Introducing New Control Paradigms in Basic Control Education – YOULA Parameterization

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Abstract – *Course material of basic control theory has been overviewed and updated recently at the Faculty of Electrical Engineering and Informatics, BME. In the theoretical material the concept of the* YOULA *parameterization has been introduced which gives a new insight into controller design. New lecture notes were written both for the theoretical material and for the MATLAB laboratory exercises. An example demonstrates the design procedure and the robustifying effect of the filters in case of plant/model mismatch.*

Keywords: control education, YOULA parameterization, YOULA regulator, sensitivity

1. Introduction

At the Faculty of Electrical Engineering and Informatics, Budapest University of Technology and Economics, control theory is taught as a basic discipline for all students specialized in informatics. The subject offers fundamental knowledge in analysis and design of continuous and sampled data control systems. The course material has been overviewed and updated recently. Newer ideas for controller design as YOULA parameterization has also been introduced which gives a new insight into controller design. It is shown that control algorithms like *PID*, dead-beat and Smith predictor control can be considered as special cases of YOULA parameterization. New university lecture notes were written providing the theoretical material [1, 2] and the related MATLAB exercises.

2. Understanding control concepts, introducing the idea of YOULA parameterization

In the sequel it will be shown how the concept of closed-loop control is introduced and how it is related then to the YOULA parameterization [1]. Control theory deals with the analysis and design of closed loop control systems. The main control structure is based on negative feedback. The goal in control of physical plants is to track the output signal according to a reference signal and to reject the effect of the disturbances. There are requirements set to the performance of the control system. First it has to be stable, then, it has to meet the quality specifications set for steady-state accuracy, dynamic properties such as overshoot, settling time, etc. The control signal has to be inside its technical limits. The control system has to be not very sensitive to measurement noises and to plant/model mismatch. It has to be also technically realizable and eligible to economical and other (e.g. environmental protection) viewpoints.

The control is realized through negative feedback if the input signal (the manipulated variable) of the process is affected by the difference of the measured output signal and its desired prescribed value. The measured output value is generally noisy because of the noise y_z acting on the measurement equipment. Based on the error signal *e* the controller *C* generates the manipulated variable *u*, which modifies the output signal of the process *P*. The process itself is supposed to be stable. The output signal of the process is changing according to the dynamics of the control circuit until it reaches its desired value. The block-diagram of the closed-loop control system is given in Fig. 1. Often the reference signal is filtered by a precompensator element of transfer function *F* (denoted by dotted line in the figure).

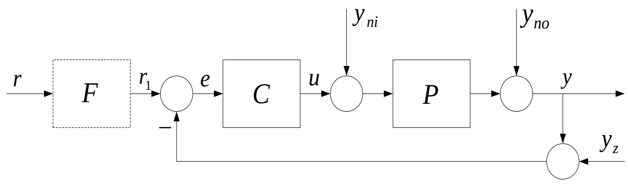


Fig.1. Closed-loop control circuit

If the disturbances and the measurement noise are not considered and the filter is supposed to be unity (*F*=1), then the open loop circuit shown in Fig. 2. is equivalent to Fig. 1. regarding reference signal tracking.

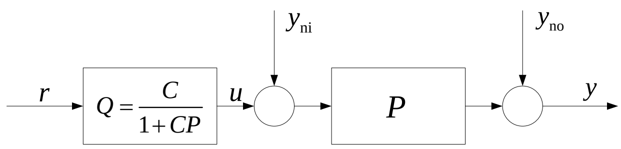


Fig.2. Equivalent open-loop structure

Here *Q* is the YOULA parameter. The classical YOULA parameterization gives a very simple way for open-loop stable processes when the regulator can be analytically designed by explicit formulas. The YOULA parameter is, as a matter of fact, a stable (by definition), regular transfer function.

$$Q(s)=\frac{U(s)}{R(s)}=\frac{C(s)}{1+C(s)P(s)} \quad \text{or shortly} \quad Q=\frac{C}{1+CP}$$

where $C(s)$ is a stabilizing regulator, and $P(s)$ is the transfer function of the stable process.

The open-loop structure shown in Fig. 2. ensures reference signal tracking but does not reject the effect of disturbances. To ensure disturbance rejection as well the open-loop control structure is extended by *IMC* according to Fig.3.

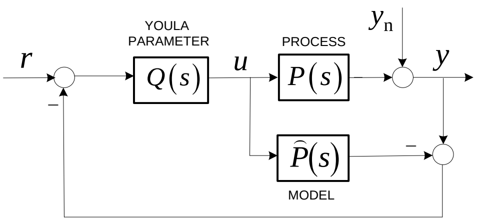


Fig. 3. YOULA parameterized control system with *IMC*

Fig. 4. shows an equivalent block diagram supposing that the model is equal to the system, $\hat{P}=P$. In this usual feedback structure the controller C is expressed by the Q YOULA parameter.

$$C(s)=\frac{Q(s)}{1-Q(s)P(s)}$$

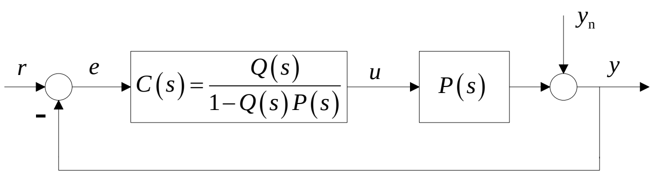


Fig. 4. The usual feedback system with the YOULA parameter in the controller

If the process P is stable, then all stable Q controllers ensure stable control system. Similar relationships are obtained for discrete systems as well where instead of the transfer functions the pulse transfer functions are considered.

The best reference signal tracking, when the output signal is exactly equal to the reference signal could be reached if the YOULA parameter is the inverse of the transfer function of the process: $Q=P^{-1}$.

But generally this condition cannot be fulfilled. The dead time of the process cannot be inverted as its inverse is not realizable. It is also not realizable if the numerator of the inverse is of higher degree than that of its denominator. Right side zeros of the transfer function cannot be inverted either, as they will produce unstable poles in the controller. For discrete systems zeros outside of the unit circle cannot be inverted, and cancellation of zeros which lie on the left side of the unit circle (or in the undesired part of the unit circle) is to be avoided as their inversion would cause intersampling oscillation.

Therefore Q can be only the inverse of the invertible part of the transfer function of the plant. Let us separate the plant transfer function to the invertible $P_+(s)$ and the noninvertible $\bar{P}_-(s)$ factors, where the latter contains also the dead time.

$$P(s)=P_+(s)\bar{P}_-(s)$$

Then $Q=P_+^{-1}$. The gain of $\bar{P}_-(s)$ has to be 1 as this determines the static gain in the forward path. Fig. 5. shows now the *IMC* control structure.

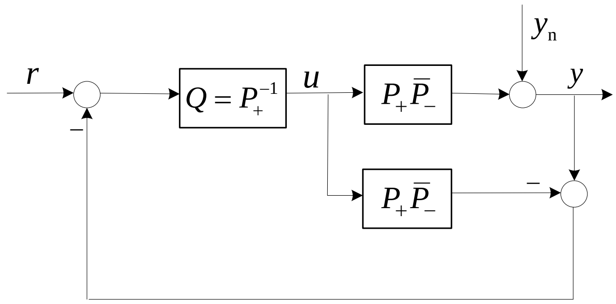


Fig. 5. Realizable YOULA parameterized *IMC* control structure

In this configuration the dynamics of reference signal tracking and disturbance rejection is the same.

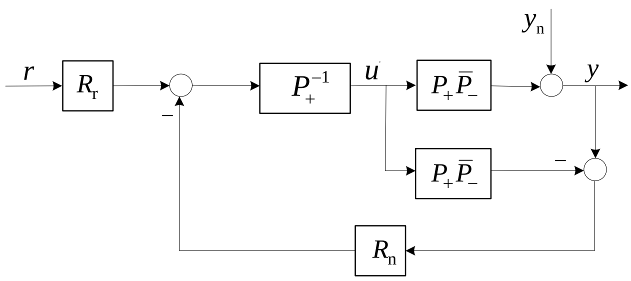


Fig. 6. YOULA parameterized control with filters

If different dynamics are required (e.g., disturbance rejection has to be faster than reference signal tracking), then reference and disturbance filters can be used with unity gain as shown in Fig. 6. This structure is called *2DF* (two-degree-of-freedom) structure.

Equivalent structures are shown in Figs. 7. and 8.

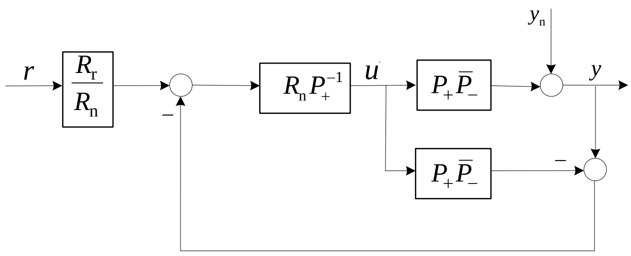
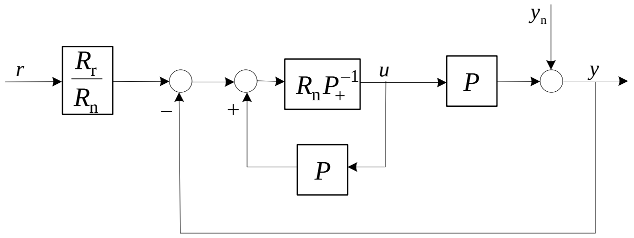


Fig. 7. Equivalent YOULA parameterized *IMC* control system

Now the YOULA parameter is $Q = R_n P_+^{-1}$.



Fig, 8. Equivalent YOULA parameterized control system

The series controller is $C = \dfrac{R_n P_+^{-1}}{1 - R_n \bar{P}_-}$. For discrete systems the relationships are similar with the z-transforms.

Besides ensuring different dynamics for reference signal tracking and disturbance rejection another role of the filters is to modify the value of the control signal u keeping it inside the allowed limits. The filters have also a robustifying effect. With their appropriate choice the control system can be done less sensitive to plant/model mismatch.

Summarizing the design procedure: The plant transfer function has to be separated into its invertible and non-invertible parts. The reference and disturbance filters have to be given as design objectives. The controller can be designed in open-loop, ensuring the best realizable reference signal tracking. Disturbance rejection is provided by enhancing the control by *Internal Model Control* (*IMC*) structure. The filters have to be chosen considering robustifying criteria.

3. Simulation example

The transfer function of the continuous plant is

$$P(s) = \frac{1}{(1+5s)(1+10s)} e^{-30s}$$

It is sampled, at the input zero order hold is applied. The sampling time is $T_s = 5$ sec. The corresponding pulse transfer function is

$$P(z) = \frac{0.1548(z+0.6065)}{(z-0.3679)(z-0.6065)} z^{-6}$$

First a *PID* controller is designed for pole cancellation (cancelling the biggest pole of the system and introducing an integrating effect instead, and cancelling also the second pole introducing a differentiation instead) and for phase margin about $\varphi_m \approx 60^o$. The pulse transfer function of the controller is

$$C(z) = 0.3074 \frac{z-0.6065}{z-1} \frac{z-0.3679}{z}$$

Let us design a YOULA controller first without filters, $R_r = R_n = 1$.

Let us separate the pulse transfer function of the plant into invertible and non-invertible parts.

$$\bar{P}_-\left(z^{-1}\right) = \frac{\left(1+0.6065\, z^{-1}\right) z^{-1}}{1.6065} z^{-6}$$

$$P_+\left(z^{-1}\right) = \frac{0.1548 \cdot 1.6065}{\left(1-0.3679\, z^{-1}\right)\left(1-0.6065\, z^{-1}\right)}$$

and the YOULA parameter is

$$Q\left(z^{-1}\right) = \frac{\left(1-0.3679\, z^{-1}\right)\left(1-0.6065\, z^{-1}\right)}{0.1548 \cdot 1.6065}$$

With first-order lag element filters $R_r(s) = \dfrac{1}{1+s}$ whose pulse transfer function is $R_r\left(z^{-1}\right) = \dfrac{0.9933\, z^{-1}}{1-0.006738\, z^{-1}}$ and $R_n(s) = \dfrac{1}{1+25s}$ whose pulse transfer function is

$$R_n\left(z^{-1}\right) = \frac{0.1813\, z^{-1}}{1-0.8187\, z^{-1}}$$

the YOULA parameter is

$$Q = \frac{0.729\left(1-0.3679 z^{-1}\right)\left(1-0.6065 z^{-1}\right) z^{-1}}{1-0.8187 z^{-1}}$$

Fig. 9. shows the output and control signal responses for *PID* and YOULA control with no filters and no plant/model mismatch. The step reference signal acts at *t*=0 sec, and a step disturbance of 0.5 amplitude acts at *t*=300 sec. It is seen that YOULA control is much faster because of the higher control signal. Fig.10. gives the responses with the filters. It is seen that in the response of the YOULA parameterized controller the dynamics is different for reference signal tracking and for disturbance rejection.

Let us consider the control behaviour in case of plant/model mismatch. The dead time of the system is 40 sec, while in the model 30 sec is considered and the controller has been designed based on this model. The *PID* controller still tolerates this uncertainty, but without the filters the YOULA controller becomes unstable (Fig. 11.). With the given filters its behaviour is acceptable (Fig. 12.).

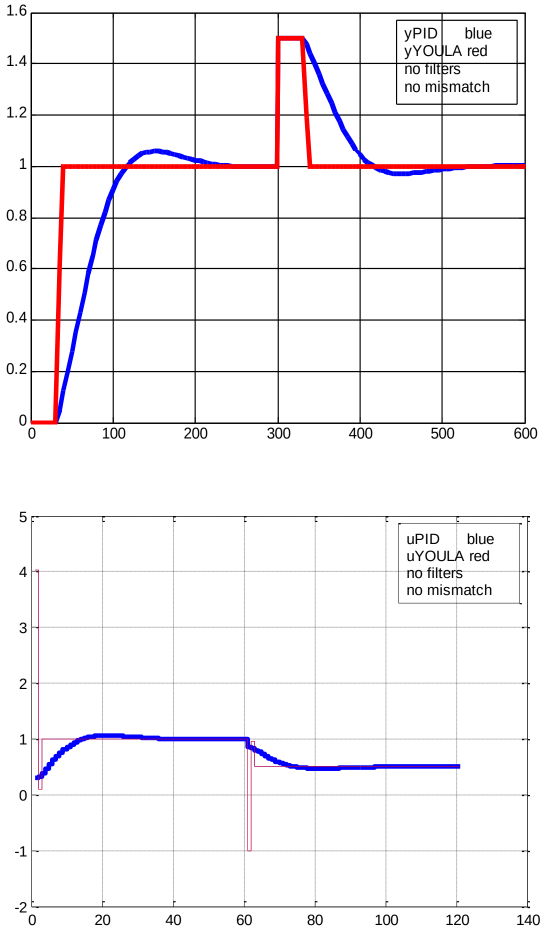


Fig. 9. Output signals (upper figure) and control signals (lower figure) for *PID* (blue) and YOULA (red) control for step input and output disturbance (no filters, no mismatch)

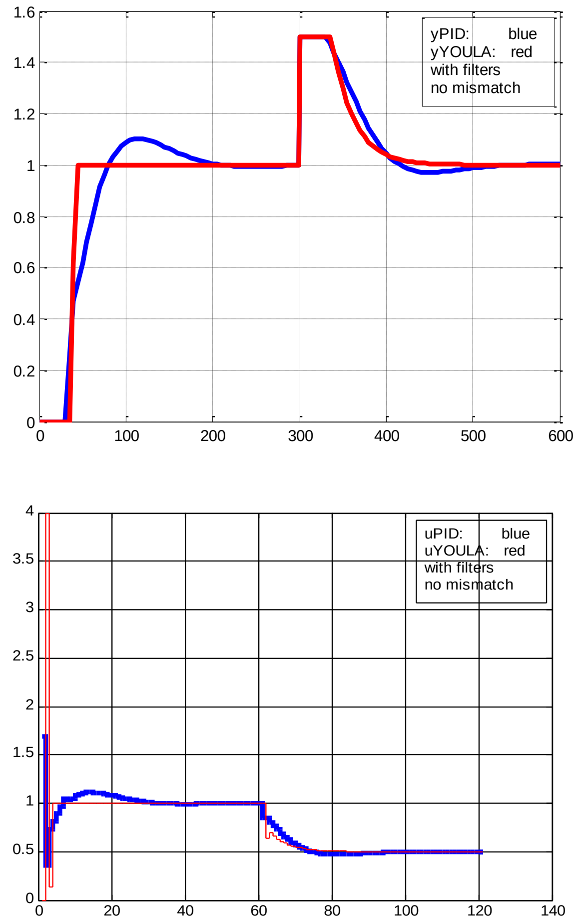


Fig. 10. Output signals (upper figure) and control signals (lower figure) for *PID* (blue) and YOULA (red) control for step input and output disturbance (with filters, no mismatch)

4. Robustification considerations for dead time mismatch

Keviczky and Bányász analyzed the relationship of performance and robustness, especially for the case of dead time mismatch [3], [4]. In this case the relative model error is

$$\frac{P-\widehat{P}}{\widehat{P}}=\frac{\Delta}{\widehat{P}}=\frac{P_{+}e^{-sT_{\mathrm{d}}}-P_{+}e^{-s\widehat{T}_{\mathrm{d}}}}{P_{+}e^{-s\widehat{T}_{\mathrm{d}}}}=e^{-\Delta T_{\mathrm{d}}s}-1$$

P is the real process and $\widehat{P}$ is its model. It is supposed that the transfer function of the process without the dead time is accurately known, $\widehat{P}_{+}=P_{+}$ and $\Delta T_{\mathrm{d}}=T_{\mathrm{d}}-\widehat{T}_{\mathrm{d}}$

For robust stability

$$\sup_{\omega}\left|e^{-j\Delta T_{\mathrm{d}}\omega}-1\right|\leq\frac{1}{\left|R_{\mathrm{n}}(\omega)\right|}$$

With first-order lag disturbance filter with time constant T_{n} this condition is expressed as

$$\left|1-\frac{\widehat{T}_{\mathrm{d}}}{T_{\mathrm{d}}}\right|<\frac{\pi}{\sqrt{3}}\frac{T_{\mathrm{n}}}{T_{\mathrm{d}}}$$

With TAYLOR expansion of the exponential term a simpler robustness condition is obtained as

$$\left|1-\frac{\widehat{T}_{\mathrm{d}}}{T_{\mathrm{d}}}\right|<\frac{T_{\mathrm{n}}}{T_{\mathrm{d}}}$$

In our example the above condition $\left|1-30/40\right|<25/40$ is fulfilled.

Fig. 13. shows the output signal for Tn=8 sec, when the required condition is not fulfilled. In this case the output signal is oscillatory, the control system does not tolerate the mismatch in the dead time. Fig. 14. gives the output signal with $T_{\mathrm{n}}=15$ and 40 sec time constants of the disturbance filter. In these cases the control performance is improved significantly.

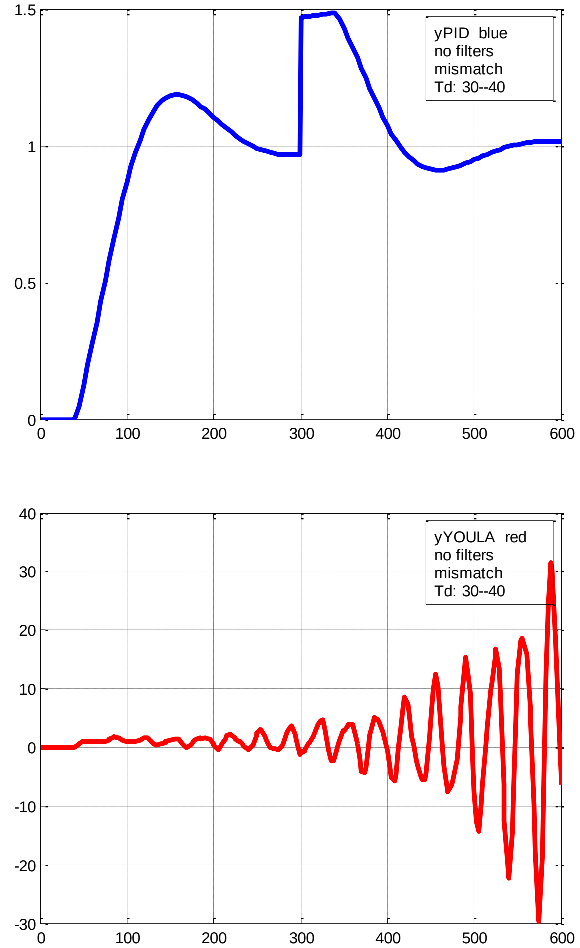


Fig. 11. With mismatch without filters *PID* control is stable, but the YOULA controller becomes unstable

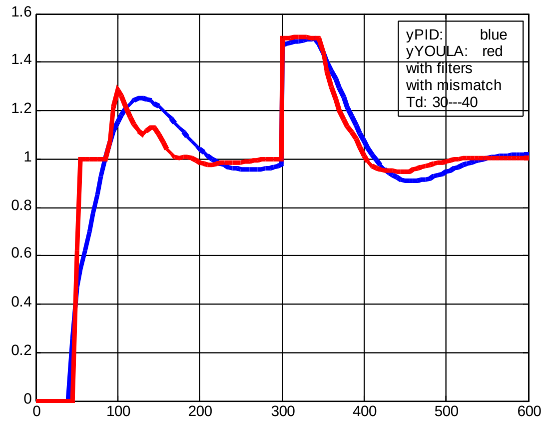
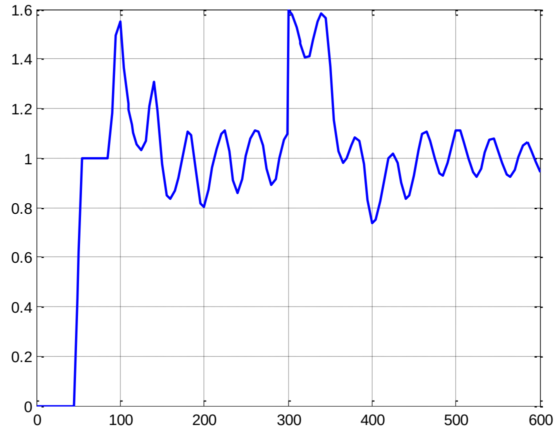


Fig. 12. With mismatch and with filters both *PID* control and the YOULA controller are stable

5. Conclusions

YOULA parameterization is a very effective control algorithm for control of stable processes. Keviczky and Bányász have researched the structure and several properties of this control paradigm. The controller can be designed in open loop providing the best realizable reference signal tracking, and extending the control system with feedback from the internal model (*IMC*) ensures disturbance rejection. Reference and disturbance filters modify the dynamic behaviour, thus the transients for reference signal tracking and disturbance rejection can be different. Appropriately chosen filters robustify the control behaviour in case of plant/model mismatch and also affect the maximum value of the control signal. It can be shown, that well known controllers as *PID*, dead beat, SMITH predictor are special cases of YOULA parameterization.



Fig, 13 With $T_n = 8$ oscillations appear in the output signal

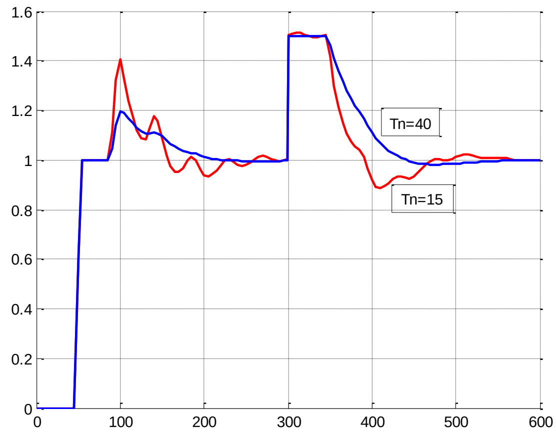


Fig.14. With appropriately chosen disturbance filter the control behaviour can be accepted

This newer approach has been introduces in control education at the Faculty of Electrical Engineering and Informatics, BME. The theory is demonstrated through examples in the computer labs using software MATLAB/SIMULINK.

6. References

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