Chromatic Ramsey number of acyclic hypergraphs

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Abstract

Suppose that T is an acyclic r-uniform hypergraph, with $r \geq 2$. We define the (t-color) chromatic Ramsey number $\chi(T,t)$ as the smallest m with the following property: if the edges of any m-chromatic r-uniform hypergraph are colored with t colors in any manner, there is a monochromatic copy of T. We observe that $\chi(T,t)$ is well defined and

$$\left\lceil \frac{R^r(T,t)-1}{r-1} \right\rceil + 1 \leq \chi(T,t) \leq |E(T)|^t + 1$$

where $R^r(T,t)$ is the t-color Ramsey number of H. We give linear upper bounds for $\chi(T,t)$ when T is a matching or star, proving that for $r \geq 2, k \geq 1, t \geq 1$, $\chi(M_k^r,t) \leq (t-1)(k-1)+2k$ and $\chi(S_k^r,t) \leq t(k-1)+2$ where M_k^r and S_k^r are, respectively, the r-uniform matching and star with k edges.

The general bounds are improved for 3-uniform hypergraphs. We prove that $\chi(M_k^3,2)=2k$, extending a special case of Alon-Frankl-Lovász' theorem. We also prove that $\chi(S_2^3,t) \leq t+1$, which is sharp for $t=2,3$. This is a corollary of a more general result. We define $H^{[1]}$ as the 1-intersection graph of H, whose vertices represent hyperedges and whose edges represent intersections of hyperedges in exactly one vertex. We prove that $\chi(H) \leq \chi(H^{[1]})$ for any 3-uniform hypergraph H (assuming $\chi(H^{[1]}) \geq 2$). The proof uses the list coloring version of Brooks' theorem.

1 Introduction

A hypergraph $H=(V,E)$ is a set V of *vertices* together with a nonempty set E of subsets of V, which are called *edges*. In this paper, we will assume that for each $e \in E$, $|e| \geq 2$. If $|e|=r$ for each $e \in E$, then H is *r-uniform*; a 2-uniform H is a graph. A hypergraph H is *acyclic* if H contains no cycles (including 2-cycles which are two edges intersecting in at least two vertices). If H is a connected acyclic hypergraph, we say that H is a *tree*. In particular, a *star* is a tree in which one vertex is common to every edge. A *matching* is a hypergraph consisting of pairwise disjoint edges, with every vertex belonging to some edge. We denote by S_k^r and M_k^r the r-uniform k-edge star and matching, respectively.

For a positive integer k, a function $c: V \to \{1,\ldots,k\}$ is called a k-coloring of H. A coloring c is *proper* if no edge of H is monochromatic under c. The chromatic number of H, denoted $\chi(H)$, is the least $m \geq 1$ for which there exists a proper m-coloring of H and in this case, we say that H is m-chromatic. Given $H=(V,E)$, a partition $\{E_1,\ldots,E_t\}$ of E into t parts is called a t-edge-coloring of H. For r-uniform hypergraphs $H_1,H_2,\ldots,H_t$, the (t-color) Ramsey number $R^r(H_1,H_2,\ldots,H_t)$ is the smallest integer n for which the following is true: under any t-edge-coloring of the complete r-uniform hypergraph K_n^r, there is a monochromatic copy of H_i in color i for some $i \in \{1,2,\ldots,t\}$. When all $H_i=H$ we use the notation $R^r(H,t)$.

Bialostocki and the senior author of this paper extended two well-known results in Ramsey theory from complete host graph K_n to arbitrary n-chromatic graphs [4]. One extends a remark of Erdős and Rado stating that in any 2-coloring of the edges of a complete graph K_n there is a monochromatic spanning tree. The other is the extension of the result of Cockayne and Lorimer [5] about the t-color Ramsey number of matchings. In [8], an acyclic graph H is defined as *t-good* if every t-edge coloring of any $R^2(H,t)$-chromatic graph contains a monochromatic copy of H. Matchings are t-good for every t [4] and in [8] it was proved that stars are t-good, as well as the path P_4 (except possibly for $t = 3$). Additionally, P_5, P_6, P_7 are 2-good. In fact, as remarked in [4], there is no known example of an acyclic H that is not t-good.

In this paper, we explore a similar extension of Ramsey theory for hypergraphs, motivating the following definition.

Definition 1. *Suppose that T is an acyclic r-uniform hypergraph. Let $\chi(T,t)$ be the smallest m with the following property: under any t-edge-coloring of any m-chromatic r-uniform hypergraph, there is a monochromatic copy of T.*

We call $\chi(T,t)$ the *chromatic Ramsey number of T*. It follows from the existence of hypergraphs of large girth and chromatic number that the chromatic Ramsey number can be defined only for acyclic hypergraphs.

2 New results

First we note that $\chi(T,t)$ is well-defined for any r-uniform tree T and any $t \geq 1$, as an upper bound comes easily from the following result.

Lemma A. ([10],[12]) If H is r-uniform with $\chi(H) \geq k+1$, then H contains a copy of any r-uniform tree on k edges.

Theorem 2. *For any r-uniform tree T, $\chi(T,t) \leq |E(T)|^t + 1$.*

Proof. Fix $t \geq 1$. Let T be an r-uniform tree with k edges and let $H = (V,E)$ be a hypergraph with $\chi(H) \geq k^t + 1$. Let $E = E_1 \dot\cup \cdots \dot\cup E_t$ be a t-coloring of its edges.

Then $\chi((V,E_1)) \cdots \chi((V,E_t)) \geq k^t + 1$ and without loss of generality, $\chi((V,E_1)) \geq k+1$. Then by Lemma A, (V,E_1) contains a copy of T. □

Since any r-uniform acyclic hypergraph T may be found in some r-uniform tree T' and $\chi(T,t) \leq \chi(T',t)$, $\chi(T,t)$ is in fact well-defined for any r-uniform acyclic hypergraph and for any $t \geq 1$. Observe the following natural lower bound of $\chi(T,t)$. Let $L(T,t,r) := \left\lceil \frac{R^r(T,t)-1}{r-1} \right\rceil + 1$.

Proposition 3. $L(T,t,r) \leq \chi(T,t)$

Proof. Let $N := R^r(T,t) - 1$. By the definition of the Ramsey number, there is a t-coloring of the edges of K_N^r without a monochromatic T. Since $\chi(K_N^r) = \lceil \frac{N}{r-1} \rceil$, the proposition follows. □

The notion of t-good graphs can be naturally extended to hypergraphs using Proposition 3. An acyclic r-uniform hypergraph T is called *t-good* if every t-edge coloring of any $L(T,t,r)$-chromatic r-uniform hypergraph contains a monochromatic copy of T. In other words, T is t-good if $L(T,t,r) = \chi(T,t)$. Note that for $r = 2$, this gives the definition of good graphs. Although it is unlikely that all acyclic hypergraphs are t-good, we have no counterexamples.

For special families of r-uniform acyclic hypergraphs, we found linear upper bounds for $\chi(T,t)$, improving upon the general exponential upper bound above. Surprisingly, most of the bounds attained do not depend on r.

2.1 Matchings

Indispensable in this section is the following well-known result of Alon, Frankl, and Lovász (originally conjectured by Erdős).

Theorem B. ([1]) For $r \geq 2, k \geq 1, t \geq 1$,

$$R^r(M_k^r, t) = (t-1)(k-1) + kr.$$

Note that special cases of Theorem B include $r = 2$ [5], $k = 2$ [13], $t = 2$ [2], [9].

We obtain the following linear upper bound for matchings using Theorem B.

Theorem 4. *For $r \geq 2, k \geq 1, t \geq 1$, $\chi(M_k^r, t) \leq (t-1)(k-1) + 2k$. Equality holds for $r = 2$.*

Proof. Let $H = (V,E)$ be an r-uniform hypergraph with $\chi(H) = p$ where $p := R^r(M_k^2, t)$. By Theorem B, $p = (t-1)(k-1) + 2k$. Consider any t-edge coloring $\{E_1, \ldots, E_t\}$ of H and any proper coloring c of H obtained by the greedy algorithm (under any ordering of its vertices). Clearly c uses at least p colors and for any $1 \leq i < j \leq p$ there is an edge e_{ij} in H whose vertices are colored with color i apart from a single vertex which is colored with j. Let $\{F_1, \ldots, F_t\}$ be a t-edge-coloring of K_p^2 defined so that $F_s := \{\{i,j\},\ 1 \leq i < j \leq p,\ e_{ij} \in E_s\}$ for each s, $1 \leq s \leq t$. From the definition of p, Theorem B (in fact the Cockayne-Lorimer Theorem suffices) implies that there is a monochromatic M_k^2 in K_p. Observe that

$$\{e_{ij} : \{i,j\} \in M_k^2\}$$

is a set of k pairwise disjoint edges in H in the same partition class of $\{E_1, \ldots, E_t\}$. This completes the proof that $\chi(M_k^r, t) \leq (t-1)(k-1) + 2k$. The lower bound $R^2(M_k^2, t) \leq \chi(M_k^2, t)$ implies equality in the $r = 2$ case. □

Next we tighten this bound, provided $r \geq 3$ and $t = 2$.

Theorem 5. *For $r \geq 3$ and $k \geq 1$, $\chi(M_k^r, 2) \leq 2k$.*

Proof. We fix $r \geq 3$ and proceed by induction on k. Suppose $k = 1$ and let H be some r-uniform hypergraph with $\chi(H) \geq 2$. Then any 2-edge-coloring of H contains a single monochromatic edge since H has at least one edge. Now suppose the theorem is true for $k-1 \geq 1$ and let $H = (V, E)$ be r-uniform with $\chi(H) \geq 2k$. Without loss of generality, H is connected. Fix some 2-edge-coloring $\{E_1, E_2\}$ of H, calling the edges of E_1 "red" and the edges of E_2 "blue". If E_1 or E_2 is empty, then Theorem 4 with $t = 1$ implies the desired bound.

So we may assume otherwise, and there exist edges $e, f \in E$ with e red and f blue. Let $s := |e \cap f|$ and $A := e \cup f$. If $H[A]$ is 2-colorable, then $\chi(H - A) \geq \chi(H) - 2 = 2(k-1)$ so by induction we find a monochromatic M^r_{k-1} matching in $H - A$. Without loss of generality, M^r_{k-1} is red and $M^r_{k-1} + e$ is a red M^r_k in H.

If $s > 1$, then $|A| = 2r - s \leq 2r - 2$ thus $H[A]$ is certainly 2-colorable and the induction works. If $s = 1$ and $H[A]$ is not 2-colorable then $H[A]$ is K^r_{2r-1}. Writing $e = \{w, u_1, \ldots, u_{r-1}\}$ and $f = \{w, v_1, \ldots, v_{r-1}\}$, the edge $g = \{w\} \cup \{u_1, u_3, \ldots\} \cup \{v_2, v_4, \ldots\} \in E(H)$. Without loss of generality, g is red and $|g \cap f| = 1 + \lfloor (r-1)/2 \rfloor \geq 2$ since $r \geq 3$. So the previous case applies to the red edge g and blue edge f. Finally, if $s = 0$ and $H[A]$ is not 2-colorable there must be $g \in H[A]$ that intersects both e and f. Then either e, g or f, g is a pair of edges of different color that intersect, and a previous case can be applied again. □

Corollary 6. *$\chi(M^3_k, 2) = 2k$.*

Proof. The upper bound is given by Theorem 5. The lower bound comes from Proposition 3 and Theorem B:

$$L(M^3_k, 2, 3) = \left\lceil \frac{k - 1 + 3k - 1}{2} \right\rceil + 1 = 2k \leq \chi(M^3_k, 2).$$

□

Corollary 7. *For $r \geq 3$, $\chi(M^r_2, 2) = 4$.*

Proof. As in Corollary 6, the upper bound comes from Theorem 5 and the lower bound from

$$L(M^r_2, 2, r) = \left\lceil \frac{2r - 1}{r - 1} \right\rceil + 1 = 4.$$

□

It is worth noting that Corollary 7 does not extend Theorem B to the chromatic Ramsey number setting for $r \geq 4$. Indeed, for $r = 4$, the lower bound $\lceil \frac{2r}{r-1} \rceil + 1$ of Proposition 3 is 4 and the bound $\lceil \frac{1+2r}{r-1} \rceil$ derived from Theorem B is 3.

2.2 Stars

Theorem 8. *For $r \geq 2, k \geq 1, t \geq 1$, $\chi(S^r_k, t) \leq t(k-1) + 2$.*

Proof. Fix $t, k \geq 1$ and let $p := t(k-1)+2$. Suppose that H is r-uniform with $\chi(H) \geq p$ and its edges are t-colored. By Lemma A, $\chi(S^r_{p-1}, 1) \leq p$, so we can find a copy of S^r_{p-1} in H. By the pigeonhole principle, k of the edges of S^r_{p-1} have the same color, and together they are a monochromatic copy of S^r_k. □

How good is the estimate of Theorem 8? Notice first that for $t = 1$ it is sharp.

Proposition 9. $\chi(S^r_k, 1) = k + 1$

Proof. Consider the complete hypergraph $K = K^r_{k(r-1)}$. Clearly, $\chi(K) = k$ and S^r_k is not a subgraph of K, as its vertex set is too large. □

If $t = 2$, Theorem 8 gives $\chi(S^r_k, 2) \leq 2k$. For $r = 2$ and odd k, this is a sharp estimate. For $k = 1$, this is trivial; for $k \geq 3$, the complete graph K^2_{2k-1} can be partitioned into 2 $(k-1)$-regular subgraphs. However, for even $k \geq 2$, $\chi(S^2_k, 2) = 2k - 1$.

An interesting problem arises when $T = S^r_2$ with $r \geq 3$, as Theorem 8 gives the relatively low upper bound 4. Can we decrease this bound? Namely:

Question 10. *Is* $\chi(S^r_2, 2) = 3$*?*

For $r = 3$ the positive answer (Corollary 14) comes from a more general result, Theorem 13 below. We first need a definition.

Definition 11. *Let* $H = (V(H), E(H))$ *be a hypergraph. The 1-intersection graph of* H *is denoted* $H^{[1]}$*, where* $V(H^{[1]}) = E(H)$ *and*

$$E(H^{[1]}) = \{(e, f) : e, f \in E(H) \text{ and } |e \cap f| = 1\}.$$

It is well-known that if $H^{[1]}$ is trivial, i.e., no two edges of H intersect in exactly one vertex, then H is 2-colorable ([14], Exercise 13.33). Note that the stronger statement $\chi(H) \leq \chi(H^{[1]})+1$ follows from applying the greedy coloring algorithm in any order of the vertices of H.

Question 12. *Let* $r \geq 3$*. Is it true that* $\chi(H) \leq \chi(H^{[1]})$ *for any* r*-uniform hypergraph* H*, provided* $H^{[1]}$ *is nontrivial?*

Our main result is the positive answer to Question 12 for the 3-uniform case.

Theorem 13. *If* H *is a* 3*-uniform hypergraph with* $\chi(H^{[1]}) \geq 2$ *then* $\chi(H) \leq \chi(H^{[1]})$*.*

Corollary 14. *For* $t \geq 1$*,* $\chi(S^3_2, t) \leq t + 1$*.*

The case $t = 2$ of Corollary 14 was the initial aim of the research in this paper and it was proved first by Zoltán Füredi [7]. Our proof of Theorem 13 uses his observation (Lemma 15 below) and the list-coloring version of Brooks' theorem. Corollary 14 is obviously sharp for $t = 2$; it follows from Proposition 3 that it is also sharp for $t = 3$, because $R^3(S^3_2, 3) = 6$ ([3]). It would be interesting to see whether Corollary 14 is true for any S^r_2 (in particular for $r = 4, t = 2$) as this is equivalent to the statement that r-uniform hypergraphs with bipartite 1-intersection graphs are 2-colorable.

3 Proof of Theorem 13

In this section, we use the phrase "triple system" for a 3-uniform hypergraph. The word "triple" will take the place of "edge" so that "edge" may be reserved for graphs. Our goal is to construct a proper t-coloring of H from a proper t-coloring of $H^{[1]}$. Note that a partition of $E(H)$ into classes $E_1, E_2, \ldots, E_t$ such that for any i, $1 \le i \le t$, no two edges of E_i 1-intersect is precisely a proper t-coloring of $H^{[1]}$. Let B_k denote the triple system with k edges intersecting pairwise in the vertices $\{v, w\}$, called the *base* of B_k. A B-component (also, B_k-component) is a triple system which is isomorphic to B_k for some $k \ge 1$. A K-component is either three or four distinct triples on four vertices. A triple system is connected if for every partition of its vertices into two nonempty parts, there is a triple intersecting both parts. Every triple system can be uniquely decomposed into pairwise disjoint connected parts, called components. Components with one vertex are called trivial components.

Lemma 15. *Let C be a nontrivial component in a triple system without* 1-*intersections. Then C is either a B-component or a K-component.*

Proof. If C has at most four vertices then $1 \le |E(C)| \le 4$ (where $E(C)$ is here considered as a set, not a multiset) and by inspection, C is either B_1, B_2, or a K-component. Assume C has at least five vertices and select the maximum m such that $e_1, e_2, \ldots e_m \in E(C)$ are distinct triples intersecting in a two-element set, say in $\{x, y\}$. Clearly, $m \ge 2$. Then $A = \cup_{i=1}^m e_i$ must cover all vertices of C, as otherwise there is an uncovered vertex z and a triple f containing z and intersecting A, since C is a component. However, from $m \ge 2$ and the intersection condition, $f \cap A = \{x, y\}$ follows, contradicting the choice of m. Thus $A = V(C)$ and from $|V(C)| \ge 5$ we have $m \ge 3$. It is obvious that any triple of C different from the e_i's would intersect some e_i in one vertex, violating the intersection condition. Thus C is isomorphic to B_m, concluding the proof. □

A multigraph G is called a *skeleton* of a triple system H if every triple contains at least one edge of G. We may assume that $V(H) = V(G)$. A *matching* in a multigraph is a set of pairwise disjoint edges. A *factorized complete graph* is a complete graph on $2m$ vertices whose edge set is partitioned into $2m - 1$ matchings. The following lemma allows us to define a special skeleton of triple systems.

Lemma 16. *Suppose that H is a triple system with $\chi(H^{[1]}) = t \ge 2$ and let $H_1, H_2, \ldots, H_t$ be a partition of H into triple systems without* 1-*intersections. There exists a skeleton G of H with the following properties.*

1. *$E(G) = \cup_{i=1}^t M_i$ where each M_i is a matching and a skeleton of H_i.*
2. *For $1 \le i \le t$, edges of M_i are the bases of all B-components of H_i and two disjoint vertex pairs from all K-components of H_i.*

3. *If $K^* = K_{t+1} \subset G$ then K^* is a connected component of G factorized by the M_i's and there is $e \in M_1 \cap E(K^*)$ such that e is from a B-component of H_1.*

Proof. From Lemma 15 we can define M_i by selecting the base edges from every B-component of H_i and selecting two disjoint pairs from every K-component of H_i. The resulting multigraph is clearly a skeleton of H and satisfies properties 1 and 2. We will select the disjoint pairs from the K-components so that property 3 also holds. Notice that $K^* = K_{t+1} \subset G$ must form a connected component in G because it is a t-regular subgraph of a graph of maximum degree t. Also, K_{t+1} is factorized by the M_i's because the union of t matchings can cover at most $\frac{t(t+1)}{2} = \binom{t+1}{2}$ edges of K_{t+1}, therefore every edge of K_{t+1} must be covered exactly once by the M_i's. Thus we have to ensure only that there is $e \in M_1 \cap E(K^*)$ with e from a B-component of H_1. For convenience, we say that a $K^* = K_{t+1}$ is a *bad component* if such e does not exist.

Select a skeleton S as described in the previous paragraph such that p, the number of bad components, is as small as possible. Suppose that $(x, y) \in M_1$ is in a bad component U. In other words, (x, y) is in a K-component of H_1, where $V(K) = \{x, y, u, v\}$ and $(u, v) \in M_1$. Now we replace these two pairs by the pairs $(x, u), (y, v)$ to form a new M_1. After this switch, U is no longer a bad component. In fact, either U becomes a new component on the same vertex set (if (u, v) was in U) or U melds with another component into a new component. In both cases, no new bad components are created and in the new skeleton there are fewer than p bad components. This contradiction shows that $p = 0$ and proves the lemma. □

Proof of Theorem 13. Let H be a triple system with $t := \chi(H^{[1]}) \geq 2$ and partition H into $H_1, \ldots, H_t$ so that each H_i is without 1-intersections. Let G be a skeleton of H with the properties ensured by Lemma 16.

Let G' be a connected component of G. By Brooks' Theorem, if G' is not the complete graph K_{t+1} or an odd cycle (if $t = 2$), $\chi(G') \leq \Delta(G') \leq t$.

Suppose first that t is even. Now $G' \neq K_{t+1}$ because that would contradict property 3 in Lemma 16: K_{t+1} cannot be factorized into matchings. Also, for $t = 2$, G' cannot be an odd cycle since odd cycles are not the union of two matchings. Thus every connected component of G is at most t-chromatic, therefore $\chi(G) \leq t$. Since G is a skeleton of H, this implies $\chi(H) \leq t$, concluding the proof for the case when t is even.

Suppose that t is odd, $t \geq 3$. In this case the previous argument does not work when some connected component $G' = K_{t+1} \subset G$. However, from Lemma 16, every K_{t+1}-component C_i of G has an edge $(x_i, y_i) \in M_1$ that is the base of a B-component in H_1. Define the vertex coloring c on $X = \cup_{i=1}^{m} V(C_i)$ by $c(x_i) = c(y_i) = 1$ and by coloring all the other vertices of all C_i's with $2, \ldots, t$.

Let F be the subgraph of G spanned by $V(G) \setminus X$ and define

$$Z := \{z \in V(F) : \{x_i, y_i, z\} \in E(H_1) \text{ for some } 1 \leq i \leq m\}.$$

Since for every $z \in Z$ there is a triple $T = (x_i, y_i, z) \in H_1$ in a B-component of H_1 (with base (x_i, y_i)), $(z, u) \in M_1$ is impossible for any $u \in V(G)$, since otherwise T and the triple of H_1 containing (z, u) would 1-intersect in z. Thus $d_G(v) \leq t - 1$ for $z \in Z$. Also, $d_G(v) \leq t$ for all $v \in V(F) \setminus Z$.

We claim that with lists $L(z) := \{2, \ldots, t\}$ for $z \in Z$ and $L(v) := \{1, \ldots, t\}$ for $v \in V(F) \setminus Z$, F is L-choosable. We use the reduction argument present in many coloring proofs (see, for example, the very recent survey paper [6]).

Suppose F is not L-choosable and let F' be a minimal induced subgraph of F which fails to be L-choosable. We may assume that any $z \in V(F') \cap Z$ has $d_{F'}(z) = t-1$ (otherwise we may L-choose $F' - z$, add z back and properly color it). Likewise we may assume $d_{F'}(v) = t$ for all $v \in V(F') \setminus Z$. By the degree-choosability version of Brooks' theorem (see [11], Lemma 1 or [6], Theorem 11), F' is a Gallai tree: a graph whose blocks are complete graphs or odd cycles.

Let A be a block of F'. Then $A \neq K_{t+1}$ because all K_{t+1}-components of G are in X. Since all vertex degrees in F' are t or $t-1$, A is either an odd cycle (if $t = 3$) or A is a K_t. A must contain an edge $e \in M_1$. Otherwise $M_2, \ldots, M_t$ would cover the edges of A, a contradiction in either case. If A is an endblock then by the degree requirements, either

$$V(A) \cap (V(F) \setminus Z) = \{w\}$$

where w is the unique cut point of A or $V(A) \subset Z$. In both cases an endpoint of e must be in Z. Then there exists some triple $\{x_i, y_i, z\} \in H_1$ which 1-intersects with the triple of H_1 containing e, a contradiction, proving that F is L-choosable.

Let $c' : V(F) \to \{1, \ldots, t\}$ be an L-coloring of F. We extend c from X to $V(H)$ by setting $c(v) := c'(v)$ for all $v \in V(F)$. Observe that c properly colors all edges of G except for the edges of the form (x_i, y_i) which are monochromatic in color 1. Since G is a skeleton, every triple of H is properly colored except possibly the triples in the from (x_i, y_i, x).

We claim that $c(x) \neq 1$. Suppose to the contrary that $c(x) = 1$. If $x \in X$ then $x \in \{x_j, y_j\}$ for some $j \neq i$, but this is impossible because the bases $(x_i, y_i), (x_j, y_j)$ are from different B-components of H_1. If $x \notin X$ then $x \in Z$ from the definition of Z. However, $1 \notin L(x)$ for $x \in Z$ and this proves the claim.

Therefore c is a proper t-coloring of H and this completes the proof. □

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