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Geometry of Potential Models*

Introduction

We aim to rethink and systematize accessibility potential models, and to investigate what they measure and under what conditions, what characteristics they possess by way of analogy with a gravitational field and what features they “inherit”.

The question is what has potential, what a particular potential value means, and why we use this value to describe social space. What conclusions can be drawn from a potential model? To answer this latter question, the values forecast by the models were compared with traffic data.

Potentials measure the position of a spatial domain compared to the rest of the areas, and the impacts of the mass distribution of the particular spatial division at the same time. In the present paper we make an effort to filter out these effects, and to disaggregate potentials.

Space in geometry and in physics, gravity

Geometric models

In mathematics, three theoretically different geometric analysis methods can be distinguished: axiomatic, group-theoretical and differential-geometric.

The axiomatic construction of geometry begins with the selection of the object set (e.g. pairs of numbers or points in Euclidean space). In this set basic elements, basic relations (e.g. adaptation) are then defined. Subsequently, general statements are generated in support of which the particular system is introduced. These are axioms. If axioms are true in an object set, then this object set is referred to as a model of the geometry provided by the geometric system. So axioms – howtever strange this may be – need to be justified in constructing a model. The model and “reality” can be connected by this moment.

Geometric space in physics

Model creation, the description of space by models is not a characteristic of geometry or mathematics only. It is right to ask in physics as well what geometric relations can be applied to describe physical phenomena.

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According to János Bolyai “*the law of gravitation also seems to be closely related to the size, structure and quality of space.* This means the recognition that there should be an internal coherence between physical gravitational field and the geometrical structure of space (Oláh-Gál 2008).

When detailing his theory, Einstein relied on the non-Euclidean concept of space, and used its interpretation and symbols further developed by B. Riemann more than two decades after Bolyai. L. Infeld, one of the colleagues of Einstein writes the following: “The interpretation of the gravitational field as geometric space is one of the greatest and most revolutionary achievements ever in the history of physics. A world without masses, electrons and electromagnetic space is an empty world, a false idea. However, if masses, charged particles and electromagnetic space emerge, then the gravitational field emerges too. If the gravitational field emerges, then our world bends. Its geometry is Riemann’s geometry and not the Euclidean one (Gábos 2004).”

Gravitational force, force field, field strength, potential

According to the law of universal gravitation or the gravitation law of Newton (1686) any two point-like bodies mutually attract each other with a force directly proportional to the product of the masses of the bodies and inversely proportional to the square of the distance between them:

$F = \gamma \cdot \frac{m_1 \cdot m_2}{r^2}$, where γ is the proportionality factor, the gravitation constant (independent from place and time).

If the radius vector drawn from mass point 2 to mass point 1 is indicated by $\vec{r}$, then the unit vector pointing from 1 to 2 is $-\vec{r}/r$, and so the gravitational force on mass point 1 by mass point 2 is:

$$\vec{F}_{1,2} = -\gamma \cdot \frac{m_1 \cdot m_2}{r^2} \cdot \frac{\vec{r}}{r} \cdot$$

The negative sign in the formula expresses that body i attracts body j (Budó 1970).

Generally, in case of a gravitational field from any body or any system of bodies, the force on a body with mass m is proportional to m: $\vec{F} = m\vec{K}$.

The $\vec{K}$ vector quantity of the gravitational field, which – disregarding the dimension – means a force on a body with unit mass, is referred to as (gravitational) field strength. Field strength usually depends on place and maybe time t: K = K(x, y, z, t).

The fundamental concept of the force field comes from Faraday (around 1840), who replaced the concept of “distance action” concerning electronic and magnetic force effects between distant bodies as well as in vacuum, accepted until that time, with the principle of proximity effect or spatial effect. Accordingly, force effects between separate bodies are always mediated by space: the direct reason for the force by a body in place A on a body in place B is that the field strength by the body in place A differs from zero in place B.

A force field can be completely defined if field strength $\vec{K}$ can be given by direction and size in every point of the particular domain. Many force fields, however, including the gravitational field, can be described much more simply as well, by only one scalar function instead of three, the "potential". The potential is related to field strength as work or potential energy to force. So the value of the potential in some point P of the gravitational field:

$$U_P = -\int_0^P K_s ds,$$

i.e., not taking into consideration dimension, equals work against gravitation forces while taking a body with unit mass from "zero point" O (in any way) to point P (Budó 1970).

Accessibility potential models

Potential model in regional analyses

The regional analysis tools of spatial interactions are potential models, which show the position advantage of each area compared to other areas, quantifying the advantage ensured by them (Schürmann–Spiekermann–Wegener 1997). According to other definitions accessibility "shows the character of spatial interaction", and "is the attraction of a node, taking into account the mass of other nodes and the costs of access to them on the network" (Bruinsma–Rietveld 1998). There is no generally agreed definition of accessibility; many different indicators with differing methodologies are used in empirical studies (for example Ingram 1971, Morris–Dumble–Wigan 1978, Handy–Niemeier 1997). On the one hand, the main task of accessibility tests is to ensure a measuring tool appropriate for evaluating the accessibility to all origin and destination points and on the other hand, for explaining the differences measured in accessibility (Chapelon 1997).

Both gravitation and potential models are based on the fact that the behaviour of spatial groups of people is determined by certain laws, and these laws are the same as physicists' laws determining the behaviour of groups of molecules. Human beings are certainly not like molecules but the behaviour of people and bodies are similarly subject to gravitation law. On the basis of this analogy the investigation of people's behaviour based on physical laws is also referred to as "social physics" (Carrothers 1956). Therefore a common feature of the models is that potential interaction between two settlements, areas etc. is inversely proportional to the distance between them. The other similarity is that any person in the examined settlements generates the same amount of interaction as anybody else. That is, the amount of interaction between two areas is directly proportional to the masses of units in the analysis.

In literature on accessibility, indicators are divided basically into three groups. There are models based on infrastructure, activities and usefulness. Accessibility potential models belong to the group based on activities (for more details see Tóth–Kincses 2007). In this study, potential models based on gravitation analogy are examined in more detail. In these cases – similarly to Newton's law of gravitation –interaction between masses in social space (population, economic volume) is usually described by the function of the value directly proportional to mass and inversely proportional to the power of the distance between them (for example, these types of models are applied in the analyses of attraction zones).

If two points in space, i and j are given, with masses P_i and P_j respectively, and the difference between them is d_{ij}, then the following hypothesis can be made for the intensity of interaction between them (G):

$$G = c \cdot \frac{P_i \cdot P_j}{f(d_{ij}^k)}, \text{ where c and k are constants.}$$

With this formula, space can be divided, on every point of which it can be decided which of two close mass points has a more intensive impact on it.

Relations, interactions, as in inter-connections in physics, are not limited to pairs of points (or masses) in society either, many other points have an impact on each point. Mass points generate space around themselves, thus creating force field. Social space – on the analogy of gravitation (electric, magnetic) field – is attempted to be approximated by potential models. The general form of a potential in a particular point of social space:

$$T_i = \sum \frac{P_j}{f(d_{ij})},$$

where P_j is the active mass assigned to point j in the examined space, and d_{ij} is the distance between points i and j.

Some characteristics of potential models

Selection of area of analysis

It seems that every point of Earth has an impact on the potential of all other points. Certainly it does not mean that researchers take into account the data of all territorial units when making calculations, but one should be aware that selection affects the form of potential surface (Lukermann and Porter 1960, Houston 1969). An additional essential criterion is that the area of analysis should form a relatively closed socio-economic system.

Territorial division

Territorial division is an important issue from the point of view of potential analyses too. The problem originates primarily from making calculations not at the level of individuals in statistical analyses but applying the characteristics of groups of individuals using some administrative or statistical groupings. The difficulty in compiling data limits the selection of levels as well. If data are available, it is worthwhile to make calculations with different numbers and sizes of territorial units, for the problem of modifiable territorial units also means a relevant aspect of analysis in this respect (Dusek 2004). By applying smaller territorial units, one can have a more detailed, while in case of larger units, a smoother potential surface.

Mass factor

The interaction ability of the different areas principally depends on the scale of their socio-economic activities. To ensure that the potential can appropriately show the interaction ability of the different areas a mass factor should reasonably quantify the level of the particular activity. Its selection differs from one task to another. In the most often applied approach, population is used as mass of analysis, either unweighted or weighted by some socio-economic factor (e.g. qualifications, income). There are investigations in which population is replaced by the volume of retail sales or the income of households. In the models, mass can mean almost any extensive quantity describing social space. Discussions on the application of mass factors have lost importance since there is a close correlation between most of such factors, so their selection has a relatively small impact on the calculated potential. The selection and application of the factor of distance may be much more important (Houston 1969).

Distance

First of all, the introduction of the factor of distance in socio-geographic analyses is due to the fact that spatial separation hinders the co-operation among the many different areas; consequently, its quantification in some way is recommended. Naturally, the most simple application of the model is the use of linear distances. However, in respect of accessibility indicators, the distance, costs or time of travel by some mode of transport is always considered. The distance to be covered between two points is referred to as the spatial resistance factor (Tóth–Kincses 2007).

Distances between the particular "masses" are also considered differently in the different models. Several approaches are known which apply the reciprocal value of distance or some power of that (see among others Hansen 1959, Davidson 1977, Fortheringham 1982). Thus, there are models applying squared, exponential (Wilson 1971, Dalvi–Martin 1976, Martin–Dalvi 1976, Song 1996, Simma–Vritic–Axhausen 2001, Schürmann–Spiekermann–Wegener 1997), Gaussian (Ingram 1971, Guy 1983) and log-logistic (Bewley–Fiebig 1988, Hilbers–Veroen 1993) resistance factors. The approximations of distances with different functions are owing to the effort made to achieve the most favourable fit when examining spatial structures. For even better approximations these non-linear resistances are even transformed (examples include the Box-Cox transformation, which makes the residuals of the regression homoscedastic[1], and transforms them by approximating them to normal distribution (Box–Cox 1964)). Although these models are based on a gravity analogy, the form of the potential is different, and the meaning of factors in the formula is not always the same either. However, there is a determined relationship between field strength and the potential:

$$\vec{K} = -\mathrm{grad}U \Rightarrow \qquad K_x = \frac{\partial U}{\partial x}; \quad K_y = \frac{\partial U}{\partial y}.$$

So one can work with different types of potentials (from that induced by the analogy of gravitation), but in this case the effects of force are also different among the sources of space. These models differ from one another in that attraction forces remain above a pre-set threshold value within different distances. However, the extent to which they describe the real balance of forces between social masses is already another question.

Generally, it can be said that if the force field remains central (i.e. the effects of force depend only on distance besides mass), to describe space it is then not necessary to know

1 Equal variance with two samples.

the value and direction of field strength in every point of space. In case of central forces (resulting from irrotationality) space can be described by a single scalar function, the potential. In this case there is no need for vectors to be added even if there are several sources (masses), the values can be added mathematically as scalars.

Self-potential

In the analysed space the degree of the location-dependent potential in a particular point of space does not only depend on the distance of masses from it and their size but also on the size of the force field the particular point can generate around itself (Frost–Spence 1995, Bruinsma–Rietveld 1998). Consequently, inherent, internal and external potentials are distinguished in such potential analyses (Nemes Nagy 1998, 2005). The distinction between these latter two factors stems from the distinction between the forces of the area of analysis in a narrower sense and of the space influencing it from outside. So the potential is calculated by adding these three factors.

When calculating the self-potential of an area, it is assumed that it is not only transport from one territorial unit to another that can be a factor improving accessibility but also that of transport within the different areas/settlements. That is, it can be stated that the different products/services do not need to be transported to other areas if they can be sold within the particular area too. Leaving out of consideration the role of self-potential may lead to misleading results. It is easy to see that the accessibility of central settlements of agglomerations and settlement groups in Hungary would always be lower in such cases than for the rest of the settlements in the settlement groups of large towns.

When calculating self-potential – similarly to other potential analyses – the area of a particular area is taken into account (possibly inner area instead of the administrative one). Considering the area as a circle, the area's radius is calculated, which is regarded as proportional to distances on public roads within the different settlements, so it is also referred to as its own distance. This distance is used in models applying linear distances, while in those applying network distances this distance is recalculated in support of some average speed/costs etc. and substituted in the formula.

Calculation of potential

Scalar addition requires linear superposition among the different members in the definition of potential. There is no interaction among the different effects, they do not amplify or attenuate one another but each has its solitary impact independently from the others (there are only two-body forces and no multi-body forces), and then these independent members are added. Larger masses do not "suppress" the attraction impact of other areas and are independent from that according to the formula. Though this is a very important feature in physics, it is uncertain that social space also has this characteristic.

The location-dependent accessibility potential can be calculated as the sum of inherent and internal potentials according to the following formula:

$$P_i = \sum_i A_i = SA_i + \sum_i BA_i \ ,$$

where ΣA_i is all the accessibility potentials of area i, SA_i is the own and BA_i is the internal potential. Furthermore, there are approaches which take into account "external potentials" in addition to the area of analysis.

In most accessibility analyses the value of the potential is not calculated in all points of space, instead, the calculation is made for towns, and the resulting data are extrapolated using methods of geographical information science to the areas the centres of which are the particular towns (Baradaran–Ramjerdi 2001). This approach is slightly strange in physics, where the potential describes space created by masses and is a function that assigns a number to each point in space. That is, the potential is a point characteristic describing space, not masses (population or income of settlements or micro-regions etc.).

Test of model creation, relation between model and real space

In describing the characteristics of potential models, different model frameworks can be constructed based on differing definitions of sets of objects (points in space), and sources, masses, basic elements (definition of lines with linear distance or distance on public roads) and basic relations of space.

In the foregoing, the structure of the different models was discussed, avoiding the questions (based on geometric interpretation) of how realistically the potential structure, generated with the definition of its basic elements and relatios, defines the space, and to what extent the volumes of spatial flows can be “confronted” with the values of the models, i.e. whether or not the axioms are fulfilled. Namely, the consequences drawn from the models can be applied to real social space only in this case. Axioms here fulfil the role of a bridge between real life and the models.

The data of the Hungarian Public Roads Non-profit PLC show the annual average daily traffic passing through a section of public road. National pubic road cross-section traffic counts are implemented – in line with international practice – by sampling procedure. This method of counting makes it possible – with knowledge of fluctuations in traffic over time – to determine average daily traffic in some cross-section from relatively few data (from small samples, from the results of counts lasting for a short time), with appropriate accuracy and reliability.

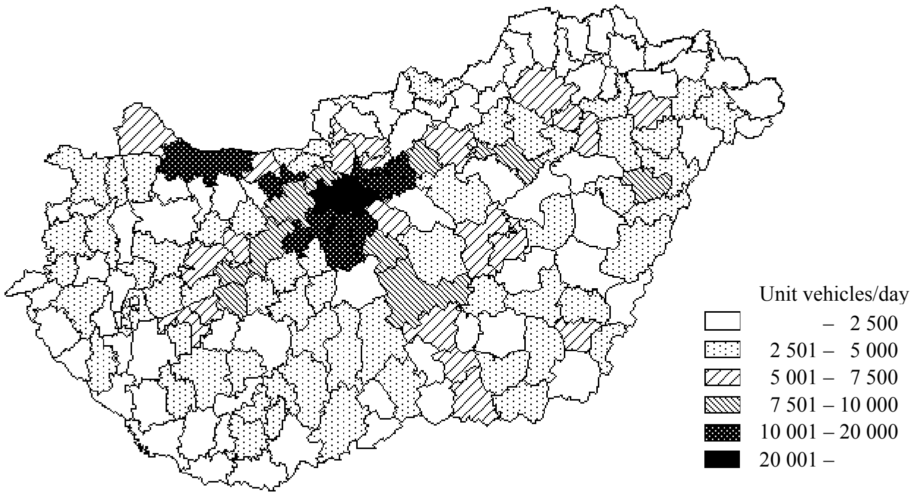
The essence of national cross-section traffic counts is that counts are carried out at a large number of stations, based on samples, distributed all over the year, on 5 different occasions (covering a period from 6 a.m. to 6 p.m.). The averages obtained from these values (g_x) multiplied by intra-day (a_x), day (b_i) and month (c_i) factors describing the regularity of traffic are the most reliable spatial values (with a probability of p=95) of average daily turnover calculated for the year as a whole:

$$ADT = \frac{1}{n} * \sum_{i=1}^{n} g_x * a_x * b_i * c_i ,$$

where n is the number of days counted, g_x is traffic observed during a count lasting x hours, a_x is the intra-day factor (traffic counted in a particular part of the day relative to traffic over 24 hours), b_i is the day factor (a number belonging to each day of the week, which changes daily traffic into a monthly average value), and c_i is the month factor (a number belonging to each month of the year, to transform monthly average traffic into annual average traffic).

Figure 1

Average daily traffic calculated for a year as a whole, 2008



Traffic (flow) data for 2004 and 2008 were compared using twelve different potential models. Incomes and resident population were applied as mass factors. The models applied linear (c_1), square (c_2), „e-ad" (c_3) and „e-ad" Box-Cox (c_4), Gaussian (c_5) and log-logistic (c_6) resistance factors (details of potential models: Tóth–Kincses 2007).

Table 1

Dimensions of analysis

Dimension	Notes
Source	Accessibility is calculated and interpreted from the point of view of all people in our investigation, and the many different social groups, as well as the differing travel purposes of different travellers are not distinguished.
Purpose	The purpose to be achieved is quantified by the population and income of the particular micro-region. This "mass" factor (component) quantifying the purpose to be achieved is included in the applied models, without adjustment.
Resistance	The spatial resistance factor in the present case means theoretical accessibility time measured between the centres of micro-regions on public roads, in minutes. The applied resistance factor can be linear, square, exponential, Box-Cox, Gaussian and log-logistic.
Restrictions	When using roads between two micro-regions, restrictions on the particular section are the maximum speed for the type of the road.
Geographic scope	On defining the area of analysis, the territory of Hungary was taken into consideration. Undoubtedly, although accessible targets outside Hungary also affect domestic potentials, external impacts had to be left out of consideration, since road network maps of an appropriate level of detail were available to us only for Hungary.
Mode of transport	The aspects of passenger and freight transport were not distinguished in the analysis.
Spatial level	The basic spatial level of our research is the level of micro-regions (LAU 1 level).
Dynamics	Population, income and public road network on 1 January 2004 and 2008 were taken into account in the research.

The models involved in the investigation were the following potentials (the calculations were also made with models allowing for the agglomeration impact, achieving similar results):

$$C_1 = \frac{p_i}{c_{ii}} + \sum_j \frac{p_j}{c_{ij}} \qquad C_2 = \frac{p_i}{c_{ii}^2} + \sum_j \frac{p_j}{c_{ij}^2}$$

$$C_3 = \frac{p_i}{e^{\beta C_{ii}}} + \sum_j \frac{p_j}{e^{\beta C_{ij}}} \qquad C_4 = \frac{p_i}{e^{\beta\left(\frac{c_{ij}^{\lambda}-1}{\lambda}\right)}} + \sum_j \frac{p_j}{e^{\beta\left(\frac{c_{ij}^{\lambda}-1}{\lambda}\right)}}$$

$$C_5 = \frac{p_i}{\frac{-c_{ii}^2}{w * e^u}} + \sum_j \frac{p_j}{\frac{-c_{ij}^2}{w * e^u}} \qquad C_6 = \frac{p_i}{1+e^{a+b\ln c_{ii}}} + \sum_j \frac{p_j}{1+e^{a+b\ln c_{ij}}},$$

where c_{ij} indicates the time needed to traverse the distance between micro-regions i and j on road, while p_i the relevant social mass of micro-region i, and a, b and w are constants of the particular spatial structure.

In the table which depicts how the potential models fit together, the results of models calculated with income and population are presented, using data from 2004 and 2008. It can be shown that somewhat better fitting can be achieved with income than with population data, although the difference is not significant. On the basis of our micro-regional tests, models applying the log-logistic resistance factor (C_6) can be considered as the best accessibility potential model. It is to be noted, however, that one cannot be sure that using a different territorial division would have led to this result.

Table 2

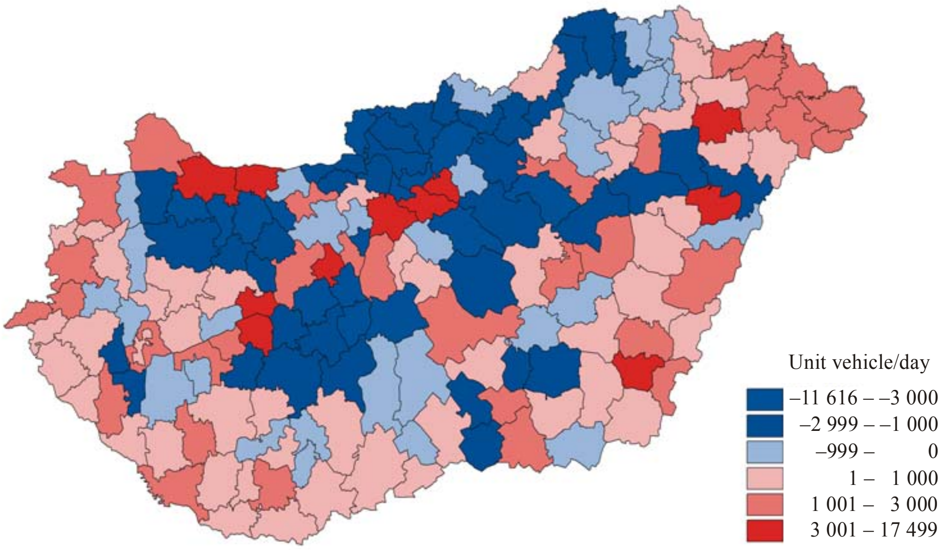
Fitting of models – based on gravitation analogy – to traffic data (R^2)

Denomination	C_1	C_2	C_3	C_4	C_5	C_6
Population, 2004	0.43	0.26	0.55	0.52	0.19	0.63
Income, 2004	0.42	0.24	0.56	0.53	0.18	0.73
Population, 2008	0.45	0.45	0.56	0.52	0.13	0.69
Income, 2008	0.46	0.45	0.58	0.55	0.11	0.72

Regression values – with a few exceptions – indicate moderate correlation. There are no large disparities between the explanatory powers of models. It can be seen that conclusions drawn from potential models need to be treated with caution, for the correlation between them and social space is not strong enough to do so in specific cases. In the following, using the best fitting model, using residuals, it was examined where significant spatial disparities existed between potential space and the flows.

Figure 2

Differences between traffic and trends calculated from potential (C_6)



One can see that in the blue areas of the country traffic is lower than the value expected by the model, while in the area of the capital, close to large towns and next to the border and Lake Balaton, the volume is larger than what could be expected by the potential. This is also logical, as these areas can be highlighted target areas, which has not been integrated into any of the models.

Hence these can be the following steps of improving the potential models so that the models can also become better and better in a mathematical sense, and the conclusions drawn from them can be applicable to the whole social space.

Relation between space and masses, disaggregation of potential

The topology of geometry of the accessibility potential showed that whatever models are used, it is a common feature of them that they measure the impacts of spatial structures, spatial division, of the location of the different spatial domains and of the distribution of masses at a time. The location of a spatial domain is basically defined by geographical position, which is somewhat modified by accessibility (depending on mode of transport). That is, in case of a particular potential value, it cannot be stated whether that results from favourable/unfavourable (settlement, spatial) structure or position, or the location of masses, spatial size or the impact of own mass. In this chapter we aim to disaggregate these impacts, to describe parts as a proportion of total potential values, and to present spatial disparities.

According to our idea on the gravitation field of social masses, therefore, an arbitrary division of space is given (settlement, micro-regional structure etc.), and then a distribution of masses according to this division (masses being either quanta or tokens "assigned" to the particular spatial structure). The value of the potential in a particular point is defined by the sum of these two impacts (internal potential) and that of the impact of own mass and own spatial size (self-potential).

In an arbitrary point of space, by the effect of the potential caused only by the division of space we mean the value which would be generated if masses were equal in all delimited spatial units. The mass distribution impact in an arbitrary point of space is the difference between the value of internal potential and spatial structure potential in the particular point. The impacts of spatial size and own mass can be interpreted in an analogous way in case of self-potentials:

$$U_i^{total} = U_i^{mass\_distribution} + U_i^{spatial\_structure} + U_i^{own\_mass} + U_i^{spatial\_size}$$, where

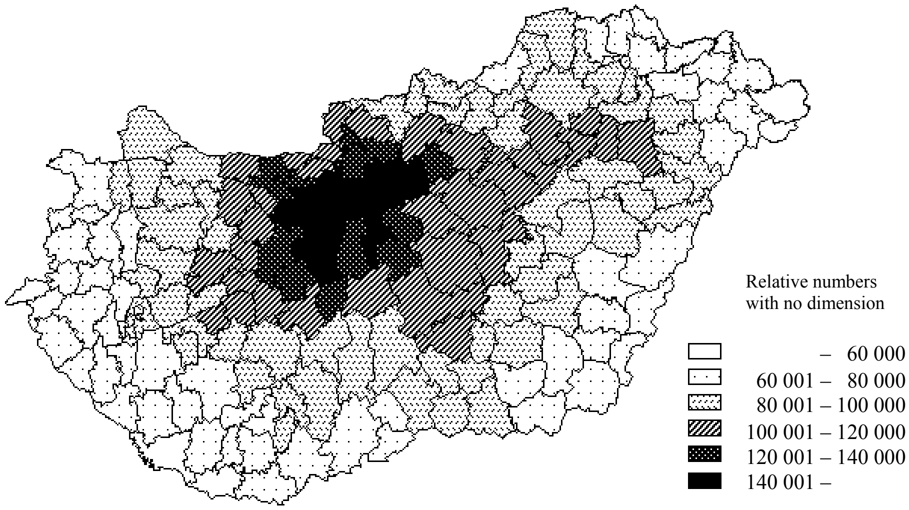
$$U_i^{spatial\_structure} = \sum_j \frac{\frac{\sum_{k=1}^{n} m_k}{n}}{f(d_{ij})}, \quad U_i^{mass\_distribution} = U_i^{internal} - U_i^{spatial\_structure},$$

$$U_i^{spatial\_size} = \frac{\frac{\sum_{i=1}^{n} m_i}{n}}{f(d_{ii})}, \qquad U_i^{own\_mass} = U_i^{self} - U_i^{spatial\_size}.$$

In the next example, the starting point of calculations was the micro-regional data series of resident population in Hungary (1 January 2008). The division of the above potential was made by the model applying linear resistance factor, using distances on public roads. The impacts of masses beyond Hungary and the cross-border effect of internal masses were not taken into account here either.

Figure 3

Potential values of resident population accessibility by micro-regions



The most important result of the split of the potential is that total potential depends on spatial division to the highest extent. This – as already mentioned – depends on the topographical situation of areas on the one hand and on the other, the accessibility factor modifying it, due to which the picture of the role of spatial division does not grow concentrically towards the borders but is somewhat distorted. The proportion of spatial division within total potential is in the 55% and 119% range. With areas along the border – having low total potential in national comparison – primarily determined by this factor, it can be stated that these micro-regions have a disadvantage already owing merely to their location, which cannot really be offset by changes in either accessibility or mass distribution or own mass. Location advantages have their impact mainly in the central part of the country.

The case of mass distribution is completely different. There are areas from where smaller-than-average masses are accessible relative to the total potential of the particular area. This means that the structural location of these micro-regions would result in higher potential values, but the distribution of masses, unfavourable to them, induces a negative effect. Examples include micro-regions with low populations, situated in a bloc along the south-western border of Hungary, or Budapest, the total potential of which is lowered by nearly 7% by the distribution of accessible masses. The effect of mass distribution varies between 43% and –21% at the national level. In essence, the role of mass distribution gradually declines moving away from the micro-regions of the Budapest agglomeration." It is worth to observe that the role of mass distribution is only positive in the Győr micro-region out of regional centres, i.e. the volume of masses available from here is very considerable. However, the rest of the regional centres primarily stand out from the potential space because of their own mass, and the further development of these is substantially hindered by relatively small masses being accessible from them.

Within the self-potential of areas the least important factor as measured against total potential is the area size of the particular area. Its share of total potential ranges between 1% and 6%. The role of own mass is positive in 45 cases from the point of view of the total potential of a particular micro-region, and negative in respect of the rest. The proportion of own masses is between 34% and –4% of the total potential of micro-regions. The most positive shares can be seen for Budapest and major cities, while the negative ones are seen in case of predominantly border-side micro-regions with low populations.

Figure 4

Role of spatial division as a proportion of potential values of resident population accessibility by micro-regions

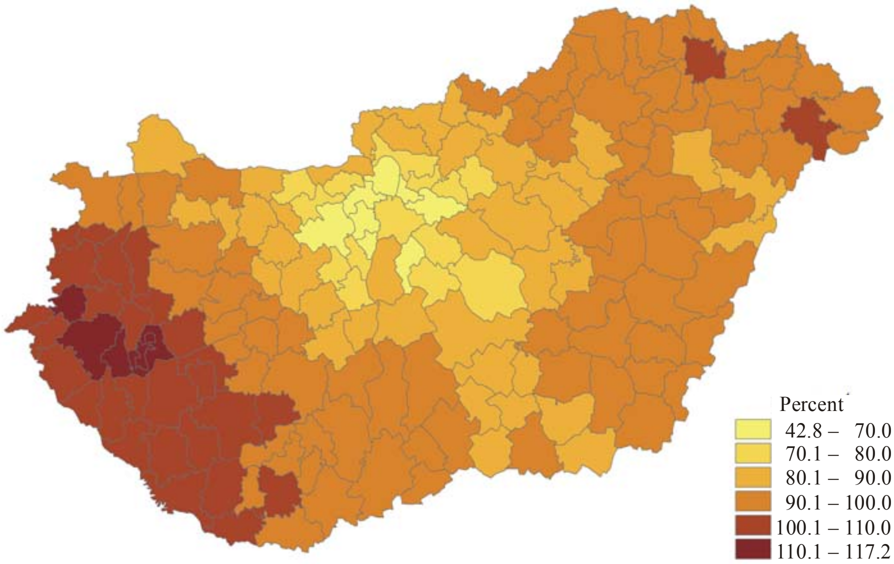


Figure 5

Mass distribution as a proportion of potential values of resident population accessibility by micro-regions

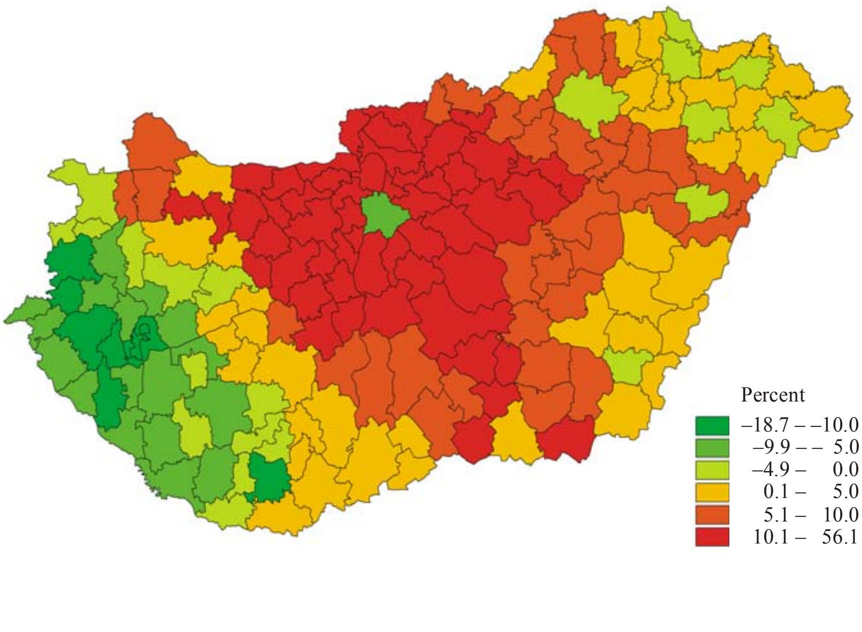


Figure 6

Role of area size as a proportion of potential values of resident population accessibility by micro-regions

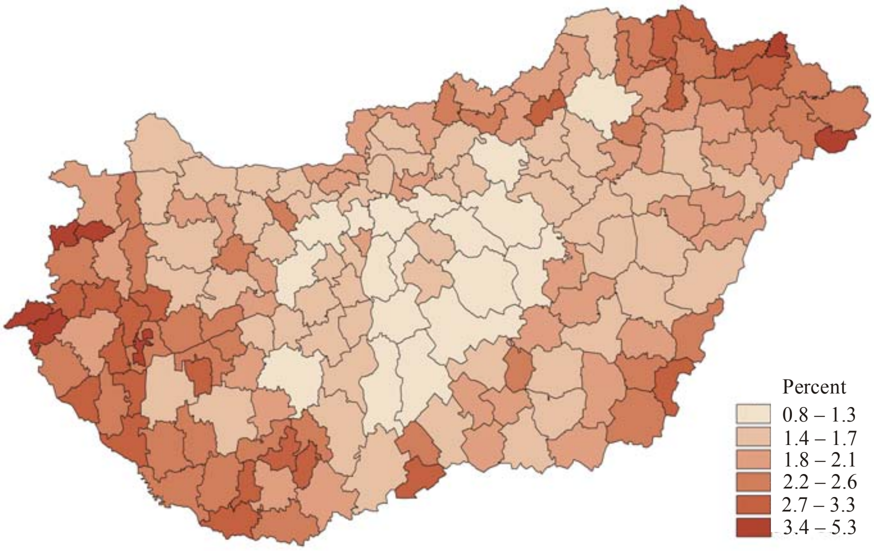
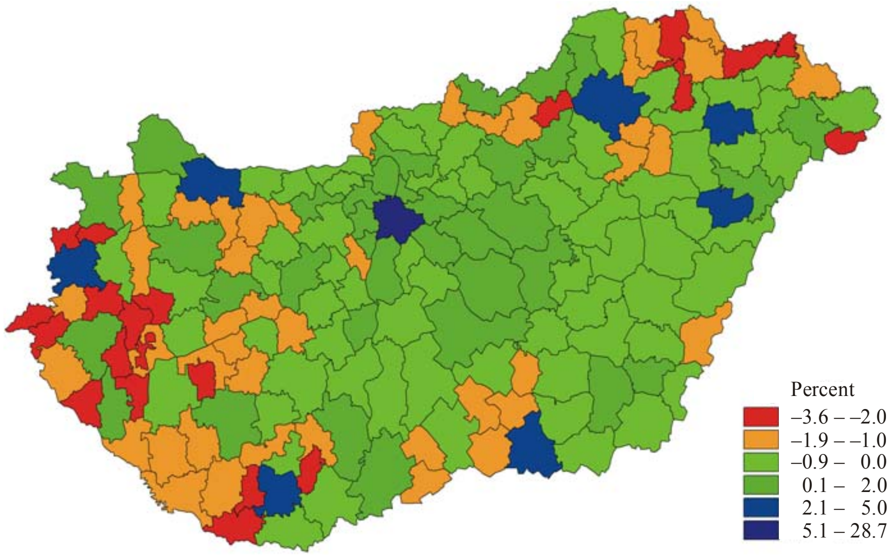


Figure 7

Own mass of areas as a proportion of potential values of resident population accessibility by micro-regions



Summary

By discussing the geometry of accessibility potential models and in support of gravity analogies, we attempted to make a short review of the application and applicability of the models. We aimed to present what may be concluded from different potential models. To answer this question, the values of the models were compared to traffic data. We found that regression values showed moderate correlation. There are no large disparities between the explanatory powers of the models. We stated that the conclusions drawn from the potential models needed to be treated with caution, for the relationship between social space and the models is not strong enough for this in particular cases.

According to the analysis of the potential models, it is not possible to state in case of a particular potential value if it results from favourable/unfavourable (settlement, spatial) structure or position, or the location of masses, area size or the impact of own mass. Therefore we attempted to disaggregate these effects of the potentials by the mathematical method detailed in the study. We pointed out that total potential depended on spatial division to the highest extent, but the impact of mass distribution as well as of own mass may also be important influential factors.

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Keywords: regional analysis methods, potential model, spatial relations.

Abstract

With the help of the description of geometry of potential methods and the gravity models, we tried to briefly explain these models' application and applicability. Our aim was to show what may be concluded from different potential models. To answer this question, traffic data were compared to the values predicted by the models. We found that the values of the regression showed a moderately strong correlation. We could determine that the conclusions drawn from the potential models should be treated with reservations.

According to the analysis of the potential models it is not possible to determine in case of a potential value whether it results from positive/negative (municipal, spatial) structure or position, or the location of masses, area size or the impact of its own mass. Thus, the mathematical method described in this study tried to separate the potentials. We found that accessibility potential was dependent on spatial subdivision to the highest degree, but important factors were the effect of mass distribution as well as of own mass.