# Application the Projective Geometry in Manufacturing Design of Worms

Dr. Balajti, Zsuzsa<sup>1, a</sup>, Abel, Jozsef<sup>2, b</sup>

<sup>1</sup>Department of Descriptive Geometry, University of Miskolc, Hungary,

<sup>2</sup>UNIX Car Parts Company, Hungary,

<sup>a</sup>balajtizs@abrg.uni-miskolc.hu, <sup>b</sup>abel.jozsef16@gmail.com

Keywords: worm, design, exact production, projective geometry

**Abstract.** The mathematical describing of the production process in mechanical engineer work in the Euclidean space model and the base of the analytical describing of the projective space model is practically identical in the form, which makes it reasonable to discuss of the production geometry to approach of the projective geometrical negotiation in our work.

It is a fact that in one of the cases, using the approach of a projective geometrical connection and the mathematical-kinematical model resulted in expansion in the field of production precision, specifically considering the examination of the production of the conical worm by grinding wheel. The abstraction of the production geometry on projective space model has a few results in case of conical worms. The elliptical errors in production of the conical worm with arched or anything profile by grinding wheel can be eliminated by this method to achieve the constant pitch, the deformation-free of profile and others.

### Introduction

In manufacturing practice of worms there exist approximating solutions, however, this work is aimed at increasing the number of these processes for the purpose of manufacturing accuracy, for the advances in science and technology provide a possibility for that and simultaneously create a demand as well.

Now days the technology is the computer-aided or fully automated systems that take over the designing processes of products, technology and tools of production [2]. The computer support of engineering work is the most important condition and potential for increasing the efficiency of production and enhancing the quality of the products [3, 6].

The computer-aided production geometry of worms bases on Euclidean space model up to now. This paper contains a novel design of production of cylindrical and conical worms respected on structure of projective space model.

## **The Production of Worms**

A great number of areas in the machine industry use helical surfaces - in the form of worm drive pairs, driving pins, screw pumps, screw compressors, toothing tools, etc. – and accordingly, a great number of institutions and companies are involved in their design, production, certification and application [4, 7, 8, 9].

One of the less known types of toothed drives suitable to torque transfer between orthogonal skew axes, and having significant load carrying capacity, is the spiroid drive [6].

However the helical surfaces are correctly designed without exception, the production is making difference from the theoretical surfaces in certain cases. In case of the production of cylindrical worms by the grinding wheel is moving parallel with axis of the cylinder, so the manufacture is based in correct geometrical design. The production process results a disagreement between the theoretical and manufactured conical worm surfaces in the frequently applied production.

One of the reasons of the difficulty of the manufacturing precision expansion is it that the grinding wheel is moving parallel with the generating line of the cone in case of the manufacturing of the worm by grinding wheel (Fig. 1).

The angle between the motion path of the grinding wheel and the round axes of the cone is the half angle of the cone (Fig. 1).

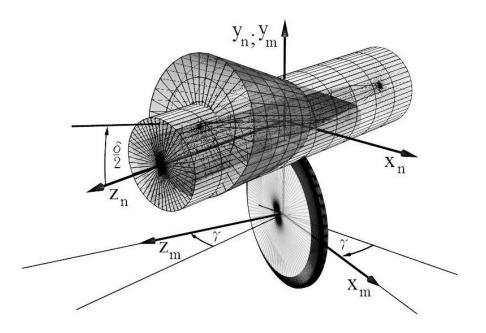


Fig. 1 Manufacturing of the cylindrical and the conical worm on a generating line

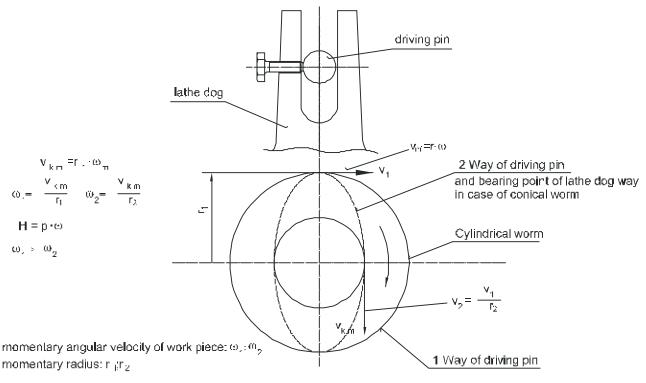


Fig. 2

The Mathematical describing of the connection between the grinding wheel and cylindrical and conical worms

The grinding wheel is moving with elliptically changing distance from the turning axes of the cone in case of the manufacturing of the conical worm, how it is shown in Fig. 2.

The profile distortion and the changing pitch are the caused production errors.

Keeping out these errors can be reconsidered by the production geometry based on the projective geometry, where the cylinder and the cone is the surface of the same category.

Basing on the above statements can be considering the same axe of the conical and cylindrical worms.

#### The Projective Geometry Space Model and the Production

The production geometry of conical worm is based on the description of the relation movement back to back of the tool and the work piece [1].

The mathematical theoretical discussion of the production in case of conical worms is correct in our literature [6]. The description of the moving pass can be described in the coordinate systems of the tool or of the work piece. The transformation matrix between of the tool and the work piece can be described by quadruple matrix [5].

The conversion from the coordinate system  $S_m(x_m, y_m, z_m)$  with  $O_m$  origin point of the tool to the coordinate system  $S_n(x_n, y_n, z_n)$  with  $O_n$  origin point of the work piece can be written by the matrix  $M_{nm}$ .

Coordinates of  $\mathbf{r}_n$  velocity vector of the M moving point in the coordinate system  $S_n$  can be counted from coordinates of  $\mathbf{r}_m$  velocity vector of the same M moving point in the  $S_m$  coordinate system by the next equation

$$\mathbf{r}_{\mathrm{n}} = M_{\mathrm{nm}} \cdot \mathbf{r}_{\mathrm{m}} \tag{1}$$

where

$$M_{nm} = \begin{bmatrix} \cos(x_n, x_m) & \cos(x_n, y_m) & \cos(x_n, z_m) & d_x \\ \cos(y_n, x_m) & \cos(y_n, y_m) & \cos(y_n, z_m) & d_y \\ \cos(z_n, x_m) & \cos(z_n, y_m) & \cos(z_n, z_m) & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
(2)

and  $d_x$ ,  $d_y$ ,  $d_z$  are coordinates of the O<sub>n</sub> point in the  $S_m(x_m, y_m, z_m)$  coordinate system. The rotating and linear moving can be handling in one matrix  $M_{nm}$ .

The euclidean (x, y, z) coordinates are suited to homogeneous ( $x_1$ ,  $x_2$ ,  $x_3$ ,  $x_4$ ) coordinates with the next conditions:

$$\mathbf{x} = \frac{x_1}{x_4}, \ \mathbf{y} = \frac{x_2}{x_4}, \ \mathbf{z} = \frac{x_3}{x_4},$$
(3)

where  $x, y, z \in R$  and  $x_1, x_2, x_3, x_4 \in R$ .

The points of the Euclidean space are easily recognizable with  $x_4 = 1$  replacement.

## The Worms Designed By Application of Projective Geometry

The projective space is the Euclidean space completed with the infinite plane.

The points with  $(x_1, x_2, x_3, x_4)$  coordinates with the condition  $x_1, x_2, x_3, x_4 \in R$  give us the projective space.

The parallel lines of the projective space have an intersection point on the infinite plane. It means that the cylinder is a cone with an infinite center point in the projective space.

The cylinder can be transformed into the cone by a projective transformation. In this case the axes of the cone and the cylinder are fitting on a line, as it can be seen on Fig. 3.

It can be seen that the production of the conical worm based on the theory of projective geometry gives an increase in the manufacturing precision with elimination of geometrical errors, so the rotation about on an axe is a circle, but it is not an ellipse. It is showed on Fig. 4.

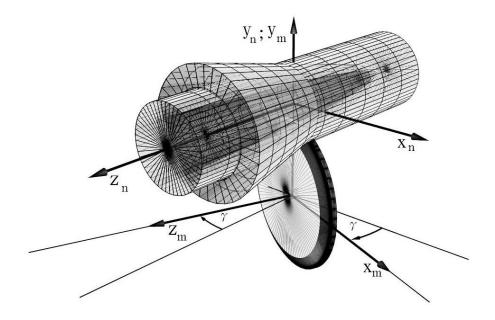


Fig. 3 Manufacturing of the cylindrical and the conical worms fitting an axe

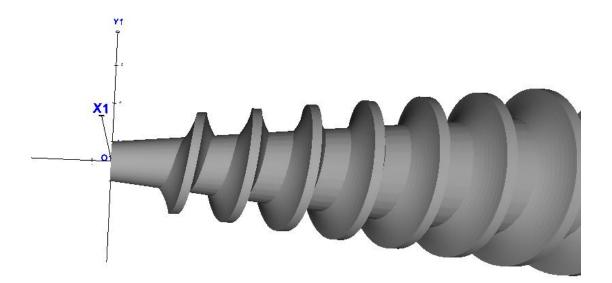


Fig. 4 Conical worm having a circle profile in axial section

## **Summary**

The mathematical model of the manufacturing of worms in the projective space can eliminate the profile and pitch errors of the manufacturing of the conical worms. It is better method to use a new theorem to get more exactly solution to approach high quality. The machine production development based on projective geometry discussion gives a more results to production precision of worms.

The mathematical, mechanical and technical development of the production, the machining process controlled by CNC, and the abstract of the production geometry by projective geometry in case of a few special worms, so spiroid, the mathematical description and the realization show many advances. The simulation of these method was developed without manufacture cost using a suitable equation in projective geometry.

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