

Nonlinear Soft Tissue Mechanics Based on Polytopic Tensor Product Modeling

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Abstract—Achieving reliable force control is one of the main design goals of robotic teleoperation. It is essential to grant safe and stable performance of these systems, regarding HMI control, even under major disturbing conditions such as time delay or model parameter uncertainties. This paper discusses the systematic derivation of polytopic qLPV model from the nonlinear dynamics of typical soft tissues of the human body based on recent experimental results. The derivation is based on the Tensor Product (TP) Model Transformation. The presented method is a crucial step in laying the foundations of adequate force control in telesurgery. The proposed approach could form the basis of LMI-based controller design.

Index Terms—TP Model Transformation, qLPV modeling, soft tissue modeling, telesurgery control

I. INTRODUCTION

Surgical robots, as Cyber-Physical Systems (CPS), are one of the finest examples of advanced Human–Machine Interfaces (HMI). Many types of surgical manipulations have a certain degree of autonomy implemented in these systems, however, the human operator (surgeon) is still present as an integral part of the control loop. Thus, cognitive skills are exploited during the interventions, although the teleoperation systems dominantly use visual feedback over force/haptic feedback [1]. Haptic feedback based force control is actively studied in master–slave teleoperation structures, since the sensory capabilities of the human operators can be increased with a successful and reliable implementation. Long distance telesurgery also carries the difficulties originating from time-delay, which can induce instability in force-controlled systems, especially in the case of contact with hard surfaces [2]. To overcome these issues, several approaches have been studied in recent years.

One of the most successful approaches are the model based control methods. Providing a reliable mechanical model of the human body (especially for soft tissue, such as organs or skin), can enhance the available force controllers [3].

This work focuses on the derivation of the mechanical model of soft tissue, under certain surgical manipulations [4]. The discussed approach fits the concept of the quasi Linear Parameter Varying (qLPV) modeling, the polytopic model representations and the Linear Matrix Inequality (LMI) based

control design methods. The main goal of this work is to integrate the nonlinear mathematical model of the process of tool–tissue interaction into the modern modeling approach of qLPV/LMI-based control theory. The systematic derivation of the model and the illustrative numerical example will guide the reader through the transformation of the nonlinear system equations into a polytopic TP representation.

It is important to note that the presented soft tissue model was created based on physical considerations, as it was presented in [4]. TP Model Transformation can be considered as a gateway between the traditional model representations and the polytopic modeling. It can be proven that mathematically correct stability analysis can be achieved when LMI-based control design is taken into consideration. In the particular case of this study, the derived model would be utilized on the slave side of the teleoperation system, integrated in a cascade controller assembly [5]. This cascade structure supports the realization of force control in extreme scenarios, such as inter-continental or inter-planetary teleoperation [6].

II. TENSOR PRODUCT MODEL TRANSFORMATION

The Tensor Product Model Transformation was first introduced by Baranyi in 2003 [7], [8]. A summary of this approach and its applicability for qLPV control theory was published in 2013 [9]. It carries the original idea of transforming an arbitrary function into TP form, if the transformation is mathematically possible. The original function can be in a closed form or represented by soft computing techniques. If the exact mathematical transformation is not possible, TP transformation can still be used for creating an approximate TP function with reduced complexity, but also cutting back from the accuracy of the original model. In this study, TP Model Transformation is utilized to reformulate the analytically given parameter-dependent system matrix of a qLPV model into polytopic model form.

As a result from the transformation, polytopic structures are created, which can be further manipulated to improve the achievable control performance by decreasing the conservativeness of the polytopic model based design [10], [11].

In order to have a better understanding of this approach, the fundamental definitions of the TP Model Transformation are listed and explained in this Section based on [9].

Definition 1: (LPV/qLPV model): Consider the following Linear Parameter Varying model:

$$\begin{bmatrix} \dot{\mathbf{x}}(t) \\ \mathbf{y}(t) \\ \mathbf{z}(t) \end{bmatrix} = \mathbf{S}(\mathbf{p}(t)) \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \\ \mathbf{w}(t) \end{bmatrix}, \quad (1)$$

with input $\mathbf{u}(t)$, disturbance input $\mathbf{w}(t)$, measured output $\mathbf{y}(t)$, performance output $\mathbf{z}(t)$ and state vector $\mathbf{x}(t)$. The $\mathbf{S}(\mathbf{p}(t)) \in \mathbb{S}$ system matrix can be partitioned to $\mathbf{A}(\mathbf{p}(t))$, $\mathbf{B}(\mathbf{p}(t))$, $\mathbf{C}(\mathbf{p}(t))$, etc. system matrices and it is defined over a hyper-rectangular parameter domain

$$\mathbf{p}(t) \in \Omega = [a_1, b_1] \times [a_2, b_2] \times \dots \times [a_N, b_N] \subset \mathbb{R}^N. \quad (2)$$

If the parameters in $\mathbf{p}(t)$ are not independent from the $\mathbf{x}(t)$ state variables, it is called quasi-LPV (qLPV) model.

In this work, the soft tissue model from [4] can be implemented as a (1) qLPV model and it is created taking the nonlinearities of the system into consideration.

The finite element polytopic model representation is a suitable tool for LMI-based controller design, which is defined as follows.

Definition 2: (Finite element polytopic model): The (1) LPV/qLPV model, where the system matrix is given as convex combinations of vertex system matrices, as

$$\mathbf{S}(\mathbf{p}) = \sum_{r=1}^R w_r(\mathbf{p}) \mathbf{S}_r \quad \forall \mathbf{p} \in \Omega, \quad (3)$$

where

$$\sum_{r=1}^R w_r(\mathbf{p}) = 1, \quad w_r(\mathbf{p}) \geq 0 \quad \forall r, \mathbf{p} \in \Omega. \quad (4)$$

The term finite means that R is bounded.

Definition 3: (Finite element polytopic TP model): The (1) LPV/qLPV model, where the system matrix is given as convex combinations of vertex system matrices, and the weighting functions are decomposed to product of univariate ones:

$$\mathbf{S}(\mathbf{p}(t)) = \sum_{j_1=1}^{J_1} \sum_{j_2=1}^{J_2} \dots \sum_{j_N=1}^{J_N} \prod_{n=1}^N w_{j_n}^{(n)}(p_n(t)) \mathbf{S}_{j_1, j_2, \dots, j_N}. \quad (5)$$

Applying the compact notation based on tensor algebra (Lathauwer's work [12]) one has:

$$\mathbf{S}(\mathbf{p}(t)) = \mathcal{S} \boxtimes_{n=1}^N \mathbf{w}^{(n)}(p_n(t)), \quad (6)$$

where the core tensor $\mathcal{S} \in \mathbb{S}^{J_1 \times J_2 \times \dots \times J_N}$ is constructed from the vertex system matrices $\mathbf{S}_{j_1, j_2, \dots, j_N} \in \mathbb{S}$ and the row vector $\mathbf{w}^{(n)}(p_n(t))$ contains scalar weighting functions $w_{j_n}^{(n)}(p_n(t))$, ($j_n = 1 \dots J_N$), that represents convex combinations as (4) for all n .

Remark 1: The polytopic TP model (6) is a special class of polytopic models, where the weighting functions are decomposed to the tensor product of univariate functions.

Definition 4: (TP Model Transformation): TP Model Transformation is a numerical method that transforms the LPV/qLPV models to polytopic TP model, so that the LMI methods developed for polytopic model based control can be applied to the resulting model.

Detailed description of TP Model Transformation and application examples can be found in [7]. It gives a trade-off between the accuracy of the resulting model and the number of required vertexes for the LMI control design. The methodology is also capable of manipulating (optimizing) the polytopic model within a compact framework.

There exists various types of polytopic TP forms for LPV/qLPV models. In this work, the MVS-type polytopic model is considered that is defined below:

Definition 5 (MVS Polytopic TP model): The (6) polytopic TP model, where the $\mathcal{S} \in \mathbb{S}^{J_1 \times \dots \times J_N}$ core tensor is constructed from the $\mathbf{S}_{j_1, \dots, j_N}$ matrices, in such a way that the $(\mathcal{S})_{j_n=j}$ n -mode subtensors construct the minimal volume enclosing simplex for the

$$\mathcal{S} \times_n \mathbf{w}_{j_n}^{(n)}(p_n) \quad (7)$$

trajectory for all $n = 1..N$.

Further reading about the TP Model Transformation, the MVS-type polytopic TP model generation and manipulation methods can be found in [7], [8], [9], [10], [11].

III. MECHANICAL DESCRIPTION OF SOFT TISSUES

In recent years, research activities in the field of robotic surgery have gained much attention, which is a direct consequence of the rapid development of interventional systems [13]. Grabbing, cutting and indentation are among those types of manipulations, which require tools of high precision and sophisticated control. Understanding the behavior of soft tissues under these manipulations is crucial in order to achieve high performance of haptic feedback tools [14].

Rheological soft tissue models are often used for modeling basic surgical manipulation tasks, such as grasping or indentation [15]. There is a rich literature about experimental measurement data for force response on soft tissue indentation tests in both compression [16] and relaxation phases [17], which can serve as a good reference for comparing the validity of various models. A comprehensive study has been published by Yamamoto on different rheological soft tissue models by carrying out point-to-point palpation [18]. A nonlinear viscoelastic model was introduced by Troyer et al., which could be implemented in finite element modeling algorithms, creating a rheological-based hybrid soft tissue model [19]. A complex model of porcine liver has been introduced by Leong et al. in [20], which was improved and validated in [4] and [21]. The schematic figure of these mass–spring–damper models are shown in Fig. 1.

The nonlinear Wiechert model, originally proposed by Takacs et al., introduces progressive stiffness characteristics to the mass–spring–damper model of soft tissues through the spring elements:

$$k_j(x) = K_j e^{\kappa_j x(t)} \quad (8)$$

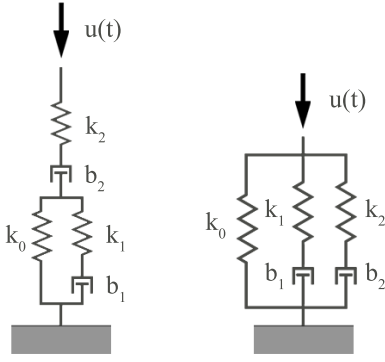


Fig. 1. Two basic combinations of the mass-spring-damper viscoelastic models: the Maxwell-Kelvin model (left) and the Wiechert model (right).

for $j = 0, 1, 2$, where x denotes the elongation of the spring element, k_j and κ_j are mechanical parameter estimated from experimental data. The proposed model has 3 Degrees of Freedom (DoF), where the virtual mass points are denoted by x_j , $j = 0, 1, 2$, and are placed on the tissue surface and at the connection of the spring and damper elements, respectively.

Taking $u(t)$, the deformation rate as the input of the model, the nonlinear system of differential equations describing the tissue mechanics can be written in state space form:

$$\begin{aligned} \dot{x}_0(t) &= u(t), \\ \dot{x}_1(t) &= \frac{1}{b_1} K_1 (x_0(t) - x_1(t)) e^{\kappa_1(x_0(t) - x_1(t))}, \\ \dot{x}_2(t) &= \frac{1}{b_2} K_2 (x_0(t) - x_2(t)) e^{\kappa_2(x_0(t) - x_2(t))}, \end{aligned} \quad (9)$$

where b_i , $i = 1, 2$ are the linear damping parameters of the model, also taken from experimental data. The output $y(t)$ of the model is the reaction force $F(t)$ exerted due to the compression, which can be written as follows:

$$y(t) = K_0 x_0(t) e^{\kappa_0 x_0(t)} + K_1 (x_0(t) - x_1(t)) e^{\kappa_1(x_0(t) - x_1(t))} + K_2 (x_0(t) - x_2(t)) e^{\kappa_2(x_0(t) - x_2(t))}. \quad (10)$$

The estimated parameter values from compression experiments on $20 \times 20 \times 20 [mm]$ cubic shaped specimens are shown in Table I. These values were used in the TP Model Transformation and numerical simulations.

IV. THE POLYTOPIC TP MODEL

In order to create an appropriate qLPV model that can be used for LMI-based controller design, first of all a goal for the control effort has to be defined. Here the goal is to control the position of the instrument tip by tracking the desired value $x_d(t)$, which in mathematical sense could be written as $x_0(t) = x_d(t)$, where $x_0(t)$ denotes the value of tissue surface deformation.

The corresponding control design methods address the regulation of the qLPV model's state to zero by state feedback or output feedback. That is, the qLPV model should be formulated to represent the error dynamics.

For these reasons, the following state variables $\Delta x_0(t) = x_0(t) - x_d(t)$, $\Delta x_1(t) = x_0(t) - x_1(t)$ and $\Delta x_2(t) = x_0(t) - x_2(t)$ are used in the qLPV model, and its output similarly, as $\Delta y(t) = y(t) - y_d(t)$, where $y_d(t)$ stands for the desired force output

$$y_d(t) = K_0 x_d(t) e^{\kappa_0 x_d(t)}. \quad (11)$$

Then the following qLPV model can be constructed

$$\begin{bmatrix} \Delta \dot{\mathbf{x}}(t) \\ \Delta y(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A}(\mathbf{p}(t)) & \mathbf{B}_u & \mathbf{B}_w \\ \mathbf{C}(\mathbf{p}(t)) & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \mathbf{x}(t) \\ u(t) \\ w(t) \end{bmatrix}, \quad (12)$$

where

$$\begin{aligned} \mathbf{p}(t) &= \begin{bmatrix} e^{\kappa_1 \Delta x_1(t)} & e^{\kappa_2 \Delta x_2(t)} & \frac{x_0(t) e^{\kappa_0 x_0(t)} - x_d(t) e^{\kappa_0 x_d(t)}}{x_0(t) - x_d(t)} \end{bmatrix}, \\ \mathbf{A}(\mathbf{p}) &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{K_1}{b_1} p_1 & 0 \\ 0 & 0 & -\frac{K_2}{b_2} p_2 \end{bmatrix}, \mathbf{B}_u = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{B}_w = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \\ \mathbf{C}(\mathbf{p}) &= [K_0 p_3 \quad K_1 p_1 \quad K_2 p_2], \quad w(t) = \dot{x}_d(t). \end{aligned}$$

The fact that the desired state appears in the system matrix, shows well the nonlinear property of the system: its settling behaviour changes with the $x_d(t)$ desired state. Because the $\Delta x_0(t)$ error variable changes with the desired state, the $\dot{x}_d(t)$ signal appears in the qLPV model and it is considered as disturbance.

Using the qLPV model (12), the MVS polytopic TP model can be obtained for the parameter dependent system matrix

$$\mathbf{S}(\mathbf{p}) = \begin{bmatrix} \mathbf{A}(\mathbf{p}) & \mathbf{B}_u & \mathbf{B}_w \\ \mathbf{C}(\mathbf{p}) & 0 & 0 \end{bmatrix} \quad (13)$$

considering the parameter values and domain from Table I.

The transformation yields to an exact polytopic TP model form, where

$$\begin{aligned} \mathbf{S}(\mathbf{p}) &= \mathcal{S} \boxtimes_{n=1}^3 \mathbf{w}^{(n)}(p_n(t)) = \\ &= \mathcal{S} \times_1 \mathbf{w}^{(1)}(p_1(t)) \times_2 \mathbf{w}^{(2)}(p_2(t)) \times_3 \mathbf{w}^{(3)}(p_3(t)) = \\ &= \sum_{j_1=1}^2 \sum_{j_2=1}^2 \sum_{j_3=1}^2 w_{j_1}^{(1)}(p_1) w_{j_2}^{(2)}(p_2) w_{j_3}^{(3)}(p_3) \mathbf{S}_{j_1, j_2, j_3}, \end{aligned} \quad (14)$$

the core tensor \mathcal{S} contains the $2 \times 2 \times 2$ vertex systems and the corresponding weighting functions are shown in Fig. 2.

In order to validate the polytopic TP model, numerical simulations were carried out to compare the force response functions to the original nonlinear differential equations. Simulations results in both the tissue relaxation and constant compression rate phases are shown in Fig. 3 and Fig. 4, respectively. As expected, the simulations indicate identical dynamic behaviour for both cases, as the polytopic TP model is capable of representing the analytic qLPV model.

V. DISCUSSION

The presented polytopic qLPV modeling methodology opens up new possibilities for addressing the dynamic and stability-related behavior of complex, nonlinear and parameter-dependent systems, such as the physical interaction of robots

TABLE I
PARAMETER ESTIMATION RESULTS FROM FORCE RELAXATION AND CONSTANT COMPRESSION RATE TESTS.

K_0	K_1	K_2	b_1	b_2	κ_0	κ_1	κ_2	p_1	p_2	p_3
[N/m]	[N/m]	[N/m]	[Ns/m]	[Ns/m]	[m ⁻¹]	[m ⁻¹]	[m ⁻¹]	[-]	[-]	[-]
2.03	0.438	0.102	5073	39.24	909.9	1522	81.18	0.9..213482	0.9..2.10592	0.9..13203.7

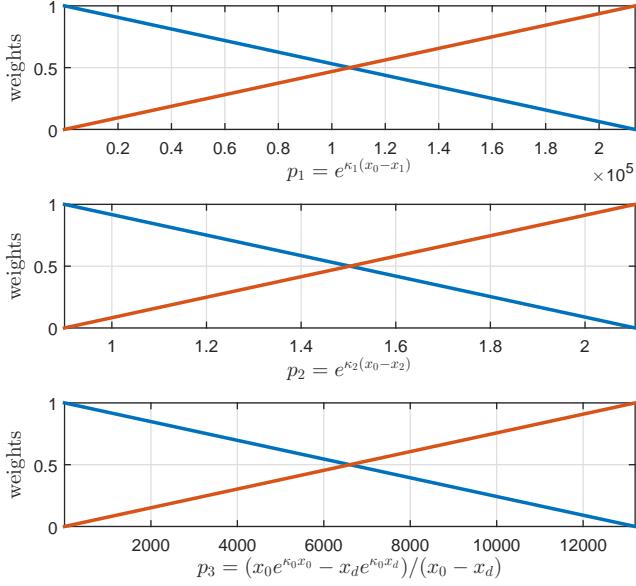


Fig. 2. Weighting functions of the MVS polytopic TP model.

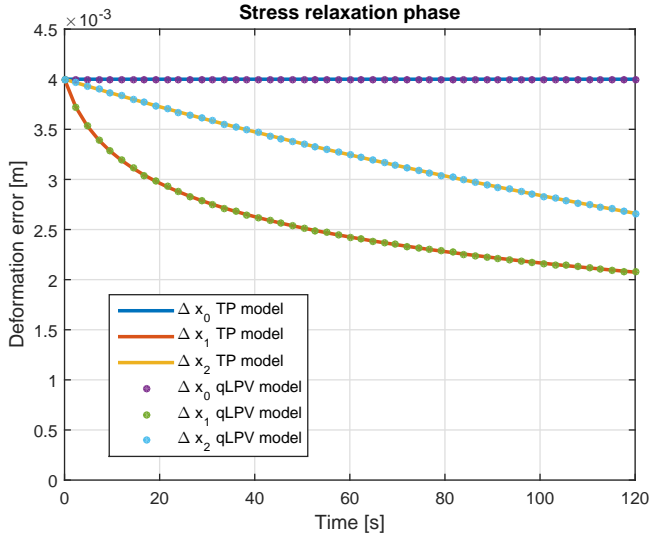


Fig. 3. Comparison of the original nonlinear model and the TP model in the tissue relaxation phase.
 $u(t) = 0, \mathbf{x}(t=0) = [0.004 \ 0 \ 0]^T$.

with biological tissues. Through LMI-based optimization, control synthesis can be performed according to predefined closed

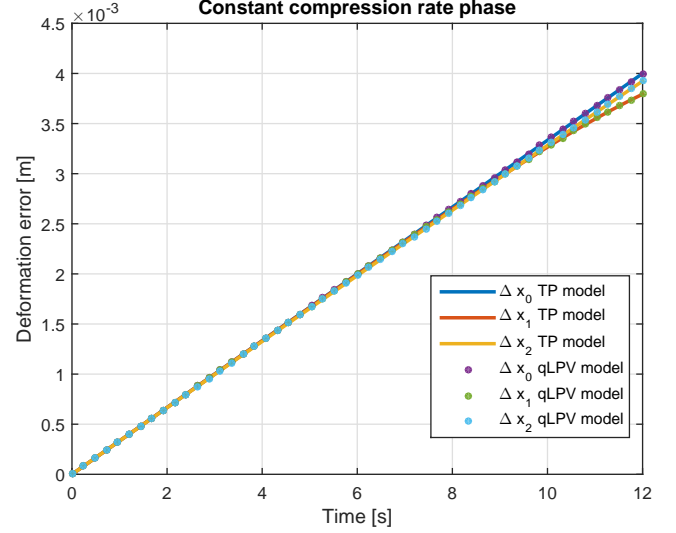


Fig. 4. Comparison of the original nonlinear model and the TP model in the constant compression rate deformation phase.
 $u(t) = 20 \text{ mm/min}, \mathbf{x}(t=0) = [0 \ 0 \ 0]^T$.

loop performance requirements. The polytopic TP model representation that is derived in this study, allows for addressing force control problems in robotic surgical devices. The control goal formulated in section IV can be handled using static and dynamic output feedback or state feedback control schemes as well. The criteria for optimal and/or robust control in LMI-based design can be addressed over a given parameter domain that is relevant to the application.

Using TP Model Transformation, the presented nonlinear soft tissue model can be transformed into a representation that directly fits to LMI-based controller design. As it was shown, the model can represent the behavior of soft tissues in the case of compression tests, which is an important step towards its implementation into model based position/force control problems. The qLPV model defined in (12) is written in an appropriate form for such controller design, where the way of defining the desired state is part of the modeling. For simplification reasons, $x_d(t) = 0[m\ m]$ was assumed in the open-loop simulation.

In this study, the reformulation of an existing system model is discussed in order to determine a representation that will serve as a basis for the design of closed loop control. The structure of the derived qLPV model and the corresponding polytopic form allows for applying well known control schemes and specifying meaningful objective functions for the purpose of LMI-based optimization. Investigation of the viable

closed loop structures and the actual control design will be addressed in future works.

VI. CONCLUSION

Robotic surgery and teleoperation control are examples of the most interesting areas in the domain of force control. This work focused on the demonstration of the potential use of TP Model Transformation in convex polytopic modeling for addressing soft tissue dynamics under surgical manipulation. The illustrative example used the nonlinear Wiechert model, a recently introduced rheological tissue model as a representation of this behavior. The model has been rewritten to an appropriate qLPV form and has been transformed using the TP Model Transformation, which can be later used in LMI-based controller design methods. Simulation results showed that the behavior of the model is equivalent to the simulation results using the qLPV system representation, indicating that the conversion from the analytical to numerical formulation can be done with a negligible loss of information. Encouraged by the presented results, our future work focuses on the implementation of the discussed model through Linear Matrix Inequality based control design methods.

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