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Depth of Investigation of Dipole-Dipole, Noncolinear and Focused Geoelectric Arrays

S. Szalai (Hungarian Academy of Sciences), A. Novák* (Hungarian Academy of Sciences) & L. Szarka (Hungarian Academy of Sciences)

SUMMARY

Investigation depth of various DC geoelectric arrays has always been in the focus of interest of geoelectricians. According to its classical definition (Roy and Apparao 1971), the depth of investigation is the depth of the maximum response due to a horizontal thin-sheet embedded in a half-space, by using a given geoelectric array. On basis of the graph of the thin-sheet response as a function of the depth (from the so-called „depth of investigation characteristics” or DIC function) Edwards (1977) found more realistic to compute the medium depth than the depth of the maximum response. DIC functions have been known so far only for simple colinear arrays, the dipole equatorial array and two focused arrays. Here we provide a summary about the depth of investigation values of various dipole-dipole arrays (for parallel, perpendicular, radial, azimuthal ones), and for the most important noncolinear and focused arrays. Depth of investigation values are computed from both approaches. DIC functions (obtained by a new analytical formula) are also presented, as illustrations. The analytical formula can be used to compute DIC function of any surface geoelectric array. A systematic interpretation of the resulting depth of investigation values provides simple but useful thumb-rules for practical applications.
INTRODUCTION

Investigation depth of various DC geoelectrical arrays has always been in the focus of interest of geoelectricians (e.g., Knödel et al. 1997). According to one of the possible and wide-spread definitions, the depth of investigation is the depth of a horizontal anomalous thin-sheet embedded in a half-space, where the thin-sheet produces a maximum response, in case of the given array (Roy and Apparao 1971). (Both the thin-sheet and the half-space are assumed to be homogeneous and isotropic.) The so-called depth of investigation characteristics (DIC) function is usually presented as a function of the depth of the thin-sheet (in Fig. 1). On basis of the same DIC function, Edwards (1977) computed an alternative depth value, the so-called medium depth. By definition, the integrated response (that is the depth integral of the DIC function) is the same above and beneath the medium depth. Edwards (1977) found the medium depth concept in a better agreement with field experiences than the depth of the maximum response.

DIC functions have been known so far only for simple colinear arrays, the dipole equatorial array, and two focused arrays (Roy and Apparao 1971, Barker 1989). In this paper we present DIC functions for several dipole-dipole arrays (parallel, perpendicular, radial and azimuthal ones) and DIC functions also for several noncolinear- and focused arrays, whose depth of investigation values are unknown. On basis of the DIC functions we compute investigation depth values from both the Roy-Apparao and the Edwards concepts. The mathematical basis of the computation for dipole-dipole arrays is given in the paper by Szalai and Szarka (2000); while the DIC functions of the other arrays were analytically computed by a new, general formula, which can be applied for any surface geoelectric array.

Figure 1: Principle of the depth of investigation characteristic (DIC) function. R: array length, $z^*/R$: depth of the maximum response (Roy and Apparao 1971), $z_e/R$: median depth (Edwards 1977).

1. DEPTH OF INVESTIGATION OF DIPOLE-DIPOLE ARRAYS

The dipole-dipole DIC functions were computed on basis of the paper by Szalai és Szarka (2000), after some transformation. (It is easier to compute dipole-dipole DIC functions from the special dipole-dipole formula than from the general one.) The DIC functions of radial, parallel, azimuthal (tangential) and perpendicular dipole-dipole arrays (shown in Figure 2) were computed in the interval $0^\circ \leq \vartheta \leq 90^\circ$ at every 10 degrees. In Figures 3 and 4 DIC functions of the radial and parallel dipole-dipole versions are presented.

Figure 2: Dipole-dipole arrays
Altogether three dipole-dipole arrays among the four ones shown in Figure 2 have similar DIC functions in a respect that the depth of investigation is independent of \( \vartheta \). The exception is the parallel array. The depth of investigation of the perpendicular dipole-dipole array is approximately \( z^*/R = 0.20 \), the depth of investigation of the radial dipole-dipole array is approximately \( z^*/R = 0.195 \), while of the azimuthal (tangential) array is \( z^*/R = 0.25 \), as it is summarised in Table 1.

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(*) \( z^*/R \) does not exist

Table 1:
**Depth of investigation (\( z^*/R \)) of the different dipole-dipole arrays**

In case of the parallel dipole-dipole arrays the depth of investigation is constant if \( 0° \leq \vartheta \leq 40° \). It is again constant (but different than before), if \( 80° < \vartheta \leq 90° \). In the interval \( 40° < \vartheta \leq 80° \) there is a drastic, non-monotonous transition, as shown in Figure 5 and in Table 1.
Disregarding this transition zone, the depth of investigation values of dipole-dipole arrays form two distinct groups: in the first, „axial-like” group (including the radial-, the perpendicular dipole-dipole arrays and the parallel array if $\theta \leq 40^\circ$) the depth of investigation is about $z^*/R \approx 0.195$, while in the second, so-called „equatorial-like” group (including the azimuthal dipole-dipole array and the parallel array if $80^\circ < \theta < 90^\circ$) the depth of investigation is about $z^*/R = 0.25$.

2. DIC FUNCTIONS AND DEPTH OF INVESTIGATION VALUES OF NONCOLINEAR AND FOCUSED ARRAYS

DIC functions of non-dipole arrays was determined by a new analytical formula, which can be applied generally for any surface geoelectric array. As examples, we present here NDIC functions of several noncolinear arrays, namely of the asymmetrical equatorial dipole array, the square-$\alpha$ array and the Baker array (in Fig. 6), and of two focused arrays: the unipole Wenner-$\beta$ and the unipole Wenner-$\gamma$ ones (in Fig. 7). (In the NDIC function the DIC is normalised with the response of the homogeneous half-space by using the given array.)

In Table 2 we give a summary about the depth of investigation values of 24 different geoelectric arrays, by applying both the maximum- and medium depth concepts. Only several of the depth of investigation values had been known before. As it can be seen, it is still the pole-pole array, which has the largest depth of investigation among all the studied arrays. Its depth of investigation is much larger than that of the focused arrays.

CONCLUSIONS

We have derived a direct analytical formula to compute NDIC function and depth of investigation values of various dipole-dipole arrays and any other colinear-, noncolinear- and focused geoelectric arrays. We systematically present the depth of investigation values for 24 various arrays, computed from both the depth of the maximum response (Roy and Apparao 1971), and the median depth (Edwards 1977) concepts. The depth of investigation is a simple but useful parameter for a systematic characterisation of geoelectric arrays.

About the dipole-dipole arrays we have concluded that from the point of view of the depth of investigation value, they form two basic groups: the dipole axial- and the dipole equatorial ones. It is remarkable that the depth of investigation is the largest for the pole-pole arrays: it is much more than that of the focused arrays.
REFERENCES

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Table 2: $z^*/R$ and $z_e/R$ depth of investigation values of 24 different surface geoelectric arrays ($z^*/R$: Roy and Apparao 1971, $z_e/R$: Edwards 1977), together with the (x,y) position of A, B, M and N electrodes. α10, β45, γ45, γ40, γ10 are preliminary names, referring to the type (α, β, γ) and the electrode positions(numbers). The focused arrays are presented separately (they may be either colinear or noncolinear).

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