THE LOCAL SPECTRAL RADIUS OF A NONNEGATIVE ORBIT OF COMPACT LINEAR OPERATORS

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Abstract. We consider orbits of compact linear operators in a real Banach space which are nonnegative with respect to the partial ordering induced by a given cone. The main result shows that under a mild additional assumption the local spectral radius of a nonnegative orbit is an eigenvalue of the operator with a positive eigenvector.

1. Introduction and the main result

Let $(X, \|\cdot\|)$ be a real Banach space. The symbol $\mathcal{B}(X)$ denotes the space of bounded linear operators in X equipped with the operator norm. By the spectrum of $T \in \mathcal{B}(X)$, denoted by $\sigma(T)$, we mean the spectrum of the complex extension $T_{\mathbb{C}} : X_{\mathbb{C}} \to X_{\mathbb{C}}$, where $X_{\mathbb{C}}$ is the complexification of X, the set of formal pairs $x + iy$ with x, $y \in X$ equipped with the norm $\|x + iy\| = \max_{0 \le t \le 2\pi} \|(\sin t)x + (\cos t)y\|$, and $T_{\mathbb{C}}(x + iy) = Tx + iTy$. A similar remark holds for the eigenvalues of $T \in \mathcal{B}(X)$. The set of eigenvalues, the so-called point spectrum of $T \in \mathcal{B}(X)$, is denoted by $\sigma_p(T)$. The spectral radius of $T \in \mathcal{B}(X)$ is defined by $r(T) = \max_{\lambda \in \sigma(T)} |\lambda|$. Note that according to Gelfand's formula

$$r(T) = \lim_{n\to\infty} \sqrt[n]{\|T^n\|}$$

for every $T \in \mathcal{B}(X)$. An operator $T \in \mathcal{B}(X)$ is called compact if the closure of $T(B)$ is compact, where B denotes the unit ball in X. The space of compact operators in X will be denoted by $\mathcal{K}(X)$. It is known that if $T \in \mathcal{K}(X)$, then every nonzero element of the spectrum $\sigma(T)$ is an eigenvalue of T. Hence

$$r(T) = \max_{\lambda \in \sigma_p(T)} |\lambda| \qquad \text{whenever } T \in \mathcal{K}(X) \text{ and } r(T) > 0. \tag{1.1}$$

A set $K \subset X$ is called a *cone* if conditions (i), (ii), and (iii) below hold.

(i) K is a nonempty, convex and closed subset of X,

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(ii) $tK \subset K$ for all $t \geq 0$, where $tK = \{tx \mid x \in K\}$,
(iii) $K \cap (-K) = \{\theta\}$, where $-K = \{-x \mid x \in K\}$.

Let $K \subset X$ be a cone. Then K induces a partial ordering $\leq_K$ on X by $x \leq_K y$ if and only if $y - x \in K$. An element $x \in X$ is called *K-nonnegative* if $\theta \leq_K x$. An operator $T \in \mathcal{B}(X)$ is called *K-nonnegative* if $\theta \leq_K x$ implies $\theta \leq_K Tx$. Clearly, $x \in X$ is K-nonnegative if and only if $x \in K$, and $T \in \mathcal{B}(X)$ is K-nonnegative if and only if $T(K) \subset K$. Recall that a cone $K \subset X$ is *total* if $\mathrm{cl}(K - K) = X$, where $\mathrm{cl}(K - K)$ denotes the closure of the set $K - K = \{\, x - y \mid x,\, y \in K \,\}$.

Krein and Rutman [5] proved the following result about the spectral radius of positive compact operators (see also [4] and [8] for secondary sources).

Theorem 1.1. *Suppose that $(X, \|\cdot\|)$ is a real Banach space and $K \subset X$ is a total cone. Assume also that $T \in \mathcal{K}(X)$ is a K-nonnegative operator with $r(T) > 0$. Then there exists $v \in K \setminus \{\theta\}$ such that $Tv = r(T)v$ so that $r(T) \in \sigma_p(T)$.*

Note that in the finite dimensional case $X = \mathbb{R}^n$ and $K = \mathbb{R}^n_+$ Theorem 1.1 implies the Perron-Frobenius theorem stating that the spectral radius of a nonnegative $n \times n$ matrix A is always an eigenvalue of A with a nonnegative eigenvector (see, e.g., [1]).

In this paper we will prove an analogue of Theorem 1.1 for the local spectral radius corresponding to a nonnegative orbit of $T \in \mathcal{K}(X)$. By an orbit of $T \in \mathcal{B}(X)$, we mean a sequence $\{T^n x\}_{n=0}^{\infty}$, where $x \in X$ is a given vector. The *local spectral radius of $T \in \mathcal{B}(X)$ at $x \in X$* is denoted by $\bar{r}(T; x)$ and is defined by

$$\bar{r}(T; x) = \limsup_{n \to \infty} \sqrt[n]{\|T^n x\|}. \tag{1.2}$$

Note that $\bar{r}(T; x)$ describes the exponential growth of the orbit $\{T^n x\}_{n=0}^{\infty}$. Its logarithm is the Lyapunov exponent. Define also

$$\underline{r}(T; x) = \liminf_{n \to \infty} \sqrt[n]{\|T^n x\|}. \tag{1.3}$$

Evidently, $\underline{r}(T; x) \leq \bar{r}(T; x)$. It is known that if T is merely bounded, then the last inequality may be strict (see [6] for details and further related results). However, if T is compact, then $\underline{r}(T; x) = \bar{r}(T; x)$ for all $x \in X$ and the following analogue of Conclusion (1.1) holds.

Theorem 1.2. *Suppose that $(X, \|\cdot\|)$ is a real Banach space and $T \in \mathcal{K}(X)$. Then for every $x \in X$ the limit*

$$r(T; x) = \lim_{n \to \infty} \sqrt[n]{\|T^n x\|} \tag{1.4}$$

exists. Furthermore, if $r(T; x) > 0$ for some $x \in X$, then $r(T; x)$ is the modulus of one of the eigenvalues of T, that is,

$$r(T; x) = |\lambda| \qquad \text{for some } \lambda \in \sigma_p(T).$$

Theorem 1.2 follows from Corollary B.3 from Appendix B of [2] and from the Riesz-Schauder theory of compact linear operators.

Our main result is the following theorem.

Theorem 1.3. *Suppose that $(X, \|\cdot\|)$ is a real Banach space, $K \subset X$ is a cone and $T \in \mathcal{K}(X)$. Assume also that for some $x \in X$ conditions*

$$T^n x \in K \qquad \text{for all } n = 0, 1, 2, \dots \tag{1.5}$$

and

$$r(T; x) > 0 \tag{1.6}$$

hold. Then there exists $v \in K \setminus \{\theta\}$ such that $Tv = r(T; x)v$ so that $r(T; x) \in \sigma_p(T)$.

Note that in contrast with Theorem 1.1, in Theorem 1.3 we do not require the K-nonnegativity of operator T. We assume merely that the orbit $\{T^n x\}_{n=0}^{\infty}$ is K-nonnegative (see (1.5)). For the finite dimensional analogue of Theorem 1.3 for Poincaré difference equations, see [7].

2. Proof

Before we present the proof of Theorem 1.3, we recall some facts from the spectral theory of bounded linear operators (see Chapter VII of [3]) and we establish some lemmas.

Suppose that $(X, \|\cdot\|)$ is a complex Banach space and $T \in \mathcal{B}(X)$. Let σ be a closed isolated subset of the spectrum $\sigma(T)$. The *spectral projection (Riesz idempotent) of T* corresponding to σ is denoted by P_σ and is defined by

$$P_\sigma = \frac{1}{2\pi i} \int_\Gamma (z - T)^{-1}\, dz, \tag{2.1}$$

where Γ is any positively oriented Jordan system such that

$$\sigma \subset \operatorname{ins} \Gamma \qquad \text{and} \qquad \sigma(T) \setminus \sigma \subset \operatorname{out} \Gamma,$$

where $\operatorname{ins} \Gamma$ and $\operatorname{out} \Gamma$ denote the inside of Γ and the outside of Γ, respectively (see Chapter VII, Section 6.9 of [3]). The bounded linear operator P_σ has properties

$$P_\sigma^2 = P_\sigma, \tag{2.2}$$

and

$$P_\sigma T = T P_\sigma. \tag{2.3}$$

The set $P_\sigma(X)$ is called the *generalized eigenspace* of T corresponding to σ. It follows from (2.3) that T maps $P_\sigma(X)$ into itself and if $T_\sigma = T|_{P_\sigma(X)}$, the restriction of T onto $P_\sigma(X)$, then the spectrum of $T_\sigma : P_\sigma(X) \to P_\sigma(X)$ is σ, that is,

$$\sigma(T_\sigma) = \sigma. \tag{2.4}$$

Assume in addition that $T \in \mathcal{K}(X)$, i.e. T is compact. Then every $\lambda \in \sigma(T)\setminus\{0\}$ is an eigenvalue of T and for every $\epsilon > 0$ the set $\{\lambda \in \sigma(T) \mid |\lambda| \geq \epsilon\} \subset \sigma_p(T)$ is finite. It is known that each $\lambda \in \sigma(T) \setminus \{0\}$ is a pole of the resolvent $(z - T)^{-1}$ and the generalized eigenspace $P_{\{\lambda\}}(X)$ is finite dimensional (see Corollary 7.8 of [3]). Now suppose that $\sigma \subset \sigma(T)\setminus\{0\}$ is a finite set. By the Residue Theorem, we have

$$P_\sigma = \sum_{\lambda\in\sigma} P_{\{\lambda\}}$$

and therefore the generalized eigenspace $P_\sigma(X)$ is also finite dimensional. Furthermore, since $0 \notin \sigma$, in this case operator $T_\sigma : P_\sigma(X) \to P_\sigma(X)$ is invertible.

As noted on p. 799 of [8], if $T \in \mathcal{B}(X)$ and

$$\sigma(T) = \sigma_1 \cup \sigma_2 \cup \cdots \cup \sigma_k, \tag{2.5}$$

where $\sigma_1, \sigma_2, \ldots, \sigma_k$, are pairwise disjoint closed isolated subsets of $\sigma(T)$, then X can be decomposed into the direct sum

$$X = P_{\sigma_1}(X) \oplus P_{\sigma_2}(X) \oplus \cdots \oplus P_{\sigma_k}(X). \tag{2.6}$$

If $(X, \|\cdot\|)$ is a real Banach space and $T \in \mathcal{B}(X)$, then the spectral projection P_σ can be defined as in (2.1) for the complex extension $T_{\mathbb{C}}$. If we consider only sets $\sigma \subset \sigma(T)$ which are symmetric with respect to the real axis, then the restriction of P_σ onto X is a projection operator on X and the set $P_\sigma(X)$ is called the *generalized real eigenspace* of T corresponding to σ. Finally, if each σ_j, $1 \leq j \leq k$, in (2.5) is symmetric with respect to the real axis, then the splitting result (2.6) remains valid for real Banach spaces.

Now we establish a lemma will play an important role in the proof of Theorem 1.3.

Lemma 2.1. *Let $(X, \|\cdot\|)$ be a real Banach space and $T \in \mathcal{K}(X)$. Suppose that for some $x \in X$ conditions (1.5) and (1.6) hold. Define*

$$\sigma_- = \{\, \lambda \in \sigma(T) \mid |\lambda| < r(T;x) \,\}, \tag{2.7}$$

$$\sigma_0 = \{\, \lambda \in \sigma(T) \mid |\lambda| = r(T;x) \,\}, \tag{2.8}$$

and

$$\sigma_+ = \{\, \lambda \in \sigma(T) \mid |\lambda| > r(T;x) \,\} \tag{2.9}$$

so that

$$\sigma(T) = \sigma_- \cup \sigma_0 \cup \sigma_+ \tag{2.10}$$

and

$$X = P_{\sigma_-}(X) \oplus P_{\sigma_0}(X) \oplus P_{\sigma_+}(X). \tag{2.11}$$

Let

$$K_0 = K \cap P_{\sigma_0}(X). \tag{2.12}$$

Then there exists $x_0 \in K_0$ with $\|x_0\| = 1$ such that

$$T^n x_0 \in K_0 \qquad \text{for all } n = 0, 1, 2, \ldots. \tag{2.13}$$

The proof of Lemma 2.1 will be based on the following simple result.

Lemma 2.2. *Let $(X, \|\cdot\|)$ be a Banach space and $T \in \mathcal{B}(X)$. Then for every $x \in X$, we have*

$$\bar{r}(T;x) \leq \max_{\lambda\in\sigma(T)} |\lambda|. \tag{2.14}$$

If $\dim X < \infty$ and $T \in \mathcal{B}(X)$ is invertible, then for every $x \in X \setminus \{\theta\}$, we have

$$r(T;x) \geq \min_{\lambda\in\sigma(T)} |\lambda| > 0. \tag{2.15}$$

Proof. Conclusion (2.14) follows immediately from the obvious inequality

$$\|T^n x\| \leq \|T^n\|\|x\|, \qquad x \in X,\ n \geq 0,$$

and Gelfand's spectral radius formula.

Suppose that $\dim X < \infty$ and $T \in \mathcal{B}(X)$ is invertible. Then $\sigma(T) = \sigma_p(T)$, $0 \notin \sigma(T)$, and the spectrum of T^{-1}, the inverse of T, is given by

$$\sigma(T^{-1}) = \{\, \lambda^{-1} \mid \lambda \in \sigma(T)\,\}.$$

Hence

$$r(T^{-1}) = \max_{\lambda\in\sigma(T)} |\lambda|^{-1} = \frac{1}{\min_{\lambda\in\sigma(T)} |\lambda|}.$$

This, combined with Gelfand's spectral radius formula, yields

$$\lim_{n\to\infty} \sqrt[n]{\|T^{-n}\|} = \frac{1}{\min_{\lambda\in\sigma(T)} |\lambda|},$$

where $T^{-n} = (T^{-1})^n$ for $n \geq 0$. Choose $\epsilon > 0$. The previous limit relation implies

$$\|T^{-n}\| < \left(\frac{1}{\min_{\lambda\in\sigma(T)} |\lambda|} + \epsilon\right)^n$$

for all large n. This, together with the inequality

$$\|x\| = \|T^{-n}T^n x\| \leq \|T^{-n}\|\|T^n x\|, \qquad x \in X,\ n \geq 0,$$

implies

$$\|T^n x\| \geq \frac{1}{\|T^{-n}\|}\|x\| \geq \left(\frac{1}{\min_{\lambda\in\sigma(T)} |\lambda|} + \epsilon\right)^{-n} \|x\|$$

for all $x \in X$ and n sufficiently large. The last inequality implies that if $x \in X \setminus \{\theta\}$, then

$$r(T;x) \geq \left(\frac{1}{\min_{\lambda\in\sigma(T)} |\lambda|} + \epsilon\right)^{-1}.$$

(The existence of the limit $r(T;x)$ (see (1.4)) follows from Theorem 1.2 since every linear operator in a finite dimensional Banach space is compact.) Letting $\epsilon \to 0$ in the last inequality, we obtain (2.15). □

Remark 1. The result of Lemma 2.2 is not really new. Conclusion (2.15) is in fact a corollary of Theorem 1.2 of [7]. We have included the above proof only for the readers' convenience.

Now we give a proof of Lemma 2.1.

Proof of Lemma 2.1. Define

$$y_n = \frac{T^n x}{\|T^n x\|}, \qquad n \geq 0. \tag{2.16}$$

By virtue of (1.6), the sequence $\{y_n\}_{n=0}^{\infty}$ is well-defined. Clearly, $\|y_n\| = 1$ for all $n \geq 0$ and hence

$$\|P_{\sigma_0} y_n\| \leq \|P_{\sigma_0}\| \|y_n\| = \|P_{\sigma_0}\|, \qquad n \geq 0.$$

This shows that $\{P_{\sigma_0} y_n\}_{n=0}^{\infty}$ is a bounded sequence in the generalized eigenspace $P_{\sigma_0}(X)$. Since T is compact and $r(T; x) > 0$, the set $\sigma_0 \subset \sigma(T) \setminus \{0\}$ is finite. As noted before, this implies that $\dim P_{\sigma_0}(X) < \infty$. Therefore the bounded sequence $\{P_{\sigma_0} y_n\}_{n=0}^{\infty}$ has a convergent subsequence. Consequently, there exist $x_0 \in P_{\sigma_0}(X)$ and a sequence $n_k \to \infty$ as $k \to \infty$ such that

$$\lim_{k \to \infty} P_{\sigma_0} y_{n_k} = x_0. \tag{2.17}$$

We claim that

$$\lim_{k \to \infty} y_{n_k} = x_0. \tag{2.18}$$

In view of (2.17) and the relation

$$y_{n_k} = P_{\sigma_-} y_{n_k} + P_{\sigma_0} y_{n_k} + P_{\sigma_+} y_{n_k}, \qquad k \geq 0,$$

in order to prove (2.18), it is enough to show that

$$\lim_{n \to \infty} P_{\sigma_-} y_n = \theta \tag{2.19}$$

and

$$\lim_{n \to \infty} P_{\sigma_+} y_n = \theta. \tag{2.20}$$

By virtue of (2.3) and (2.16), we have for $n \geq 0$,

$$P_{\sigma_-} y_n = \frac{T^n P_{\sigma_-} x}{\|T^n x\|} = \frac{T^n_{\sigma_-} P_{\sigma_-} x}{\|T^n x\|},$$

where $T_{\sigma_-} = T|_{P_{\sigma_-}(X)}$. This, together with (1.2) and (1.4), implies

$$\limsup_{n \to \infty} \sqrt[n]{\|P_{\sigma_-} y_n\|} = \limsup_{n \to \infty} \frac{\sqrt[n]{\|T^n_{\sigma_-} P_{\sigma_-} x\|}}{\sqrt[n]{\|T^n x\|}} = \frac{\bar{r}(T_{\sigma_-}; P_{\sigma_-} x)}{r(T; x)}. \tag{2.21}$$

From Conclusion (2.14) of Lemma 2.2 and conditions (2.4) and (2.7), we obtain

$$\bar{r}(T_{\sigma_-}; P_{\sigma_-} x) \leq \max_{\lambda \in \sigma(T_{\sigma_-})} |\lambda| = \max_{\lambda \in \sigma_-} |\lambda| < r(T; x).$$

The last inequality implies that the limsup in (2.21) is less than one. Therefore $P_{\sigma_-}y_n \to \theta$ exponentially as $n \to 0$ and hence (2.19) holds.

Now we prove (2.20). First we show that $P_{\sigma_+}x = \theta$. Suppose by way of contradiction that $P_{\sigma_+}x \neq \theta$. We have for $n \geq 0$,

$$P_{\sigma_+}y_n = \frac{T^n P_{\sigma_+}x}{\|T^n x\|} = \frac{T^n_{\sigma_+} P_{\sigma_+}x}{\|T^n x\|}, \tag{2.22}$$

where $T_{\sigma_+} = T|_{P_{\sigma_+}(X)}$. The compactness of T and the definition of σ_+ imply that $\sigma(T_{\sigma_+}) = \sigma_+ \subset \sigma(T) \setminus \{0\}$ is a finite set. Consequently, the generalized eigenspace $P_{\sigma_+}(X)$ is finite dimensional and $T_{\sigma_+} : P_{\sigma_+}(X) \to P_{\sigma_+}(X)$ is invertible and compact. From (2.22), by the application of Theorem 1.2, we find that

$$\lim_{n\to\infty} \sqrt[n]{\|P_{\sigma_+}y_n\|} = \lim_{n\to\infty} \frac{\sqrt[n]{\|T^n_{\sigma_+} P_{\sigma_+}x\|}}{\sqrt[n]{\|T^n x\|}} = \frac{r(T_{\sigma_+}; P_{\sigma_+}x)}{r(T; x)}. \tag{2.23}$$

Since $P_{\sigma_+}x \neq \theta$, Conclusion (2.15) of Lemma 2.2 and (2.9) imply that

$$r(T_{\sigma_+}; P_{\sigma_+}x) \geq \min_{\lambda\in\sigma(T_{\sigma_+})} |\lambda| = \min_{\lambda\in\sigma_+} |\lambda| > r(T; x).$$

Hence the limit in (2.23) is greater than one. This implies that $\|P_{\sigma_+}y_n\| \to \infty$ exponentially as $n \to \infty$ contradicting the fact that

$$\|P_{\sigma_+}y_n\| \leq \|P_{\sigma_+}\|\|y_n\| = \|P_{\sigma_+}\|, \qquad n \geq 0.$$

Thus, $P_{\sigma_+}x = \theta$ which implies that

$$P_{\sigma_+}y_n = \frac{T^n P_{\sigma_+}x}{\|T^n x\|} = \theta, \qquad n \geq 0.$$

Hence (2.20) holds. As noted before, this completes the proof of (2.18).

By virtue of (2.16) and (2.18), we have

$$\|x_0\| = \lim_{k\to\infty} \|y_{n_k}\| = 1.$$

As shown before, $x_0 \in P_{\sigma_0}(X)$. Further, T and hence each T^n, $n \geq 0$, maps $P_{\sigma_0}(X)$ into itself. Consequently, $T^n(x_0) \in P_{\sigma_0}(X)$ for every $n \geq 0$. Thus, in order to prove (2.13), it remains to show that

$$T^n x_0 \in K \qquad \text{for every nonnegative integer } n. \tag{2.24}$$

Let n be a fixed nonnegative integer. By virtue of (2.16), (2.18), the continuity of T and hence of T^n, we have

$$T^n(x_0) = T^n\big(\lim_{k\to\infty} y_{n_k}\big) = \lim_{k\to\infty} T^n y_{n_k} = \lim_{k\to\infty} \frac{T^{n+n_k}x}{\|T^{n_k}x\|}.$$

From assumption (1.5) and the cone property (ii), it follows that

$$\frac{T^{n+n_k}x}{\|T^{n_k}x\|} \in K \qquad \text{for every } k \geq 0.$$

From this, taking into account that K is a closed set, we see that $T^n(x_0)$ as a limit of the above sequence from K also belongs to K. Since $n \geq 0$ was arbitrary, this proves (2.24). □

Now we are in a position to give a proof of Theorem 1.3.

Proof of Theorem 1.3. Adopt the notation of Lemma 2.1. Define

$$C = \{\, x \in K_0 \mid \|x\| \leq 1 \text{ and } T^n x \in K_0 \text{ for all } n = 0, 1, \dots \,\}$$

with K_0 as in (2.12). By the application of Lemma 2.1, we conclude that there exists $x_0 \in C$ such that $\|x_0\| = 1$. It is easily verified that C is a convex closed subset of $P_{\sigma_0}(X)$. As noted in the proof of Lemma 2.1, the subspace $P_{\sigma_0}(X)$ is finite dimensional and if $T_{\sigma_0} = T|_{P_{\sigma_0}(X)}$, then $T_{\sigma_0} : P_{\sigma_0}(X) \to P_{\sigma_0}(X)$ is invertible.

Define an operator $F : C \to P_{\sigma_0}(X)$ by

$$F(x) = \frac{(1 - \|x\|)x_0 + \|x\| T_{\sigma_0} x}{\|(1 - \|x\|)x_0 + \|x\| T_{\sigma_0} x\|}, \qquad x \in C.$$

The cone property (iii), the fact that $x_0 \neq \theta$ and the invertibility of operator $T_{\sigma_0} : P_{\sigma_0}(X) \to P_{\sigma_0}(X)$ imply that F is well-defined. Clearly, F is continuous on C. Further, the definition of C and the cone properties (i) and (ii) imply that $F(C) \subset C$. By Brouwer's fixed point principle, there exists $v \in C$ such that $F(v) = v$. Since $\|v\| = \|F(v)\| = 1$, it follows that $T_{\sigma_0} v = \rho v$, where $\rho = \|T_{\sigma_0}(v)\|$. Thus, ρ is a nonnegative eigenvalue of $T_{\sigma_0} : P_{\sigma_0}(X) \to P_{\sigma_0}(X)$. Since the spectrum of T_{σ_0} coincides with σ_0, we have $\rho = |\rho| = r(T; x)$. Thus, $r(T; x)$ is an eigenvalue of T with eigenvector $v \in K$. □

Remark 2. Under the hypotheses of Theorem 1.3, the set K_0 defined by (2.12) is a cone in the finite-dimensional space $P_{\sigma_0}(X)$. Consequently, Theorem 1.3 can also be deduced from Conclusion (2.13) of Lemma 2.1 and Theorem 1.3 of [7]. Note that the above short proof is independent of Theorem 1.3 of [7]. It uses only Brouwer's fixed point theorem.

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