Research Note

Progress in Agricultural Engineering Sciences 11(2015), 85–94
DOI: 10.1556/446.11.2015.3

Sinkage and Rolling Resistance of Wheels

Some new results on an old problem

GY. SITKEI

Abstract. Our poor prediction capability on the sinkage and rolling resistance of wheels originates in the unreliable description of soil properties and the soil-wheel interaction. Some improvements in both have shown that the theoretical calculations may give results comparable with experimental observations. To determine soil properties, the "two wheel measuring method" is suggested. Further improvement possibilities have also been outlined.

Keywords: hardness distribution, rolling resistance, sinkage, contact area, dimensionless number, two wheel measuring method

1. Introduction

The prediction on sinkage and rolling resistance of wheels encounters many problems also today. There are many proposed calculation methods and formulae to predict wheel performance, indicating that we have no one accurate method which would fulfil the general requirements.

Bekker [1, 2] in his pioneering works has derived the basic equations for calculating sinkage and rolling resistance. These equations are based on the engineering mechanics extended and applied to soil. In order to use these equations, the load carrying capacity of soils is required. The first equation for this was proposed by Berstein [3] which was a simple linear equation as a function of sinkage.

Remembering Boussinesq theory of the elastic half space [4], it turned out that the simplest load carrying capacity equation assumes the form [5]:

$$p_{av} = k\left(\frac{z}{d}\right)^n \tag{1}$$

where p_{av} is the average pressure under an indenter,
d is the diameter of the indenter,
z is the vertical soil deformation,
k is the load carrying capacity factor.

The exponent *n* characterizes the deformation and compaction behaviour of soil under vertical loading. It is mainly influenced by the particle distribution of soil. Boussinesq theory and Eq. (1) is valid only in the case if the soil is homogeneous as a function of depth. Sadly, that is never the case. The various tillage operations and subsequent settling create quite different local load carrying capacities as a function of the depth. *Figure 1* shows distributions measured with cone penetrometer on a differently tilled sandy loam soil. The upper loose layer has a low load carrying capacity while the deeper layers, depending on the tillage operations, have increasing load carrying capacity at different depths.

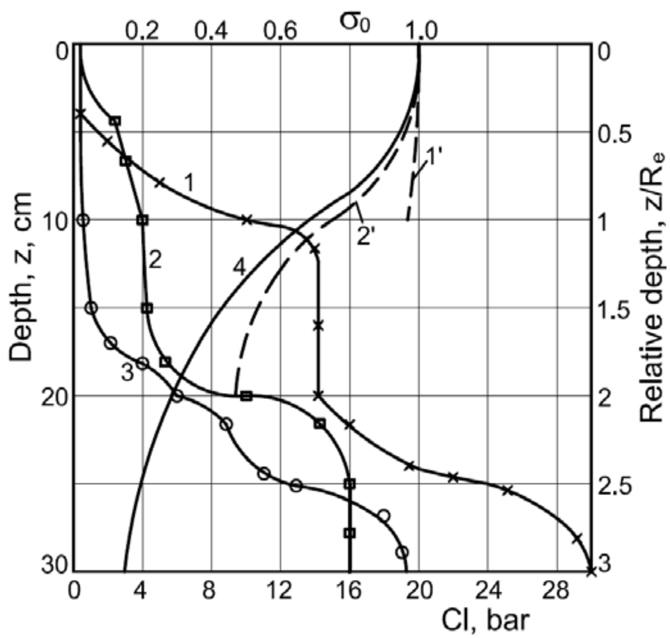

Fig. 1. Hardness distribution in soil after different tillage operations.
CI – Cone Index, σ_0 - average contact pressure, R_e – equivalent radius of the contact surface, 1 – stubble field, 2 – disk-harrowed, 3 – chisel tilled, 4 – pressure distribution in the homogeneous half space, 1′, 2′ – expected pressure distribution in stubble field and after disk-harrowing

A firm layer, however, fundamentally modifies the stress distribution in the soil body [6, 7]. This important result is almost unknown to researchers on the field, therefore, it is recalled in *Figure 2.* It is striking that, in the case of a firm layer, the pressure is 60% higher compared to homogeneous half space.

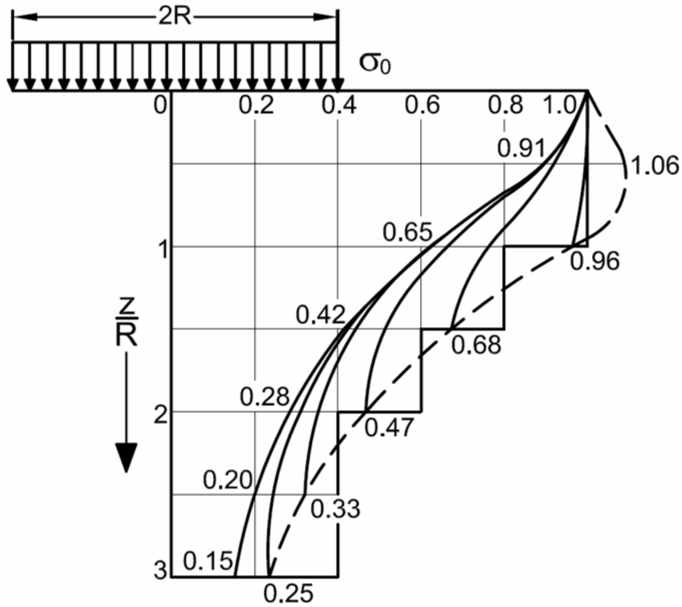

Fig. 2. Distortion of stress distribution under a circular plate if rigid layer is placed in given depth

In *Figure 1* we can see several layers with stepwise increasing hardness at different depths, depending on the tilling operation and soil settling. These steps in hardness are acting quite similar to the rigid layer shown in *Figure 2*. The evaluation of the developing load capacity based on the penetrometer readings seems to be a very difficult task. The seemingly loose upper layer under load will be compacted and its load carrying capacity may increase rapidly. The compaction of soils considerably depends on the initial density, the particle size distribution, the local moisture content, and on the hardness profile under the contact surface, therefore, its reliable calculation is hardly possible [8, 9]. A more purposeful approach is today to use circular plates (10–30 cm dia.) or tyres of different width as proposed in 1961 [10]. The loading surface should be large enough to ensure similar loading conditions to those of real tyres. In this way, we integrate the different hardness zones into a resultant or equivalent load carrying capacity factor k and exponent n. A simultaneous registration of hardness distribution may help to explain and, later perhaps, to generalize the obtained experimental results.

Due to the above mentioned circumstances, Eq. (1) in the strict sense of the word is not valid for real cases. However, accepting the validity of Eq.

(1), considerable improvements can be achieved in that way that the diameter of the indenter is related to the contact area of the wheel and the variation of contact surface as a function of sinkage will be taken into account. In the following, this approach is described and its results are demonstrated.

2. Theoretical Considerations

The equivalent diameter of a wheel contact surface d_e can be expressed by the wheel diameter D and width b in the following form [11]:

$$d_e = C \cdot D = const\sqrt{b/D} \cdot D = const\sqrt{b \cdot D} \tag{2}$$

where the constant depends on the relative wheel sinkage.

Using measurement results, the contact surface of tyres F can be approximated in the following dimensionless form:

$$\frac{F}{b \cdot D} = 0.18 + 0.75\left(\frac{z}{D}\right)^{0.8} \tag{3}$$

and the equivalent diameter of the contact surface is given by the relation

$$d_e = \left[\frac{4}{\pi}\left(0.18 + 0.75\left(\frac{z}{D}\right)^{0.8}\right)\right]^{0.5} \cdot \sqrt{b \cdot D} \tag{2a}$$

The average pressure in the contact surface p_{av} is given by dividing the wheel load G with the contact surface

$$p_{av} = \frac{G}{F} = \frac{G}{b \cdot D\left[0.18 + 0.75\left(\frac{z}{D}\right)^{0.8}\right]}. \tag{4}$$

Eq. (4) is valid in the case if G means the nominal load at a current inflation pressure p_i given by the manufacturer. In other cases, a proper correction should be made.

The average tyre pressure is in equilibrium with the average soil pressure and equating Eqs. (1) and (4) yields:

$$\frac{z}{D} = \sqrt{\frac{4b}{\pi D}} \left[0.18 + 0.75 \left(\frac{z}{D} \right)^{0.8} \right]^{\frac{n-2}{2n}} \cdot \left(\frac{G}{bkD} \right)^{1/n} \quad (5)$$

Please note that in Eq. (1), the diameter of the indenter is identical with the equivalent diameter of the contact surface according to Eq. (2a). Eq. (5) establishes an analytical relation between the well-known dimensionless quantities

$$\frac{z}{D} = f\left(\frac{b}{D}, \frac{G}{bkD} \right)$$

taking into account also the influence of the exponent *n*.

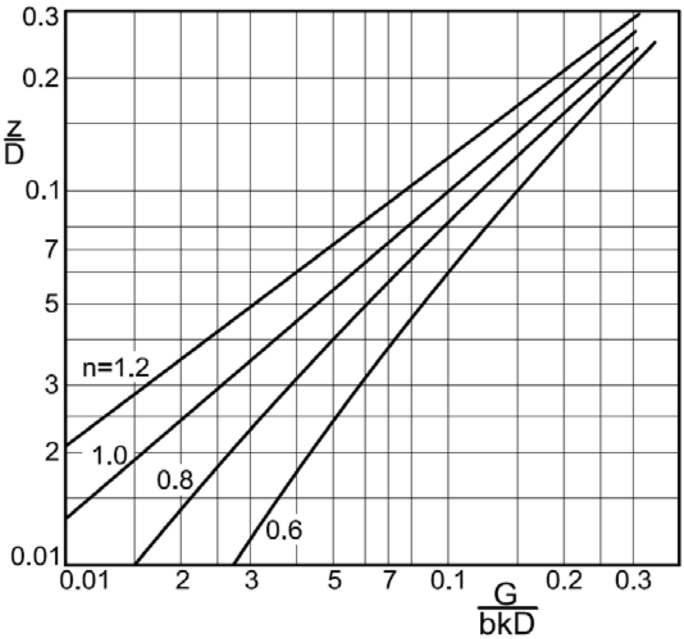

Fig. 3. Relative sinkage of wheels as a function of load number for different exponent *n*

Eq. (5) is implicit for *z/D*, its evaluation is demonstrated in *Figure 3* using real exponent values. It is striking that the exponent *n*, characterising soil behaviour, influences the relative sinkage considerably. As the specific load increases, the difference continuously decreases and at very high sinkages even vanishes.

The coefficient of rolling resistance, keeping in mind Eq. (2), is integrated in the common way and yields the following equation [11]:

$$f = \frac{b \cdot k \cdot D}{C^n \cdot D(n+1)} \left(\frac{z}{D} \right)^{n+1}. \tag{6}$$

Using Eqs. (2a) and (5), the above equation takes the form:

$$f = \frac{0.587}{n+1} \cdot \frac{1}{\left[0.18 + 0.75 \cdot (z/D)^{0.8}\right]^{0.5+1/n}} \cdot \left(\frac{G}{bkD} \right)^{1/n}, \tag{7}$$

which is illustrated in *Figure 4.* The influence of the exponent *n* is the same as in *Figure 3,* but its influence is vanishing at higher relative wheel loads. Combining Eqs. (5) and (7) gives another relationship for the coefficient of rolling resistance, directly related to the relative wheel sinkage

$$f = \frac{1}{n+1} \frac{1}{0.18 + 0.75 (z/D)^{0.8}} \left(\frac{z}{D} \right), \tag{8}$$

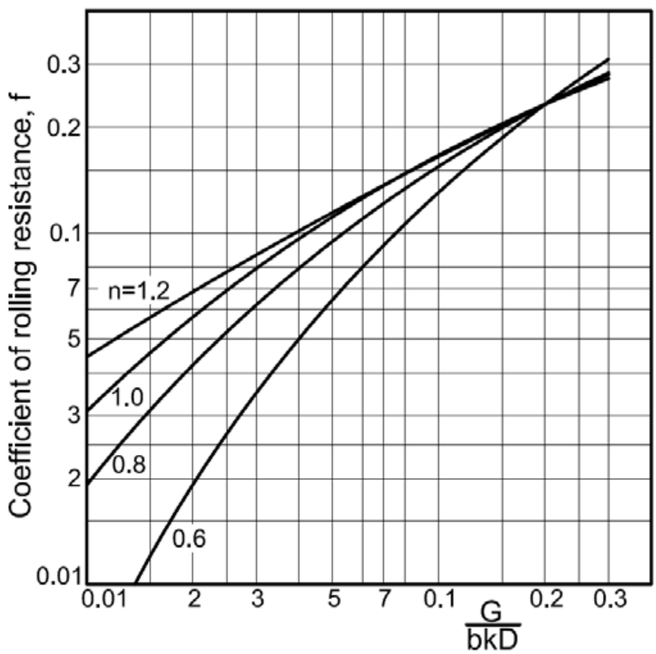

Fig. 4. Coefficient of rolling resistance vs. load number for different exponent *n*

which is plotted in *Figure 5*. Constant lines of the similarity load number *G/bkd* are also included. A closer inspection of the obtained results reveals that the soil behavior characterized by the exponent *n* has a decisive influence on the sinkage and rolling resistance. Due to the inclusion of a more accurate account of contact surface, the curves are not straight and this means that the resultant exponents are not constant as a function of relative sinkage *z/D* or the load number *G/bkD*.

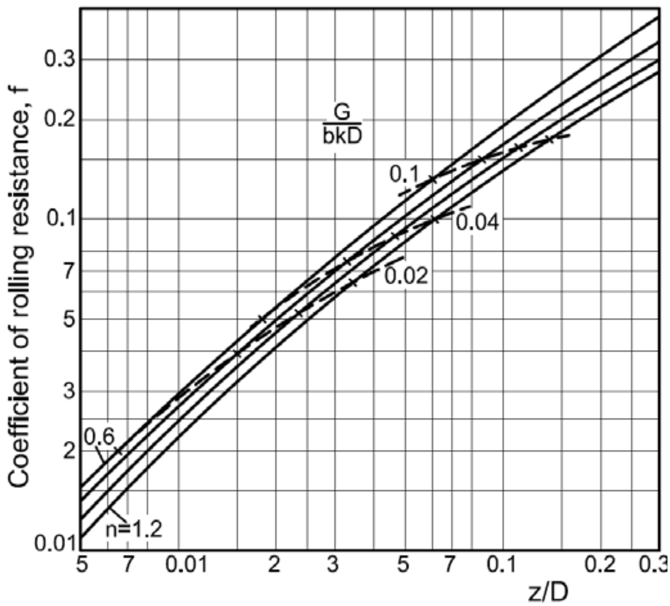

Fig. 5. Variation of rolling resistance as a function of relative sinkage and exponent *n*

For quick estimations, Eq. (8) can be simplified to the following explicit equation in respect to *z/D*

$$f = \frac{0.745}{n^{1.2}} \left(\frac{z}{D} \right)^{1-0.3n} . \tag{8a}$$

Taking the common exponent value of 0.8, Eq. (8a) yields

$$f = 0.97 \left(\frac{z}{D} \right)^{0.76} ,$$

which is in good agreement with experimentally obtained equations supplemented with the resistance due to tyre deformation ($f_0 = 0.03$), that is

$$f = 0.03 + 0.97\left(\frac{z}{D}\right)^{0.76}.$$

This result shows that the rolling resistance of tyres can theoretically be estimated with acceptable accuracy. The necessary and sufficient conditions are, however, the knowledge of reliable values for the load carrying capacity factor k and the exponent n.

Plate-sinkage studies require heavy measuring equipment and, therefore, such measurements have been and are very rare. The handy cone penetrometer, at the same time, is not suitable to measure reliable load bearing capacity. At the first ISTVS Conference in Turin, 1961, F. Pavlics has introduced the Bevameter B-100, utilizing two rigid wheels of different width [10]. The author's opinion is that this principle would deserve special attention to elaborate a new measuring device using pneumatic tyres today. Using a vehicle with a known tyre size and load, the measured rut depth determines the load carrying capacity with acceptable accuracy, but the exponent n, sadly, not. The complete characterisation of soil requires two different wheel widths with the following evaluation method.

Keeping in mind Eqs. (2a) and (3), the equivalent diameter of the contact surface is

$$d_e = \sqrt{\frac{4}{\pi} b \cdot D\psi} \qquad \text{with} \qquad \psi = 0.8 + 0.75 \cdot \left(\frac{z}{D}\right)^{0.8}.$$

Using Eq. (1), the average pressure p_{av} is expressed by the ratio of load G and contact surface F for both wheels in the following form:

$$\frac{G_1}{b_1 D\psi_1} = k\left(\frac{z_1}{\sqrt{\frac{4}{\pi} b_1 D\psi_1}}\right)^n$$

and

$$\frac{G_2}{b_2 D\psi_2} = k\left(\frac{z_2}{\sqrt{\frac{4}{\pi} b_2 D\psi_2}}\right)^n, \tag{9}$$

which can easily be solved for k and n. The wheel load may be equal or unequal. As an example, on a sandy loam soil (disk-harrowed, see *Figure 1*) measurements were conducted with two different tyres:

D = 76 cm, b_1 = 17.5 cm, G_1 = 4400 N, p_i = 1.5 bar, z_1 = 5 cm

D = 76 cm, b_2 = 25 cm, G_2 = 4400 N, p_i = 0.85 bar, z_2 = 4 cm

Tyres in both cases were subjected to the nominal load, which explains the use of different inflation pressures. The following two equations are obtained:

$$1.2517 = k \cdot 0.2363^n$$

$$0.9248 = k \cdot 0.1625^n$$

Solution of these equations yields: k = 4.02 bar and n = 0.808.

In the future, it will be a great challenge to establish relationships between hardness distribution as a function of depth and load carrying capacity in a user-friendly form. It cannot often enough be stressed that the size of contact surface fundamentally influences the interaction between tyre and soil and, therefore, the use of relative depth co-ordinate is compulsory (see *Figure 1*). The relative co-ordinate is related to the half width or radius of contact surface.

The initial hardness distribution and the relative location $\left(z/R_e\right)$ of the hard zone influence the wheel sinkage considerably. The sinkage originates from vertical compaction and side motion of soil. Calculations show that for shallow hard pan ($z/R_e = 1$, see *Figure 1*) the contribution of lateral flow is not more than 20–30%, but for deep hard pan $\left(z/R_e = 2.5\right)$ it may be as high as 40–60%. Without understanding these phenomena, further progress will hardly be possible. Furthermore, in the possession of an appropriate processing method, the cone penetrometer could be a convenient measuring device for mapping soil density distribution. Because the direct penetrometer reading (CI) does not correlate with the local soil density, therefore, the definition of a resistance factor for the given cone seems to be indispensable [12].

3. Conclusions

Based on the theoretical considerations and new derivations, the following main conclusions may be drawn:

– the main shortcomings of the existing calculation methods are founded on the poor mechanical characterization of soils as a function of depth, and by missing variables influencing contact pressure and sinkage,

- new derivations taking the effect of wheel sinkage and the exponent *n* into account have shown that more accurate calculations may give fully acceptable results,
- for better characterization of soils, instead of rather complicated plate-sinkage measurements, the "two-wheel measuring method" is proposed,
- penetrometer readings are suitable for mapping hardness distribution as a function of depth and, perhaps in the near future, also for density determination [13].
- in the future, the most important challenge would be a more accurate description of the load carrying capacity of soils with hardness distribution.

References

[1] Bekker, G., *Theory of land locomotion.* Ann Arbor, 1956

[2] Bekker, G., *Off-the-road locomotion.* Ann Arbor, 1960

[3] Berstein, E., Probleme zur experimentellen Motorflugtechnik. *Der Motorwagen,* 1913

[4] Timoshenko, S. and L. Goodier, *Theory of elasticity.* Mc. Graw Hill, New York, 1951

[5] Саакян, С., О закономерности сопротивления почвы вдавливанию. (On the resistance of soils to vertical pressure), *Сборник трудов по земл. мех.* т. III. 1956

[6] Егоров, К., Распределные напрежений, в основани конечной толщини. (Stress distribution in foundation of finite thickness). *Сборник "Механика грунтов,* 43, Гостройиздат, 1961

[7] Егоров, К., К вопросу деформации основания конечной толщини. (On the deformation of foundations of finite thickness) *Сборник "Механика грунтов,* 34, Гостройиздат, 1958

[8] Sitkei, G., Unconfined compression of agricultural soils with viscoelastic behavior. *Proc. of 11th ISTVS Conf.* 1993, pp. 322–331

[9] Sitkei, G., A non-linear viscoelastic plastic model describing compaction process. *Proc. of 2nd IMACS/IFAC Conf.* 1997. pp. 105–112

[10] Pavlics, F., The new Bevameter B-100. *Proc. of 1st ISTVS Conf.* 1961

[11] Laib, L. (Ed.), Terepen mozgó Járművek. (Off-road-vehicles), Szaktudás Kiadó, Budapest, 2002

[12] Разоренов, В. Пенетрационные испытания гйрунтов. (Penetrometer measurements on soils), Москва, Стройиздат, 1980

[13] Pillinger, G., Deformation and damping of soils under tires. PhD Diss. Gödöllő, 2015