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Frege's Definition of Number:
No Ontological Agenda?

Abstract. Joan Weiner (2007) has argued that Frege's definitions of numbers constitute linguistic stipulations that carry no ontological commitment: they don't present numbers as pre-existing objects. This paper offers a critical discussion of this view, showing that it is vitiated by serious exegetical errors and that it saddles Frege's project with insuperable substantive difficulties. It is first demonstrated that Weiner misrepresents the Fregean notions of so-called *Foundations*-content, and of sense, reference, and truth. The discussion then focuses on the role of definitions in Frege's work, demonstrating that they cannot be understood as mere linguistic stipulations, since they have an ontological aim. The paper concludes with stressing both the epistemological and the ontological aspects of Frege's project, and their crucial interdependence.

1 THE PROBLEM

It is indisputable that Frege's logicist project, including the development of his logical calculus, had an epistemological aim, namely to prove the *a priori* and analytic status of the arithmetical truths, and thus to prove that they are deducible from the laws of logic. More problematic, and a subject of recent debate, is the question concerning the status of definitions within this project.[1] Frege dismisses previous attempts at the definition of number, and replaces them with new definitions. In addition, in several passages he describes definitions as arbitrary conventions. So it seems as if Frege is not interested in capturing with his definitions the pre-existing meanings of arithmetical symbols, but in stipulating new ones. But how can this revisionary project be brought into harmony with the epistemological aim, i.e. how can arbitrary

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[1] Some key contributions to this debate are (Benacerraf 1981), (Weiner 1984), (Picardi 1988), (Kemp 1996). For more details, see the recent overview in Shieh (2008).

definitions contribute to proving the logical status of *the* truths of arithmetic, i.e. of antecedently existing truths?

In the most recent contribution to this debate Joan Weiner (2007) offers a radically new solution: Frege was a more thorough revisionist than the dilemma above presents him. His revisionism affected not only his conception of definition, but also of sense, reference and truth. Prior to his work, numerals did not have a determinate sense and reference, and arithmetical statements were not strictly speaking true. According to Weiner, Frege did not believe that concept-script systematisation is unveiling the true nature of numbers and the true referents of numerals, but only that it introduces stricter semantic and inferential constraints of precision stipulating the sense and reference of numerals and arithmetical statements for the first time. Thus talk about numbers as objects and strict arithmetical truth is only possible as a system-internal discourse, and concept-script systematisation is a normative linguistic precisification serving an epistemological aim, with no ontological and semantical discoveries about pre-systematic arithmetic and its language. In particular, definitions carry no content-preserving and ontological commitment.

2 WEINER'S ARGUMENT

Weiner offers a wealth of substantive and exegetical considerations in favour of her view, focusing most explicitly on the role of definitions within Frege's work, especially in the *Foundations of Arithmetic*. She investigates what requirements a definition (of number, numerals etc.) must satisfy in order to qualify as adequate or faithful to prove the truths of arithmetic from primitive truths (Weiner 2007, 683). One such obvious requirement seems to be the following:

> *The obvious requirement*: A definition of an expression must pick out the object to which the expression already refers or applies (ibid. 680).

Weiner denies there is any evidence in Frege's writings for this requirement. Definitions are not preserving the putative pre-systematic reference of numerals. Still, they must be faithful to pre-systematic arithmetic in some sense, since systematisation is not meant to transform arithmetic into some 'new and foreign science' (ibid. 687). Her explanation is as follows: 'Faithful definitions must be definitions on which those sentences that we take to express truths of arithmetic come out true and on which those series of sentences that we take to express correct inferences turn out to be enthymematic versions of gapless proofs in the logical system' (ibid. 690, 790). In other words, what systematisation preserves is truth-related and inference-related content. For example, regarding truth-related content a definition of '0' and '1' is unacceptable, if it presents as true a sentence

which in pre-systematic arithmetic is taken to express a falsehood, namely '0=1'. Thus faithful definitions must cover for what are taken to be the well known properties of numbers.[2] Regarding inference-related content, faithful definitions must preserve inferences that we take to be valid, for example 'If Venus has zero moons and the Earth one, then given that 0<1, the Earth has more moons than Venus' (ibid. 686). Thus faithful definitions must cover for all applications of number, including those in applied arithmetic.[3]

Frege is therefore not concerned with preservation of reference, and not even simply with preservation of putative truths, rather of what Weiner calls '*Foundations*-content'. This is 'some sort of content connected with inferences' (ibid. 692). *Foundations*-content partly points back to the judgeable content of *Begriffsschrift*, which was defined by Frege as content that has only 'significance for the inferential sequence' (1879, x.). But *Foundations*-content also partly anticipates the later notion of sense, i.e. *Sinn* (Weiner 2007, 689-1), for two reasons. First, the judgeable content of a term, she claims, is not its referent (ibid. 690, fn. 17), just as much as sense is not reference. Second, a term hitherto considered non-empty will not cease to have *Foundations*-content if we discover it is empty, for the discovery will lead to a re-evaluation of our pre-systematic beliefs and inferences, a re-evaluation still involving the term itself (ibid. 690). Equally, a fictional term like 'Hamlet' has *Foundations*-content, since there are speakers who think it enables them to express truths and correct inferences (ibid. 691). Hence, it is not required for a term to have a referent in order to have *Foundations*-content, and this brings *Foundations*-content in the vicinity of *Sinn*.

Thus, what concept-script systematisation achieves is preservation of *Foundations*-content. However, this should not be understood in the trivial sense of 'preservation', as if something outside of the system is identical with something in the system. As it transpires from Weiner's argument, preservation of *Foundations*-content means rather something like 'normative transformation of pre-system content into systematic content'. As quoted above, systematisation involves the process of proving *within the logical system* the truth of the pre-systematic sentences taken to express truths as well as the correctness of pre-system inferences taken to be correct. But proving 'in the logical system' is a highly normative process, guided essentially by *two precision requirements* that distinguish sharply the system from the pre-system: the gapless proof requirement, i.e. all proofs are absolutely gapless, and the sharpness requirement, i.e. genuine concepts must have sharp boundaries.[4] Essentially, this 'preservation' is to be understood as a creative process of precisification of pre-systematic language,

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[2] (Weiner 2007, 688.) Cf. (Frege 1879, §70).
[3] (Weiner 2007, 689.) Cf. (Frege 1879, §19).
[4] See (Weiner 2007, 701), (Frege 1884, §62, §74), (Frege 1903, §56).

which is indeterminate and vacillating (ibid. 697), as himself Frege seems to suggest about expressions quite generally (see (Frege 1906a, 302–3)). 'Frege's task is to replace imprecise pre-systematic sentences with precise systematic sentences [of arithmetic]' (Weiner 2007, 710). This suffices to make sense of Frege's epistemological aim, the core of his logicist project.

This view has some intriguing implications and corollaries, the most important of which shall be briefly summarised. More details evidence will be presented during the discussion further below.

(a) *Foundations*-content is close to sense, but not identical with it. To have a determinate sense, an expression must have a definition satisfying the precision requirements. But pre-systematic expressions don't have such a definition, hence they don't have a sense and, by extension, no reference.[5] Sense and reference (*Sinn* and *Bedeutung*) are therefore only system-internal features of expressions.[6] There is no evidence to the contrary in Frege's writings. In particular, Frege never says that terms of pre-systematic language have a reference or require one in order to have a use (ibid. 706f.).[7] The absurdity of this view is merely apparent, for fixing the sense and reference of a term is only the ideal end of a science, once it comes to fruition in a system (ibid. 709f.)[8].

(b) If pre-systematic terms don't have a determinate reference, then given compositionality, pre-systematic sentences don't have a determinate reference either, i.e. a truth-value. We only 'take them to express truths' (ibid. 690f.). This does not mean there is nothing 'right' about them (ibid. 710), but only that their rightness does not satisfy the constraints imposed by systematisation. We must distinguish between different notions of truth, as Frege does, i.e. *pre-systematic truth* and *strict truth* (ibid. 709f.).[9] Pre-systematic truth is one of the aforementioned faithfulness requirements a definition (of number etc.) must satisfy.

(c) Since pre-systematic arithmetical expressions do not have a determinate reference, ordinary arithmetical predicates like 'is a number' do not have a reference either. Hence, the concept of number is not already fixed prior to Frege's definitions (ibid. 696). Quite the opposite: in *Foundations* (§100) Frege stresses the arbitrary, stipulative character of definitions (ibid. 695ff.), and he does so again in the important posthumous

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[5] The alternative of having an indeterminate sense (and reference) is excluded for Frege, since there is no such thing for him. See for instance (1903, §56).

[6] This claim has been advanced before. See e.g. (Stekeler-Weithofer 1986, 8.,10.).

[7] Weiner's argument seems to come close here to a Wittgensteinian theory of meaning as use, although Wittgenstein is not mentioned.

[8] See also (Frege 1914, 242).

[9] 'Pre-systematic truth' is not Weiner's term, but my terminological correlate to her label 'regarding a sentence as true' (ibid. 706, 709).

text "Logic in Mathematics" (Frege 1914). Here he claims that a determination of sense is either decompositional, in which case it is a self-evident axiom, or it is a mere stipulation ('constructive definition'). Since Frege does not seem to present his *Foundations* definitions as self-evident (1884, §69), they must be stipulations, stipulations which precisify *Foundations*-content and thus transform arithmetic into a system of science.

(d) A cursory reading of Frege's writings might induce one to assume that he thinks numbers are pre-existing, language-independent objects whose nature his definitions aim to capture. Call this *the ontological thesis*. Weiner rejects this thesis.[10] On her view Frege makes no claim that numbers existed prior to his definitions, or else he would have to say that the definitions are (or articulate) discoveries about pre-existing objects. But they are linguistic stipulations, not ontological discoveries. Frege does not claim that it is part of the nature of numbers to be extensions, but is interested only in the linguistic question 'Are the assertions we make about extensions assertions we can make about numbers?', which he answers by means of a linguistic principle par excellence, the context principle (Weiner 2007, 698f.). As Frege writes: 'I attach no decisive importance to bringing the extensions of concepts into the matter at all' (1884, §107).

Weiner's interpretation is certainly intriguing and original. Nevertheless, it is vitiated by serious exegetical errors, and it saddles Frege's theory of numbers with insuperable substantive difficulties. I will first show that Weiner misrepresents so-called *Foundations*-content, sense and reference, and the notion of truth in Frege's work (sections 3-5). Then I will focus on the role of Fregean definitions, demonstrating that they have, *pace* Weiner, an ontological point, and that they are not mere stipulations. The paper concludes with stressing both the epistemological and the ontological aspects of Frege's project, and their crucial interdependence.

3 '*FOUNDATIONS*-CONTENT'?

We can start with the notion of *Foundations*-content, on which Weiner bases her rejection of the obvious requirement. Is there really a notion of content in the *Foundations* closely related, although not identical to sense? There is no decisive evidence. Frege uses the term loosely. It may mean various things such

[10] Weiner has defended this anti-ontological stance in previous work. See for instance (Weiner 1990, ch. 5.)

as 'sense of a sentence', i.e. a judgement or thought (1884, x., §3, §70, §106), 'sense of a recognition judgement' (ibid. §106, 109), 'judgeable content'(ibid. §62, §74), 'reference' (ibid. §74fn.), and also just conventional meaning and use (ibid. §60). 'Content' in the *Foundations* is simply not a technical term, which fits the prolegomenous character of the book. There is nowhere an argument specifying that an expression could have a content but lack a referent, i.e. that 'Hamlet' has *Foundations*-content. In one place Frege claims the exact opposite in fact: 'the largest proper fraction' has no content and is senseless, because no object falls under it (ibid. §74fn.). He makes a similar point about 'the square root of -1' (ibid. §97). Moreover, the predominant and philosophically significant role of 'content' in the *Foundations* is found in Frege's repeated requests to specify a content of arithmetical judgements in such a way that they turn out to express identities, and thus to secure the objecthood of numbers, given his acceptance of the principle that identity is an essential mark of objecthood (1884, §62f.).[11] Thus we have positive evidence that content in the *Foundations* is closer to reference than to sense.

At the very least, is Foundations-content not related to Begriffsschrift content and insofar only inferential, not referential? But there is no dichotomy here. Judgeable content is so intimately tied to reference that it affects the most basic formation rules of the notation in *Begriffsschrift*. Frege stipulates that any expression following the content stroke must have a judgeable content. That the relation between a judgeable content and its expression is one of being designated, is visible from at least two facts: that the expression of a judgeable content is a designator starting with the definite article, paradigmatically the nominalised form of a proposition ('the circumstance that there are houses', 'the violent death of Archimedes at the capture of Syracuse')[12], and that the expression of a judgeable content can flank the sameness of content sign, i.e. the identity sign. Hence, the expression of a judgeable content is a name of the judgeable content and no formula in concept-script is even syntactic if such a name fails to designate anything.[13]

Weiner argues that since 'Phosphorus = Phosphorus' is derivable from the law of identity, while 'Hesperus = Phosphorus' is not, the two names cannot have the same *Begriffsschrift*-content (Weiner 2007, 690). But this example is uncongenial to Frege's concerns: 'Phosphorus' and 'Hesperus' are names of *un*judgeable content, while in *Begriffsschrift* he is interested only in names of judgeable content—content that can be asserted. Hence, unlike with names of unjudgeable content, there is no room for distinguishing between what a name

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[11] See also (Rumfitt 2003, 198.)

[12] (Frege 1879, §2-3.)

[13] Frege refers to the expression of a judgeable content flanking the identity sign explicitly as a name (Frege 1879, §8, passim).

of a judgeable content refers to and its inferential content. Therefore, once it has been established (by a synthetic judgement) that two names name the same judgeable content, 'A' and 'B' are intersubstitutable for the purposes of inference ('≡' functions as an identity, but also as a stipulative sign; see (Frege 1879, §8, §23)), just as much as, vacuously, 'A' and 'A'. The difference between two names of the same judgeable content is neither a referential nor an inferential difference, but only of a different mode of determination of the same inferential content, as Frege explicitly states (Frege 1879, §8). Claiming that *Foundations*-content is somehow connected to *Begriffsschrift* content, while arbitrarily revising Frege's own view of the latter, is question-begging.

4 SENSE AND REFERENCE

It is equally unwarranted to treat sense and reference (*Sinn* and *Bedeutung*) as system-internal features which do not apply to ordinary expressions. There are enough passages to the contrary, for example when Frege writes that 'The moon is the reference [*Bedeutung*] of "the moon"' (1919, 255) or when he explains, in a lecture, that '[i]f we claim that the sentence "Aetna is higher than Vesuvius" is true, then the two proper names do not just have a sense [*Sinn*] […], but also a reference [*Bedeutung*]: the real, external things that are designated'[14]. In "Introduction to Logic" he writes: 'If we say "Jupiter is larger than Mars", what are we talking about? About the heavenly bodies themselves, the references [*Bedeutungen*] of the proper names "Jupiter" and "Mars"' (1906b, 193). In *Grundgesetze* he points out that the notion of reference (*Bedeutung*) cannot be (genuinely) explained, since any such explanation would presuppose knowledge that some terms have a reference (Frege 1893, §30); hence reference must predate the setup of any system. Since to have a reference implies having a sense, it follows that those expressions of natural language that have reference also have a sense. Neither feature is therefore exclusively system-internal for Frege. Hence, if it is insisted that Frege's definitional project is to be described as preserving so-called *Foundations*-content, then this will involve preservation of pre-systematic content that has a referential component, and this will not be content of a wholly different kind from the content described in *Grundgesetze* split into sense and reference (see (Frege 1893, x.)).

[14] Carnap 2004. 150.

5 KINDS OF TRUTH?

Weiner claims that Frege allows for many notions of truth, including pre-systematic and strict truth. But nowhere does Frege make such a claim. He argues, in "Thoughts", that truth is undefinable, on the basis that any attempted definition would have to analyse truth into constituent properties (of the truth bearer), of which in turn it would have to be *true* that they apply in a particular case in order to make the definition applicable (1918a, 60). This circularity suggests not only the indefinability of truth, but also its simplicity, and thus its univocity. Frege intimates in "Thoughts" additional arguments— very plausible ones—against Weiner's claim. Thus he distinguishes sharply between 'taking something to be true' ('*Fürwahrhalten*') and 'proving the true' ('*Beweis des Wahren*'). The former arises through psychological laws, while the latter belongs to the laws of truth, which are not the object of psychology, but only of logic (ibid. 58f.). Since Weiner characterises pre-systematic truth in terms strongly resembling *Fürwahrhalten*, e.g. 'what we take to express truth' or 'what we regard as true', it would follow, absurdly, that Frege takes truths of pre-systematic arithmetic to belong to the realm of the psychological, and pre-systematic arithmetic to psychology. Equally, if pre-systematic truth is vacillating and vague, it would seem to be a predicate coming in degrees. But Frege specifies that truth does not allow for 'more or less' (ibid. 60). Finally, Frege speaks on repeated occasions about the truths of arithmetic or mathematics as such (e.g. (1884, §3, §11, §14, §17, §109)). But he nowhere qualifies them as merely pre-systematic, to be distinguished from the truths arrived at within the system.

Quite generally, the truth of a thought is timeless (e.g. (1884, §77, 1918a. 74)). Hence, either a pre-systematic notion of truth is timeless as well, in which case it is not clear how this squares with its indeterminacy and vagueness, or a pre-systematic assertion does not express a thought at all. Of course, Weiner can retort that this is indeed so: a pre-systematic assertion only expresses *Foundations*-content. But then *Foundations*-content is assertable. And it is negatable, thinkable, judgeable etc. However, *Foundations*-content is not really judgeable, since judging is 'acknowledging the truth of a thought' (ibid. 62). Judgeable is only that to which strict truth can apply. But then *Foundations*-content is not assertable either, for to assert is to make manifest a judgement (ibid.). And if it is not judgeable, *Foundations*-content lacks the essential association with judgeable content Weiner claims it has, and thus it does not have an inferential character either, for to infer is to judge (1879-1891, 3). Pre-systematic arithmetical proofs and arguments could not be counted as valid, if we continue this line of thought. In conclusion, the distinction between

pre-systematic and strict truth, in conjunction with the notion of *Foundations*-content, leads to catastrophic consequences.[15]

6 THE OBJECTIVITY OF DEFINITIONS: THE MATERIAL MODE

Doubts about Weiner's interpretation also arise if we look more carefully at what Frege says about and what he does with his definitions in the *Foundations*. There are passages in the *Foundations* whose wording stress the stipulative character of definition, e.g. when Frege speaks about fixing ('*festsetzen*') the sense or meaning of expressions, or refers to definitions as stipulations ('*Festsetzungen*').[16] But his phraseology indicates an objective side to definition as well. Thus he speaks of the need to ascertain, find out ('*feststellen*'), explain the sense of an equation (ibid. x., p. 73, §62, §106), which is inaccurately translated as 'to fix/ to define the sense' by Austin, or of the need to find or attend to ('*aufsuchen*') a judgeable content which can be transformed into an identity whose sides contain the new numbers (1884, §104).[17]

The objectivity of definitions is also manifest in Frege's tendency to adopt the material mode and define objects, as opposed to mere expressions. This is confirmed by many passages in the *Foundations* (1879, §7, §8, §9, §10, §18, §67). In two places he speaks explicitly of 'the definition of an object' (1879, §67, §74). A case in point is the very passage in §100 which Weiner takes as decisive evidence in favour of her thesis that Fregean definitions are creative linguistic stipulations. She claims that Frege is telling us that the meaning of 'the square root of -1' is not fixed prior to our definitions, but only fixed for the first time by the definitions (Weiner 2007, 695). However, let us look at the full context:

> We should be equally entitled to choose as further square roots of -1 a certain quantum of electricity, a certain surface area, and so on; but then we should naturally have to use *different symbols* to signify these *different roots*. That we are able, *apparently*, to create in this way as many square roots of -1 as we please, is not so astonishing when we reflect that the meaning of the square root of -1 is not something which was already unalterably fixed before we made these choices, but is decided for the first time by and along with them' (1884, §100; my italics).

[15] One additional problem: if we accept the distinction, what are we to do with Weiner's own arguments, which are formulated in pre-systematic philosophical prose, and not in concept-script? Are they not valid either? Are her conclusions not strictly true? Both a 'yes' and a 'no' answer invalidate her theory.

[16] E.g. §7, §65, §67, §68, §75, §104, §109.

[17] See also §106, where Frege reports that he has established that numbers are not collections of things or properties.

Clearly, Frege is discussing here the possible definition of an object. This is why he talks about different *symbols* having to be assigned to each square root of -1, *once* each square root of -1 has been chosen. So the definitional choice is not a linguistic one. In addition, Frege's tone is verging on sarcasm here, as it is in the embedding discussion.[18] He is certainly not telling us that a definition entitles us, in virtue of its creative powers, to introduce as many square roots of -1 as we want, rather he is presenting his opponent's point of view ('apparently').[19] The latter is a sort of reformed formalist, who has accepted Frege's anti-formalist arguments (up to §100), according to which the introduction of signs alone will not bring complex numbers into existence. Therefore, the reformed formalist sets out to supplement his definition of a complex number with the assignment of a random object, to 'fix' the 'meaning' of the complex number. Frege is spelling out the absurd consequences of this approach, viz. the possibility of a plurality of such assignments.[20] Weiner's misunderstanding of this passage is twofold: Frege is not propounding *his* own view of definition of a linguistic *symbol* as creative stipulation, but his imaginary *opponent's* view of definition of an *object* as a random ontic assignment. There is no evidence in *Foundations* that Frege takes definitions to be arbitrary, creative definitions.

Weiner falls prey to a similar misunderstanding when she discusses an eligibility condition of primitive truths. In "On Formal Theories of Arithmetic" (1885), Frege argues that every definition must come to an end, hitting upon indefinable primitives, the original building blocks of science (*Urbausteine*), which are expressed in axioms (1885, 96). Weiner takes this to be a semantic point: the eligibility condition 'is that the *expression* of the primitive truth should include only simple, undefinable *expressions*. For these simples are the ultimate building blocks of the discipline' (Weiner 2007, 682, my italics). But this is not Frege's point. Again, Frege speaks here in the material, not the formal mode, concerned with defining the objects themselves, in this case the objects of geometry: 'It will not be possible *to define an angle* without presupposing knowledge about the straight line. Of course, what a definition is based on might itself have been defined previously' (1885, 104, my italics). The primitives terminating such a chain of definitions are not expressions, but undefinable objects 'whose

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[18] In the same context he remarks: 'Let the Moon multiplied by itself be -1. This gives us a square root of -1 in the shape of the Moon' (1884, §100). This was hardly written with a straight face, given Frege's usual predilection for sarcasm and irony. See also his related attack against Kossack (1884, §103): 'We are given no answer at all to the question, what does 1 + i really mean? Is it the idea of an apple and a pear, or the idea of toothache and gout? Not both at once, at any rate, because then 1 + i would not be always identical with 1 + i.'

[19] Dummett is another author who misunderstands this passage. See (Dummett 1991, 178f).

[20] And he shows a few pages on that the formalist origin of this kind of reasoning leads to misconstruing the subject matter of arithmetic as synthetic and even as synthetic a posteriori (1884, §103).

properties are expressed in the axioms' (ibid.).[21] It is in the material mode that Frege goes on to explain that the building blocks of arithmetic must be of purely logical nature (or else we cannot account for the universality of arithmetic), and to describe the terminological replacement of 'set' with 'concept' as not being a mere renaming, but of importance for the actual state of affairs (1885, 104f.).

7 THE OBJECTIVITY OF DEFINITIONS: FREGE'S PLATONIC REALISM

Another challenge to Weiner's interpretation is Frege's Platonism, which he maintains with respect to arithmetical truth, arithmetical objects and logical objects, and which is manifest in the importance existence proofs play for his definitions. Frege explains the definition of 1 by reference to the semipiternality and apriority of the truth of the propositions it helps to derive, e.g. '1 is the immediate successor of 0'; no physical occurrence, including 'subjective' ones concerning the constitution of our brains, could ever affect the truth of this theorem (1884, §77). A mere linguistic constraint of precisification cannot explain this semipiternality; the truth-value of 'Tom is alive' will continue to depend on contingent, empirical facts even if we cut the boundaries of the embedded expressions sharp.[22] What explains the semipiternality is the specific objectivity of the content of arithmetical propositions, the nature of arithmetical objects. This nature is grounded in the nature of logical objects, given that number-statements are ultimately about relations between logical objects (correlations of (extensions of) second-order concepts). Far from explaining logical objects as the result of linguistic constraints and stipulations, he ascribes to them ontological objectivity: they are simply there, ready to be discovered by us. Thus judgeable contents, the paradigmatic logical objects of the early work, are for Frege as objective as any mind- and language-independent object, like the sun (1879-1891, 7), although not physical.[23] Definitions could not play any creative role at this ontic level, and they certainly don't play it in concept-script, where a definition is merely an abbreviation: it stipulates that a simple sign is to have the same judgeable content as a more complex one (1879, §24). The existence

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[21] We can call the expressions designating such primitives also 'primitives', but only me-tonymically.

[22] Frege's eternalist theory truth, maintained elsewhere, does not matter here, since that theory is not concerned with the justification of the truth-value of a proposition, but with the question about the proper bearer of truth.

[23] Elsewhere he says something similar about the relation between an object and its concept: 'To bring an object under a concept is merely to recognise a relation that already existed beforehand' (1984. 198). Also: 'Our relation to logical truths and mathematical structures is inessential to their nature and existence' (1984. 371). The beautiful passage in the preface to *Grundgesetze*, according to which the laws of logic 'are boundary stones set in eternal founda-tions, which our thought can overflow, but never displace' (1893, xvi.), is also relevant here.

of judgeable contents and of names of judgeable contents thus precedes any definition in concept-script.

Frege claims the same kind of objectivity for numbers in the *Foundations*, i.e. independence of the mind, language, stipulations. Corresponding passages are legion (e.g. §26, §60, 62), and it is hard to see how mere precision requirements imposed by system construction could make sense of them. Take the remarkable passage in which he compares the objectivity of number with that of the North Sea, pointing out the independence of both from our arbitrary stipulations (1884, §26).[24] If we were to slightly change the meaning of 'North Sea' today, whatever true content (thought) has been expressed until now by 'The North Sea is 10,000 miles in extent' would not become false. The North Sea is out there, objective, ready to be discovered by us. Presumably, then, there are correct and incorrect definitions of the North Sea, if the North Sea is independent of the definition of 'the North Sea': a correct definition will pick out precisely the North Sea. *Pace* Weiner, what this suggests is that there is an element of discovery in at least a subclass of definitions: the correct ones pick out a pre-existing object in a precise and determinate manner. This is entirely compatible with what Frege says elsewhere about numbers, namely that we discover them in the concepts (1884, §48, §58), and about the similarity between the mathematician and the geographer: neither can create 'things at will; [both] can only discover what is there and give it a name' (1884, §96).[25] It is unclear how Weiner's interpretation can cope with such passages. They are not mentioned in her article.

As is visible from his rejection of empiricism in logic and mathematics, the objectivity Frege claims for logical and arithmetical entities is actually stronger than that of physical entities. Numbers are abstract objects, ready to be recognised by us, but without physical properties, including spatiality and temporality. Our recognition of abstracta is itself situated in time (and presumably space), but abstracta are not, as he explains using the example of the equator. The equator has not been created in the sense that nothing positive could be said about it prior to its alleged creation (1884, §26). This example is not wholly fortunate, since the equator is a dependent abstractum (prior to the creation of the Earth there was indeed nothing positive to say about the equator), but Frege's point is on firm grounds with respect to self-subsistent abstracta like numbers: there is something positive to say about them at all times (with the appropriate linguistic usage in place). This renders the temporalisation of the truth of statements concerning numbers (e.g. '"Numbers are extensions" is not

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[24] Note, and ignore, that Frege's discussion of the arbitrariness of stipulations is confusingly embedded in a discussion against psychologism.

[25] Frege also compares the mathematician with the botanist who determines something objective when he determines the number and colour of a plant's petals. See also (1893, xiii.)

a true statement prior to Frege's definitions formulated in 1879'), as entailed by Weiner's argument, both objectionable and unFregean.

Frege's drive towards a Platonist ontology is also manifest in another respect. In the *Foundations* Frege does not merely provide us with a definition of, say, 0, and content himself with the fact that it satisfies the sharpness requirement. Instead, he gives us various *existence proofs*, e.g. that if 0 is the number of an empty concept *F* and an empty concept *G*, then there must be a relation φ bringing *F* and *G* under one-one correlation (1884, §75), or that there is something which immediately succeeds 0 (1884, §77). Equally, he justifies his diagnosis that the formalists have only introduced empty signs instead of new number-words on their failure to prove the existence of the new numbers (1884, §92ff.). The great importance of existence proofs comes out in the sharp contrast Frege draws, on several occasions, between defining a concept by means of the properties an object must have to fall under the concept and proving that something does fall under the concept (see 1884, §74, 1893, xiv.). This shows that definitions, on his account, can never bring objects into existence, but only specify concepts hitherto lacking a designator.

8 NUMBERS AS EXTENSIONS

Frege's project thus clearly has an ontological aim, to discover what is already there. Weiner's denial of this point even with respect to the *Foundations* is not convincing. She claims that there is no argument in the *Foundations* that numbers are really extensions, and points at §69 and §107 for evidence of this. Concerning §69, she seems to suggest that Frege avoids addressing the ontological question 'Are numbers extensions?', and asks instead a linguistic question, motivated by the context principle, namely whether the assertions which we make about extensions are assertions we can make about numbers. But Frege does not believe that in answering the linguistic question he is eschewing the ontological question, or dismissing it as out of place. On the contrary, his interest in language has an explicit ontological agenda: 'There is no intention of saying anything about the symbols; no one wants to know anything about them, except insofar as some property of theirs directly mirrors some property in what they symbolise' (1884, §24). The context principle, as formulated in the *Foundations*, is not employed as an anti-ontological tool, but is called to dispel the psychologistic prejudice that an expression can stand for an entity only when, taken in isolation, we associate a mental idea with it (1884, §59). Thus the context principle actually serves the ontological agenda: numbers are self-subsistent entities because their expressions can be construed as singular terms, not in isolation, but at least in the sentential contexts of their most paradigmatic arithmetical use, which is also ontologically significant (equations understood as identities).

Frege's remark that he does not attach decisive importance to bringing in extensions (1884, §107) is also not evidence against his ontologism. The opposite is true. Frege brings in extensions in the course of offering his 'explicit' definition of number. This definition is given expressly in response to the 'Julius Caesar' problem, namely that all recognition statements of the form 'N$_\tau F(\tau)$ = a' must have a sense, i.e. that we must decide for any object a whether the statement is true or not (1884, §66-8, §107). But of course, recognition statements are identity statements, and their truth is a criterion for the objecthood of the content of the signs flanking the identity sign (1884, §62, §107). Hence, what Frege is most concerned with here is, again, an ontological issue: to specify, or at least sketch, a logicist criterion for the objecthood of numbers. He brings in extensions of concepts for this, which are logical objects on his account. The wariness he exhibits in §107 is therefore certainly not about the need to import *some* suitable objects to underpin his definition: such an import is essential, not indifferent to his definition of number. We can see this from the defence of his own suggestion that one could write 'concept' instead of 'extension of the concept' in his definition of number (1884, §68fn): by substituting 'concept' for 'extension of the concept' in the definition 'the Number which belongs to the concept F is the extension of the concept "equinumerous to the concept F"' the word 'concept' would be preceded by the *definite article*, and the whole phrase ('the concept') would be thus still a singular term, determining numbers as objects.[26] The wariness is rather about the fact that the phrase 'extension of the concept' is itself left undefined ('presupposed') in the *Foundations*, and hence that the 'Julius Caesar' problem remains open; for to decide whether N$_\tau F(\tau)$ = a we will have to be able to decide sharply whether a is a certain extension. So Frege's wariness has an ontological motivation, that of securing a sharp objecthood criterion for numbers, which is not achieved just by employing the notion 'extension of the concept'. This interpretation is confirmed once we look at Frege's mature solution to the problem, as offered in *Grundgesetze*, where he brings in extensions as value-ranges, not as *defined*, but as *primitive* objects.[27] This is obviously an ontological move, moreover one entirely untouched by the stipulative role of definitions. Weiner's insistence on the stipulative role of definitions, as allegedly ruling out an ontological agenda, is out of focus.

[26] See Frege (1892, 48.) See also the illuminating discussion in (Burge 1984, 274-84.)
[27] See Dummett (1991, 159.)

9 CONCLUSION

I hope to have shown two things in this paper. First, Weiner's interpretation of the notions of '*Foundations*-content', sense, reference and truth is extremely problematic. Second, the obvious requirement and the ontological thesis are very likely correct. Frege is a Platonic realist. He aims to define and analyse pre-existing arithmetical objects and concepts. A full defense of this view would have to look in more detail at the role of definitions in both *Foundations* and *Grundgesetze*, and the relation between the two, which cannot be done here.[28] In any case, I think it is more than probable, given the discussion above, that we have little chance to understand Frege's project of defining number, if we neglect his ontological agenda.

One final remark is called for. While Weiner is wrong in underplaying Frege's ontological agenda, she is surely on more firm ground in stressing the epistemological aim of his project. However, this also needs qualification. Weiner sees Frege realising the epistemological aim by means of a semantic undertaking, the sharpening of pre-systematic arithmetical language. But it is unclear how such a sharpening, by itself, would ever satisfy Frege's Cartesian craving for absolute certainty. Consider his repeated insistence that the *Foundations* have only established a probable thesis (1884, §87, §90), while his demand for total proof aims 'to place the truth of a proposition beyond all doubt' (1884, §2), to give it 'absolute certainty that it contains no mistake and no gap' (1884, §91), to raise the probability that arithmetical truths are analytic and *a priori* to a certainty (1884, §109) etc.[29] Clearly, vagueness of concepts is not the only source of doubt and error; a thinker might have sharpened all his concepts and still not be able to reach more than probable knowledge. '*X* is a sharp concept, but it is uncertain whether *y* falls under *X*' is not incoherent. Adding the gapless proof requirement does not yield the desired certainty either, as we still need to access the unshakeable ground on which the derived propositions rest, the axioms expressing primitive truths (*Urwahrheiten, Urgesetze*, §2-4).[30] Frege has his eyes set on more than just increased conceptual and proof-technical rigour, to be achieved by mere stipulations. Instead, he formulates a programme of genuinely reductive analysis: an arithmetical truth has found its epistemological classification if we can trace its proof back to the primitive truths (1884, §4), whose number we have reduced to a minimum (1884, §2). Since primitive truths are truths evident without further proof, they must involve an

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[28] See my forthcoming article investigating this.

[29] See also his talk about the 'unconditional assurance against a proof or a gap' (1884, §91fn.) and 'the secure ground under our feet (1903, §62).

[30] Frege's simile is that of the 'Unerschütterlichkeit eines Felsblockes', which is best translated as 'unshakeability of a boulder'. This places Frege's metaphor in the gravitational orbit of Descartes' *fundamentum inconcussum*.

indubitable source of knowledge. Hence, Frege's epistemological project has a foundationalist and rationalist agenda. Moreover, there is no tension between Frege's foundationalism and his ontological agenda. On the contrary, the former presupposes the latter. The truths about logical objects are self-evident because of their nature: 'In arithmetic we are not concerned with objects which we come to know as something alien from without through the medium of the senses, but with objects given directly to our reason and, as its nearest kin, utterly transparent to it. And yet, or rather for that very reason, these objects are not subjective fantasies. There is nothing more objective than the laws of arithmetic' (1884, §105). In fact, Platonism is the basis of all knowledge: 'If there were nothing firm, eternal in the continual flux of all things, the world would cease to be knowable, and everything would be plunged in confusion' (1884, vii)[31]. At the ultimate level, epistemological questions are intimately bound with ontological ones.

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