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POTENTIALITY OF APPLICATION THE PROJECTIVE GEOMETRY IN WORM DESIGN

In the procedure of the analytical geometrical describing of the production process in mechanical engineer work and the projective space is practically identical, which makes it reasonable to discuss of the production geometry to approach of the projective geometrical negotiation.

It is a reality that in one of the cases, using the approach of a projective geometrical connection and the mathematical-kinematical model resulted in expansion in the field of production precision, specifically considering the examination of the production of the conical worm. The abstract of the production geometry on projective geometry has a few results. The elliptical errors can be eliminated by this method way, to achieve the constant pitch.

INTRODUCTION

This article contains a novel design of the production geometry of cylindrical and conical worms. In practice there exist approximating solutions, however, this work is aimed at increasing the number of these processes for the purpose of manufacturing accuracy, for the advances in science and technology provide a possibility for that and simultaneously create a demand as well.

The production of the mechanical technology is the computer-aided or fully automated systems that take over the designing processes of products, technology and tools of production. Now days the computer geometry support of engineering work is the most important condition and potential for increasing the efficiency of production and enhancing the quality of the products [2, 5].

A great number of areas in the machine industry use helical surfaces - in the form of worm drive pairs, driving pins, screw pumps, screw compressors, toothing tools, etc. – and accordingly, a great number of institutions and companies are involved in their design, production, certification and application [3, 7, 8].

One of the less known types of toothed drives suitable to torque transfer between orthogonal skew axes, and having significant load carrying capacity, is the spiroid drive [5], how those are visible in Figure 1.

However the helical surfaces are designed right, the production of worm in the practice is making difference from the theoretical surfaces. The production process in the finding literature discusses the theoretical and practical problems in different way because of the differences in technical conditions.

In case of production of worms the grinding wheel is moving parallel with the axis of the cylinder in case of manufacturing the helical worms by a grinding wheel. The mathematical theoretical discussion in case of conical worms is correct in literature [5].

Figure1 – Spiroid drive

The technical possibility in case of the expansion of the production precision is showing a big difficulty, the literature of which discusses the examination of the comparison of the manufactured worms in the practice and the planed worms.

One of the reasons of the difficulty of the manufacturing precision expansion is it that the grinding wheel is moving parallel with the generated line of the cone in case of the manufacturing the worm by a grinding wheel (Figure 2).

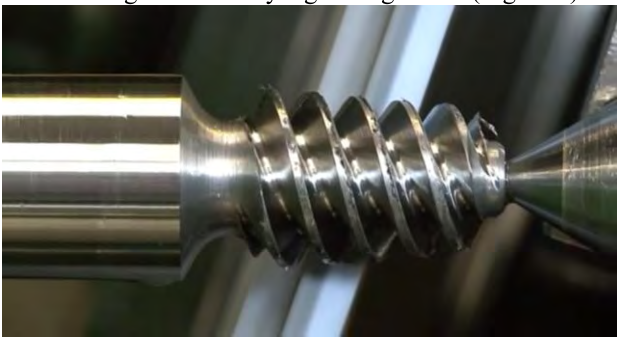

Figure 2 – Manufacturing the conical worm by grinding wheel

The angle between the moving line of the grinding wheel and the round axes of the cone is the half angle of the cone. (Figure 3.)

The grinding wheel is continuously changing distance from the turning axes of the conical worm in case of the showed manufacturing of the conical worm (Figure 4.).

That results different production errors, for example profile distortion and changing pitch.

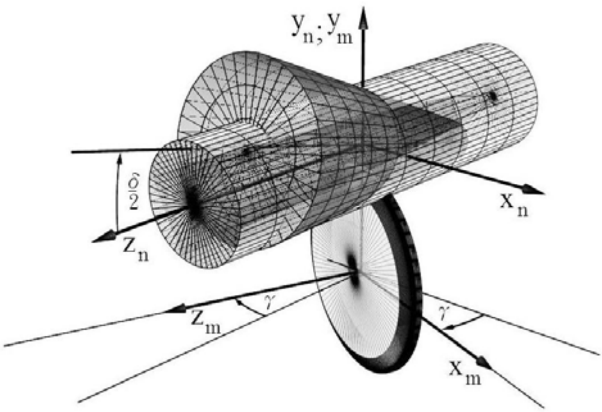

Figure 3 – Manufacturing of the cylindrical and the conical worm on a generating line

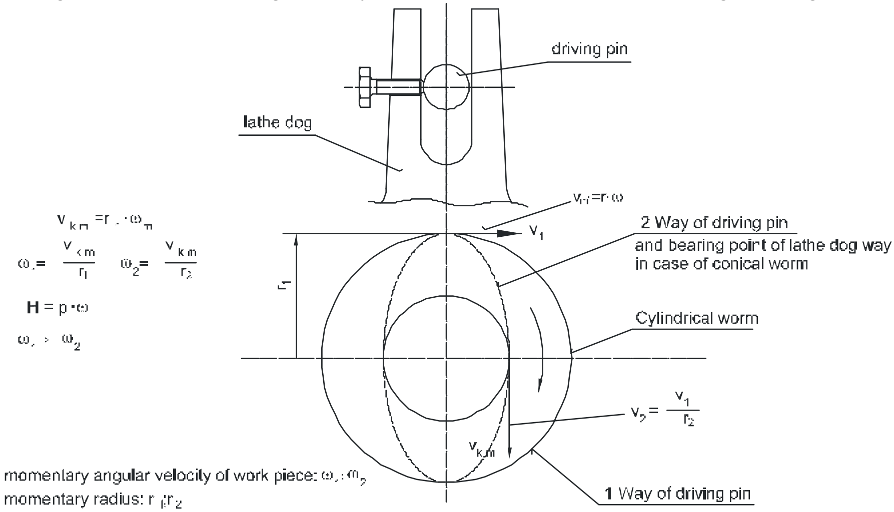

Figure 4 – The Mathematical describing of the connection between the grinding wheel and cylindrical and conical worms

Keeping out these errors can be reconsidered by the production geometry based on the projective geometry. Basing on the above statements the discussion and description of the solution can be useful as we can build on the equivalence of the conical and cylindrical worms.

1. THE PRODUCTION AND THE PROJECTIVE GEOMETRY

The production geometry is based on the description of the relation movement back to back of the tool and the work piece [1].

The pass the moving is possible in the coordinate systems of the tool and the work piece. The transformation matrix between of the tool and the work piece can be described by quadruple matrix.

The conversion from the coordinate system $S_m(x_m, y_m, z_m)$ with O_m origin point of the tool to the coordinate system $S_n(x_n, y_n, z_n)$ with O_n origin point of the work piece can be the matrix M_{nm}.

The conversion from the coordinate system $S_m(x_m, y_m, z_m)$ with O_m origin point of the tool to the coordinate system $S_n(x_n, y_n, z_n)$ with O_n origin point of the work piece can be the matrix M_{nm}.

The coordinates $\mathbf{r}_n$ in the S_n coordinate system of the moving M point can be counted from the coordinates $\mathbf{r}_m$ in the S_m coordinate system by the next equation

$$r_n = M_{nm} \cdot r_m \tag{1}$$

where

$$M_{nm} = \begin{bmatrix} \cos(x_n, x_m) & \cos(x_n, y_m) & \cos(x_n, z_m) & d_x \\ \cos(y_n, x_m) & \cos(y_n, y_m) & \cos(y_n, z_m) & d_y \\ \cos(z_n, x_m) & \cos(z_n, y_m) & \cos(z_n, z_m) & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

and (d_x, d_y, d_z) is the coordinates of O_n in the $S_m(x_m, y_m, z_m)$. The rotating and the shifting is being handled together in a matrix M_{nm}.

The alternate of the (x, y, z) Euclidean coordinates are the (x_1, x_2, x_3, x_4) homogeneous coordinates of P point on the surface of the work piece with the next conditions:

$$\mathrm{x} = \frac{x_1}{x_4}, \ \mathrm{y} = \frac{x_2}{x_4}, \ \mathrm{z} = \frac{x_3}{x_4} \tag{3}$$

where $\mathrm{x}, \mathrm{y}, \mathrm{z} \in R$ and $x_1, x_2, x_3, x_4 \in R$.

The points of the Euclidean space are easily recognized with the $x_4 = 1$ replacement.

It follows that the production geometry can be placed on the basis of projective geometry.

2. THE WORMS DESIGNED BY APPLICATION PROJECTIVE GEOMETRY

The projective space is the Euclidean space completed with the infinite plane.

The points with (x_1, x_2, x_3, x_4) coordinates with the condition $x_1, x_2, x_3, x_4 \in R$ give us the projective space.

The parallel lines of the projective space have an intersection point on the infinite plane. The cylinder is a cone with an infinite center point in the projective space.

The cylinder and the cone can be corresponded together; the projective transformation connection can be created.

Such projective connecting is the central connecting, which place the cone and the cylinder of the Euclidean place to an axe, as you can see it on Figure 5.

The production of the conical worm based on the theory of projective geometry gives an increase in the manufacturing precision with elimination of geometrical errors, so the rotation about on an axe is a circle, but it is not an ellipse.

It can calculated from contacting points in based on Figure 6.

The used coordinate systems:

K_0 (x_0, y_0, z_0)	Stationary coordinate system affixed to machine tool
K_{1F} (x_{1F}, y_{1F}, z_{1F})	Rotating coordinate system affixed helicoid surface
K_1 (x_1, y_1, z_1)	Coordinate system connected to linear moving table
K_{2F} (x_{2F}, y_{2F}, z_{2F})	Rotating coordinate system fixed to tool
K_2 (x_2, y_2, z_2)	Stationary coordinate system fixed to tool
K_k (x_k, y_k, z_k)	Auxiliary coordinate system

The used marks:

α (^{o})	Forming, tilting angle (in degrees) of tool into profile of helicoidal surface in the characteristic section, eg grinding of involute helicoidal surface using plane face surface wheel
γ (^{o})	Lead angle (in degrees) on the worm's references surface
φ_1 (^{o})	Angular displacement of a helicoid surface
φ_2 (^{o})	Angular displacement of a tool
ω_1 (s^{-1})	Angular velocity of worm
ω_2 (s^{-1})	Angular velocity of tool

The point on the worm surface is the

$$\mathbf{r}_{1F} = \mathbf{r}_{1F}\left(\eta, \vartheta\right) \tag{4}$$

in the coordinate system $K_{1F}(x_{1F},y_{1F},z_{1F})$.

The normal vector is on the surface

$$\mathbf{n}_{1F} = \frac{\partial\ \mathbf{r}_{1F}}{\partial \eta} \times \frac{\partial \mathbf{r}_{1F}}{\partial \vartheta} \tag{5}$$

In this case the P_{1a} kinematic projection matrix, for direct method:

$$\mathbf{P}_{1a} = \mathbf{M}_{1F,2F} \cdot \frac{d}{dt} \cdot \mathbf{M}_{2F,1F} \tag{6}$$

$\underline{M}_{1F,\ 2F}$ is the coordinate transformation matrix (transforms K_{2F} to K_{1F}), and the $\underline{M}_{2F,1F}$ is the coordinate transformation matrix (transforms K_{1F} to K_{2F}).

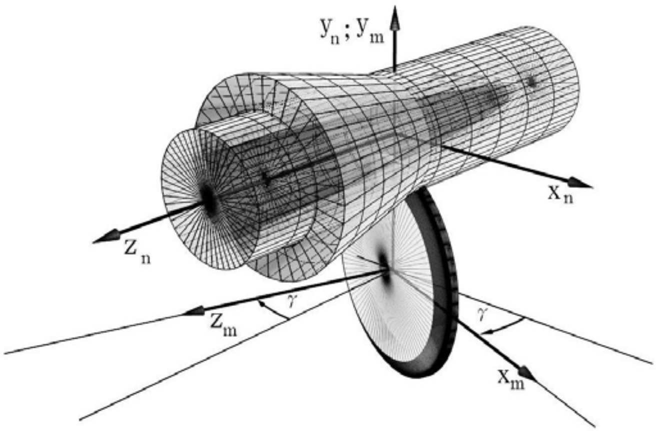

Figure 5 – Manufacturing of the cylindrical and the conical worms to an axe

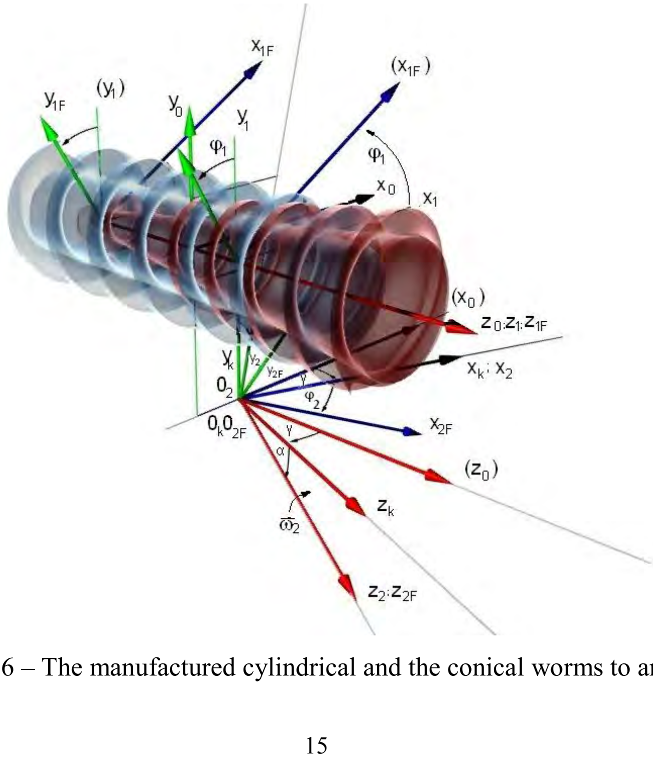

Figure 6 – The manufactured cylindrical and the conical worms to an axe

$$
\mathbf{P}_{1a} = \begin{bmatrix}
0 & -1-i\cdot\cos\alpha\cdot\cos\gamma & \begin{array}{l} +i\cdot\cos\alpha\cdot\sin\gamma\cdot\sin\varphi_1 \\ -i\cdot\sin\alpha\cdot\cos\varphi_1 \end{array} & \begin{array}{l} -\left(a_0+i\cdot c\cdot\cos\alpha\cdot\cos\gamma+p_r+i\cdot z_{ax}\cdot\cos\alpha\cdot\sin\gamma\right)\cdot\sin\varphi_1 \\ +\left(i\cdot p_a\cdot\cos\alpha\cdot\sin\gamma+p_r\right)\cdot\varphi_1\cdot\sin\varphi_1 \\ -\left(i\cdot a_0\cdot\cos\alpha\cdot\cos\gamma-c-i\cdot z_{ax}\cdot\sin\alpha\right)\cdot\cos\varphi_1 \\ -i\cdot\left(p_a\cdot\sin\alpha-p_r\cdot\cos\alpha\cdot\cos\gamma\right)\cdot\varphi_1\cdot\cos\varphi_1 \end{array} \\
1+i\cdot\cos\alpha\cdot\cos\gamma & 0 & \begin{array}{l} +i\cdot\sin\alpha\cdot\sin\varphi_1 \\ +i\cdot\cos\alpha\cdot\sin\gamma\cdot\cos\varphi_1 \end{array} & \begin{array}{l} +\left(i\cdot a_0\cdot\cos\alpha\cdot\cos\gamma-c-i\cdot z_{ax}\cdot\sin\alpha\right)\cdot\sin\varphi_1 \\ +i\cdot\left(p_a\cdot\sin\alpha-p_r\cdot\cos\alpha\cdot\cos\gamma\right)\cdot\varphi_1\cdot\sin\varphi_1 \\ -\left(a_0+i\cdot c\cdot\cos\alpha\cdot\cos\gamma+p_r+i\cdot z_{ax}\cdot\cos\alpha\cdot\sin\gamma\right)\cdot\cos\varphi_1 \\ +\left(i\cdot p_a\cdot\cos\alpha\cdot\sin\gamma+p_r\right)\cdot\varphi_1\cdot\cos\varphi_1 \end{array} \\
\begin{array}{l} -i\cdot\cos\alpha\cdot\sin\gamma\cdot\sin\varphi_1 \\ +i\cdot\sin\alpha\cdot\cos\varphi_1 \end{array} & \begin{array}{l} -i\cdot\sin\alpha\cdot\sin\varphi_1 \\ -i\cdot\cos\alpha\cdot\sin\gamma\cdot\cos\varphi_1 \end{array} & 0 & \begin{array}{l} -\left(p_a+i\cdot p_r\cdot\cos\alpha\cdot\sin\gamma\right)\cdot\varphi_1 \\ -i\cdot a_0\cdot\cos\alpha\cdot\sin\gamma-i\cdot c\cdot\sin\alpha+p_a+z_{ax} \end{array} \\
0 & 0 & 0 & 0
\end{bmatrix} \tag{7}
$$

$$
\mathbf{P}_{1a} = \mathbf{M}_{1F,2F}\cdot\frac{d}{dt}\cdot\mathbf{M}_{2F,1F}
$$

The next equation gives us the contacting points:

$$
\mathbf{n}_{1F}\left(\eta,\vartheta\right)\cdot\mathbf{v}_{1F}\left(\eta,\vartheta\right)=0 \tag{8}
$$

These points give us the base of the examinations on the worm surfaces [4].

The contact points can be seen on the Figure 7. in concrete case an spiroid worm gear drive. The contact points were calculated by computer programme. The computer programme was made in C programme language.

The velocity vector can be seen in the next form:

$$
\mathbf{v}_{1F}{}^{(12)} = \begin{bmatrix}
0\cdot x_{1F} & -(1+i\cdot\cos\alpha\cdot\cos\gamma)\cdot y_{1F} & \begin{array}{l} +(i\cdot\cos\alpha\cdot\sin\gamma\cdot\sin\varphi_1 \\ -i\cdot\sin\alpha\cdot\cos\varphi_1)\cdot z_{1F} \end{array} & \begin{array}{l} -\left(a_0+i\cdot c\cdot\cos\alpha\cdot\cos\gamma+p_r+i\cdot z_{ax}\cdot\cos\alpha\cdot\sin\gamma\right)\cdot\sin\varphi_1 \\ +\left(i\cdot p_a\cdot\cos\alpha\cdot\sin\gamma+p_r\right)\cdot\varphi_1\cdot\sin\varphi_1 \\ -\left(i\cdot a_0\cdot\cos\alpha\cdot\cos\gamma-c-i\cdot z_{ax}\cdot\sin\alpha\right)\cdot\cos\varphi_1 \\ -i\cdot\left(p_a\cdot\sin\alpha-p_r\cdot\cos\alpha\cdot\cos\gamma\right)\cdot\varphi_1\cdot\cos\varphi_1 \end{array} \\
(1+i\cdot\cos\alpha\cdot\cos\gamma)\cdot x_{1F} & +0\cdot y_{1F} & \begin{array}{l} +(i\cdot\sin\alpha\cdot\sin\varphi_1 \\ +i\cdot\cos\alpha\cdot\sin\gamma\cdot\cos\varphi_1)\cdot z_{1F} \end{array} & \begin{array}{l} +\left(i\cdot a_0\cdot\cos\alpha\cdot\cos\gamma-c-i\cdot z_{ax}\cdot\sin\alpha\right)\cdot\sin\varphi_1 \\ +i\cdot\left(p_a\cdot\sin\alpha-p_r\cdot\cos\alpha\cdot\cos\gamma\right)\cdot\varphi_1\cdot\sin\varphi_1 \\ -\left(a_0+i\cdot c\cdot\cos\alpha\cdot\cos\gamma+p_r+i\cdot z_{ax}\cdot\cos\alpha\cdot\sin\gamma\right)\cdot\cos\varphi_1 \\ +\left(i\cdot p_a\cdot\cos\alpha\cdot\sin\gamma+p_r\right)\cdot\varphi_1\cdot\cos\varphi_1 \end{array} \\
\begin{array}{l} -(i\cdot\cos\alpha\cdot\sin\gamma\cdot\sin\varphi_1 \\ -i\cdot\sin\alpha\cdot\cos\varphi_1))\cdot x_{1F} \end{array} & \begin{array}{l} -(i\cdot\sin\alpha\cdot\sin\varphi_1 \\ +i\cdot\cos\alpha\cdot\sin\gamma\cdot\cos\varphi_1)\cdot y_{1F} \end{array} & +0\cdot z_{1F} & \begin{array}{l} -\left(p_a+i\cdot p_r\cdot\cos\alpha\cdot\sin\gamma\right)\cdot\varphi_1 \\ -i\cdot a_0\cdot\cos\alpha\cdot\sin\gamma-i\cdot c\cdot\sin\alpha+p_a+z_{ax} \end{array} \\
 & & 0 &
\end{bmatrix} \tag{9}
$$

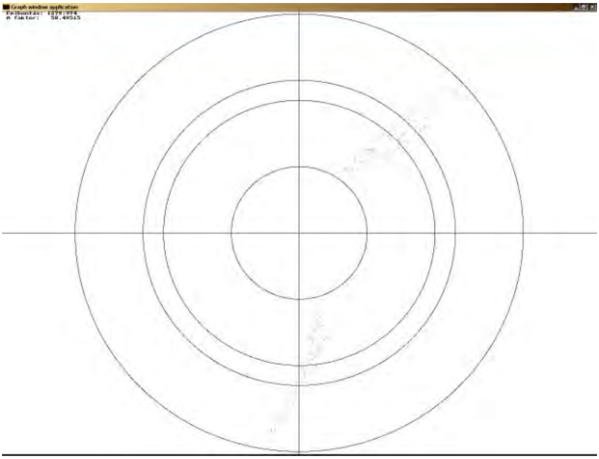

Figure 7 – Contact points on a frontal section of the conical worm

SUMMARY:

In the procedure of the manufacturing of worms in same way, we can get the same unsolved errors. So it is better method to use a new theorem to get more exactly solution to approach high quality. Machine production development based on projective geometry discussion gives a more results to production precision [1, 4].

The mechanical and technical development of the production, for example the machining process controlled by CNC, and the abstract of the production geometry by projective geometry in case of a few special worms, so spiroid, the mathematical description and the realization show an advance.

References: 1. Dr. Balajti, Zsuzsa: The Connection Between the Production and Projective Geometry, microCAD 2011. International Scientific Conference, Section L: Production Engineering and Manufacturing Systems, Miskolc, 31. March – 1. April 2011., pp. 1-4., ISBN 978-963-661-965-7.,2. Kovács, Lehel: Computer graphics, In: Scientia Issue (in Hungarian), Cluj Napoca, 2009., ISBN: 978-973-1970-14-1. p.: 384.3. Kundrak J., Perepelicia B A, Sashkova N V: Analiticheskoe opredelenie kinematicheskikh geometricheskikh parametrov pri tochenii i rastachivanii poligonnykh poverkhnostejj, REZANNIE I INSTRITUMENT, pp.: 43-47., 1987. 4. Litvin, Faydor L., Fuents, Alfonso: Gear Geometry and Applied Theory,Cambidge University Press, ISBN 0 521 815177, 2004., p.: 800., 5. Óváriné Balajti, Zsuzsa: Development of the Production Geometry of Kinematic Drive Pairs, PhD dissertation (in Hungarian) p.: 170., University of Miskolc, 2007.,6. Perepelica B A: Otobrazsenija affinogo posztrantszva b teorii formorazovanyija poverhnoctej rezanijem, Harkov, Izdatelyelsztvo Pri Harkovszkom Goszudarsztvennom Juniverszitete Izdatelszkogo Obegyínyenyija"Visa Skola", p..152, 1981.7. Perepelica B A, Kundrák J: How to use the multiparameter mapping to describe the kinematics of metal cutting ,In: Result of the scientific cooperation between the Polytechnical University of Kharkov and the University of Miskolc: Jubilee Conference, Miskolc, Magyarország, 1994.03.01-1994.03.02.,Miskolc: University of Miskolc, pp. 95-98(ISBN:963 661 238 2, 8. Sztankovics I, Kundrák J: Mathematical Model of Machined Surfaces in Rotational Turning and Skiving, Proceedings of the 13th International Conference on Tools, Miskolc, Magyarország, Miskolc: Bíbor Kiadó, ISBN:978-963-9988-35-4, 2012.03.27-2012.03.28., pp. 83-88.

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