Accurate Frequency Domain Modelling of an Electric Furnace

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***Abstract*—Controllers of industrial furnaces may operate differently in different temperature ranges. The controller has different parameter sets for each of these ranges. The operation of controllers is switched according to the temperature. It is desirable to change the parameters continuously following the temperature. The continuous change of parameters instead of mode switching may decrease the switching transients and lead to more accurate temperature control.**

A laboratory-scale electric furnace is investigated in this paper. The main focus is on the temperature dependent behavior of the furnace. The objective is to build up a temperature dependent model that is capable to describe the furnace in a wider temperature range.

I. Introduction

Electric furnaces behave differently in different temperature ranges. This means that the furnace can not be modelled with a simple linear model around a given operation point, but at least a temperature-dependent model or a set of models, depending on the temperature, should be used to describe the furnace. Even if the models at different operating points are well known, the controller of such furnaces must use several parameter sets, one for each operating point. This causes a transient when the controller is switched from one setpoint to the other.

However, if the furnace could be identified with only one temperature dependent model, and the control of the furnace could be based on this model, the switching transients could be eliminated.

In this paper a small laboratory-purpose electric furnace is investigated with the aim of describing one temperature dependent model of the furnace for a wide temperature range. After introducing the preliminary measurements and its consequences, the enhancement of the measurement technique is shown. The very long-term measurements executed will be described in this paper. The analysis of measurement data and a possible temperature dependent model will be shown as well.

Manuscript received July 6, 2007. This work was supported by the Hungarian Scientific Fund (OTKA) TS49743.

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II. Preliminaries

In our experiments a small, laboratory-scale heat-treating furnace (with a volume of 1.4 liters) is investigated.

During the first measurements the furnace was embedded into a control loop.

The setpoint temperature of the controller was changed so that it covered the whole range of interest. The applied ON/OFF controller produced an ON/OFF signal at its output. The input of the furnace was this switching signal that switched the heating element to the mains. The output was the furnace temperature as shown in Fig. 1.

The conditions of usual, accurate frequency domain system identification are met when the excitation is periodic and the measured variances can be reduced by averaging [1]. Unfortunately, as we will see in the next section, this basic condition cannot be fulfilled in our case.

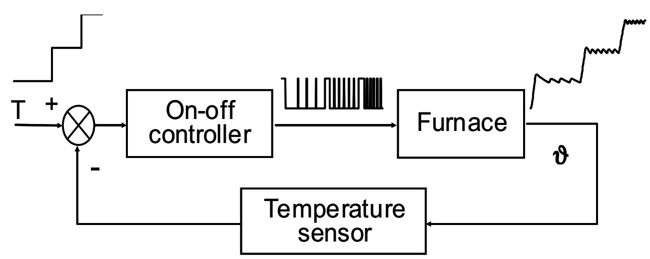

Fig. 1. Closed-loop measurement setup of the furnace. The on-off controller converts the applied setpoint temperature to a pulse sequence. This pulse sequence involves the control action taken by the controller in order to suppress the disturbing effects of the environment.

The bandwidth of the transfer function of the furnace is very low (~10 mHz). To precisely identify the transfer function in this range, a very long measurement time is needed (a few hours or even days). In the closed-loop setup the ON/OFF controller suppresses the changing effects of the environment. However, during such a long time, the changes of the environment can be significant. These changes will be suppressed by the closed-loop controller, but at the same time the periodicity of the train pulses will be distorted. Even a small distortion of the pulse sequence smears the power spectrum.

III. Measurement redesign

As seen in the preceding section, the smeared power spectrum weakens the signal-to-noise ratio (SNR) of the measurements, so the possible precision is poor. As a consequence: precise identification of a furnace cannot be done in a straightforward manner with a closed-loop measurement setup. This is due to several causes:

- the *controller* influences the measurements: suppresses the disturbing noises of the environment, at the same time, however, the switching signal looses its periodic nature,
- the linear model of the furnace depends on the *setpoint (temperature),*
- the *disturbing effects* of the environment are not known, so no correction is possible,
- the timing of observations is imprecise due to the way of present implementation originally designed for digital control only.

IV. Novelties in measurement implementation

In order to build an exact model of the furnace the above mentioned disturbances must be handled.

- The controller could only be eliminated, if it is ensured that the energy input to the process is limited. From earlier closed-loop experiments we can estimate the necessary filling-factor to heat the system to a given temperature, and keep the temperature around this operating point. The temperature dependency of the system can be analyzed by applying switching signals with different filling-factors in an open-loop setup. These measurements will give information about the system at different operating points.
- A simple square wave is not an optimal excitation, because we have no influence on the spectrum. Instead, we can use a discrete interval binary sequence excitation (DIBS). The advantage of DIBS is that we have a direct influence on the power spectrum of the switching signal [2]. The design algorithm of DIBS signals is well known and is implemented in MATLAB [3].
- The timing of observations was almost unusable in the closed-loop measurement setup. This is because the objective of a controller design does not cover a specification for the timing; the controller was originally not intended for the use of accurate data acquisition.
- Because the records of our measurements are long, the slowly changing effects of the environment must be taken into consideration. One of these effects is the changing of the weather (i.e. the movement of the air changes the rate of thermal losses or the sun shines stronger, etc.). Another important distortion arises from the slow changes of the main supply (line power). This change consists of the daily fluctuation of the power and the burst-like load of heavy current customers. From experience, these changes in the mains amplitude can reach even 10% at the measurement site. In order to get a reasonable model, the temperature of the environment and the applied input power must be measured, too.

By implementing a data acquisition device which is designed for high-quality measurement of an electric furnace in open-loop setup, all the above mentioned points could be taken into consideration. The block diagram of the implemented device is shown in Fig. 2. To have some insight into the effect of the changes of the environment, beside the furnace temperature the room temperature and the input power to furnace is measured as well.

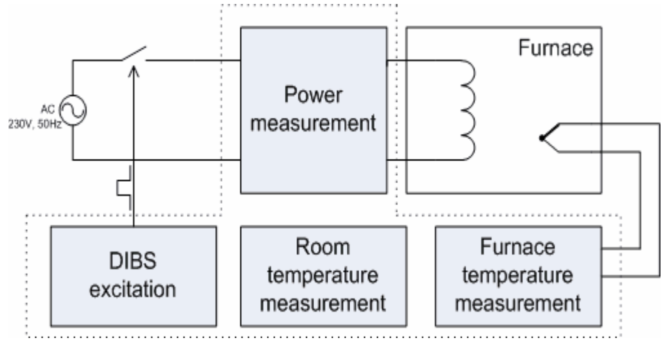

Fig. 2. Block diagram of the enhanced measurement setup. The DIBS excitation signal switches the heating element of the furnace to the mains (ON/OFF). Beside the furnace temperature the exact value of the consumed power and the environment temperature are measured as well.

After having estimated the bandwidth of the transfer function, we have aimed a frequency resolution of 0.2 mHz. This means that one cycle of the periodic measurements should last 5000 seconds. If we wish to reduce measurement variances by averaging, this measurement cycle must be continuously repeated.

V. Measurement results

As described in the previous section, the furnace was excited at different setpoints with impulse sequences having different filling-factors. As our first measurements have shown, the interesting part of the furnace transfer function falls between 0 and 20 mHz in frequency. To have a good frequency resolution (0.2 mHz) in this range periodic measurements with a cycle-time of 5000 seconds have been executed. This means that one simple period of the measurements has lasted about 1 hour and 23 minutes. The following table shows the summary of the executed measurements with different excitation sequences.

Table I.
Summary of executed measurements

Filling-factor [%]	Average temp. [ºC]	Meas. duration [s]	Meas. duration [hh:mm:ss]	Cycle time [s]	Number of periods
5	133.4	179 201	49:46:41	5000	35
10	237.9	522 145	145:02:25	5000	104
15	323.0	51 648	14:20:48	5000	10
20	421.6	251 553	69:52:33	5000	50
25	535.8	47 264	13:07:44	5000	9
30	560.2	515 297	143:08:17	5000	103
40	711.2	201 953	56:05:53	5000	38
50	805.1	220 289	61:11:29	5000	44

Measurements with filling-factors from 5% to 50% have been executed. However, it was seen that the measured data with 50% filling-factor has much greater variance as the other data, so some modifications on the measurement setup is needed. This will be done later. At this point we have restricted our investigation up to 40% filling-factor.

As Table I shows the different excitations were applied for different length in time, however there are some extra long measured sequences (5%, 10%, 20%, 30%, 40%). These data may serve as a very good basis for the model-fitting.

After computing the frequency response functions (FRF), however, it can be seen that even the shorter measurements (15%, 25%) fit very well to the others, so they can be used for modelling and validation as well. The following sections show some measurement results with their time series plotted.

A. Measurement with 5% filling-factor

The measurement with 5% filling-factor has taken 175.000 seconds, which was enough for 35 full periods. The measured environment and furnace temperatures are shown in Fig. 3.

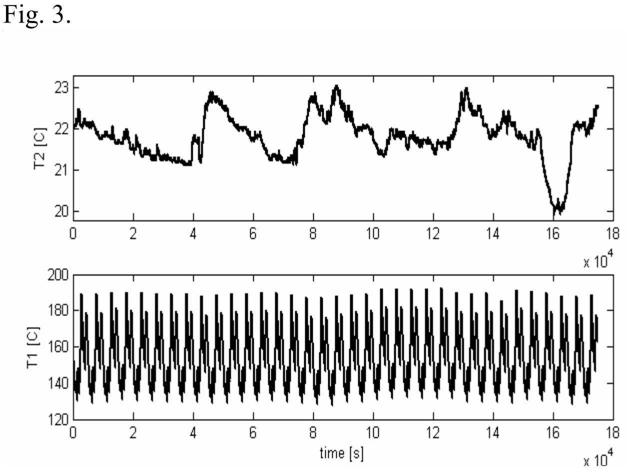

Fig. 3. The measured environment (*T2*) and furnace (*T1*) temperatures using an excitation signal with 5% filling-factor.

B. Measurement with 10% filling-factor

Using a DIBS excitation signal with 10% filling-factor the furnace has reached an average temperature of 237.9 °C. This was the longest among all measurements; it took 522000 seconds (145 hours). The temperature data can be seen in Fig. 4.

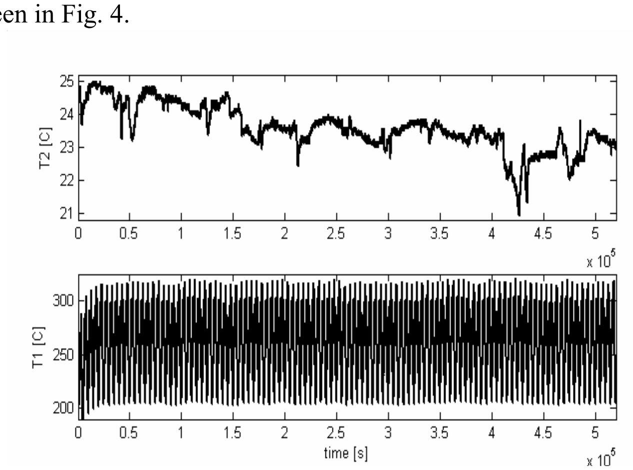

Fig. 4. The measured environment (*T2*) and furnace (*T1*) temperatures using an excitation signal with 10% filling-factor.

The collected 104 full periods make it possible to reduce the measurement variance by a factor of 10.

C. Measurement with 20% filling-factor

The measurement with 20% filling-factor took almost 70 hours. Instead of showing the whole dataset, in this case only two consecutive periods are plotted on Fig. 5. Beside the environment (T_2) and furnace (T_1) temperatures, the consumed power from the main (*P*), and the ON/OFF switching signal are plotted, as well. During the ON state of the input signal, the nominal power of the furnace (1000 watts) should be consumed by the heating element. (In order to show the fluctuation of the consumed power, the power data (*P*) is zoomed in Fig. 5.) In an ideal case the consumed power should be either 0 (OFF) or the nominal 1000 watts (ON). However, as the zoomed power data in Fig. 5 shows, the measured power values differ from the expected nominal values significantly and even the difference is changing. For the whole 70 hours this fluctuation has reached 10%.

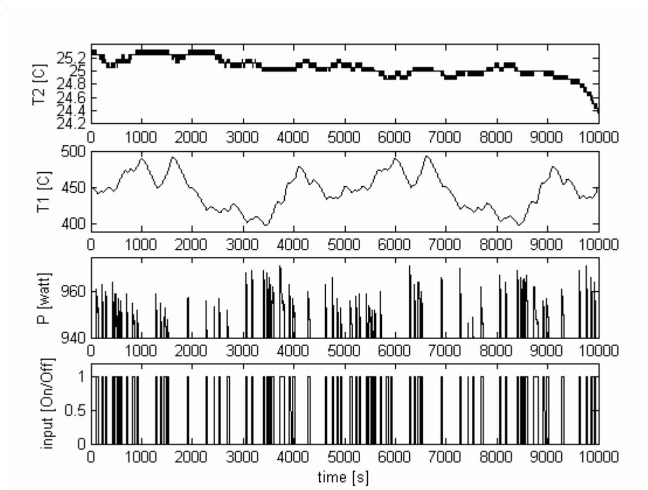

Fig. 5. Two periods of the measured environment (*T2*) and furnace (*T1*) temperatures, consumed power (*P*) (vertically zoomed to 940-975 watts) and the input switching signal when using an excitation signal with 20% filling-factor

VI. Analysis of the Measured Data

The collected data have been analyzed in the frequency domain. The time series have been segmented and the segments converted using the discrete Fourier transform (DFT). After conversion the Fourier coefficients of the segments have been averaged. And the averaged spectrum data were used to fit models in the Laplace domain [4].

After evaluating all the measured data in this way, the comparison of identified transfer functions resulted that a shifting change in the Bode amplitude plot can be recognized. This change can be seen even in the frequency response functions (FRF) of the measurements, as shown in Fig. 6.

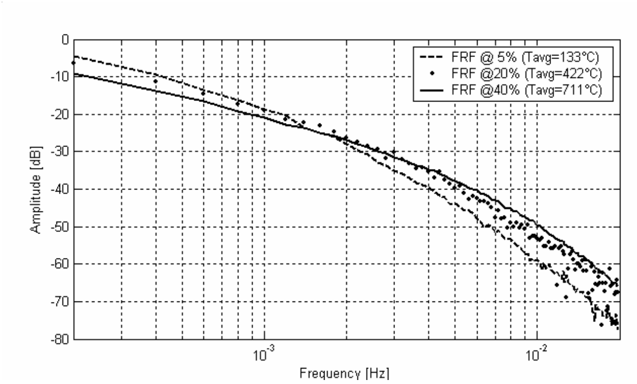

Fig. 6. Amplitude of the measured frequency response functions (FRF) of the furnace using excitation signals with 5% (dashed line), 20% (dots) and 40% (solid line) filling-factors.

As Fig. 6 shows the power throughput of the system at higher temperatures (at higher filling-factors) is somewhat higher than that at lower temperatures. However, around DC-frequency the relation is the opposite: the lower filling-factor results higher throughput in the FRF. Fig. 6 shows not all of the measurement results. This is to clearly visualize the differences between the FRFs of the selected measurements. However, it should be pointed out, that those measurements that are not shown on Fig. 6. (excitations with 10%, 15%, 25%, and 30%) behave similarly to these data. Based on this graph a temperature dependent change can be recognized in the frequency response functions.

The frequency axis of the figures goes only from 0.2 mHz to 20 mHz. The DC-measurements were not used for model-fitting. And above 20 mHz the measured FRFs contain greater noise. This is because using the DIBS excitation we have forced the input signal to have almost 80% of its power on spectral components under 20 mHz. Above 20 mHz the transfer function of the furnace has values less than 80 dB. Omitting these small values causes only minor modelling error. Hence, the fitting of transfer function to the measured data above 20 mHz is not necessary.

VII. MODEL-FITTING AT THE SETPOINT TEMPERATURES

For practical reasons we have fixed the order of models to be 2/3 (2nd order numerator and 3rd order denominator). From experience we have seen that a model with an order of 2/3 matches the measured FRFs already well.

At 5% filling-factor ($T_{AVG} = 133{,}4°C$) the model is:

$$H_{133}(s) = \frac{0.04917s^2 - 0.009888s + 0.0009716}{3097s^3 + 150.6s^2 + 1.183s + 0.0006407}. \quad (1)$$

At 20% filling-factor ($T_{AVG} = 421{,}6°C$) the model is:

$$H_{422}(s) = \frac{0.1037s^2 - 0.01782s + 0.001788}{2823s^3 + 167s^2 + 2.592s + 0.001773}. \quad (2)$$

At 40% filling-factor ($T_{AVG} = 711{,}2°C$) the model is:

$$H_{711}(s) = \frac{0.1143s^2 - 0.02339s + 0.002479}{2389s^3 + 183.8s^2 + 4.435s + 0.00383}. \quad (3)$$

The models were created with the Frequency Domain System Identification Toolbox of Matlab minimizing the Estimator for Linear Systems (ELiS) cost function [3], [4].

VIII. MODELLING THE TEMPERATURE DEPENDENCY

After having identified the linear models at some chosen setpoint temperatures, one may try to estimate the behavior of the furnace between the setpoints based on the identified models. To have a closer insight into the change of the FRF between the setpoints, Fig. 7 shows the amplitude of the difference of FRFs belonging to measurements with 5% and 40% filling-factors.

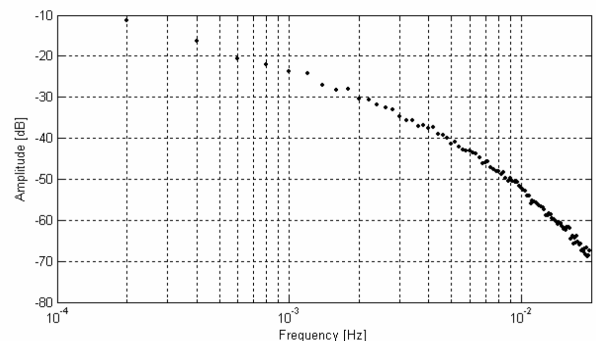

Fig. 7. The amplitude of difference of the FRFs measured with 5% and 40% filling-factors

Observing the difference function in Fig. 7, it can be stated that the temperature dependent behavior of the electric furnace between 155°C and 711°C can be modelled by an additional transfer function.

One possible solution is to combine the setpoint models linearly with the weighted sum of the setpoint transfer functions. This can be considered as a "linear interpolation" of models between the setpoints. The models at temperatures T_1 and T_2 will be combined to get the model at a temperature T_X ($T_1 \le T_X \le T_2$):

$$H_{TX}(s) = w_1 H_{T1}(s) + w_2 H_{T2}(s) \quad (4)$$

where $H_{T1}(s)$ is the setpoint model at temperature T_1, $H_{T2}(s)$ is the setpoint model at temperature T_2 and $H_{TX}(s)$ is the estimated model at temperature T_X. This kind of model combination preserves stability of the system.

The weighting factors w_1 and w_2 are calculated as

$$w_1 = \frac{T_X - T_1}{T_2 - T_1}, \quad (5)$$

and

$$w_2 = \frac{T_2 - T_X}{T_2 - T_1}. \quad (6)$$

As the setpoint models have an order of 2/3, the resulting model combination will have an order of 5/6.

For example, combining the identified models at 5% (mean temperature: 133°C) and 20% (mean temperature: 422°C) the resulting model at 15% (mean temperature: 323°C) can be obtained with the following formula:

$$H_{323}(s) = \frac{323-133}{422-133} H_{133}(s) + \frac{422-323}{422-133} H_{422}(s). \tag{7}$$

IX. Validation

A. Validation of simulated data at 15%

As we have many data sets measured, we could easily validate the results of the above described model combination method.

To combine the model we have used only the identified models at 5% and 20%. All the other measured data were not used. So the correctness of the model $H_{323}(s)$ can easily be tested by comparing the measured data using 15% filling-factor ($T_{AVG} = 323°C$) to the simulated data. The simulation was generated by applying the measured excitation signal to the input of the combined model $H_{323}(s)$. The simulated and measured FRFs can be seen in Fig. 8.

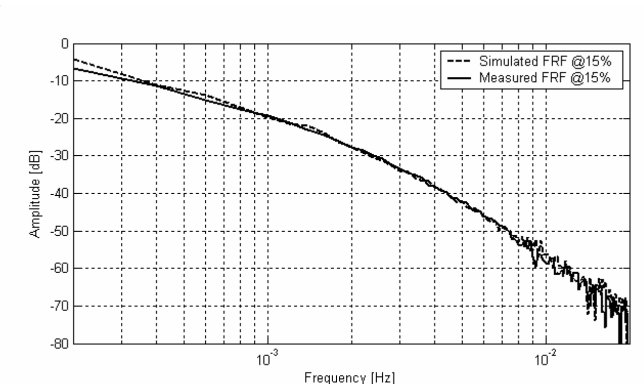

Fig. 8. Simulated (dashed line) and measured (solid line) FRFs at 15% filling-factor. The combined model was tested whether it gives similar FRF to the measured data at 15%. These measurement data were not used in the identification.

As Fig. 8 illustrates, the simulated FRF of the combined model for 323°C matches very well to the FRF of the real measured data.

B. Validation of simulated data at 30%

Another combined model was tested at 30% filling-factor ($T_{AVG} = 560{,}2°C$). This model can be combined using the setpoint models at 20% and 40% filling-factors.

According to (4), the setpoint models can be combined with the following formula:

$$H_{560}(s) = \frac{560-422}{711-422} H_{422}(s) + \frac{711-560}{711-422} H_{711}(s). \tag{8}$$

To validate the model $H_{560}(s)$ the measured data set at 30% filling-factor can be used. These data were not used in the identification. The simulated and measured FRFs can be seen in Fig. 9.

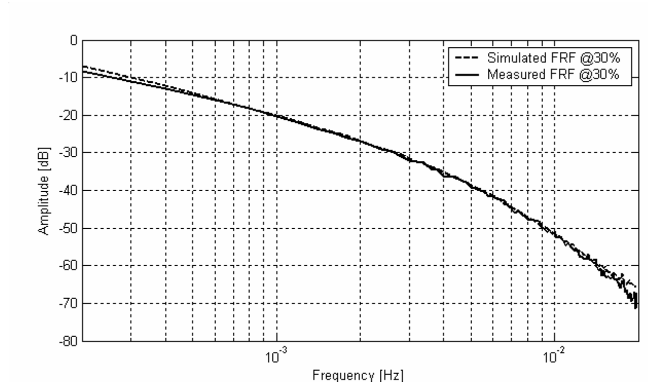

Fig. 9. Simulated (dashed line) and measured (solid line) FRFs at 30% filling-factor. The simulated data fits very well to the measurement.

In this test we have used the setpoint models at 20% and 40% to combine the model at 30%. As Fig. 9 shows the simulation results and measurement data coincide very well.

X. Conclusion

The conditions of accurate furnace measurements, the important parameters and a possible measurement setup have been defined. Based on the accurate measurements, high quality frequency domain furnace models were computed.

Analyzing the FRFs of the measurements it is stated that the transfer function of an electric furnace changes with temperature. This change has been measured and quantified. The measured change can be modelled. Using some setpoint models the behavior of the furnace between the setpoints is described. Measurements have confirmed that the setpoint models can be “linearly interpolated”.(This can be seen only on two data sets in this paper, however, all simulations match the measurements very well.)

Subsequently it is seen that the very low variance caused by the very long measurements is gratuitous. Using a few periods (10-15), we have already enough reduction in the variance to fit high quality models. The very long measurements were needed because this was not known before.

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