Multiple fuzzy reasoning approach to fuzzy mathematical programming problems*

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Abstract

We suggest solving fuzzy mathematical programming problems via the use of multiple fuzzy reasoning techniques. We show that our approach gives Buckley's solution [1] to possibilistic mathematical programs when the inequality relations are understood in possibilistic sense.

Keywords: Compositional rule of inference, multiple fuzzy reasoning, fuzzy mathematical programming, possibilistic mathematical programming, fuzzy implication

1 Preliminaries

Yager [4] introduced an approach to making inferences in a knowledge-based system which uses mathematical programming techniques. In this paper we apply multiple fuzzy reasoning (MFR) techniques to solve mathematical programming problems with fuzzy parameters. We show that our approach yields Buckley's solution [1] to possibilistic mathematical programs, when the inequality relations are understood in possibilistic sense. We shall use the following inference rules:
The compositional rule of inference

Antecedent 1:	x and y have property W
Fact:	x has property P
Consequence:	y has property Q

where P and Q are fuzzy sets, W is a fuzzy relation and Q is obtained by the sup-min composition of P and W, i.e.

$$Q(y) = (P \circ W)(y) = \sup_x \min\{P(x), W(x, y)\}.$$

MFR scheme

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$$\begin{array}{ll} \text{Antecedent 1:} & x \text{ and y have relation } W_1 \\ & \ldots \\ \text{Antecedent m:} & x \text{ and y have relation } W_m \\ \text{Fact:} & x \text{ has relation } P \\ \hline \text{Consequence:} & y \text{ has relation } Q \end{array}$$

where

$$Q(y) = (P \circ \min_{i=1,\ldots,m} W_i)(y) = \sup_x \min\{P(x), \min_{i=1,\ldots,m} W_i(x,y)\}$$

2 The new approach

Consider the fuzzy mathematical programming (FMP) problem

$$\begin{aligned} g(\tilde{c}, x) &\to \tilde{\max} \\ f_1(\tilde{a}_1, x) &\leq \tilde{b}_1 \\ &\ldots \\ f_m(\tilde{a}_m, x) &\leq \tilde{b}_m \end{aligned} \tag{1}$$

where $\tilde{c} = (\tilde{c}_1, \ldots, \tilde{c}_k)$ and $\tilde{a}_i = (\tilde{a}_{i1}, \ldots, \tilde{a}_{in})$ are vectors of fuzzy quantities (i.e. fuzzy sets of the real line $\mathbb{R}$), $\tilde{b}_i$ is a fuzzyquantity, $x = (x_1, \ldots, x_n)$ is a vector of decision variables, $g(\tilde{c}, x)$ and $f_i(\tilde{a}_i, x)$ are defined by Zadeh's extension principle, and the inequality relation $\leq$ is defined by a certain fuzzy relation. Now, by using FMR techniques, we shall determine a fuzzy quantity, $\tilde{\max}$, which satisfies the inequality

$$g(\tilde{c}, x) \leq \tilde{\max}, \ \forall x \in \mathbb{R}^n,$$

under the constraints $f_i(\tilde{a}_i, x) \leq \tilde{b}_i$, $i = 1, \ldots, m$. We consider the inequality relation $\leq$ as a fuzzy relation on $\mathbb{R}$ and for every $x \in \mathbb{R}^n$ determine $\tilde{\max}_x$ from the following MFR scheme

$$\begin{array}{ll} \text{Antecedent 1:} & f_1(\tilde{a}_1, x) \leq \tilde{b}_1 \\ \ldots & \\ \text{Antecedent } m\text{:} & f_m(\tilde{a}_m, x) \leq \tilde{b}_m \\ \text{Fact:} & g(\tilde{c}, x) \\ \hline \text{Consequence:} & \tilde{\max}_x \end{array}$$

where, according to the MFR inference rule,

$$\tilde{\max}_x = g(\tilde{c}, x) \circ \min_{i=1,\ldots,m} (f_i(\tilde{a}_i, x) \leq \tilde{b}_i).$$

It is clear that $\tilde{\max}_x$ is a fuzzy set realizing the inequality $g(\tilde{c}, x) \leq \tilde{\max}_x$ under the premises $f_i(\tilde{a}_i, x) \leq \tilde{b}_i$, $i = 1, \ldots, m$. Finally, we define the solution, $\tilde{\max}$, of the FMP problem (P1) as

$$\tilde{\max} = \sup_x g(\tilde{c}, x) \circ \min_{i=1,\ldots,m} (f_i(\tilde{a}_i, x) \leq \tilde{b}_i)$$

It is clear that $\tilde{\max}$ is a fuzzy set realizing the inequality $g(\tilde{c}, x) \leq \tilde{\max}$, $\forall x \in X$, under the premises $f_i(\tilde{a}_i, x) \leq \tilde{b}_i$, $i = 1, \ldots, m$.

Remark 2.1 *It can be shown that our approach (i) yields Buckley's solution [1] to possibilistic mathematical programs, when the inequality relations are understood in possibilistic sense; (ii) under well-chosen inequality relations and objective function coincides with those ones suggested by Delgado et al [2], Ramik and Rimanek [3] and Zimmermann [5].*

3 Illustration

Example 3.1 *We illustrate our approach on simple FLP problem, where the inequality relation $\leq$ is defined by the Gödel implication:*

$$x \leq y = x \to y = \begin{cases} 1 & \textit{if } x \leq y \\ y & \textit{otherwise} \end{cases}$$

The FLP problem	The adequate MFR scheme	
$x_1\tilde{a}_1 \leq \tilde{b}_1$	Antecedent 1:	$x_1\tilde{a}_1 \leq \tilde{b}_1$
$x_2\tilde{a}_1 \leq \tilde{b}_2$	Antecedent 2:	$x_2\tilde{a}_1 \leq \tilde{b}_2$
$x_1\tilde{a}_1 + x_2\tilde{a}_1 \to \widetilde{\max}$	Fact:	$x_1\tilde{a}_1 + x_2\tilde{a}_1$
	Consequence:	$\widetilde{\max} = \tilde{b}_1 + \tilde{b}_2$

References

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