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Period and light-curve fluctuations of the *Kepler* Cepheid V1154 Cygni

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ABSTRACT

We present a detailed period analysis of the bright Cepheid-type variable star V1154 Cygni (V1154 Cyg; $V = 9.1$ mag, $P \approx 4.9$ d) based on almost 600 d of continuous observations by the *Kepler* space telescope. The data reveal significant cycle-to-cycle fluctuations in the pulsation period, indicating that classical Cepheids may not be as accurate astrophysical clocks as commonly believed: regardless of the specific points used to determine the $O - C$ values, the cycle lengths show a scatter of 0.015–0.02 d over 120 cycles covered by the observations. A very slight correlation between the individual Fourier parameters and the $O - C$ values was found, suggesting that the $O - C$ variations might be due to the instability of the light-curve shape. Random-fluctuation tests revealed a linear trend up to a cycle difference 15, but for long term, the period remains around the mean value. We compare the measurements with simulated light curves that were constructed to mimic V1154 Cyg as a perfect pulsator modulated only by the light travel time effect caused by low-mass companions. We show that the observed period jitter in V1154 Cyg represents a serious limitation in the search for binary companions. While the *Kepler* data are accurate enough to allow the detection of planetary bodies in close orbits around a Cepheid, the astrophysical noise can easily hide the signal of the light-time effect.

Key words: techniques: photometric – planets and satellites: detection – stars: individual: V1154 Cyg – stars: variables: Cepheids.

1 INTRODUCTION

Cepheids are luminous, F- and G-type supergiant stars, exhibiting radial pulsations driven by the κ-mechanism with periods from a few days up to about 100 d. Owing to their high luminosity and well-defined period–luminosity relations, they are primary distance indicators. The pulsation period is one of the most important parameters of a Cepheid variable and it is assumed to be stable on short time-scales. However, on the evolutionary time-scales, Cepheid periods are subject to variations but these period changes become detectable only over several decades or even longer time-scales (Szabados 1983; Turner, Abdel-Sabour Abdel-Latif & Berdnikov 2006).

Observations of Cepheids with ground-based instruments provide very poor light-curve sampling, usually one to two data points per night which is mainly due to the long pulsation period of these variables. In addition, except whole Earth campaigns, observations usually cover a few weeks at most. The three phases of the Optical Gravitational Lensing Experiment (OGLE; Soszynski et al.

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2008a,b, 2010) and massive compact halo objects (MACHO; Alcock et al. 1999) projects produced years of observations of Cepheids in the Magellanic Clouds, but again, their light curves contain one point per night. While this is perfectly enough to characterize the general variability of the few thousand Cepheids, detailed insight into the light-curve shape changes or short-term period fluctuations is prevented by the daily sampling. Poleski (2008) has successfully analysed the period changes of Cepheids based on the OGLE and MACHO data sets and found variability on the scale of a few years. The near-continuous observations of *Kepler* give us a unique opportunity to study these variables in more detail than never before.

Cepheids are often regarded as clockwork-precision, regular pulsators, and with a few exceptions (V473 Lyr: Burki, Mayor & Benz 1982, Polaris: Turner et al. 2005; Bruntt et al. 2008; Spreckley & Stevens 2008) this is usually a valid approximation to the limits set by the sampling and precision of ground-based observations. State-of-the-art one-dimensional hydrocodes (Florida–Budapest: Kolláth et al. 2002, Warsaw: Smolec & Moskalik 2008) have been supporting this view, because after reaching their limit cycle (in case of a single-mode pulsation) they are able to repeat pulsational cycles essentially forever. This of course assumes that no evolutionary or additional processes acting on longer time-scales are included in the set of equations solved numerically.

Photometric time-series data obtained by space-based telescopes have been available on a few Cepheids: Spreckley & Stevens (2008) analysed photometric data of Polaris observed with the Solar Mass Ejection Imager (SMEI) instrument onboard the Coriolis spacecraft; independently, Bruntt et al. (2008) also studied Polaris using the SMEI and *Wide Field Infrared Explorer* (*WIRE*) star tracker photometry; Berdnikov (2010) studied the periodicity of eight Cepheids based on SMEI photometry; while the existing *WIRE* data on δ Cephei are currently being analysed (Bruntt 2007). Although the precision of these data is superior to that of their ground-based counterparts, the 0.0001 mag relative accuracy still hides the subtle effects to be discovered from the *Kepler* data.

In this paper, we present the first evidence that the pulsation period of V1154 Cygni (V1154 Cyg) is jittering, i.e. it is not as stable as the present models predict. In Section 2, we briefly describe the data used, with details of pixel-level photometry of the saturated *Kepler* observations, the methods of the analysis and simulated study about the timing accuracy of the *Kepler* data. In Section 3, our findings on the period fluctuations in V1154 Cyg are described, while the implications are discussed in Section 4. We conclude with a short summary in Section 5.

2 DATA ANALYSIS

The data we use in this paper were provided by the *Kepler* space telescope. The telescope was launched in 2009 March and designed to detect transits of Earth-sized planets. A detailed technical description of the *Kepler Mission* can be found in Koch et al. (2010) and Jenkins et al. (2010a,b).

The *Kepler* space telescope observes 105 square degrees area of the sky in the constellations Cygnus and Lyra, and has two observational modes, sampling data either in every 58.9 s (short cadence whose characteristics was analysed by Gilliland et al. 2010) or 29.4 min (long cadence), providing near-continuous time series for hundreds of thousands of stars. The data are divided into quarters: the commissioning run, Q0 (∼10 d), then Q1 (∼34 d) followed by Q2, Q3, Q4, Q5, Q6 and Q7 (∼90 d for each); when this paper was written ∼600 d of observations were available to us.

In the field of view of *Kepler*, there is only one genuine Cepheid variable so far (Szabó et al. 2011a), V1154 Cyg (KIC 7548061), which has been observed in long-cadence mode in all quarters and short-cadence mode in Q1, Q5, Q6. V1154 Cyg has a mean V brightness of ∼9.1 mag and a period of ∼4.925 d. Previous observational studies of this Cepheid are listed and discussed by Szabó et al. (2011a).

2.1 Cepheid pixel photometry

In Szabó et al. (2011a), we demonstrated that the apparent large amplitude variations (especially in Q2) are not intrinsic to the star, because some of the flux was lost from the assigned *optimal aperture* (Bryson et al. 2010) of V1154 Cyg. The usual output of the *Kepler* photometer is the integrated flux (light curve). However, to correct for this instrumental effect we have to rely on additional information to which we had no access at the time of writing the Szabó et al. (2011a) paper.

Besides the extremely precise measurements of *Kepler* it is equally important that recently the individual pixel time-series data have been made available[1] by the *Kepler* Team. In a number of cases the pixel data (1) provided crucial information on the photocentre of the transit variations (Batalha et al. 2010) thereby confirming planetary candidates, (2) helped to devise a custom aperture for RR Lyrae (Szabó et al. 2010), which is too bright and is located very close to the edge of one of the CCDs, thereby providing a way to observe this important prototype Blazhko variable with *Kepler*, or (3) in a recent paper (Szabó et al. 2011b) based on pixel data we were able to determine which component of a close visual common proper motion binary is transited by a 'hot' brown dwarf or planet.

We have downloaded all the long-cadence target pixel files (Q0–Q7) of V1154 Cyg. We assigned a much larger aperture than the original optimal aperture to sum the pixels to ensure that we do not lose flux, see Fig. 1. It is worth noting that V1154 Cyg is heavily saturated on the *Kepler* CCDs, so an elongated aperture is applied. It is also noticeable that the central (saturated) column is not contained in its entirety with the original optimal aperture. We took care to exclude additional stars captured in the downloaded pixels. Later it proved to be a wise decision, because by adding up its pixels the additional star centred at (3,13) turned out to be a previously unknown variable star showing frequency peaks in the [2,3] c/d frequency range.

The resulting light curve is very similar to the *Kepler* light curve, except that the spurious amplitude drop has disappeared. Although light-curve sections of different Qs had to be shifted vertically because of the different sensitivities of the CCDs (occurring due to the quarterly roll of the space telescope), we did not have to adjust the amplitudes which is reassuring. The only exception is Q0 (commissioning phase containing only two pulsational cycles), where inspection of the downloaded pixels confirmed that some flux was lost in the maximum phases, because the flux spilled out of the downloaded pixels, and this affected the pulsation amplitude and the $O - C$ values as well.

2.2 The $O - C$ diagram

We studied the period stability of V1154 Cyg applying the classical $O - C$ diagram method (Sterken 2005). Thanks to the continuous observations of *Kepler*, we could determine the $O - C$ values for

[1] Mikulski Archive for Space Telescopes, http://archive.stsci.edu/kepler/

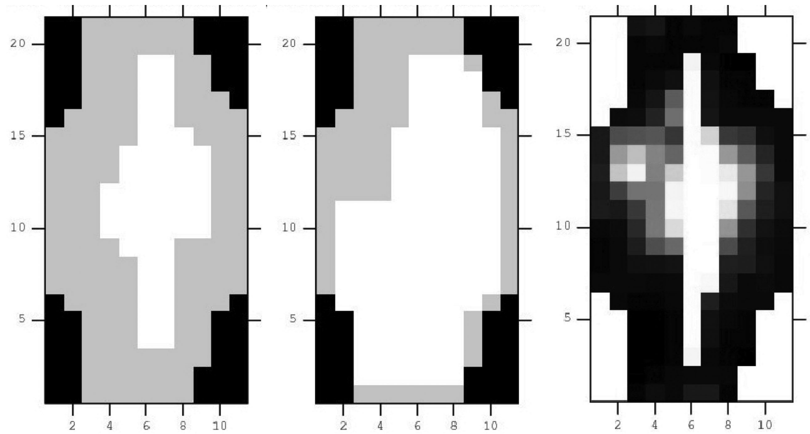

Figure 1. Left: original optimal aperture mask in Q2 (white), all the downloaded pixels (grey), pixels that were not downloaded (black). The axes label row and column pixel numbers. Middle: our custom mask designed to retain all the flux. Right: flux distribution around one of the maxima of V1154 Cyg in Q2 (grey-scale). We deselected the pixels assigned to the near neighbour of V1154 Cyg.

almost all cycles, except those with gaps. For a thorough analysis, we used four different methods to calculate the $O - C$ values that are described below.

The first and second methods were used to determine the times of maxima and minima of the light curve by fitting 10th-order polynomials around the extrema. The calculated $O - C$ diagrams for the times of maxima and minima are shown in the top two panels of Fig. 2 plotted with black dots. The average error of the $O - C$ values is ± 0.0021 d, which was determined by applying our method to a synthetic light curve (see Section 2.3) generated using the Fourier

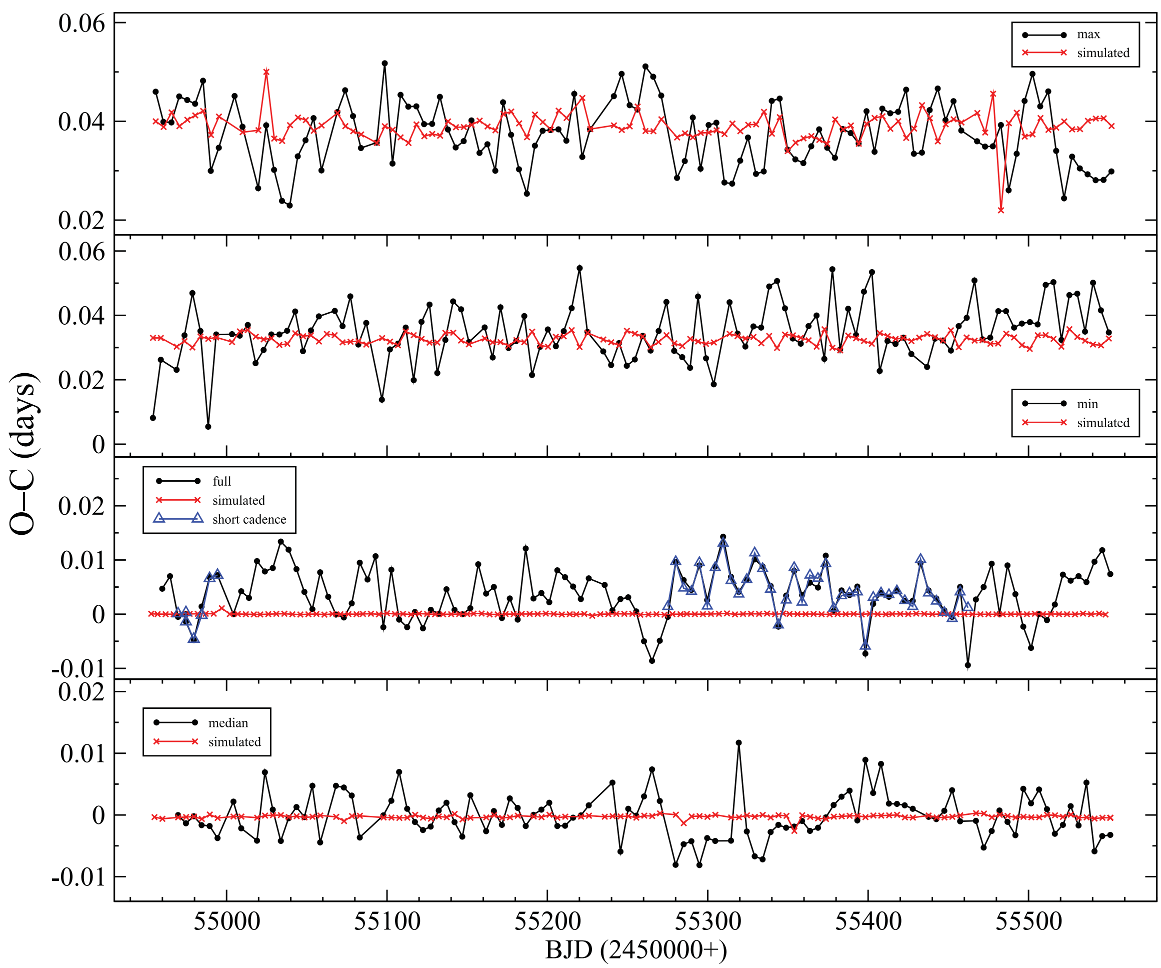

Figure 2. The $O - C$ diagram of V1154 Cyg calculated with four different methods (black dots) using the long-cadence data, while the blue triangles represent the $O - C$ diagram based on the short-cadence data. The full description of the four methods is given in Section 2.2. The red crosses are the resulted $O - C$ diagrams of synthetic data, see Section 2.3.

parameters of V1154 Cyg and averaging the deviations between the analytically derived maxima/minima and the maxima/minima calculated by our method.

The third method was used to measure phase shifts of each pulsational cycle and transform this to an $O-C$ diagram. This was done by fitting eighth-order Fourier polynomial to the phase diagram of one pulsational cycle and used it as a master curve – allowing only vertical and horizontal shifts – to fit every other phase diagram. Combining the phase shifts and the period with the epoch, we calculated the $O-C$ values (with an average error of ± 0.0012 d which was adopted from the standard error of the fit of the master curve) which are plotted in the third panel from top of Fig. 2 with black dots.

We also determined the moments of the median brightness (mid-value of the adjacent minimum and maximum brightness) on the ascending part of the light curve. Their $O-C$ diagrams are plotted with black dots in the bottom panel of Fig. 2. Assuming a 0.0001 relative uncertainty in the value of both the minimum and maximum brightnesses of a given epoch number, the moment of the median brightness can be determined with an unprecedented precision of 0.0001 d, owing to the steepest brightness variation during this particular phase of the pulsation cycle. The significantly smaller uncertainty of the median $O-C$ values testifies that, for photometric observations of low-amplitude Cepheids, it is the point of the median brightness on the ascending branch that is the preferred light-curve feature for following the behaviour of the pulsation period.

For the calculation of the $O-C$ diagrams (O stands for the observed, C stands for calculated times of extrema), we used the following ephemeris:

$$C = \mathrm{HJD}\ 245\,4969.704\,32 + 4.925\,454 \cdot E, \quad (1)$$

where C is the predicted time of the maxima (i.e. C), while E refers to the cycle number. The $O-C$ values show a scatter of about ± 0.015 d ($\cong$20 min) for the full and the median, and about ± 0.02 d ($\cong$30 min) for the maxima and the minima. The scatter of the $O-C$ diagram is far larger than we expect from a Cepheid variable (see Section 3 and Table 2).

We also calculated the $O-C$ diagram using the available Q1, Q5 and Q6 short-cadence data by applying the third method. The results are plotted with blue triangles in the third panel of Fig. 2. There is no significant difference between the resulted $O-C$ diagrams whether it is based on the long-cadence and the short-cadence data, giving supporting evidence that the detected period jitter of V1154 Cyg does not originate from the data sampling. As it is expected, however, the average error of the short-cadence $O-C$ values is significantly smaller than the long-cadence one: ± 0.0001 d.

2.3 Timing accuracy of *Kepler* data

To check the reliability of the detected $O-C$ variations with various methods, we designed a numerical experiment, based on synthetic data of *Kepler* quality, describing a hypothetic stable light variation (no changes in light curve or pulsation phase). The model was a sixth-order Fourier polynomial, fitted to the entire light curve. In such a way, the model described the average light-curve shape of V1154 Cyg.

To produce artificial data, bootstrap of *Kepler* noise was added to the model with null signal. The noise was taken from the time series of V1154 Cyg: the residuals were cycle-by-cycle fitted by a Fourier polynomial, and the set of residuals were sampled with a replacement. Then, artificial data were sent into the same algorithms as in the case of the observations. Since the measured distribution of $O-C$ regards a null-signal event, it exactly reflects the accuracy of timing measurement in *Kepler* time series of V1154 Cyg.

$O-C$ of artificial data were non-Gaussian, and they exhibited significantly heavier wings than a normal distribution. This is because two to four points are severe outliers, and they reflect probably the effect of uneven sampling (i.e. gaps in data). However, the method of the median brightness and the fit of the full light curve still results in standard deviations of 0.000 18 and 0.000 21 d, respectively, very small values compared to the measured effect.

The methods of maximum and minimum brightness exhibit a significantly larger scatter of $\approx$0.01 d, but after neglecting the four most distant outliers, the scatter decreases to 0.002 and 0.003 d, respectively. This scatter is larger than in the case of the median brightness or fitting the full light curve, compared to measuring at the maximum or minimum of the light curve when the brightness variation is slow. However, these values are still many factors smaller than the detected signal. In summary, we conclude that the variation of $O-C$ diagrams cannot be interpreted with numerical fluctuations, and that the measured $O-C$ patterns are real.

In Fig. 3, we explain the timing method using median brightness. The upper panel shows a segment of the light curve, while the lower panels zoom in two regions of the same light curve. The error bar of the individual data points (0.1 mmag) can still hardly be seen in the lower-right panel, which spans 8.5 mmag in the vertical axis. Here we compare the best-fitting linear function (dashed line) through the two consecutive data points that bracket the median brightness. The dotted line corresponds to the limiting case where the linear function is shifted vertically exactly to the lowest limits of the error bars, keeping the fit still consistent with the data. The time lag between the dashed and the dotted lines is 24 s; solutions with a larger time lag would misfit at least one of the data points. Hence we take this measurement as an estimate of the accuracy.

The accuracy in timing of photometric minima and maxima is worse because the light curve is flat at the extrema, and the error bars enable somewhat larger time uncertainties.

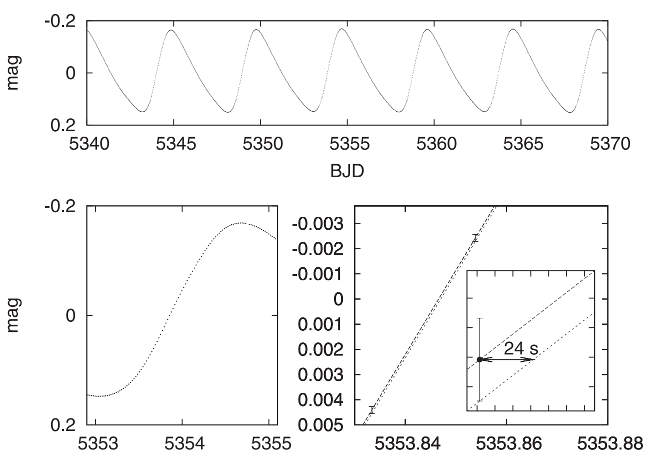

Figure 3. Demonstration of the accuracy of timing the median brightness. The upper panel shows six cycles of the light curve. The lower-left panel is a magnification of one individual ascending branch and the lower-right panel is a zoom into the region between two consecutive data points that bracket the median brightness. The dashed line in the small inset corresponds to the best-fitting linear function, while the dotted line corresponds to the lowest limit that still goes through both errorbars. The time lag between the two lines is an estimate of the accuracy, in this case 24.

 425, 1312–1319

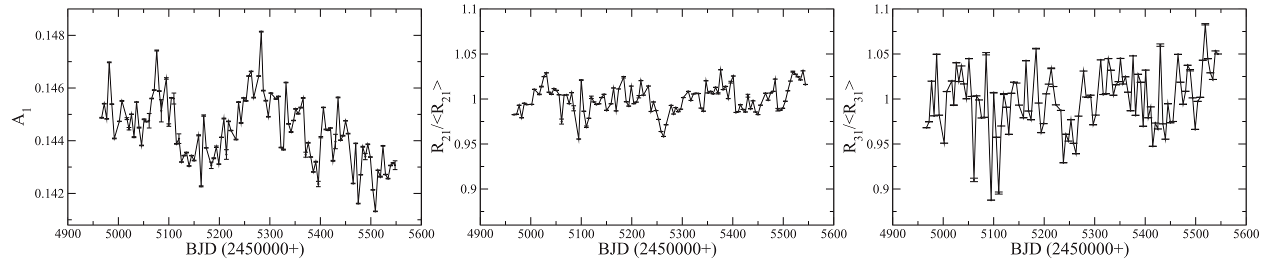

Figure 4. From left to right: the amplitude (A_1) change and the relative variations of R_{21} and R_{31} with average errors of ±0.000 042, ±0.0001 and ±0.0012, respectively.

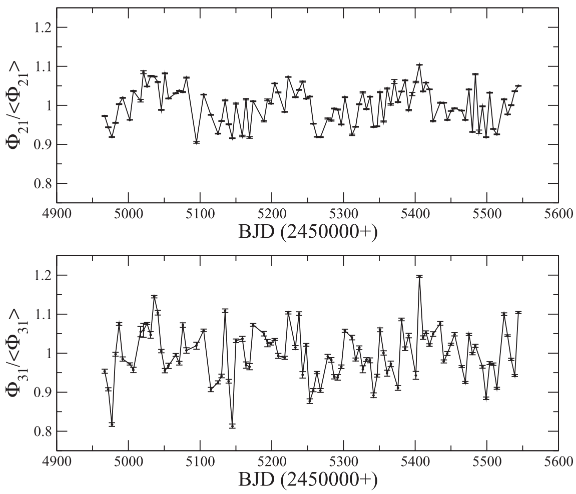

Figure 5. The relative phase variations of ϕ_{21} (top) and ϕ_{31} (bottom), with average errors of ±0.0013 and ±0.0051, respectively.

2.4 Fourier parameters

We studied the change of the light-curve shape as a possible cause of the scatter in the $O-C$ diagram by examining the temporal variation of the Fourier parameters of the light curve. For this, we fitted an eighth-order Fourier polynomial at the primary frequency and its harmonics:

$$m = A_0 + \sum_{i=1}^{8} A_i \cdot \sin(2\pi i f t + \phi_i), \tag{2}$$

where m is the magnitude, A is the amplitude, f is the frequency, t is the time of the observation, ϕ is the phase and index i runs from 1 to 8. Then, we characterized the light-curve shapes with the Fourier parameters (Simon & Teays 1982), of which we show particular results for $R_{21} = A_2/A_1, R_{31} = A_3/A_1$ as well as $\phi_{21} = \phi_2 - 2\phi_1$ and $\phi_{31} = \phi_3 - 3\phi_1$.

Fig. 4 shows the amplitude change and the relative variations of R_{21} and R_{31}, while Fig. 5 shows the relative variation of ϕ_{21} and ϕ_{31}. All of these parameters show significant cycle-to-cycle variations. The detailed study of these variations in the $O-C$ and the Fourier parameters is described in the next section.

3 RESULTS

3.1 Correlations between $O-C$ data sets

Different determinations of $O-C$ points show loose correlations, no correlation or even anticorrelations with each other. Results of a Spearman's rank correlation test (Lupton 1993) are summarized in Table 1; for each compared data set pairs, the top row shows Spearman's ρ and the p-value is given below. This test is like a generalization of the correlation coefficient: the value of ρ measures how well the dependence between two variables can be described with a general monotonic function. The value of ρ is 1 or −1 if there is a perfect fit with an increasing or decreasing function, respectively; values in between express the relative weight of residuals which cannot be explained by a monotonic law. The value of p shows how probable is that the suspected connection between the variables is due to a false recognition of simple numerical fluctuations. That is, an intermediate value of ρ still safely expresses the slight monotonic connection between the variables if p is very small. The significant correlations are as follows:

Table 1. Results of a Spearman's rank correlation test of different $O-C$ data sets. The sign *ns* means no correlation with a confidence exceeding 85 per cent.

	Minimum	Maximum	Full
Medium	*ns*	0.42	−0.51
		$p = 5 \times 10^{-6}$	$p = 2 \times 10^{-8}$
Minimum	–	−0.14	*ns*
		$p = 0.15$	
Maximum	–	–	−0.73
			$p = 2 \times 10^{-16}$

$$(O-C)_{\rm Max} = 0.7(2) \times (O-C)_{\rm Med} + 1148.6(9)$$

$$(O-C)_{\rm Full} = -0.6(1) \times (O-C)_{\rm Med} + 5(5)$$

$$(O-C)_{\rm Full} = -0.52(4) \times (O-C)_{\rm Max} + 600(50)$$

$$(O-C)_{\rm Max} = -0.11(7) \times (O-C)_{\rm Min} + 1010(90)$$

(errors of the last digits are given in parentheses, absolute terms are in seconds).

The most interesting is the general lack of strong dependences between the differently defined $O-C$ values derived for the same pulsation cycles. In particular, the minimum is practically not connected to any other $O-C$ measures, while $O-C$ of the maxima shows relatively good (but still somewhat stochastic) dependence on the $O-C$ of the whole cycle. Another remarkable feature is the anticorrelation between the $O-C$ of the whole cycle and that of the median brightness or the maximum.

This behaviour suggests two conclusions. At first, the scatter in $O-C$ values is somehow reproducible; at least certain kinds of $O-C$ values can be predicted to some accuracy if one knows the value of another $O-C$. Therefore, the scatter we observed in any $O-C$ is not due to some unrecognized stochastic error e.g. coming from the sampling and/or the evaluation of data. Perhaps the phase of the pulsation is changing, or there are slight local irregularities in the light curve. But the second finding is that the process which acts on the $O - C$ values changes rapidly; this is why there are no strict correlations between the different determinations of $O - C$. These together suggest that the cause of the observed behaviour of $O - C$ data is the irregular local fluctuations of the light-curve shape. We shall explore this hypothesis in the next subsections.

3.2 Fourier parameters explain $O - C$ points

Local variations of the light curve can be described by higher order Fourier coefficients. Therefore, one expects that if changes in the light-curve shape were the primary reason for $O - C$ variations, then there would be connections between the Fourier parameters and the $O - C$ values. If linear models are taken into account, only approximate fits are expected because the light-curve shape depends on the Fourier parameters in a very complex and non-linear way. This was really observed: we generally found very slight correlations between the individual Fourier parameters and the $O - C$ values.

Another possibility is a multilinear fit of the $O - C$ points using the Fourier parameters. Taking the first eight amplitude and phase coefficients, and additionally, the height of the minimum and the maximum of the individual cycles, we found that the times of the mean amplitude can really be predicted with 90 per cent accuracy via a linear – and therefore rather heuristic – model (i.e. the scatter decreases by 10 per cent meanwhile the degree of freedom decreases by only 17 per cent, from 126 to 108). The accuracy was less impressive in the other cases, e.g. it was only 30 per cent when fitting the times of minima (i.e. the residuals decreased only by 30 per cent after the fit).

The probable interpretation is that the $O - C$ variations are really due to the instability of the light-curve shape, because the light-curve shape alone is the best predictor of $O - C$ values. The existence of less impressive fits does not contradict much this interpretation, because the light-curve shape is a non-linear function of our parameter space, and it has been foreseen that the linear fit will not always work. In fact, good fits can be expected only in the case when the parameters are combined by fortune in such way that the local light-curve irregularities can be effectively described with a multilinear approximation, as it happened for the times of median brightness.

3.3 Eddington–Plakidis test

The $O - C$ residuals (Fig. 2) were analysed for the presence of random fluctuations in the pulsation period using the method described by Eddington & Plakidis (1929).

Eddington & Plakidis (1929) made a formalism to test for random, cycle-to-cycle fluctuations in period. They proposed that the average value $\langle u(x)\rangle$ is given as

$$\langle u(x)\rangle^2 = 2\alpha^2 + x\varepsilon^2, \quad (3)$$

where $\langle u(x)\rangle$ is the mean absolute difference between the $O - C$s which are x cycles apart, α is the mean observational error in measuring the time of maximum or minimum light and ε is the mean fluctuation in period from one cycle to the next (Percy & Kolin 2000). These errors each are assumed to be accidental and uncorrelated. Hence a plot of $\langle u(x)\rangle^2$ against x should be a straight line with slope ε^2 and intercept $2\alpha^2$.

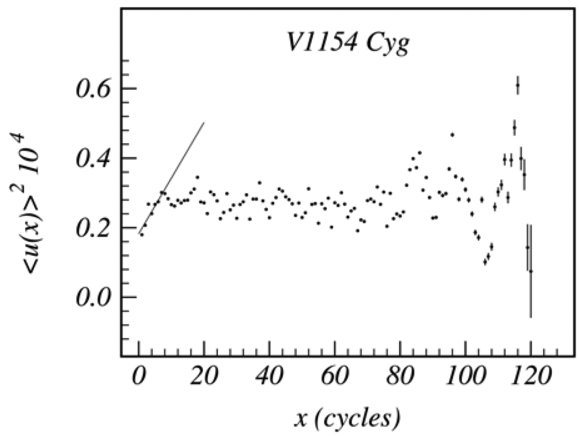

Figure 6. The result of the Eddington–Plakidis test. We fit a linear trend up to $x = 15$, which indicates fluctuations in the period or light-curve shape. See the text for the full description of this effect.

Table 2. Coefficient solutions of the Eddington–Plakidis test for four different methods.

Method	ϵ	ϵ_{err}	α	α_{err}
Full	0.0013	0.000 54	0.0030	0.000 85
Medium	0.0008	0.000 42	0.0023	0.000 66
Minimum	0.0015	0.000 58	0.0063	0.001 41
Maximum	0.0015	0.000 48	0.0043	0.001 05

In practice, this means that if there is a linear trend in the plot of $\langle u(x)\rangle^2$ at low x (from several to several tens cycles), then it is assumed that random period fluctuations are present in the pulsations of a given star and this segment of the plot is used for a linear fit. In other words, all wave-like fluctuations in the $O - C$ residuals (after the removal of all trends) are assumed to be attributed to random fluctuations in the pulsation period or in the light-curve shape.

As is shown in Fig. 6, a linear trend is revealed up to a cycle difference $x \sim 15$. The mean random-fluctuation parameters are summarized in Table 2 for the four different methods.

These results lead us to the conclusion that in short terms (up to a cycle difference $x \sim 15$), the period and/or the light-curve shape changes randomly. However, for long term, there is an internal clock in V1154 Cyg which keeps the period around the mean value. This feature is also supported by the fact that the $O - C$ diagram of V1154 Cyg covering 150 00 d indicates a constant mean pulsation period (see fig. 13 in Szabó et al. 2011a).

A similar Eddington–Plakidis test was performed for SMEI photometric data of eight bright Cepheids by Berdnikov (2010). He found random period fluctuations, in some cases superimposed on period changes due to stellar evolution, but the period and its changes could not be followed on a time-scale of individual pulsation cycles unlike this study of V1154 Cyg.

4 IMPLICATIONS OF THE RESULTS

If the results presented here for V1154 Cyg are common among Cepheids, then the consequences might affect other areas of research.

4.1 Cepheids models

Numerical hydrodynamical models do not show such hectic period variations from cycle to cycle. This is primarily because these codes contain a simplified description of turbulent convection (Florida–Budapest code: Kolláth et al. 2002; Warsaw code: Smolec & Moskalik 2008), or before the early 1990s contained radiative energy transfer only. Using this type of physics the pulsation usually reaches its limit cycle (or alternatively double-mode pulsation), and then stays in this state essentially forever, because these codes are very good at conserving energy and momentum. Hence, the pulsation period is practically constant. Any deviation from this precise, regular machinery is due to choosing imperfect numerical scheme or coding errors. The above-mentioned codes are free from these problems.

Present-day models are not capable of dealing with complex, magnetohydrodynamic (MHD) interaction (Stothers 2009) that could cause regular or irregular modulations of the stellar structure (hence in the period), because of the enormous difficulties of implementing and computing three-dimensional MHD for full stellar envelopes. However, Buchler, Goupil & Kovács (1993) developed a method based on the amplitude equation formalism to describe convective and turbulent random fluctuations and hence stochastic interaction between convection and pulsation. This mechanism would cause fluctuations in the period, but would not deform the shape of the pulsation.

Deviations from this precise clockwork mechanism do appear in certain high-luminosity models (Buchler & Kovács 1987; Kovács & Buchler 1988; Moskalik & Buchler 1990, 1991; Buchler & Moskalik 1992), namely they can show period doubling or a series of period doubling bifurcations en route to chaos. These types of dynamical phenomena also cannot be responsible for the observed period variations.

Evolutionary changes are usually happening on a much longer time-scale and in a regular manner; in addition the physics driving the evolution is usually missing from these models designed to follow only the pulsation.

Clearly, a more precise (preferably three-dimensional) description of turbulent convection would be essential to model the observed large cycle-to-cycle period 'jitter'.

4.2 Light-curve distortion

We can speculate that a stable global pulsation can appear as 'jittered' or distorted provided that the stellar atmosphere is not quasi-static or spherically symmetric due to large-scale turbulent cells for example. This effect provides no feedback to the global pulsation, but perturbs the observed light curve. Changes causing local perturbations of the optical depth may also induce apparent phase disturbances. This effect may be present in all cool giant variable stars although with different magnitude. However, the detailed modelling of these mechanisms are beyond the scope of this paper.

4.3 Searching for companions

The usually assumed stability of the pulsation period is a fundamental ingredient in finding low-mass or substellar companions around Cepheids. In light of the period jittering found in V1154 Cyg, it is worth checking what kind of sensitivity can be expected for a perfect pulsator observed by *Kepler*. For this, we have calculated several simulations as follows. First, we took a short subset of the long-cadence light curve covering one pulsation cycle to fit a fourth-order Fourier polynomial that represented the ideal light-curve shape of the star. Then, we took each time stamp of the combined Q0–Q7 long-cadence light curve to calculate the analytic template, which was phase-modulated by a sinusoidal term: the period of the modulation was set to 200 d, while the amplitude was set to representative values between 0.1 and 60 s. The flux errors reported by the *Kepler* pipeline were used to add a random error to each point of the simulation. This way the properties of the random errors matched those of the original data. The simulated light curves were then analysed with the fourth method used in the $O-C$ analysis (see Section 2.2), namely with the local-phase determination using a master curve represented by a Fourier polynomial.

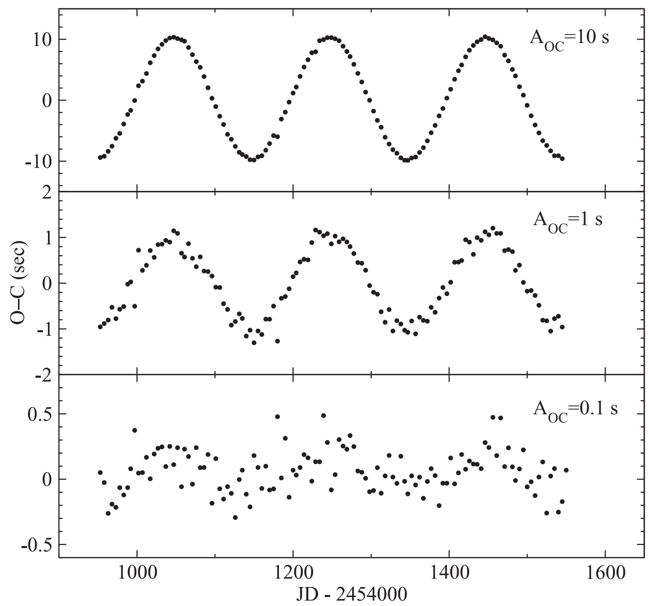

Figure 7. The $O-C$ diagram of three model light curves with sinusoidal phase modulations (modulation semi-amplitudes shown in the upper-right corner in each panel). See the text for details.

In Fig. 7, we show the results for three simulations. The amplitudes of the phase modulations were 10, 1 and 0.1 s, which can be connected to the mass ratio of the companion and the Cepheid using the third Kepler law and assuming a mass for the Cepheid. Ignoring the sin i ambiguity of the unknown inclination angle (i.e. assuming an edge-on view of the system), the semi-amplitude of the $O-C$ diagram depends on the Cepheid mass and the mass ratio as follows:

$$A_{O-C} = \frac{1}{c}\left(\frac{G}{4\pi^2}\right)^{1/3} \frac{M_1^{1/3}\left(1+\frac{M_2}{M_1}\right)^{1/3}}{\left(1+\frac{M_1}{M_2}\right)} P_{\rm orb}^{2/3}, \quad (4)$$

where M_1 and M_2 are the masses of the Cepheid and its companion, respectively, $P_{\rm orb}$ is the orbital period, G is the gravitational constant and c is the speed of light. For a 5 $M_\odot$ Cepheid and a 200 d orbit, the given three $O-C$ amplitudes correspond to a companion mass of about 50, 5 and 0.5 $M_{\rm Jup}$.

The three simulated $O-C$ diagrams in Fig. 7 clearly show that *Kepler*'s precision could be enough to detect planet-sized companions on short-period orbits around Cepheids, if the stars were perfect clocks. However, the astrophysical noise in V1154 Cyg is way too high to allow that kind of detections of substellar companions.

5 SUMMARY

The period variations for Cepheid V1154 Cyg have been examined based on 600 d of photometric observations with the *Kepler* space telescope. In order to analyse the best quality light curve we performed pixel photometry on the available individual pixel time-series data. We calculated $O - C$ values for all cycles using four different methods: times of maxima and minima, median brightness and phase shifts of all pulsational cycles. We also determined the Fourier parameters of each cycle.

We found very slight correlation between the individual Fourier parameters and the $O - C$ values, suggesting that the $O - C$ variations might be due to the instability of the light-curve shape. A random-fluctuation test revealed linear trend up to a cycle difference 15 but for long term the period remains around the mean value.

Finally, we showed that if the period jitter is common among other Cepheids then it needs to be taken into account in the hydrodinamic models and it sets a limit to detect substellar companions around Cepheids.

Subtle variations in the pulsation period and shape of the light curve may be present in the pulsation of other Cepheids. Indirect evidence supporting the behaviour mentioned here was given by Klagyivik & Szabados (2009) who found that the photometric phase curves of some small-amplitude Cepheids in our Galaxy show a wider scatter than expected from the measurement errors even if data are folded on the correct value of the pulsation period.

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