $SU(3)$ gauge theory have been fixed in,[3,4] and this choice has been explained as well. Here ω_{eff} is a scale separating the low- and high frequency-momentum regions, while ω' and $\bar{\omega}$ are given by the relations

$$\omega' = \sqrt{\omega^2 + 3\Delta^2} = \sqrt{\omega^2 + m'^2_{eff}}, \quad m'_{eff} = \sqrt{3}\Delta = 1.17\ GeV, \tag{5}$$

and

$$\bar{\omega} = \sqrt{\omega^2 + \frac{3}{4}\Delta^2} = \sqrt{\omega^2 + \bar{m}^2_{eff}}, \quad \bar{m}_{eff} = \frac{\sqrt{3}}{2}\Delta = 0.585\ GeV, \tag{6}$$

respectively. The so-called gluon mean number[8,9]

$$N_g \equiv N_g(\beta, \omega) = \frac{1}{e^{\beta\omega} - 1}, \quad \beta = T^{-1}, \tag{7}$$

which appears in the integrals (2)-(4), describes the distribution and correlation of massless gluons in the GM. Replacing ω by $\bar{\omega}$ and ω' we can consider the corresponding gluon mean numbers as describing the distribution and correlation of the corresponding massive gluonic excitations in the GM, see integrals $P_3(T)$ and $P_4(T)$ in Eqs. (4). They are of NP dynamical origin, since their masses are due to the mass gap Δ^2. All three different gluon mean numbers range continuously from zero to infinity.[8,9] We have the two different massless excitations, propagating in accordance with the integral (2) and the first of the integrals (4). However, they are not free, since in the PT $\Delta^2 = 0$ limit they vanish (the composition $[P_2(T) + P_3(T) - P_4(T)]$ becomes zero in this case). The gluon mean numbers are closely related to the pressure. Its exponential suppression in the $T \to 0$ limit and the polynomial structure in the $T \to \infty$ limit are determined by the corresponding asymptotics of the gluon mean numbers. The low- and high-temperature expansions for the gluon pressure (1) have been derived in.[6,7]

It is worth emphasizing that the effective scale ω_{eff} is not an independent scale parameter. From the stationary condition at zero temperature[3] and the scale-setting scheme at non-zero temperature[4] it follows that

$$\omega_{eff}^2 = (0.4564)^{-1}\Delta^2, \tag{8}$$

so it is expressed in terms of the initial fundamental scale parameter - the mass gap. Its introduction is convenient from the technical point of view in order to simplify our expressions which otherwise would be too cumbersome.

Let us note that the term (3) describes the same massive gluonic excitations $\bar{\omega}$ (6), but their propagation, however, suppressed by the α_s-order. We can consider it as a new massive excitation in the GM, denoted it as $\alpha_s \cdot \bar{\omega}$. In fact, the term $P^s_{PT}(T)$ is NP, depending on the mass gap Δ^2, which is only suppressed by the α_s order. When the interaction is formally switched off, i.e., letting $\alpha_s = \Delta^2 = 0$, the above-defined composition $M(T)$ becomes zero, as it follows from Eqs. (4), and thus the gluon pressure (1) itself. This is due to the normalization condition of the free PT vacuum to zero valid at non-zero T as well.

As mentioned above, the gluon pressure (1) has been calculated and discussed in.[6,7] It is shown in Fig. 1 and its numerical values in the form of the corresponding Table are present in,[10] where one can find the explicit expansions of its asymptotics as well. It has a maximum at some "characteristic" temperature, $T_c = 266.5\ MeV$. Below T_c the gluon pressure is exponentially suppressed in the $T \to 0$ limit, in accordance with the low-temperature asymptotic of the gluon mean number (7), as noted above. Close to T_c and at moderately high temperatures up to approximately $(4-5)T_c$ the exact functional dependence on the mass gap Δ^2 and the temperature T of the gluon pressure (1) remains rather complicated. This means that the NP effects due to the mass gap are still important up to rather high temperature. The gluon pressure $P_g(T)$ has a polynomial character in integer powers of T up to T^2 at high temperatures. So it is in agreement with the corresponding asymptotic of the gluon mean number (7). At very high temperature its expansion is as follows:

$$P_g(T) \sim B_2 \alpha_s \Delta^2 T^2 + [B_3 \Delta^3 + M^3]T, \quad T \to \infty, \tag{9}$$

up to unimportant here parametersB_2, B_3 and M^3. The term $\sim T^2$ has been first introduced and discussed in the phenomenological equation of state (EoS)[11] (see also[4,6,7,10,12–19] and references therein). On the contrary, in our approach both terms $\sim T^2$ and $\sim T$ have not been introduced by hand. They naturally appear on a general ground as a result of the explicit presence of the mass gap from the very beginning in the gluon analytical EoS (1).

Concluding, let us note that the first term $\Delta^2 T^2$ in the gluon pressure (1) plays a dominant role in the region of moderately high temperatures approximately up to $(4-5)T_c$. In the limit of very high temperatures it is exactly cancelled by the term coming from the composition $M(T)$ in Eq. (1), as it has been established in.[7] In other words, the $\sim \Delta^2 T^2$ behavior of $P_g(T)$ is replaced by $\sim \alpha_s \Delta^2 T^2$ behavior at very high temperature, as it should be, in principle. It would be very surprised if a pure NP contribution were survived in the limit of very high temperature, while for its PT counterpart/correction it would be expected/possible. At the same time, the second purely NP term $\sim T$ is suppressed in comparison with the first PT term in the very high temperature limit in Eq. (9), indeed.

3. The full GP EoS

From Fig. 1 it clearly follows that the gluon pressure (1) will never reach the general Stefan-Boltzmann (SB) constant $3P_{SB}(T)/T^4 = 24\pi^2/45$ at very high temperatures. Let us remind that the very high-temperature behavior

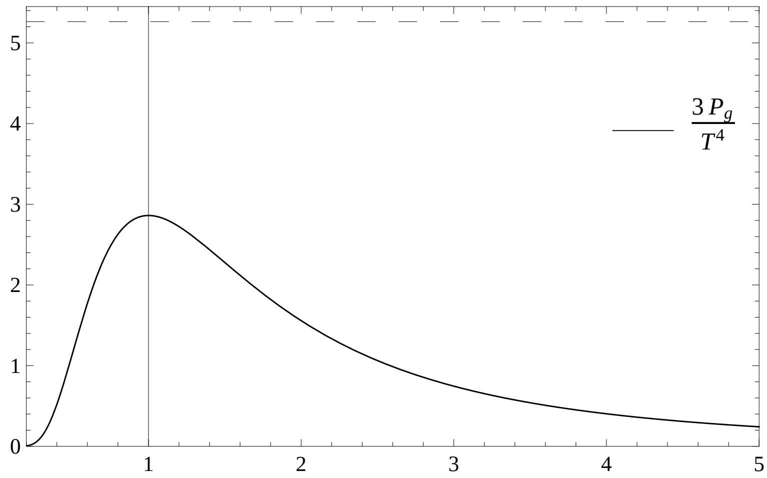

Fig. 1. The gluon pressure (1) scaled (i.e., divided) by $T^4/3$ is shown as a function of T/T_c (solid curve). It has a maximum at $T_c = 266.5\ MeV$ (vertical solid line). The horizontal dashed line is the general SB constant $3P_{SB}(T)/T^4 = 24\pi^2/45$.

$(T \to \infty)$ of all the thermodynamic quantities is governed just by the SB ideal gas limit, when the matter can be described in terms of non-interacting (i.e., free) massless particles (gluons). That is not a surprise, since the SB term has been canceled in the gluon pressure from the very beginning due to the normalization condition of the free PT vacuum to zero.[4,7] Analytically this cancelation/subtraction at high temperatures (above T_c) has been shown in,[7] where it has also been shown that the massless (but not free) gluons may be present at low temperatures (below T_c) in the GM. However, their propagation in this region cannot be described by the SB term itself. All this means that the SB pressure has been already subtracted from the gluon pressure, but in a very specific way, i.e., the above-mentioned normalization condition is not simply the subtraction of SB term. The gluon pressure (1) may change its continuously falling off regime above T_c only in the near neighborhood of T_c in order for its full counterpart to reach the corresponding SB limit at high temperatures. The SB term is valid only at high temperatures, nevertheless, it cannot be added to Eq. (1) above T_c, even multiplied by the corresponding $\Theta((T/T_c) - 1)$-function. The problem is that in this case the pressure will get a jump at $T = T_c$, which is not acceptable. The full pressure is always a continuous growing function of temperature at any point of its domain. This means that we should add some other terms valid below T_c in order to restore a continuous character of the full pressure across T_c. This can be achieved by imposing a special continuity condition on these terms valid just at T_c. Moreover, the gluon pressure $P_g(T)$ itself should be additionally multiplied by the functions which are always negative below and above T_c. This guarantees the positivity of the full pressure below T_c, while above T_c this guarantees the approach of the full pressure to the SB limit in the AF way, i.e., slowly and from below. These terms will also contribute to the condition of continuity for the full pressure. All these problems make the inclusion of the SB term into EoS highly non-trivial. The most general way how this can be done is to add to the both sides of Eq. (1) the term $[\Theta((T/T_c)-1)H(T)+\Theta((T_c/T-1))L(T)]$, valid in the whole temperature range, and the auxiliary functions $H(T)$ and $L(T)$ are to be expressed in terms of $P_{SB}(T)$ and $P_g(T)$ (see below).

The previous Eq. (1) then becomes

$$P_{GP}(T) = P_g(T) + \Theta\left(\frac{T_c}{T} - 1\right) L(T) + \Theta\left(\frac{T}{T_c} - 1\right) H(T), \qquad (10)$$

and its left-hand side here and below is denoted as $P_{GP}(T)$ (the abovementioned full counterpart). The gluon plasma (GP) pressure (10) is continuous at T_c if and only if

$$L(T_c) = H(T_c), \qquad (11)$$

which can be easily checked. Due to the continuity condition (11), the dependence on the corresponding Θ-functions disappears at T_c, and the GP pressure (10) remains continuous at any point of its domain. The role of the auxiliary function $L(T)$ is to change the behavior of $P_{GP}(T)$ from $P_g(T)$ at low (L) temperatures below T_c, especially in its near neighborhood, as well as to take into account the suppression of the SB-type terms below T_c. The auxiliary function $H(T)$ is aimed to change the behavior of $P_{GP}(T)$ from $P_g(T)$, as well as to introduce the SB term itself and its modification due to AF at high (H) temperatures above and near T_c. These changes are necessary, since in the gluon pressure $P_g(T)$ the SB term is missing (as described above), and it cannot be restored in a trivial way. So the appearance of the corresponding Θ-functions in the GP pressure (10) is inevitable together with the functions $H(T)$ and $L(T)$, playing only an auxiliary role but still useful from the technical point of view.

The gluon pressure $P_g(T)$ (1), which fixes the value of the characteristic temperature $T_c = 266.5\ MeV$, is a necessary analytical and dynamical input information for the GP pressure (10). On the other way around, the lattice pressure[14] is a main numerical input information to use in order to fix the functions $L(T)$ and $H(T)$ in Eq. (10). Our general method[6,10] how to merge analytical and lattice approaches in order to understand the physics of YM quantum gauge theories at non-zero temperature is described in some general terms below. We proposed and developed a method of analytical simulations which allows one to express such introduced auxiliary functions $L(T)$ and $H(T)$ in terms of basic known functions $P_g(T)$ and $P_{SB}(T)$, multiplied by the so-called simulating functions $\phi_l(T),\ \phi_h(T)$ and $f_l(T),\ f_h(T)$, respectively. They are to be necessary represented by the corresponding asymptotics of the gluon mean number (7) in the low and high temperature limits. This makes it possible to reproduce lattice data in any requested temperature interval and to ensure the correct SB limit for all the thermodynamic obsrvables/quantities as well.

Then we have performed the numerical simulation of the GP pressure (1) above T_c in order to fix the function $\phi_h(T)$ in accordance with the lattice pressure[14] in this region by using the Least Mean Square (LMS) method.[20] Our procedure makes it also possible to continue the lattice pressure to the region of very high temperatures. As a next step, we have performed the numerical simulation of the GP pressure (1) below T_c in order to fix the three free fitting parameters, which necessarily appear in the simulating function $f_l(T)$. This has been done in accordance with lattice data[14] in this region, but only very close to T_c. Our procedure makes it also possible to continue the calculation of the GP pressure to very low temperatures, where convincing lattice data does not exists at all. Thus, we can predict the behavior of the lattice pressure curve up to zero temperature, knowing only its behavior very close to T_c.

"Sewing" together such obtained two parts with the help of the relation (11), we are coming to the analytical expression reproducing the lattice pressure[14] in the whole temperature range $[0,\infty)$ as a function of the mass gap Δ^2 as follows:

$$
\begin{aligned}
P_{GP}(T) &= P_g(T) \\
&+ \quad \Theta\left(\frac{T_c}{T}-1\right)\left[(0.015732e^{-\mu_1((T_c/T)-1)} + 0.003884e^{-\mu_2((T_c/T)-1)})P_{SB}(T) - e^{-\mu(T_c/T)}P_g(T)\right] \\
&+ \quad \Theta\left(\frac{T}{T_c}-1\right)\left[(1-\alpha_s(T))P_{SB}(T) - \phi_h(T)P_g(T)\right],
\end{aligned}
\tag{12}
$$

where

$$\mu = 0.0001, \quad \mu_1 = 39.1, \;\mu_2 = 3.4, \tag{13}$$

while

$$\alpha_s(T) = \left(0.22037 \frac{1}{t} - 0.033 \frac{\ln t}{t^2}\right), \tag{14}$$

$$t = 1 + 0.1929 \ln(T/T_c), \quad T \geq T_c = 266.5 \; MeV, \tag{15}$$

and

$$\phi_h(T) = 1.55 + 0.8482 \left(\frac{T_c}{T}\right)^3, \quad \phi_h(T_c) = 2.3982. \tag{16}$$

The GP pressure (12) is completely known now, since the SB pressure is $P_{SB}(T) = (8\pi^2/45)T^4$ and the gluon pressure $P_g(T)$ is also exactly known, Eq. (1). For the numerical evaluation of the GP pressure (12) in detail see.[10] For simplicity, in what follows we will omit the subscript "GP" in the GP pressure (12), i.e., we will put $P_{GP}(T) \equiv P(T)$. The same will be done in the notation for the trace anomaly relation below. According to such obtained analytical expression, the corresponding lattice pressure is exponentially suppressed at low temperatures, it is continuous across T_c and approaches its SB limit at high temperatures, i.e., satisfying thus to all thermodynamics limits. In other words, the GP pressure (12) is, in fact, the lattice pressure[14] analytically expressed as a function of the mass gap and temperature and properly continued to the regions of very low and high temperatures, see Fig. 2.

3.1. *Trace anomaly relation*

A thermodynamic quantity of special interest is the thermal expectation value of the trace of the energy momentum tensor. Equivalently, it is known as the interaction measure and defined as follows:

$$I(T) = \epsilon(T) - 3P(T), \tag{17}$$

where $\epsilon(T)$ is the energy density, which in its turn defined as $\epsilon(T) = T(\partial P(T)/\partial T) - P(T)$. So knowing the GP pressure (12), one can calculate any other thermodynamic quantity/observable, see our works.[6,10] The importance of this thermodynamic observable is that it is very sensitive to

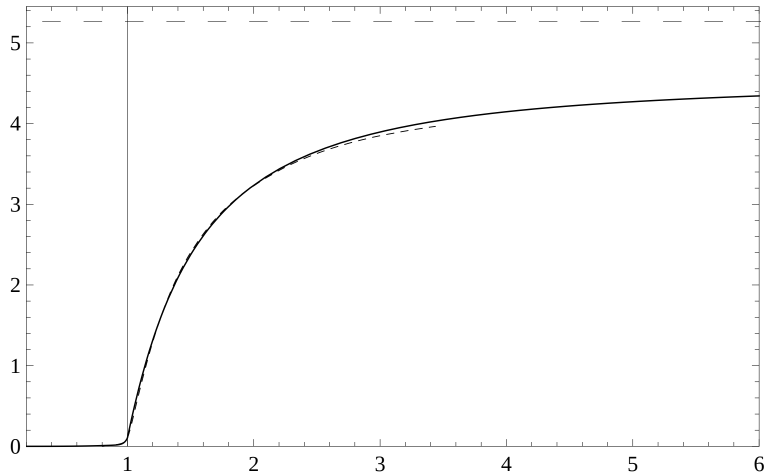

Fig. 2. The GP pressure (12) is shown as a function of $a = T/T_c$ (solid curve). The lattice curve[14] for $SU(3)$ pressure is also shown (dashed curve). The horizontal dashed line is the general SB constant. Both pressures are scaled in the same way, i.e., divided by $T^4/3$.

the NP effects, since the corresponding pure PT contributions are exactly cancelled in the composition (17). This can be explicitly shown by using the GP pressure (12) above T_c, and the relation $3P_{SB}(T) = \epsilon_{SB}(T)$, see below. Properly scaled it is shown in Fig. 3. The rapid rise of the peak (due to the latent heat (LH) in the energy density) is exactly placed at T_c, and it is about 2.5. In all lattice calculations it peaks at about $1.1T_c!$,[14,15,17,18,21] and it is about 2.6, and almost coincides with our value in.[18] The wrong position of the lattice trace anomaly peak can be due to an ultraviolet cut-off, the finite volume effects, etc. In this connection let us indeed remind that in lattice simulations at any temperature it is necessary finally to go to the continuum (physical) limit, namely lattice spacing goes to zero and then the infinite volume limit should be taken. These are nothing else but the removal of the ultraviolet and infrared cutoffs which is the part of the renormalization procedure.[22,23] It seems to us that our analytical method resolves this $SU(3)$ lattice thermodynamics artefact.

Just above T_c and up to rather high temperatures $(4 - 5)T_c$ the NP effects due to the mass gap are still important in the trace anomaly. Fig. 3 demonstrates rather complicated dependence of the trace anomaly on the mass gap and temperature in this interval. The trace anomaly equation (17), divided by T^4 is

$$\frac{I(T)}{T^4} = -\frac{1}{3} T\alpha_s'(T)(SB) - 0.55\frac{[TP_g'(T) - 4P_g(T)]}{T^4}, \quad T > T_c, \tag{18}$$

where $(SB) = 24\pi^2/45 \approx 5.2638$ is the above-mentioned SB general constant/limit. So, indeed the main contribution to the trace anomaly comes from the second NP term in Eq. (18), and it is not a simple power-type fall off. It is mainly due to the complicated dependence of the gluon pressure $P_g(T)$ on the mass gap and the temperature in this region, where it cannot be approximated by some simple power-type expression. However, this is possible to do in the limit of very high temperatures, approximately above $(4-5)T_c$. Substituting the asymptotic (9) and its derivative into the previous equation and doing some algebra, one obtains

$$\frac{I(T)}{T^4} \sim -\frac{1}{3} T\alpha_s'(T)(SB) + 1.1\bar{B}_2\alpha_s \left(\frac{T_c}{T}\right)^2 + 1.65\bar{B}_3 \left(\frac{T_c}{T}\right)^3, \quad T \to \infty, \tag{19}$$

while $\bar{B}_2$ and $\bar{B}_3$ are unimportant here constants.

Concluding, let us briefly discuss the size of the discontinuity in the energy density, the above-mentioned LH. It is

$$\epsilon_{LH} = 1.41 \tag{20}$$

in dimensionless units (for its definition and analytical/numerical evaluation, respectively, see our work[10]). It is worth emphasizing that the same value (20) comes from the independent calculations of the energy density and the trace anomaly, as it should be, since the pressure itself is a continuous function across T_c, i.e., $\epsilon_{LH} = \Delta(\epsilon - 3P)/T_c^4 = \Delta\epsilon/T_c^4$ (here, obviously, Δ is not the mass gap). This means that the first-order phase transition in the GP is analytically confirmed for the first time, in complete agreement with thermal $SU(3)$ YM lattice simulations[14,17,18,24] (and references therein). The reason of such sharp changes at T_c in the derivatives of the GP pressure is that its exponential rise below T_c is changing to the polynomial rise above T_c in order to reach finally the SB limit. The value (20) is in fair agreement with lattice ones in[14,18,21,25–27] (and references therein). This agreement is not a trivial thing, since, we have adjusted our analytical numerical simulations with those of lattice ones in[14] only for the pressure. First of all, the energy density being derivative of the pressure, nevertheless, is an independent thermodynamic observable, having a discontinuity at T_c, while the pressure is a continuous function across T_c. Secondly, the lattice results heavily depend on how the continuum limit is to be taken and on other details of the above-cited lattice simulations. For example, the lattice data points closest to T_c for the entropy density may still be affected by an upward finite-volume effect[27] .

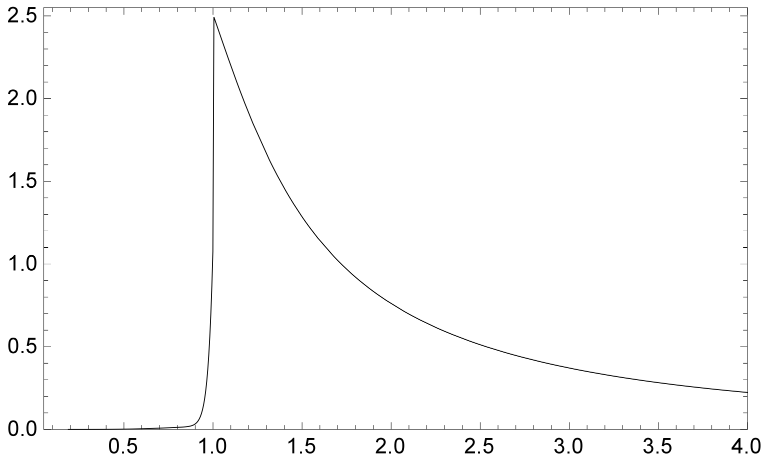

Fig. 3. The trace anomaly (17) divided by T^4 is shown as a function of T/T_c.

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