Interpolation methods for digital elevation models

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***Abstract*—A grid elevation model stores the elevations in the nodes of a grid. If we would like query the elevation in an arbitrary position, we have to use any interpolation method which calculate the elevation from the surrounding nodes of the grid model.**

This article describes a new method for the interpolation and compare this described method to some other well known interpolation and surface subdivision method.

I. Introduction

One type of the Digital Elevation Models (DEM) is called GRID, where the heights of the terrain are stored relative to the nodes of a square lattice. This can be interpreted as the product of the terrain surface (as an two-dimensional function) and an two-dimensional Dirac-comb. As a result, the model can not represent the details whose scale is smaller than 2T, where T is the period of the Dirac-comb. [1]

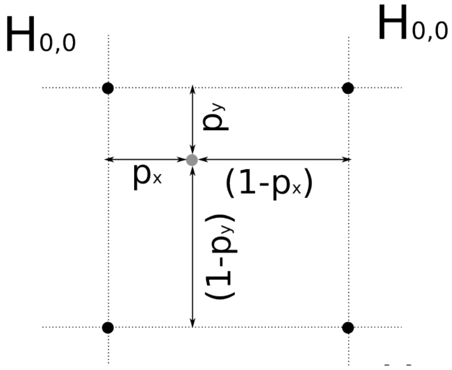

Figure 1. Denotations in the equation of bilinear transformation

The data storing of the GRID is analogous to the raster images, because in both cases the data is a two-dimensional array (a matrix). The advantage of this method is that we can determinate simple and quickly the neighboring vertices of the lattice from an given position. The disadventage is the non-conformity to the details of the surface.

Because the logical structure of the GRID is analogous to the raster images, we can store this elevation models by methods of the raster data, for example GeoTIFF files. (ISO 12639:1998)

II. Interpolation in the GRID

If a position is in the area of the used lattice, but not exactly in a vertex of the GRID model, we have to use an interpolation or approximation method to calculate the elevation from the elevation of the neighboring vertices. One of the simplest method divides the squares of the lattice to two triangles. Three points of this triangles clearly define a plane, and this plane determines the elevation.

Another classic methods is the bilinear surfaces. We can fit an bilinear surface to the four points of a square. We need not divide to triangles the squares for the interpolation.

If the position of a looking point is in a square of the lattice, we need the elevation of four nodes of this square denoted $H_{0,0}$, $H_{0,1}$, $H_{1,0}$ and $H_{1,1}$. The position of the looking point within this square marks by two number between 0 and 1, denoted p_x and p_y. This numbers are coordinates in the normalized coordinate system of the square.

The elevation of the looking point is:

$$H=H_{0,0}\left(1-p_x\right)\left(1-p_y\right)+H_{0,1}\,p_x\left(1-p_y\right)$$
$$¿+H_{1,0}\left(1-p_x\right)p_y+H_{1,1}\,p_x\,p_y¿$$

This is equal to the altitude which is received when two auxiliary point are calculated in two parallel edge of the square in line with the looking point, and the elevation of the looking point is calculated are calculated between this auxiliary points.

III. Catmull-Clark subdivision

The Catmull-Clark subdivision [2] is a common method in the computer graphics for increasing the number of the polygons of a mesh surface. The method used once creates a half length edge mesh. If the method is used i times repeatedly, the edge length of the returned mesh will be 2^i part of the original lattice.

The first step of the method calculates the coordinates of the new vertices situated in the centers of the original squares as the average of the coordinates of the vertices of the polygon. The second step calculates the new vertices near the middle points of the edges of the original mesh as the average of neighboring points: the vertices of the neighboring polygons from the first step and the two endpoint of the edge from the original mesh. The third step calculate new position for the original vertices as the average of the original position (½ weight) and the neigboring points from the first and second step (1/2n weight, where n is the number of the points of neighboring polygons and vertices).

This method is widely implemented in the world of the computer graphics. Any mesh surface can be smoothed in this way.

A. Application in GRID DEM

The Catmull-Clark subdivision can be used with square lattices of the GRID digital elevation models, because the surface of this model is considered as a special case of the mesh surface. This mesh contains only square polygons, and the horizontal position of the vertices of the mesh form a lattice.

The size of a lattice subdivided once will be (2n-1)x(2m-1), where the original lattice contains nxm vertices. The length of the edges will be the half of the original edges. The horizontal position of same points are the same as the horizontal position of the original vertices, and other part of the new vertices are in the middlepoint of the original edges and centers of the original squares.

The height of the elements of the new, subdivided model are calculated by the principles of the Catmull-Clark Subdivision, but in this case the heights of the vertices are calculated only, the horizontal positions are determined by the lattice implicity.

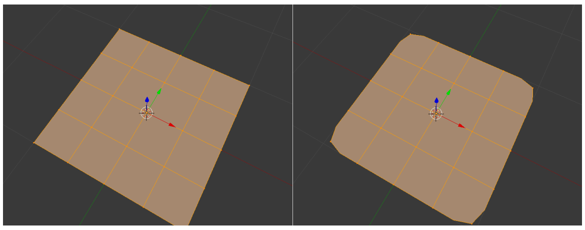

Figure 2. The rounded corner of the surface in the classical Catmull-Clark method.

The result of this solution is somewhat different from the result of the classical Calmull-Clark subdivision, because the corners of the surface are not rounded.

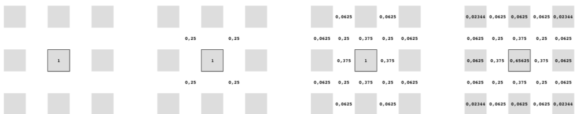

Figure 3. Calculating the weight of a vertex step by step

The Figure 4. shows the final weight of an vertex of the original grid. The gray background cells are the vertices which have also been in the previous grid, the framed cell is the studied vertex. The Figure 5. and 6. shows the weights of the multiple subdivision. Because the result is symmetrical, the Figure 6. shows only a quarter of the table.

0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000
0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000
0,000000	0,000000	0,023438	0,062500	0,062500	0,062500	0,023438	0,000000	0,000000
0,000000	0,000000	0,062500	0,250000	0,375000	0,250000	0,062500	0,000000	0,000000
0,000000	0,000000	0,062500	0,375000	0,656250	0,375000	0,062500	0,000000	0,000000
0,000000	0,000000	0,062500	0,250000	0,375000	0,250000	0,062500	0,000000	0,000000
0,000000	0,000000	0,023438	0,062500	0,062500	0,062500	0,023438	0,000000	0,000000
0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000
0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000

Figure 4. The weight of a vertex (the bordered cell) in the subdivided grid.

0,000549	0,001465	0,002930	0,005371	0,005920	0,007813	0,006836	0,007813	0,005920	0,005371	0,002930	0,001465	0,000549
0,001465	0,005859	0,012695	0,021484	0,028809	0,031250	0,031250	0,031250	0,028809	0,021484	0,012695	0,005859	0,001465
0,002930	0,012695	0,029053	0,051758	0,072266	0,085938	0,083984	0,085938	0,072266	0,051758	0,029053	0,012695	0,002930
0,005371	0,021484	0,051758	0,099609	0,149902	0,187500	0,203125	0,187500	0,149902	0,099609	0,051758	0,021484	0,005371
0,005920	0,028809	0,072266	0,149902	0,237610	0,306641	0,342773	0,306641	0,237610	0,149902	0,072266	0,028809	0,005920
0,007813	0,031250	0,085938	0,187500	0,306641	0,414063	0,464844	0,414063	0,306641	0,187500	0,085938	0,031250	0,007813
0,006836	0,031250	0,083984	0,203125	0,342773	0,464844	0,547852	0,464844	0,342773	0,203125	0,083984	0,031250	0,006836
0,007813	0,031250	0,085938	0,187500	0,306641	0,414063	0,464844	0,414063	0,306641	0,187500	0,085938	0,031250	0,007813
0,005920	0,028809	0,072266	0,149902	0,237610	0,306641	0,342773	0,306641	0,237610	0,149902	0,072266	0,028809	0,005920
0,005371	0,021484	0,051758	0,099609	0,149902	0,187500	0,203125	0,187500	0,149902	0,099609	0,051758	0,021484	0,005371
0,002930	0,012695	0,029053	0,051758	0,072266	0,085938	0,083984	0,085938	0,072266	0,051758	0,029053	0,012695	0,002930
0,001465	0,005859	0,012695	0,021484	0,028809	0,031250	0,031250	0,031250	0,028809	0,021484	0,012695	0,005859	0,001465

Figure 5. The weight of a vertex (the bordered cell) in the twice subdivided grid. Two rows and columns of zeros have been cut from the border

The impact of a vertex does not extend more distance than two units of the original grid, because

$$\lim_{n \to \infty} \sum_{i=1}^{n} \frac{1}{i} = 2$$

.

The elevation of a vertex of the subdivided grid can be calculated as the mean of the elevation of some vertices of the original grid. This method can be derived a new grid

0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000
0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000
0,000000	0,000000	0,000013	0,000034	0,000069	0,000126	0,000173	0,000275	0,000343	0,000519	0,000543	0,000706	0,000679	0,000858	0,000787	0,000916	0,000793
0,000000	0,000000	0,000034	0,000137	0,000298	0,000504	0,000767	0,001099	0,001526	0,002075	0,002567	0,002823	0,003044	0,003433	0,003727	0,003662	0,003540
0,000000	0,000000	0,000069	0,000298	0,000681	0,001213	0,001877	0,002808	0,003784	0,005249	0,006393	0,007378	0,007746	0,008904	0,009285	0,009399	0,008881
0,000000	0,000000	0,000126	0,000504	0,001213	0,002335	0,003849	0,005737	0,007996	0,010620	0,013218	0,015396	0,017143	0,018448	0,019199	0,019287	0,019165
0,000000	0,000000	0,000173	0,000767	0,001877	0,003849	0,006446	0,009842	0,013710	0,018387	0,022846	0,027317	0,030624	0,033268	0,034082	0,034973	0,034485
0,000000	0,000000	0,000275	0,001099	0,002808	0,005737	0,009842	0,015076	0,021393	0,028748	0,036392	0,043579	0,049805	0,054565	0,057465	0,058105	0,057861
0,000000	0,000000	0,000343	0,001526	0,003784	0,007996	0,013710	0,021393	0,030602	0,041901	0,053566	0,065247	0,075165	0,084167	0,089516	0,092041	0,090759
0,000000	0,000000	0,000519	0,002075	0,005249	0,010620	0,018387	0,028748	0,041901	0,058044	0,075699	0,093384	0,109863	0,123901	0,134369	0,140137	0,141846
0,000000	0,000000	0,000543	0,002567	0,006393	0,013218	0,022846	0,036392	0,053566	0,075699	0,100214	0,125538	0,149612	0,170429	0,186912	0,197693	0,201813
0,000000	0,000000	0,000706	0,002823	0,007378	0,015396	0,027317	0,043579	0,065247	0,093384	0,125538	0,159256	0,191795	0,220413	0,243641	0,260010	0,266479
0,000000	0,000000	0,000679	0,003044	0,007746	0,017143	0,030624	0,049805	0,075165	0,109863	0,149612	0,191795	0,233828	0,270226	0,301460	0,322876	0,333221
0,000000	0,000000	0,000858	0,003433	0,008904	0,018448	0,033268	0,054565	0,084167	0,123901	0,170429	0,220413	0,270226	0,316238	0,354755	0,382080	0,392944
0,000000	0,000000	0,000787	0,003727	0,009285	0,019199	0,034082	0,057465	0,089516	0,134369	0,186912	0,243641	0,301460	0,354755	0,402641	0,433472	0,448639
0,000000	0,000000	0,000916	0,003662	0,009399	0,019287	0,034973	0,058105	0,092041	0,140137	0,197693	0,260010	0,322876	0,382080	0,433472	0,472900	0,489624
0,000000	0,000000	0,000793	0,003540	0,008881	0,019165	0,034485	0,057861	0,090759	0,141846	0,201813	0,266479	0,333221	0,392944	0,448639	0,489624	0,514557

Figure 6. The weight of a vertex (the bordered cell) in the three times subdivided grid. One quarter of the symmetric table has been cut.

model whose resolution is $1/2^n$ part of the original grid.

B. *Approximation of an arbitrary point*

The method described in the last subsection can be used only where the resolution of the derived grid is $1/2^n$ part of the original grid, and entire grid is calculated. A modified type of this method can calculate the elevation in an arbitrary point.

This methos works on 6x6 sub-lattices, The sub-lattice is situated around the looking point, three rows and columns are before and after. This grid contains 5x5 squares, the looking point is within the central square.

A 7x7 subdivided grid can be calculated from such a 6x6 lattices. A 6x6 grid can be subdivided to a 11x11 grid, but the previous 6x6 grid does not contains every vertex for calculating the elevation of the two outer rows and columns.

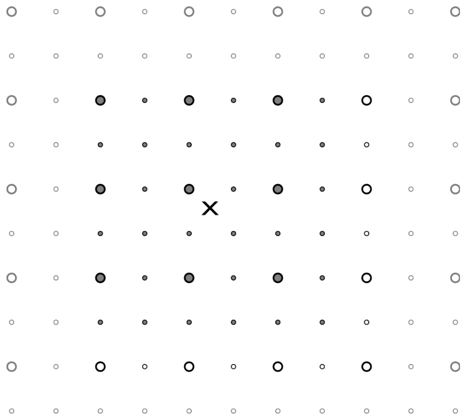

Figure 7. The 6x6 lattices. Big circles: vertex in the i step. Small circles: vertex in the i+1 step. Black border: vertex which can be calculated for the i+step from data of i step. Gray filling: vertex which will be vertex of the 6x6 lattice of the i+1 step.

The 7x7 grid contains an 6x6 subgrid whose central square contains the looking point. The method can be used recursively in this 6x6 subgrid until the resolution of the grid reaches a limit (for example 1 centimeter). The elevation of the looking point will be the closest point of the last 6x6 grid. The first 6x6 grid was a subgrid of the original elevation model.

IV. COMPARSION OF THE METHODS, CONCLUSION

A. The properties of the surfaces

The bilinear surfaces can be created an simple method, but this surface is not continuous in the junction of the squares.

The Catmull-Clark subdivision creates an approximation surface, because the original elevations of the vertices are modified in the result.

An surface created by the Catmull-Clark subdivision is continuous, because it is a B-Spline surface. [3] This surface is suitable to study the properties of the terrain surfaces depended on the second derivates of the surface for example the curvature.

B. Contour line visualization

The Figure 2. and the Figure 3. shows the contour line map of an area. Both of this maps are derived from 3 arcsec SRTM modell, and before the contour line generation resampling to a new grid with 10 meters resolution.

In the Figure 3. bilinear surfaces are used for the resampling and in the Figure 2. used Catmull-Clark subdivision. The lines of the bilinear interpolation based map have fractions because the surface of this model is not continuous. The lines of the subdivision based map are continuous.

V. SUMMARY

The Catmull-Clark subdivision can be used for approximation of GRID digital elevation model, the resampling of the grid or determine the elevation in an arbitrary horizontal position.

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[2] E. Catmull and J. Clark, *Recursively generated B-Spline surfaces on arbitrary topological meshes*, Proceedings of the 25th annual conference on Computer graphics and interactive techniques - SIGGRAPH '98". p. 85.

[3] Stam, J. (1998). *Exact evaluation of Catmull-Clark subdivision surfaces at arbitrary parameter values,* (PDF). Proceedings of the 25th annual conference on Computer graphics and interactive techniques - SIGGRAPH '98. pp. 395–404

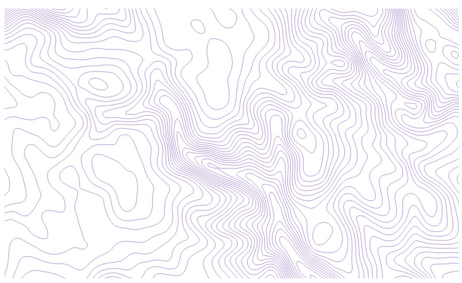

Figure 8. Contour line map from Catmull-Clark subdivided surface

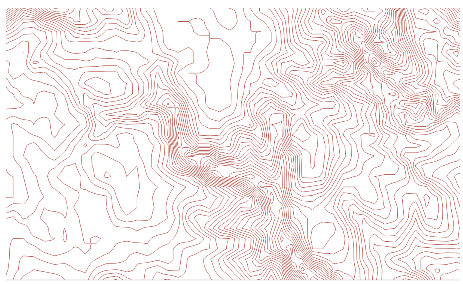

Figure 9. Contour line map by bilinear interpolation