A&A 447, 23–30 (2006)
DOI: 10.1051/0004-6361:20041129

Astronomy & Astrophysics

A new definition of the intermediate group of gamma-ray bursts

I. Horváth[1], L. G. Balázs[2], Z. Bagoly[3], F. Ryde[4], and A. Mészáros[4,5,6]

[1] Department of Physics, Bolyai Military University, Budapest, Box-12, 1456, Hungary
e-mail: horvath.istvan@zmne.hu
[2] Konkoly Observatory, Budapest, Box-67, 1525, Hungary
e-mail: balazs@konkoly.hu
[3] Laboratory for Information Technology, Eötvös University, Budapest, Pázmány P. s. 1/A, 1117, Hungary
e-mail: bagoly@ludens.elte.hu
[4] Stockholm Observatory, AlbaNova, 106 91 Stockholm, Sweden
e-mail: felix@astro.su.se
[5] Astronomical Institute of the Charles University, V Holešovičkách 2, 180 00 Prague 8, Czech Republic
e-mail: meszaros@mbox.cesnet.cz
[6] Max Planck Institute for Astrophysics, Garching, Karl-Schwarzschild-Str. 1, Postfach 1317, 85741 Garching, Germany

Received 21 April 2004 / Accepted 25 September 2005

ABSTRACT

Gamma-ray bursts can be divided into three groups ("short", "intermediate", "long") with respect to their durations. This classification is somewhat imprecise, since the subgroup of intermediate duration has an admixture of both short and long bursts. In this paper a physically more reasonable definition of the intermediate group is presented, using also the hardnesses of the bursts. It is shown again that the existence of the three groups is real, no further groups are needed. The intermediate group is the softest one. From this new definition it follows that 11% of all bursts belong to this group. An anticorrelation between the hardness and the duration is found for this subclass in contrast to the short and long groups. Despite this difference it is not clear yet whether this group represents a physically different phenomenon.

Key words. gamma-rays: bursts – cosmology: miscellaneous

1. Introduction

It is a great challenge to classify gamma-ray bursts (GRBs) into classes. Mazets et al. (1981) and Norris et al. (1984) suggested there might be a separation in the duration distribution. Using the First BATSE Catalog, Kouveliotou et al. (1993) found a bimodality in the distribution of the logarithms of the durations. This bimodality is highly pronounced, if one uses the parameter T_{90} (the time during which 90% of the fluence is accumulated, Kouveliotou et al. 1993) to characterize the durations of GRBs (McBreen et al. 1994; Koshut et al. 1996; Belli 1997; Pendleton et al. 1997). Today it is widely accepted that the physics of these two groups (also called "subclasses" or simply "classes") are different, and these two kinds of GRBs are different phenomena (Norris et al. 2001; Balázs et al. 2003). The high redshifts and the cosmological distances are directly confirmed for the long bursts only, while for the short ones there is only indirect evidence for their cosmological origin (Mészáros 2001, 2003).

Using the Third BATSE Catalog (Meegan et al. 1996) Horváth (1998) has shown that the distribution of the logarithms of the durations of GRBs ($\log T_{90}$) could be well fitted by a sum of three Gaussian distributions. He finds it statistically unlikely (with a probability $\sim 10^{-4}$) that there are only two groups. Simultaneously Mukherjee et al. (1998) report the finding (in a multidimensional parameter space) of a very similar group structure of GRBs. Somewhat later several authors (Hakkila et al. 2000; Balastegui et al. 2001; Rajaniemi & Mähönen 2002; Hakkila et al. 2003; Borgonovo 2004; Hakkila et al. 2004) included more physical parameters into the analysis of the bursts (e.g. peak-fluxes, fluences, hardness ratios, etc.). A cluster analysis in this multidimensional parameter space suggests the existence of the third ("intermediate") group as well (Mukherjee et al. 1998; Hakkila et al. 2000; Balastegui et al. 2001; Rajaniemi & Mähönen 2002). The physical existence of the third group is, however, still not convincingly proven. For example, Hakkila et al. (2000) believe that the third group is only a deviation caused by a complicated instrumental effect, which can reduce the durations of some faint long bursts. Later Hakkila et al. (2003) published another paper which had different conclusions (we discuss this greater detail later). However, the celestial distribution of the third group is anisotropic (Balázs et al. 1998, 1999; Mészáros et al. 2000a,b; Litvin et al. 2001); i.e. different from that of the long GRBs alone (Mészáros & Štoček 2003). The $\log N - \log S$ distribution may also differ from those of the other groups (Horváth 1998). Taken together this means that the existence of the third intermediate group is acceptable, but its physical meaning, importance and origin is less clear than those of the other groups. Hence, its further study is required.

Using Principal Component Analysis (PCA), Bagoly et al. (1998) have shown that there are only two major quantities necessary (called the Principal Components; PCs) to characterize most of the properties of the bursts in the BATSE Catalog. Consequently, the problem of the choice of the relevant parameters describing GRBs is basically a two-dimensional problem. For the statistical analysis the choice of two independent parameters is enough; they may be, but are not necessarily, the two principal components. This means that only two parameters, relevantly chosen, should be enough for the classification and determination of the groups. Concluding from the analysis of the clustering properties of GRBs in the BATSE 3B Catalog Mukherjee et al. (1998) identified the following measured quantities relevant for classification: duration (T_{90}), total fluence ($F_{\rm tot} = F_1 + F_2 + F_3 + F_4$) and hardness ($H_{321} = \frac{F_3}{F_1+F_2}$). ($\log H_{321}$ is highly redundant with $\log H_{32}$ ($= \log F_3 - \log F_2$) which is a linear combination of the two PCs mentioned above.)

In order to perform a statistical analysis to estimate the probable number of classes Mukherjee et al. (1998) made the apriori assumption that the observed BATSE sample is a superposition of multivariate Gaussians in the variables included in the analysis. Concerning $\log T_{90}$ Horváth (1998) showed that its distribution could be well fitted with three Gaussians. Recently, Balázs et al. (2003) has proven that the intrinsic distributions of the total fluence and duration were two dimensional Gaussians for the long and short GRBs, separately. The Gaussian fit for the observed distribution of the total fluence of long bursts, however, was poor due to the effect of the luminosity distance. The dependence of the observed fluence distribution on the luminosity distance might result in "ghost clusters" when attempting to fit with Gaussians. In the contrary, the effect of the luminosity distance was eliminated when computing hardness.

Fitting the observed distribution with the superposition of Gaussian components one had to keep the number of estimated parameters as small as possible to ensure the stability of the Maximum Likelihood procedure (e.g. in case of two dimensions and 4 components the number of parameters is 23 while the same in 3 dimensions is 39). Summarizing all these considerations we decided to use two dimensional Gaussians with the logarithmic duration ($\log T_{90}$) and hardness ($\log H_{321}$ or $\log H_{32}$, alternatively).

Based on this technique several questions should be answered concerning the intermediate group. First, will the statistical analysis, using only these two parameters, reconfirm the existence of the intermediate group? Second, if this question is answered in the affirmative, then one has to show that either further groups exist, or they do not. Using a much smaller sample Mukherjee et al. (1998) claim that only three groups are necessary. On the other hand, Cline et al. (1999) propose the existence of a fourth subgroup of very short durations. Third, one also has to define the quantities by which this third group is different. Fourth, the method – making it possible to assign a certain GRB to a given group – should also be developed. Fifth, the fraction of this third intermediate group in the whole BATSE Catalog should also be determined more exactly. Sixth, does the intermediate group really represent a third type of bursts different from both the short and long ones in its astrophysical origin?

The observational data from The Current BATSE GRB Catalog (Meegan et al. 2001) will be used to answer these questions in which there are 2702 GRBs, for 1956 of which both the hardnesses and durations are measured. The paper is organized as follows: Sect. 2 briefly summarizes the mathematics of the two-dimensional fits. Section 3 deals with these fits in the two-dimensional parameter space and confirms the reality of the intermediate group. Section 4 gives the mathematical definition of the intermediate group making it possible to determine, for any GRB, the probability that it belongs to a given group and deals with possible observational bias. Section 5 discusses the physical differences between the classes. Section 6 summarizes the conclusions of this paper.

2. Mathematics of the two-dimensional fit of *k* classes

We will study the distribution of GRBs in the $\{\log T_{90}; \log H_{32}\}$ plane. Previously, Belli (1997) used this plane to separate the bursts. She suggested that the curve $H_{32} = 2T_{90}^{0.5}$ gave a better division than the cut $T_{90} = 2$ s between the short and long GRBs.

We can assume that the observed probability distribution of the GRBs in this plane is a superposition of the distributions characterizing the different types of bursts present in the sample. Introducing the notations $x = \log T_{90}$ and $y = \log H_{32}$ and using the law of full probabilities (Rényi 1962) we can write

$$p(x,y) = \sum_{l=1}^{k} p(x,y|l) p_l. \quad (1)$$

In this equation $p(x,y|l)$ is the conditional probability density assuming that a burst belongs to the lth class. p_l is the probability for this class in the observed sample ($\sum_{l=1}^{k} p_l = 1$), where k is the number of classes. In order to decompose the observed probability distribution $p(x,y)$ into the superposition of different classes we need the functional form of $p(x,y|l)$. The probability distribution of the logarithm of durations can be well fitted by Gaussian distributions, if we restrict ourselves to the short and long GRBs (Horváth 1998). We assume the same also for the y coordinate. With this assumption we obtain, for a certain lth class of GRBs,

$$p(x,y|l) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-r^2}} \times \exp\left[-\frac{1}{2(1-r^2)}\left(\frac{(x-a_x)^2}{\sigma_x^2} + \frac{(y-a_y)^2}{\sigma_y^2} - \frac{C}{\sigma_x\sigma_y}\right)\right], \quad (2)$$

where $C = 2r(x-a_x)(y-a_y)$; a_x, a_y are the means, σ_x, σ_y are the dispersions, and r is the correlation coefficient

Table 1. Results of the EM algorithm in the $\{\log T_{90}; \log H_{32}\}$ plane. $k = 2$, $L_{\max} = 1193$.

l	p_l	a_x	a_y	σ_x	σ_y	r
1	.280	−.233	.740	.541	.259	.049
2	.720	1.488	.396	.471	.237	.128

Table 2. Results of the EM algorithm. $k = 3$, $L_{\max} = 1237$.

l	p_l	a_x	a_y	σ_x	σ_y	r
1	.245	−.301	.763	.525	.251	.163
2	.109	.637	.269	.474	.344	−.513
3	.646	1.565	.427	.416	.210	−.034

(Trumpler & Weaver 1953, Chap. 1.25). Hence, a certain class is defined by 5 independent parameters, $a_x, a_y, \sigma_x, \sigma_y, r$, which are different for different l. If we have k classes, then we have $(6k - 1)$ independent parameters (constants), because any class is given by the five parameters of Eq. (2) and the weight p_l of the class. One weight is not independent, because it holds $\sum_{l=1}^{k} p_l = 1$. The sum of k functions defined by Eq. (2) gives the theoretical function of the fit. In Balázs et al. (2003) this fit for $k = 2$ was used, and the procedure for $k = 2$ was described in more detail. However, that paper used fluence instead of hardness. Here we will make similar calculations for $k = 3$ and $k = 4$.

3. New confirmation of the intermediate group

By decomposing $p(x, y)$ into the superposition of $p(x, y|l)$ conditional probabilities one divides the original population of GRBs into k groups, at least from the mathematical point of view. Decomposing the left-hand side of Eq. (1) into the sum of the right-hand side, one needs the functional form of $p(x, y|l)$ distributions, and also k has to be fixed. Because we assume that the functional form is a bivariate Gaussian distribution (see Eq. (2)), our task is reduced to evaluate its parameters, k and p_l.

In order to find the unknown constants in Eq. (2) we use the Maximum Likelihood (ML) procedure of parameter estimation (Balázs et al. 2003). Assuming a set of N observed $[x_i, y_i]$, $(i = 1, \ldots, N)$ values (N is the number of GRBs in the sample for our case, which here is 1956) we can define the Likelihood Function in the usual way, after fixing the value of k, in the form

$$L = \sum_{i=1}^{N} \log p(x_i, y_i), \tag{3}$$

where $p(x_i, y_i)$ has the form given by Eq. (1). Similarly, as it was done by Balázs et al. (2003), the EM (Expectation and Maximization) algorithm is used to obtain the $a_x, a_y, \sigma_x, \sigma_y, r$ and p_l parameters at which L reaches its maximum value. We made the calculations for different values of k in order to see the improvement of L as we increase the number of parameters to be estimated.

Tables 1–3 summarize the results of the fits for $k = 2, 3, 4$.

Table 3. Results of the EM algorithm. $k = 4$, $L_{\max} = 1243$.

l	p_l	a_x	a_y	σ_x	σ_y	r
1	.234	−.307	.752	.524	.246	.215
2	.060	.441	.426	.637	.440	−.871
3	.060	.623	.262	.325	.325	−.095
4	.646	1.569	.426	.410	.211	−.034

Table 4. Results of the EM algorithm in the $\{\log T_{90}; \log H_{321}\}$ plane. $k = 2$, $L_{\max} = 920$.

l	p_l	a_x	a_y	σ_x	σ_y	r
1	0.276	−0.251	0.544	0.531	0.256	0.016
2	0.725	1.479	0.132	0.479	0.287	0.123

The confidence interval of the parameters estimated can be given on the basis of the following theorem. Denoting by $L_{\max}$ and L_0 the values of the Likelihood Function at the maximum and at the true value of the parameters, respectively, one can write asymptotically as the sample size $N \rightarrow \infty$ (Kendall & Stuart 1976–1979),

$$2(L_{\max} - L_0) \approx \chi^2_m, \tag{4}$$

where m is the number of parameters estimated ($m = 6k - 1$ in our case), and χ^2_m is the usual m-dimensional χ^2 function (Trumpler & Weaver 1953). Moving from $k = 2$ to $k = 3$ the number of parameters m increases by 6 (from 11 to 17), and $L_{\max}$ grows from 1193 to 1237. Since $\chi^2_{17} = \chi^2_{11} + \chi^2_6$ the increase in $L_{\max}$ by a value of 44 corresponds to a value of 88 for a χ^2_6 distribution. The probability for $\chi^2_6 \geq 88$ is extremely low ($<10^{-10}$), so we may conclude that the inclusion of a third class into the fitting procedure is well justified by a very high level of significance.

Moving from $k = 3$ to $k = 4$, however, the improvement in $L_{\max}$ is only 6 (from 1237 to 1243) corresponding to $\chi^2_6 \geq 12$, which can happen by chance with a probability of 6.2%. Hence, the inclusion of the fourth class is *not* justified. We may conclude from this analysis that the superposition of three Gaussian bivariate distributions – and *only these three ones* – can describe the observed distribution.

This means that the 17 independent constants for $k = 3$ in Table 2 define the parameters of the three groups. We see that the mean hardness of the intermediate class is very low – the third class is the softest one. Because $p_2 = 0.109$, 11% of all GRBs belongs to this group. This value is very close to those found previously (Mukherjee et al. 1998; Horváth 1998; Hakkila et al. 2000; Horváth 2002; Rajaniemi & Mähönen 2002; Horváth et al. 2004).

To test the robustness of the groups found by using this procedure we also repeated the calculations in the $\{\log T_{90}; \log H_{321}\}$ plane. Comparing the maximum values of the likelihood function (920, 980, 982) obtained by assuming $k = 2$, 3 and 4 components it is clear from Tables 4–6 that 3 Gaussian distributions are necessary and sufficient to account for the GRB sample studied ($L^3_{\max} - L^2_{\max} = 60$, $L^4_{\max} - L^3_{\max} = 2$).

Table 5. Results of the EM algorithm. $k = 3$, $L_{\max} = 980$.

l	p_l	a_x	a_y	σ_x	σ_y	r
1	0.233	−0.354	0.560	0.486	0.237	0.082
2	0.154	0.722	0.057	0.480	0.432	−0.356
3	0.613	1.588	0.174	0.404	0.249	−0.048

Table 6. Results of the EM algorithm. $k = 4$, $L_{\max} = 982$.

l	p_l	a_x	a_y	σ_x	σ_y	r
1	0.234	−0.354	0.559	0.485	0.238	0.078
2	0.148	0.704	0.062	0.447	0.432	−0.335
3	0.333	1.580	0.115	0.403	0.268	−0.141
4	0.284	1.600	0.236	0.400	0.214	0.064

Comparison of the results obtained in the $\{\log T_{90}; \log H_{32}\}$ and $\{\log T_{90}; \log H_{321}\}$ planes show that the parameters of the Gaussian distributions match each other well in the $x = \log T_{90}$ coordinate (see Tables 2 and 5).

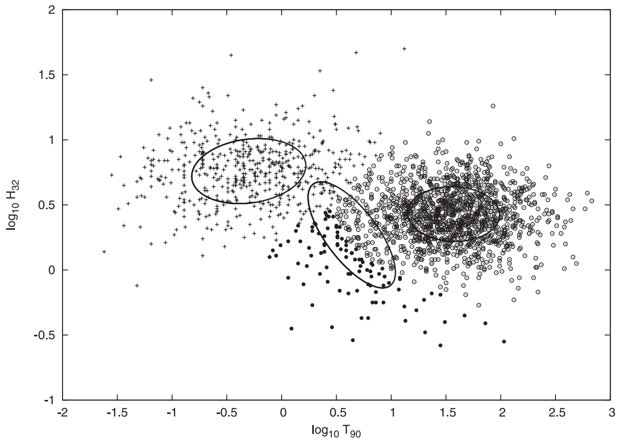

Fig. 1. Distribution of $N = 1956$ GRBs in the $\{\log T_{90}; \log H_{32}\}$ plane. The 1σ ellipses of the three Gaussian distributions are also shown, which were obtained in the ML procedure. The different symbols (crosses, filled circles and open circles) mark bursts belonging to the short, intermediate and long classes, respectively.

4. Mathematical classification of GRBs

4.1. The method

Based on the calculations in the previous paragraph we resolved the $p(x, y)$ probability density of the observed quantities into a superposition of three Gaussian distributions. Using this decomposition we can classify *any* observed GRB into the classes represented by these groups. In other words, we develop a method allowing us to obtain, for any given GRB, its three membership probabilities, which define the likelihood of the GRB to belong to the short, intermediate and long groups. The sum of these three probabilities is unity. For this purpose we define the following $I_l(x, y)$ indicator function, which assigns to each observed burst a membership probability in a given l class as follows:

$$I_l(x, y) = \frac{p_l p(x, y|l)}{\sum_{l=1}^{k} p_l p(x, y|l)}. \quad (5)$$

According to Eq. (5) each burst may belong to any of the classes with a certain probability. In this sense one cannot assign a given burst to a given class with certainty, but with a given probability. This type of classification is called a "fuzzy" classification (McLachlan & Basford 1988). Although, any burst with a given $[x, y]$ could be assigned to all classes with a certain probability, one can select that l at which the $I_l(x, y)$ indicator function reaches its maximum value. Figure 1 shows the distribution of GRBs in the $\{\log T_{90}; \log H_{32}\}$ plane, in which the classes obtained in this way are marked by different symbols. The 1σ ellipses of the three Gaussian distributions are also shown.

4.2. Application of the fuzzy classification

Inspecting Fig. 1 one can recognize immediately that the domain within the ellipse of the intermediate group is only partly populated by GRBs belonging to this class according to the classification procedure described above. The remaining part is dominated by GRBs classified as short and, in particular, as long. In other words, the ellipse of the third group contains an essential amount of GRBs, which should belong either to the long group or to the short group. Due to the "fuzzy" classification some probability was also assigned to the other classes. Based on the analytical expressions of the components, one can easily calculate the contribution of any other groups within the ellipse of a given class by summing the $I_l(x, y)$ values of different l-s for the bursts lying in this particular region.

The reliability of the classification can be characterized by counting the different classes of the GRBs lying within the 1σ ellipse of a given Gaussian component. If the classification were correct, only those GRBs would lie within the ellipse of a given l that have classes corresponding to this component. Denoting by n_l the number of GRBs within the ellipse belonging to class l one gets $n_1 = 218$, $n_2 = 174$, $n_3 = 514$. The rows of Table 7 give the number of GRBs of all classes within the 1σ ellipses of the short, intermediate and long Gaussian components. The first row shows that in the ellipse that defines the short group, there are 218 GRBs. In accordance with the fuzzy classification all have the highest probability assigning them to the short group. Similarly, the third row shows that in the ellipse, which defines the long group, there are 514 GRBs. All these, in accordance with the fuzzy classification, have the highest probability assigning them to the long group. But in the second row, which defines the 174 GRBs in the ellipse defining the intermediate group, only 47 bursts have the highest probability assigning them to the intermediate group. A further 21 (106) GRBs should belong to the short (long) class.

Table 7 demonstrates that the classifications of the short and long GRBs are very reliable, since they do not overlap the other two classes. This means that GRBs within the ellipse of the first and third class (first and third row in Table 7) were well classified as short and long, respectively. In contrast, the ellipse of the intermediate component (second row) contains a

Table 7. Number of GRBs classified by the procedure described in the text, within the 1σ ellipses of $l = 1, 2, 3$ Gaussian components.

l	Short	Interm.	Long	Total
1.	218	–	–	218
2.	21	47	106	174
3.	–	–	514	514
Total	239	47	620	906

Table 8. Cross tabulation of GRBs classified in the $\{\log T_{90}; \log H_{321}\}$ (Class_{321}) and the $\{\log T_{90}; \log H_{32}\}$ (Class_{32}) plane, respectively.

	Class_{321}			
Class_{32}	Short	Interm.	Long	Total
short	474	42	4	520
interm.	–	98	1	99
long	–	89	1247	1336
Total	474	229	1252	1955

significant number of members of the two other classes, in particular of the long group. This is caused predominantly by the closeness of the most numerous long class to the intermediate one.

There are $N - (n_1 + n_2 + n_3) = 1050$ GRBs scattered over a much larger area outside the ellipses. In this region the Gaussian components have low probabilities. The indicator function can still have a large value, however, because there are small numbers in both the nominator and denominator of the right-hand side of Eq. (5). Although the classification of these bursts is formally correct, it is less reliable than those within the ellipses.

We demonstrated the robustness of classification by comparing the results obtained from the $\{\log T_{90}; \log H_{321}\}$ and the $\{\log T_{90}; \log H_{32}\}$ planes, respectively. A cross tabulation between these two classifications is given in Table 8. One may infer from this cross tabulation that the short and long classes correspond within about 10% to the respective groups obtained from the other classification. Consequently, the robustness of the short and long group is well established. On the contrary, the population of the intermediate group is much poorer in classifying in the $\{\log T_{90}; \log H_{32}\}$ plane than in the other one. Table 8 clearly shows that classification Class_{321}, except for one, contains all GRBs assigned to the intermediate group by Class_{32}.

Classification in the $\{\log T_{90}; \log H_{321}\}$ plane indicated 42 GRBs from the short and 89 ones from the long groups, respectively. This high number of indicated bursts clearly shows that a slight variation of the parameters of the Gaussian distribution representing the intermediate group results in a drastic change in the number of classified objects in this group. However, comparing the fraction of GRBs belonging to the intermediate group according to Tables 2 and 5 one gets figures of 213 and 294, respectively.

If one assigned the burst to that group that had the maximum membership probability a slight change in the parameters of the corresponding Gaussian distribution may move the GRB to an other group. On the contrary, the fuzzy classification assigns membership probability to all of the bursts. Hence, a small variation of the parameter gives a small variation in the estimated number of bursts in the intermediate group obtained by summing the membership probabilities of all GRBs in the sample.

4.3. Effect of observational bias on the classification

Performing several classification techniques on the whole BATSE GRB sample indicates the intermediate group with a high certainty. Hakkila et al. (2003) claimed the structure of the BATSE sample identifying the intermediate group is due to a special kind of observational bias. They pointed out that it is reasonable to assume that observations of faint, long GRBs detected only the brightest part of the burst and a significant fraction was buried in the background noise. It also means an underestimation of the true duration. As a consequence the faint long bursts appear to be softer and shorter than in reality. This effect could produce the intermediate group in the sample. Detailed study of this effect, however, has proven that the existence of the intermediate group cannot be accounted for by it.

Hakkila et al. (2003) studied a further possibility which might be the reason for the existence of the intermediate group. The detection of the bursts proceeds on three timescales: 64 ms, 256 ms and 1024 ms. To record a GRB the count rate of the peak-flux has to exceed the detection threshold on at least one of these time scales. A slow faint burst, which emitted the same amount of energy as a shorter one, might be missed by the observation since the peak-flux event on the longest 1024 ms timescale was less that that of a faster one. Supposing a Gaussian distribution for the logarithmic duration of the long bursts, truncation of the slow faint GRBs results in a relative overabundance of those that lie in the short duration wing. Fitting this truncated distribution with Gaussian distributions one obtains an additional component accounting for the enhancement at the short duration wing.

In the case of bursts where the duration is shorter than the time scale of detection there is a one to one correspondence between the peak-flux and the total number of counts observed. As a consequence, the fluence and the peak-flux on this time scale are identical within a conversion factor. Let us suppose, in addition to the 64, 256 and 1024 ms timescales, we have a further one which is longer than the longest burst in the BATSE sample. Figure 2 shows the relationship between the $\log T_{90}$ duration and the σ_{F32} error of the $F_{32} = F_3 + F_2$ fluence. The horizontal dashed line marks the expected mean error of the longest burst in the sample.

Let us take a hypothetic detection timescale as long as the longest GRB in our sample. The detection is successful if the fluence is greater than 5.5-times the noise level (this is the usual BATSE trigger criterion). We marked this level by the horizontal line in the top panel of Fig. 3. A burst fulfilling this criterion would be detected independently of exceeding the trigger level on the other time scales. Following the idea of Hakkila et al. (2003) we introduced a dual timescale from 1024 ms and the longest duration in the BATSE Catalog (800 s), in contrast

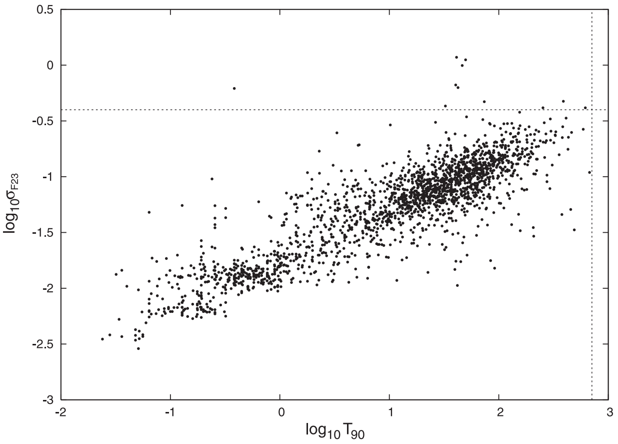

Fig. 2. Relationship between $\log T_{90}$ and the $\log \sigma_{F32}$ error of the F_{32} fluence. The vertical dashed line indicates the duration of the longest bursts in the sample. The horizontal dashed line marks the expected value of $\log \sigma_{F32}$ at the longest duration.

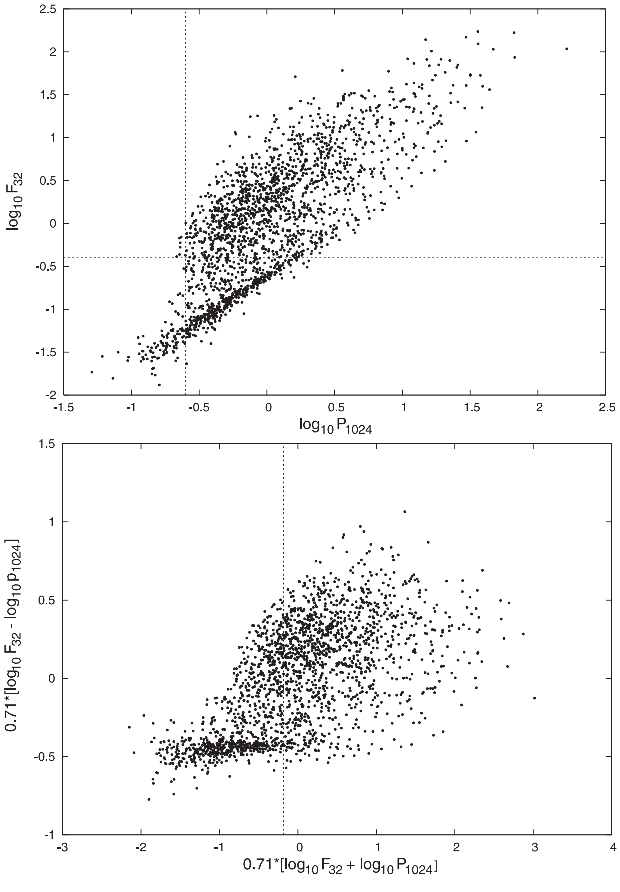

Fig. 3. Distribution of the GRBs of the BATSE Current Catalog in the $\{\log_{10} P_{1024}; \log_{10} F_{32}\}$ plane. Vertical dashed line indicates the trigger level of the P_{1024} peak flux and the horizontal one marks the expected value of $5.5 \times \sigma_{F32}$ at the longest duration (*top panel*). Distribution of points in the top panel after 45° rotation (*bottom panel*). Vertical dashed line shows the limit of completeness on the dual time scale defined in the text.

to 10 000 s of Hakkila et al. (2003). The difference between the two timescales may have an impact on the final classification.

Table 9. Results of the EM algorithm on the truncated sample. $k = 2$, $L_{\max} = 1152$. (Details of the truncation are described in the text.)

l	p_l	a_x	a_y	σ_x	σ_y	r
1	0.194	0.562	0.290	0.697	0.380	−0.574
2	0.806	1.631	0.178	0.391	0.238	−0.024

Table 10. Results of the EM algorithm on the truncated sample. $k = 3$, $L_{\max} = 1172$.

l	p_l	a_x	a_y	σ_x	σ_y	r
1	0.067	0.015	0.660	0.502	0.139	−0.064
2	0.130	0.932	0.046	0.652	0.322	−0.391
3	0.802	1.622	0.186	0.397	0.233	−0.040

Denoting with P_{1024}^{th} and F_{32}^{th} the detection threshold on the 1024 ms and the hypothetical long timescale the inequality $P_{1024}^{\mathrm{th}} + F_{32}^{\mathrm{th}} < P_{1024} + F_{32}$ defines that part of the $\{\log_{10} P_{1024}; \log_{10} F_{32}\}$ plane in which all the GRBs are detected. Replacing the inequality with the equality in the previous relationship we obtain a line of −45° slope which is the boundary of completeness in this plane. Rotating the coordinates by 45° the boundary of the area of completeness becomes a vertical line as indicated in the bottom panel of Fig. 3.

Restricting ourself to the region of completeness in the $\{0.71[\log_{10} F_{32} + \log_{10} P_{1024}]; 0.71[\log_{10} F_{32} - \log_{10} P_{1024}]\}$ plane (right of the vertical dashed line in the bottom panel of Fig. 3) we repeated the group-searching algorithm making use of this part of the BATSE sample. Tables 9–11 summarize the results of the computation.

The truncation procedure described above left 1229 GRBs in the sample. Inspecting the results given in Tables 9–11 one may infer that increasing the number of Gaussian components from $k = 2$ to $k = 3$ yielded a significant increase in the likelihood while from $k = 3$ to $k = 4$ did not. We conclude that even in this truncated sample some fraction (13%) (it was 15% in the non-truncated case) still appeared to belong to the intermediate group. Comparing the a_x parameters between Tables 5 and 10 shows that the deviations are much less than the corresponding σ_x term. It remains to show, however, what fraction of the intermediate GRBs in the whole sample was assigned to the same class in the truncated case. In Table 12 we made a cross tabulation between the classification of the whole and the truncated sample in the $\{\log T_{90}; \log H_{321}\}$ plane. This table shows that out of the 92 intermediate GRBs in the non-truncated sample 77 remain in the same class in the truncated case but 17 arrived from the other two classes (12 from the short and 5 from the long group).

4.4. Caveats

The fuzzy classification assigned three $\{p_1, p_2, p_3\}$ probabilities ($p_1 + p_2 + p_3 = 1$) to each GRB in the sample. Somewhat arbitrarily, we assigned the k class to a burst event where p_k, ($k = 1, 2, 3$) was maximal. The fraction of GRBs selected in

Table 11. Results of the EM algorithm on the truncated sample. $k = 4$, $L_{\max} = 1175$.

l	p_l	a_x	a_y	σ_x	σ_y	r
1	0.063	−0.049	0.663	0.486	0.138	0.002
2	0.072	0.939	−0.067	0.722	0.303	−0.395
3	0.041	0.592	0.271	0.259	0.291	−0.566
4	0.825	1.620	0.184	0.393	0.235	−0.046

Table 12. Cross tabulation of GRBs classified in the $\{\log T_{90}; \log H_{321}\}$ plane in the truncated and non-truncated cases, respectively.

	Non-truncated class$_{321}$			
Truncated class$_{321}$	Short	Interm.	Long	Total
short	83	5	0	88
interm.	12	77	5	94
long	0	10	1037	1047
Total	95	92	1042	1229

this way could be different to $\sum_{i=1}^{N} p_k^i/N$, the expected percentage of class "k" within the whole population. The truncation we applied in Sect. 4.3 affected the parameters of the best fitting Gaussians, consequently p_ks, and it might move some bursts into another class while others are added. The fraction of a class within the whole population, however, could be more resistent than the classification of individual objects. This fact implies that we cannot classify the individual bursts with certainty in this way.

As the fuzzy classification required a functional form for a suspected class to obtain membership probabilities, we assumed Gaussian distributions. The results reflect therefore a stochastic structure of the sample rather than isolating a group of objects with some distinct astrophysical properties. Consequently, it remained unclear at this stage whether the stochastic structure we uncovered by the EM algorithm really represents a new class of GRBs.

5. Physical differences between the mathematical classes

In Sect. 4.4 we pointed out that the mathematical deconvolution of the $p_l(x, y)$ joint probability density of the observed quantities into Gaussian components does not necessarily mean that the physics behind the classes obtained mathematically is different. It could well be possible that the true functional form of the distributions is not exactly Gaussian and that the algorithm of deconvolution formally inserts a third one only in order to get a satisfactory fit. One needs detailed investigations based on the physical (e.g. spectral) properties of the individual bursts to prove its astrophysical validity.

Recently Balázs et al. (2003) found compelling evidence that there is a significant difference between the short and long GRBs. This might indicate that different types of engines are at work. The relationship of long GRBs to the massive collapsing objects is now also observationally well established (Mészáros 2003), and the relation between the comoving and observed time scales is well understood (Ryde & Petrosian 2002). The short bursts can be identified as originating from neutron star (or black hole) mergers (Mészáros 2001). So the mathematical classification of GRBs into the short and long classes – obtained here (see Table 1 for $k = 2$) and in Balázs et al. (2003) – is also physically justified.

An important question that must be answered in this context is whether the intermediate group of GRBs, obtained in the previous paragraph from the mathematical classification, really represents a third type of burst physically different from both the short and the long ones.

The classification into the short, intermediate and long classes is based mainly on the duration of the burst. From Table 2 one may infer that these three classes differ also in the hardnesses. The difference in the hardnesses between the short and long group is well known (Kouveliotou et al. 1993). According to these data the intermediate GRBs are the softest among the three classes. This different small mean hardness and also the different average duration suggest that the intermediate group should also be a different phenomenon, that is, both in hardness and in duration the third group differs from the other two. On the other hand, no correlation exists between the hardness and the duration within the short and the long classes. More precisely, no correlation exists for the long group and a very weak correlation exists for the short group (see Table 2). Thus, these two quantities may be taken as two independent variables, and the short and long groups are different in *both* these independent variables.

In contrast, there is a strong anticorrelation between the hardness and the duration within the intermediate class. This is a surprising, new result, and because the hardness and the duration are *not* independent in the third group, one may simply say that only one significant physical quantity is responsible for the hardness and the duration within the intermediate group. Consequently, the situation is quite different here, because one needs two independent variables to describe the remaining two other groups. This is a strong constraint in modeling the third group. Hence, the question of the true nature of the physics in the intermediate group remains open, and obviously needs further detailed study.

6. Conclusions

Using the bivariate, duration-hardness fittings we obtained the following results:

- Increasing k from 2 to 3 shows that the introduction of the third group is real. This means that three groups of GRBs should exist. This confirms the earlier results of several authors.
- Increasing k from 3 to 4 shows that the introduction of the fourth group is not needed. This means that *only* three groups should exist. This result is in accordance with Mukherjee et al. (1998). Discussion of the possible biases

and also the use of two different hardnesses do not change this conclusion.
- From the fitting procedures it follows that the duration and the hardness are good quantities for the classification of GRBs. Remarkably, the intermediate class is on average even softer than the long group.
- We developed a method that makes it possible to define, for any GRB, the probabilities determining its membership of a given class. (The memberships are available by internet, Horváth et al. 2005.)
- 11%–15% of GRBs in the Current BATSE Catalog should belong to the intermediate class.
- An unusual anti-correlation between the duration and hardness might exist in the intermediate group. Hence, contrary to the other two classes, here the duration and hardness might not be independent variables, and hence the intermediate class can be different from the other two classes where the logarithmic hardness and duration are non-correlated variables. Thus, further detailed analysis has to be carried out to study this suspected behavior of the intermediate class.

All these considerations mean that we answered five questions of the six formulated in the Introduction. The question "Is the intermediate group a physically different phenomenon?" was not answered with satisfying certainty, and needs further analysis.

Acknowledgements. Thanks are due to D. L. Band, C.-I. Björnsson, J. T. Bonnell, L. Borgonovo, I. Csabai, J. Hakkila, S. Larsson, P. Mészáros, J. P. Norris, H. Spruit, G. Tusnády and R. Vavrek for valuable discussions. This study was supported by OTKA, grant Nos. T034549 and T048870; by a Research Plan J13/98: 113200004 of The Czech Ministry of Education, Youth and Sports (A.M.), and by a grant from the Wenner-Gren Foundations (A.M.). A.M. wishes to express his gratitude to host institutes in Stockholm and Garching for their warm hospitality. The detailed and constructive remarks of the anonymous referee significantly helped to improve the scientific merit of our paper.

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