Zoltán Gendler Szabó

Tasks and Ultra-tasks

Abstract. Can we count the primes? There is a near unanimous consensus that in principle we can. I believe the near-consensus rests on a mistake: we tend to confuse counting the primes with counting each prime. To count the primes, I suggest, is to come up with an answer to the question "How many primes are there?" because of counting each prime. This, in turn requires some sort of dependence of outcome on process. Building on some ideas from Max Black, I argue that—barring very odd laws of nature—such dependence cannot obtain.

1 COUNTING THE PRIMES

There are countably many primes but this does not settle the question whether the primes can be counted. "Countably infinite" is a technical term which applies to multitudes equinumerous with the natural numbers. The question I want to pursue here is whether any such multitude can be counted. I will talk about counting the primes but that is just an example—I could talk about counting any countably infinite set.

The obvious difficulty with counting the primes is that even if you could go on forever, it seems there would always remain more to count. But what if you could count faster and faster as you proceed? Then, we are taught, there would be no problem. You could call the first prime within the first 1 minute, the second within the next 1/2 minute, the third within the next 1/4 minute, … the nth within the next $1/2^{n-1}$ minute, … and so on. After 2 minutes you would have counted *each prime*. But would you thereby have counted *the primes*?

There is a difference between counting each prime and counting the primes. If you count the natural numbers you have counted each prime but you have not counted the primes. In general, if all Fs are Gs, then in counting the Gs you count each F but not necessarily the Fs. Should we perhaps say that counting the Fs is counting each F without counting *non-F*s? This won't do, as the following example illustrates. Suppose there is a birthday party at your house for your six year old and you ask him to count his guests. He counts them in the living room, in the dining room, in the kitchen, and so on for all the rooms of the house. He is lucky—the children don't move around and he doesn't miss any. He is also smart—knowing that he is not too good with large numbers, whenever he is done with a room he writes down the result, moves to the next room, and starts the count from 1 again. Now suppose you asked him after all the numbers are written down but before they are added up "Did you count the guests?" The natural response would be "Not yet." Each and only the guests have been counted but the guests have not been counted.

Why not? Because the child doesn't know how many guests there are, one might think. But this is not exactly right: coming to know the number of guests is neither necessary nor sufficient for counting them. If the child makes a mistake in adding up the numbers on the paper he will have counted the guests without coming to know their number; if he is told how many guests there are before he adds up the numbers on the paper he will know the number of guests without having counted them. To count the *F*s, I suggest, is to come up with some sort of (perhaps incorrect, perhaps poorly justified) answer to the question "How many *F*s are there?" *as a result of* counting each *F*.[1] Counting each guest in the usual way *leads to* an answer to the question "How many guests are there?"; counting each guest the way the child did it does not.

I believe the near-consensus that the primes could in principle be counted rests on a mistake: we tend to confuse counting the primes with counting each prime. It may be obvious that we can count each prime in the exponentially accelerating way described above but it is not obvious that this process leads to an answer to the question "How many primes are there?". When we count the primes below 1000 in the usual way, at each step we come to have an answer to the question "How many primes have you already counted?" When we reach what we take to be the largest prime below 1000 we *ipso facto* arrive at an answer to the question "How many primes are there below 1000?" But when we count each prime there is no last step in the count. Accordingly, it is anything but obvious that we can come to have an answer to the question "How many primes are there?" simply as a result of counting each prime one after the other.

The expressions "as a result of" and "leads to" are not among the clearest in our philosophical vocabulary. If I had an analysis of the concepts they express, I would gladly dispense with them. Unfortunately, I have no analysis and I am not optimistic about finding one anytime soon. (In section 3, I will suggest a necessary condition for a process to lead to a state.) One thing is clear: these

[1] This is compatible with the possibility of counting the *F*s when one already has an answer to the question "How many *F*s are there?" We can come up with an answer to a question as a result of a procedure even if we already have an answer to that question.

locutions express some sort of dependence, presumably a causal one, of outcome on process. I will argue that the outcome of an infinite count (having an answer to a certain cardinality question) cannot depend in the right way on the process of counting. If I am right, the conventional wisdom is wrong: the primes (or, for that matter, any other countably infinite multitude) cannot be counted.

2 BLACK'S ARGUMENT

More than half a century ago, Max Black argued that certain infinitary tasks are impossible to perform (Black 1951). When Black's article originally appeared, it produced a flurry of reactions, including Thomson 1954, where the famous lamp is discussed. (Benacerraf 1962) was seen as a decisive refutation of Black's argument, which in turn has largely faded from philosophical discussion. I think the dismissal was too quick: there is something important Black was right about, though it is not exactly what he thought he was right about.

The target of Black's criticism is the standard "mathematical" resolution of Zeno's paradox. According to this line of thought, Achilles can catch up with the tortoise by reaching the place p_0 where the tortoise had started the race, then reaching the place p_1 the tortoise had reached by the time Achilles reached p_0, then reaching the place p_2 the tortoise had reached by the time Achilles reached p_1, … and so on. This amounts to performing an infinite number of tasks: moving first to p_0, then from p_0 to p_1, then from p_1 to p_2, … and so on. If Achilles runs faster than the tortoise, the amount of time it takes to perform all these tasks one after the other is finite. Hence, we are told, there is no problem with catching up with the tortoise in this manner.

Black disagrees. His central complaint is that the "mathematical" resolution of the paradox "tells us, correctly, when and where Achilles and the tortoise will meet, *if* they meet; but it fails to show that Zeno was wrong in claiming they *could not* meet" (Black 1951, 93). Not that anyone should doubt that Achilles can catch up with the tortoise – Black certainly does not. His question is whether Achilles can catch up with the tortoise by performing an infinite series of tasks. Black thinks there is a serious difficulty with that idea "and it does not help to be told that the tasks become easier and easier, or need progressively less and less time in the doing" (Black 1951, 94).

What is the difficulty? Black asks us to imagine a mechanical scoop with an infinitely long narrow tray to its left and another to its right. The scoop is capable of moving marbles from one tray to the other at any finite speed. Let machine *Alpha* work as follows: Initially there are infinitely many marbles in the left tray and the right tray is empty. *Alpha* moves the first marble from left to right in 1 minute and then rests for 1 minute, it moves the second marble from left to right in 1/2 minute and then rests for 1/2 minute, it moves the third marble from left to right in 1/4 minute and then rests for 1/4 minute, … , it moves the *n*th marble from left to right in $1/2^{n-1}$ minute and then rests for $1/2^{n-1}$ minute, … and so on. After 4 minutes, *Alpha* stops. Black argues that it is impossible for *Alpha* to move the marbles from left to right.

Since there is nothing special about moving marbles, if Black is right about *Alpha* then all sorts of infinitary tasks are impossible to perform. Imagine that whenever *Alpha* moves a marble from left to right, Achilles traverses one of the intervals mentioned in the "mathematical" resolution to the paradox. If *Alpha* cannot move the marbles left to right by moving them one by one, then Achilles can presumably also not catch up with the tortoise by traversing the intervals one after the other. *Alpha* is a good stand-in for counting the primes as well. Imagine that a prime is written on each of the marbles and that we call a prime when *Alpha* moves the appropriate marble from left to right. If the primes can be counted at all, then presumably they can be counted in this way—at least, those who think otherwise should explain why counting the primes by calling each prime in an exponentially accelerating way is possible, but moving the marbles from one tray to another by moving each marble in the same exponentially accelerating way is not.[2] I will assume that if *Alpha* cannot move the marbles from left to right, then the primes cannot be counted either.

So why does Black think that *Alpha* cannot move the marbles from left to right? The argument is presented in a somewhat oblique fashion and it can be interpreted in more than one way; I will give my best and most charitable reconstruction. The first thing to note is that in order to move the marbles from left to right, *Alpha* has to bring it about that at some time, the marbles are on the right. (Suppose it is your custom to remove no more than a couple of books at a time from your library to your study. In the evening, you always return them. Still, over the years you have carried each of the hundreds of books from your library into the study at one time or other. Then saying "I have moved the books from the library to the study" would be false, given the most natural reading of this sentence. Similarly, if the marbles are never all on the right then *Alpha* did not move them to the right.[3]) The second thing to note is that *Alpha* cannot bring it about that the marbles are all on the right unless it brings that about

[2] It is not enough to point out that counting is a mental process, and moving marbles a physical one. Some dualists believe that there are mental processes fundamentally different from all physical ones. Even if they are right, it does not follow that infinite counts are among the mental processes that lack a physical analogue. It is beyond doubt that one can model finite counts with finite marble transfers—why think that we cannot model infinite counts with infinite marble transfers?

[3] I don't deny that "move the marbles from left to right" has a distributive reading that is synonymous with "moving each of the marbles from left to right," just as "counting the guests" has a distributive reading that is synonymous with "counting each of the guests." But the dominant readings are the collective ones, and those are the ones that I am concerned with.

when it stops. (Before *Alpha* stops there are always marbles on the left. After it stops it cannot bring anything about.) It follows that *Alpha* cannot move the marbles from left to right unless it can bring it about that they are all on the right when it stops. Black seeks to show that Alpha cannot bring this about.

Black goes on to consider two other machines, *Beta* and *Gamma*. They are similar to *Alpha* but they share their trays and they work in tandem: Initially there is a single marble in the left tray and the right tray is empty. *Beta* moves the marble from left to right in 1 minute while *Gamma* rests, then *Gamma* moves the marble from right to left in 1 minute while *Beta* rests, then *Beta* moves the marble from left to right in 1/2 minute while *Gamma* rests, then *Gamma* moves the marble from right to left in 1/2 minute while *Beta* rests, … , then *Beta* moves the marble from left to right in $1/2^n$ minute while *Gamma* rests, then *Gamma* moves the marble from right to left in $1/2^n$ minute while *Beta* rests, … and so on. After 4 minutes, *Beta* and *Gamma* stop.

We can think of *Alpha*, *Beta* and *Gamma* as having the same kind of task: to bring it about that all the marbles they are working with are on one side when they stop. *Alpha* accomplishes its task just in case it brings it about that the infinitely many marbles that are initially on the left are all on the right when it stops, *Beta* accomplishes its task just in case it brings it about that the single marble that is initially on the left is on the right when it stops, and *Gamma* accomplishes its task just in case it brings it about that the single marble that is initially on the left (but is subsequently moved to the right by *Beta*) is on the left when it stops. Now, we can reason as follows:

(1) Necessarily, *Alpha* accomplishes its task if and only if *Beta* does.
(2) Necessarily, *Beta* accomplishes its task if and only if *Gamma* does.
(3) Necessarily, either *Beta* or *Gamma* does not accomplish its task.

Therefore,

(4) Necessarily, *Alpha* does not accomplish its task.

Alpha's task was to bring it about that the marbles are on the right when it stops. Since it cannot accomplish this task, it cannot bring it about the marbles are on the right, and hence, it cannot move the marbles from left to right. This is Black's argument, as I understand it.

3 IN SUPPORT OF THE ARGUMENT

The argument is clearly valid; the question is whether it is sound. Black supports (1) by emphasizing that *Alpha* and *Beta* are intrinsically the same: if you put them side by side their moves would be entirely parallel. The difference is merely that while *Alpha* transfers an infinite number of qualitatively identical marbles, *Beta* transfers the same marble an infinite number of times. Something similar can be said in defense of (2). While there certainly is an intrinsic difference between *Beta* and *Gamma* – after a move that took $1/2^n$ minute the former rests $1/2^n$ minute, while the latter only $1/2^{n+1}$ minute – this seems negligible in the light of the fact the two machines go through an identical sequence of pairs of moves and rests. The difference is in the direction of the moves and in the order within the pairs: *Beta* moves first and rests afterwards, while for *Gamma* it is the other way around. Regarding (3), Black simply points out that after 4 minutes the marble must end up somewhere, so either *Beta* or *Gamma* must fail to accomplish its task.

How strong these considerations are depends on what kind of necessity is at stake. Physical necessity won't do—it makes the premises as well as the conclusion trivial. The workings of *Alpha*, *Beta*, and *Gamma* are all physically impossible: they require motion faster than the speed of light. Black is clear that he has logical necessity in mind, but that won't do either. It is not logically necessary for the marble to end up in just one of the trays—bilocation of an object is not only possible but perhaps even actual at the micro-physical level.

So Black's argument is uninteresting when the modality is construed as physical necessity and unsound when it is construed as logical necessity. But ordinarily when we wonder whether *Alpha* can accomplish its task we have neither the physical not the logical interpretation of the modal auxiliary in mind—we wonder whether there are more or less homely possible worlds where *Alpha* succeeds. The fact that nothing moves faster than the speed of light in the actual world and the fact that marbles are at different places at the same time in some remote possible worlds are irrelevant to the question we are after. This is not a special feature of the problem at hand: in general when we are asked whether something can be done we are supposed to ignore parochial limitations and far-fetched possibilities.[4] I interpret the question whether *Alpha* can move the marbles from left to right as asking whether it is possible—as we ordinarily understand what is possible in the sorts of contexts set by the description of the machine—for *Alpha* to accomplish its task.

Given the ordinary understanding of necessity (3) is in good shape: if *Beta* and *Gamma* both accomplish their tasks, then the marble is both on the left and on the right, which is certainly a far-fetched possibility. But the other two premises remain problematic even under the ordinary construal of the modality. The problem is that as long as the machines operate independently of each

[4] Suppose someone asks you the following: "Can you swim across this river?" If you say "No, I don't have my goggles with me" you misconstrue the question by failing to ignore parochial limitations. If you say "Yes, but I would need to learn how to swim first" you misconstrue the question by failing to ignore remote possibilities. What counts as parochial limitation or remote possibility depends on the context.

other, something can interrupt the working of one but not the other. It could happen, for example, that before the full 4 minutes elapse someone crushes *Beta* with a hammer. Then *Beta* surely does not accomplish its task but we are given no reason to think that *Alpha* fails too. Alternatively, it might be that while *Gamma* is making one of its moves and *Beta* is at rest, the latter is replaced by a duplicate machine which picks up the moving of the marble where *Beta* left off. Then *Beta* does not accomplish its task but for all we know *Gamma* might. To do justice to the intuition behind Black's argument, we must ignore not only far-fetched possible worlds but also nearby ones where something interferes with the proper functioning of the machines.

Suppose we do that; then the following case can be made for (2). If the machines work uninterrupted, the case of *Beta* and *Gamma* exhibits global symmetry. If the world is not too far-fetched, we should not expect anything to break this symmetry. While the description of the case leaves it open where the marble is located after the full 4 minutes has elapsed, reasonable guesses do not privilege any particular position: the chance that the marble should end up on the left equals the chance that it should end up on the right. Suppose then that the marble ends up on the right. Then *Gamma*, the machine whose task it was to bring it about that the marble ends up on the left, has failed. But *Beta* has not succeeded either. *Beta*'s moves did not raise the chances of the marble ending up on the right above the chances of it *not* ending up there, so the moves did not *bring it about* that the marble ended up on the right.[5] If we assume the marble ends up on the left (or somewhere other than the left or right trays), an analogous argument shows that neither *Beta* nor *Gamma* accomplishes its task.

I think these considerations show that, if we interpret the modality appropriately, premises (2) and (3) of Black's argument come out true: in nearby worlds where their work remains uninterrupted, neither *Beta* nor *Gamma* succeeds. However, many would insist, the case of *Alpha* is different. *Beta*'s work is constantly undone by *Gamma*, and vice versa, while nothing whatsoever interferes with the work of *Alpha*. This might explain how *Beta* and *Gamma* could fail while *Alpha* succeeds. While I don't think this is correct, I agree that (1) is in need of support.

We need a way to bolster the intuition that the makeup and task of *Alpha* and *Beta* are sufficiently similar that if they both work uninterrupted, the possibilities where one succeeds without the other are far-fetched. Here is a somewhat analogous problem. Consider two pebbles *Aleph* and *Beth*, the former being heavier than the latter. Suppose we drop both at the same time from the same height. There is a powerful intuition that *Aleph* will reach the ground

[5] I am assuming here only a necessary (but not sufficient) condition for a process bringing about a state: if the process *P* brings about the state *S*, then the objective chance of *S* holding after *P* is higher than the objective chance of *S* not holding after *P*.

before *Beth*. To combat this intuition, Galileo invited us to consider a case where we tie *Aleph* and *Beth* together and drop them from the same height. Call the tied up object *AlephBeth*. *AlephBeth* can be seen in two different ways—as a unit or as two separate objects. If heavier objects in general fall faster than lighter ones, *AlphBeth* as a unit would have to fall faster than *Aleph* (since its weight exceeds the weight on *Aleph*) but *AlephBeth* as two separate objects would have to fall slower than *Aleph* (since it is held back by the slower *Beth*). But it make no real difference whether we consider *AlephBeth* as one object or two, and so the intuition that *Aleph* reaches the ground before *Beth* must be mistaken.[6]

I will try to follow in Galileo's footsteps and argue that the intuition that *Alpha* might succeed while *Beta* fails is similarly mistaken. Consider *AlphaBeta*, a machine that works as follows. There are two trays on its left—one above the other—and a single tray on its right. *AlphaBeta* moves marbles from the lower left hand tray to the right hand tray. Initially there are infinitely many marbles in the upper left tray, a single marble in the lower left tray, and no marble on the right. Whenever *AlphaBeta* moves a marble, a hole opens up at the end of the upper tray and a single marble drops into the lower tray. *AlphaBeta* moves the marbles in the same pattern as *Alpha* and *Beta*, and it has the same task: to bring it about that all the marbles it is working with are on the right when it stops. It seems to me that (again, ignoring far-fetched possibilities and interruptions) the following claims are true:

(1') Necessarily, *AlphaBeta* accomplishes its task if and only if *Alpha* does.
(1'') Necessarily, *AlphaBeta* accomplishes its task if and only if *Beta* does.

One way to think about *AlphaBeta* is this: it moves infinitely many marbles from left to right in exactly the same pattern as *Alpha*. It makes no real difference whether the marbles that are waiting to be moved on the left will have dropped a bit just before the scoop picks them up. Another way to think about *AlphaBeta* is this: it moves a marble from the lower left tray to the right and while it rests, a marble shows up in the lower left tray, and this happens infinitely many times in the same pattern as with *Beta*. It makes no real difference whether the marble that shows up after each move is taken from the right hand tray or from some other place. But if both ways of thinking are legitimate then (1') and (1'') are true, and since they jointly entail (1), we now have an intuitive justification for the first premise.

In response, one could point out that there are possible laws that could guarantee that one of these machines fails without guaranteeing that the other does. For example, it could be a law that although things can move at any finite

[6] The idea of thinking about Galileo's thought experiment along these lines is from (Gendler 1998).

speed horizontally, they cannot move faster than the speed of light vertically; in a world governed by such a law *AlphaBeta* fails but *Alpha* might still succeed. Alternatively, it could be a law that after a single object oscillates between two locations infinitely many times, it goes out of existence; in a world with such a law, *Beta* fails but *AlphaBeta* might still succeed. But I assume that worlds governed by such recherché laws are beyond the scope of worlds we consider in making ordinary judgments of necessity. Note that Galileo's argument is subject to similar objections: it could be a law that whenever two objects are tied together they fuse into an extended simple, or go out of existence. All we can say in defense of the Galilean thought experiment is that such laws are far-fetched enough to be properly ignored.

I concede that there is a fairly natural law that distinguishes between *AlphaBeta* and *Beta*: the law of the continuity of motion. Let's assume that the law holds in an all nearby worlds. It follows that in nearby worlds, at the moment *Beta* and *Gamma* stop, the marble goes out of existence. (For if it existed, it would presumably have to be at a single location *l*. And at times arbitrarily close to the time when *Beta* and *Gamma* stopped, the marble had been at some fixed distance from *l*, which violates the continuity of motion.) On the other hand, there seems to be nothing that forces any of the marbles *AlphaBeta* moved from the left to the right to go out of existence. So, one might argue, in some nearby worlds *AlphaBeta* succeeds even though *Beta* fails, and so (1'') is false. But the last step in this reasoning is fallacious: even if we grant that when *AlphaBeta* stops the marbles are all on the right, *it does not follow* that *AlphaBeta* accomplished its task. There is a logical gap between the claim that *AlphaBeta* moved each marble from left to right and the claim that it moved the marbles from left to right (just as there is a logical gap between the claim that someone counted each guest and the claim that he counted the guests). Given the intuitive plausibility of (1''), I suggest that in nearby worlds *AlphaBeta* fails even though when it stops, the marbles are all on the right. So, accepting the claim that motion is necessarily continuous does not undermine the argument.

Before moving on, I'd like to restate Black's argument in terms of objective chances; this brings out the role of considerations about laws. Let P_x be a proposition describing the process machine x goes through and let S_x be a proposition describing the state the marbles must be in when x stops if x accomplishes its task. If a state is a result of a process, then the chances of the state holding must be higher than it not holding, given the process.[7] This means that *Alpha* cannot accomplish its task unless:

(5) $$Pr\,(S_{Alpha} \mid P_{Alpha}) > Pr\,(\neg S_{Alpha} \mid P_{Alpha}).$$

[7] I am assuming here only a necessary (but not sufficient) condition for a state holding as a result of a process: if the state *S* holds as a result of the process *P*, then the objective chance of *S* holding after *P* is higher than the objective chance of *S* not holding after *P*.

I am using *Pr* for objective chances set by the laws. Worlds where the laws distinguish between *Alpha* and *Beta* are remote. So, in a world relevant for assessing ordinary necessity, (5) and (6) have the same truth-value:

(6) $$Pr\,(S_{Beta} \mid P_{Beta}) > Pr\,(\neg S_{Beta} \mid P_{Beta}).$$

Now, by the logic of probability, (6) is equivalent to (7):

(7) $$Pr\,(S_{Beta}\ \&\ \neg S_{Gamma} \mid P_{Beta}) + Pr\,(S_{Beta}\ \&\ S_{Gamma} \mid P_{Beta}) > Pr\,(\neg S_{Beta}\ \&\ \neg S_{Gamma} \mid P_{Beta}) + Pr\,(\neg S_{Beta}\ \&\ S_{Gamma} \mid P_{Beta})$$

Since *Beta* and *Gamma* work in tandem they go through the very same process. Accordingly, $Pr\,(S_{Beta}\ \&\ \neg S_{Gamma} \mid P_{Beta})$ is the objective chance the marble ends up on the right and not on the left, given that these machines go through their moves. Similarly, $Pr\,(\neg S_{Beta}\ \&\ S_{Gamma} \mid P_{Beta})$ is the objective chance that the marble ends up on the left, not on the right, given that these machines go through their moves. In worlds governed by normal laws these are the same. So, in those worlds (7) and (8) have the same truth-value:

(8) $$Pr\,(S_{Beta}\ \&\ S_{Gamma} \mid P_{Beta}) > Pr\,(\neg S_{Beta}\ \&\ \neg S_{Gamma} \mid P_{Beta})$$

But (8) is false in worlds governed by normal laws: given that *Beta* and *Gamma* go through their moves, the objective chance that the marble ends up in *both* trays is certainly not higher than the objective chance that it ends up in *neither*. So, in the non-too-distant worlds relevant for assessing ordinary necessity (5) is also false, which means that *Alpha* does not accomplish its task.

4 SUPER-TASKS AND ULTRA-TASKS

The strength of Black's argument has not been widely appreciated. This is, to a large extent, his own fault: he repeatedly misstated its conclusion. He says that the argument shows that it is logically impossible to perform an infinite series of tasks. He even says that "the notion of an infinite series of act is self-contradictory" (Black 1951, 101). The argument, as I presented it, shows no such thing. We are given no reason to think that *Alpha* cannot move *each marble* from left to right—the conclusion is merely that it cannot move *the marbles* from left to right. Suppose *Alpha* moves each of the marbles and after it is done the marbles are all on the right. This could happen. But if it does, the outcome does not come about as a result of the infinite series of tasks *Alpha* performs. Perhaps it happens as a result of some interference, or as a result of some other thing *Alpha* does. Or perhaps it happens not as a result of anything in particular.

The point can be clarified by distinguishing between two notions of an infinitary task. One is that of a *super-task* – a series of tasks of type ω performed in a finite amount of time. The other is that of an *ultra-task* – a single task performed by performing a super-task.[8] Each individual move performed by *Alpha* is a task. If *Alpha* performs each of its moves it performs a super-task. What we earlier called the task of *Alpha* is an ultra-task: moving the marbles from left to right by performing this super-task. What Black's argument shows is that *Alpha* cannot perform this ultra-task.[9] And since there is nothing special about *Alpha*, a reasonable conjecture is that the same holds for ultra-tasks *tout court*: without there being some very odd laws, they cannot be performed at all.

The literature that followed Black's paper tended to focus on the question whether super-tasks are possible. The consensus appears to be that they are—and I share this view. But can one perform some other task by performing a super-task in the sense in which one can cross the street by making a series of steps or one can draw a picture by connecting a series of points? Considerations inspired by Black suggest a negative answer to this latter question. Super-tasks are *possible* but they are *inert*—when you perform a super-task you cannot thereby perform something over and above the individual tasks included in the super-task.

Hercules's second labor was to kill the Lernean Hydra, a nine-headed monster. The Hydra was a tough opponent: whenever Hercules cut off one of its heads, two new heads grew in its place. Killing such a creature by exponentially accelerating decapitation is an ultra-task, and as such, it is impossible to perform.[10] This is not to say that the supertask of cutting off infinitely many heads cannot *in principle* be performed, or that it is impossible for the Hydra to end up dead after this super-task is performed. Remarkable though it is, all this can happen: after the number of the Hydra's heads grows steadily beyond any finite limit, the Hydra suddenly finds itself headless. But this would not be a killing of the Hydra: the beast would not die as a result of what Hercules did. Setting aside the possibility of strange laws, some intervening force or miracle was also needed. Or perhaps its death was not the result of any process whatsoever.

[8] The notion of a *super-task* is due to (Thomson 1954). (Benacerraf 1962) introduced the notion of a *super-duper-task*: a series of tasks of type ω + 1. Peter Clark and Stephen Read (1984) suggested the notion of a *hyper-task*: series of uncountably many tasks. While I am almost out of adjectives, an *ultra-task* is fundamentally different from all of these. It is not a series of tasks, just one task—constituted by infinitely many.

[9] I don't think Black saw clearly the distinction between super-tasks and ultra-tasks: this is the fundamental confusion in his paper.

[10] According to Apollodorus's account, Hercules manages to kill the Hydra with the help of his trusted nephew Iolaus. Every time Hercules cut off one of the heads, Iolaus held a torch to the stump preventing the growth of the new heads. So, Hercules killed the Hydra by performing a finite series of tasks.

Catching up with the tortoise by traversing infinitely many distinct intervals would also be an ultra-task—something that cannot be done. Of course, Achilles can catch up with the tortoise and —contra Black—he can also traverse infinitely many distinct intervals in a finite time. What Achilles cannot do is perform the former task by performing the latter. The "mathematical" resolution of Zeno's paradox is wrong, even though both common sense and mathematics are vindicated. If you want an answer to the question "In virtue of what does Achilles catch up with the tortoise? (and it is by no means clear that you should want an answer to such a question) you need to appeal to something other than a super-task he performs. You can, for example, truthfully say that Achilles caught up with the tortoise by running faster that the animal, or by moving his legs one after the other, or by traversing a distance of some specific length.

5 TELICITY

I have argued that ultra-tasks are impossible (although not logically impossible) to perform. We can gain a better understanding of why this is so by examining what tasks are.

Let's start with some examples. Catching up with a tortoise, moving a marble from one tray to another, killing the Hydra—these are all tasks. One thing they all have in common is that they all happen *in* a time and are not going on *for* a time. For example, we say that Achilles caught up with the tortoise *in* two minutes, but not that he did that *for* two minutes. By contrast, we say that Achilles ran *for* an hour, but not that he ran *in* an hour. This is the classic test for telicity: it shows that 'caught up with a tortoise' a *telic* verb phrase and 'run' an *atelic* one.

Tasks are events described by telic verb phrases. Such events include a result, or *telos*, whose obtaining marks the end of the task. The *telos* of catching up with a tortoise is being lined up with the tortoise, the *telos* of moving a marble from one tray to another is for that marble to be in the latter tray, the *telos* of killing the Hydra is for the Hydra to be dead, and so on. I suggest that tasks are compound events consisting of a process leading up to a result state. Telic verb phrases describing a single task[11] say of an object that it is involved in a certain process that led to its *telos*. To catch up with a tortoise is to be doing something in a way that leads to being lined up with the tortoise, to move a marble from one tray to another is to be moving the marble from one tray in a way that leads to its being in the other, to kill the Hydra is to be killing it in a way that leads to its death, and so on. The phrase "counting the guests" in its most natural reading

[11] A verb phrase can be used to describe a single event or a multitude of events. 'John traveled in four countries' can mean that there was an event of him traveling in four countries or that four countries are such that there was an event of him traveling in them.

describes a single task; its *telos* is having an answer to the question "How many guests are there?" and the counting process leads up to the state of having this answer.

There is a distinction due to (Vendler 1967) within the category of telic verb phrases between *accomplishments* and *achievements*. The former are said to describe events that are extended in time (like stealing a car or building a house), and the latter events that are near-instantaneous (like reaching the peak or finding a key). It is sometimes said that achievement verb phrases don't allow the progressive, but this is not a reliable criterion (e.g. 'Jack was finding his key' is indeed odd, but 'Jill was reaching the peak is just fine.) I think the real difference between them is that in the case of accomplishments, the process that leads up to the *telos* is properly described with the *progressive from of the verb phrase*, while this is not so in the case of achievements. So if Mary crossed the street (accomplishment) then she was crossing it and this particular process led to her being across the street. But if Mary got across the street (achievement), then something was going on—perhaps she was crossing the street, perhaps she was being carried by someone else, perhaps she was being teleported, it does not matter—and this process, whatever it was, led to her being across the street.

If these ideas are on the right path, then it is part of the meaning of telic verb phrases that a process leads to its natural result.[12] Moving infinitely many marbles from the left hand tray to the right hand tray means moving them from the left hand tray in a way that leads to their being in the right hand tray. This is no doubt possible: one could pick up all the marbles at once from the left hand tray and place them in the right hand one. But moving each of the marbles individually would be a super-task and, I argued, such a super-task does not lead to the marbles being in the right hand tray. The marbles may each be moved from the left and they may all end up on the right, but the latter would not happen as a result of the former. It may happen as a result of something else or not as a result of anything at all.

To count the *F*s, I suggested, is to come up with an answer to the question "How many *F*s are there?" *as a result* of counting each. Now we can see that this suggestion is a consequence of a general semantic thesis about telic verb phrases and a specific proposal about the *telos* of counting. Counting the guests at a birthday party is a straightforward task; counting the primes is not. The latter is an ultra-task, and as such, impossible. You could, in principle, count each prime and at the end come to have the answer that there are infinitely many of them. But your counting would not lead to your having that answer. The "countable" multitudes cannot be counted after all.*

[12] For a sketch of a semantic account along these lines see (Szabó 2004) and (Szabó 2008).

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