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New Modelling of Computer Aided Design of Worms in the Same Axis

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Abstract

Nowadays the mechanical engineering technology is the computer-aided or fully automated systems that take over the designing processes of products, technology and tools of production. Computer support of special engineering work is the one of the most important condition for increasing the efficiency of production and enhancing the quality of the products. Development of the production of worms needs use of production geometry in case of a few special worms and mathematical description shows a challenge. Programmed mathematical model is suitable to evaluate the production geometry problems in various types of cylindrical and conical worm surfaces – with an approaching exact mathematical solution – that it can be applied for many purposes in engineering practice.

Keywords: Programmed model, production geometry, worm surfaces

1 Introduction

The first mathematical model to analysis of the worm gear driving and its manufacturing was created in University of Miskolc, in Department of Production Engineering. Many employments exist with that aim to increase the accuracy of worm's manufacturing in the today [2, 3, 5, 6]. In the new mathematical model [7] ensure attempt to enhance the accuracy of the manufacturing by grinding wheel in case the cylindrical and conical worm on the same axe with line and arc profile to eliminate the caused error of elliptical relation between the conical worm rotated with half cone angle, and the grinding wheel. The error elimination is causing the aspect of projective geometry, in which the cylinder and the cone is in one category of the surfaces.

Paper presents the worms modelling in the main program with variety geometrical parameters, which have made in the improved mathematical model to improve accuracy processing of the parts of the conical and helicodical worm gear drives. The last chapter containing an introduction of the one possible solution to calculate the curvature function of the cone.

2 The program

Developed program shows mathematical simulation of cylindrical and conical worms, which is made in C program language in the expanded mathematical and kinematical model in theorem in projective geometry [1]. C program language was designed for implementing system software, it is also widely used for developing portable application software. C is one of the most widely used programming languages of all time. C has greatly influenced many other popular programming languages, most notably C++, which began as an extension to C.

The C programming language uses libraries as its primary method of extension. In C, a library is a set of functions contained within a single "archive" file. Each library typically has a header file, which contains the prototypes of the functions contained within the library that can be used by a program, and declarations of special types of data and macro symbols used with these functions. In order for a program to use a library, it must include the library's header file, and the library has to be linked with the program, which in many cases requires compiler flags.

3 Modeling of helicodical elements of drive pairs

The helicodical surfaces are generated in program using straight or arched curves. Their size causes it cannot be allowed to give all their details as shown in [7]. Developed theory is illustrated with numerical examples and equation of the general form of the helicodical surface is derived below.

3.1 Cylindrical worms

The relative position of the general line to the axis of the cylindrical surface, and the base cylinder will determine the type of the helicoidally surface. The straight generated worm surface can be Archimedean, having a line edge in axial section (Fig. 1.), or a convolute one having a line edge in intersection with a parallel plane to the worm axis (Fig. 2.). The generated equation of the cylindrical helicodical surface is as follows:

$$\left.\begin{aligned} x_{1F} &= \xi(\eta)\cdot\cos\vartheta - \eta\cdot\sin\vartheta + \vartheta\cdot p_r\cdot\cos\vartheta \\ y_{1F} &= \xi(\eta)\cdot\sin\vartheta + \eta\cdot\cos\vartheta + \vartheta\cdot p_r\cdot\sin\vartheta \\ z_{1F} &= \zeta(\eta) + p_a\cdot\vartheta \end{aligned}\right\} \qquad (1)$$

The involute worm surface is a special case of convolute worm surface, because of having a line edge section in a parallel plane to the worm axis and tangential to the base cylinder as shown in Fig. 3.

One of the most modern types of cylindrical helicoidal surfaces is the worm generated using a different profile tool. Depending on the kinematical conditions between the worm and the tool, the arched profile can appear directly on the worm active surface, in axial or normal section. In Fig. 4., the arched profile of worm is circle in axial section with the given geometrical parameters.

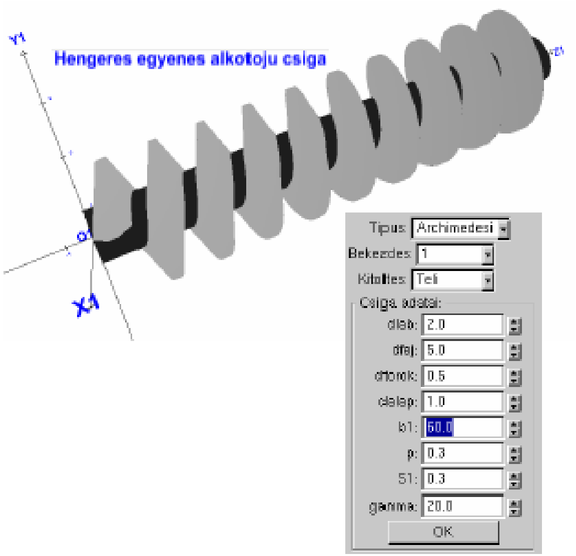

Fig. 1 *The cylindrical Archimedean worm*

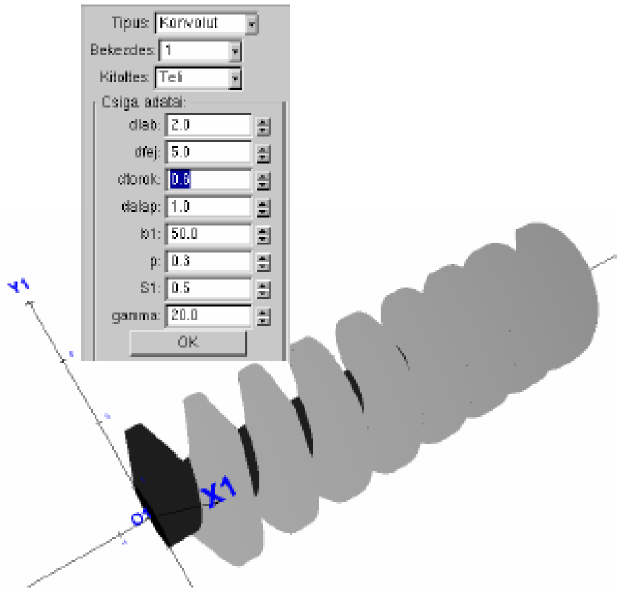

Fig. 2 *The cylindrical convolute worm*

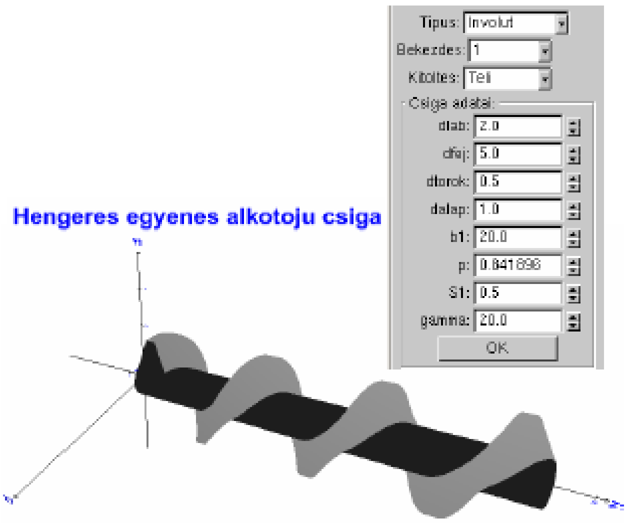

Fig. 3 *The cylindrical involute worm*

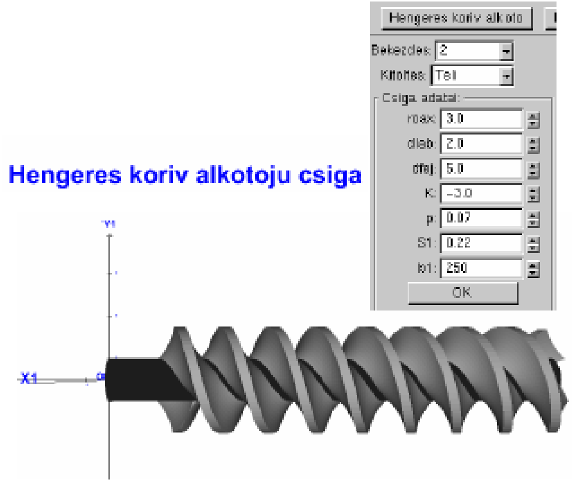

Fig. 4 *The cylindrical worm with arched profile*

The equation of the cylindrical helicoidcal surface with circle profile in axial section can be given in the next form.

$$x_{1F} = -\eta \cdot \sin \vartheta$$

$$y_{1F} = +\eta \cdot \cos \vartheta$$

$$z_{1F} = p_a \cdot \vartheta - \sqrt{\rho_{ax}^2 - (K - \eta)^2} + z_{ax} \qquad (2)$$

3.2 Conical worms

The type of the generator curve and it's placing to the axis determine the helicodical surfaces, from that give us the worms. The geometrical parameters of the conical worms give us the type of the spiroid worm [4]. The interpretation of Archimedean, involute and convolute worms in conical case can be similarly in cylindrical helicodical surfaces. The simulation of these case can be seen in Fig. 5., 6.and 7. The equation of the Archimedean worm is in the next form:

$$\mathbf{r}_{1F} = \begin{bmatrix} -B_1 \cdot \sin \vartheta \\ +B_1 \cdot \cos \vartheta \\ u \cdot \sin \beta + p_a \cdot \vartheta \\ 1 \end{bmatrix} \qquad (3)$$

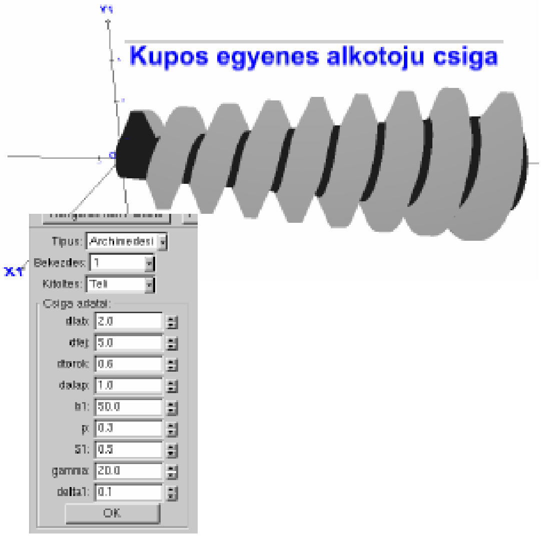

Fig. 5 *The Archimedean conical worm*

The convolute conical worm is shown in Fig. 6., while equation of the convolute conical worm is in the next form:

$$\mathbf{r}_{1F} = \begin{bmatrix} -B_1 \cdot \sin\vartheta + r_a \cdot \cos\vartheta \\ B_1 \cdot \cos\vartheta + r_a \cdot \sin\vartheta \\ u \cdot \sin\beta + p_a \cdot \vartheta \\ 1 \end{bmatrix} \qquad (4)$$

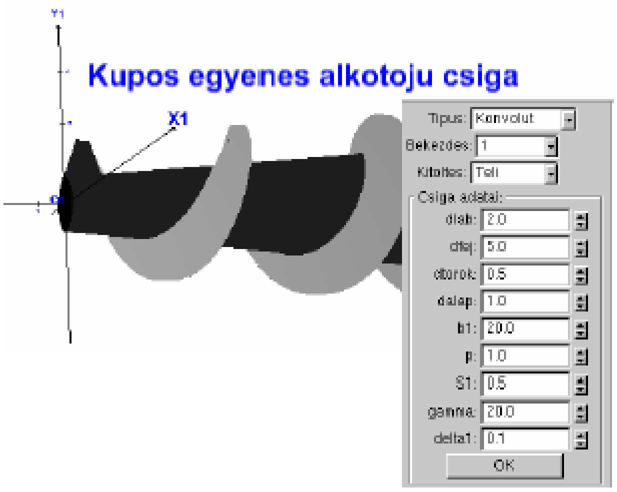

***Fig. 6** The convolute conical worm*

The conical involute worms are the special case of the convolute conical worm. The geometrical parameters were discharge suitable, as it is indicated in Fig. 7, thus, equation of the convolute conical worm is in the next form:

$$\mathbf{r}_{1F} = \begin{bmatrix} -B_1 \cdot \sin\vartheta - r_t \cdot \cos\vartheta \\ B_1 \cdot \cos\vartheta - r_t \cdot \sin\vartheta \\ u \cdot \sin\beta + p_a \cdot \vartheta \\ 1 \end{bmatrix} \qquad (5)$$

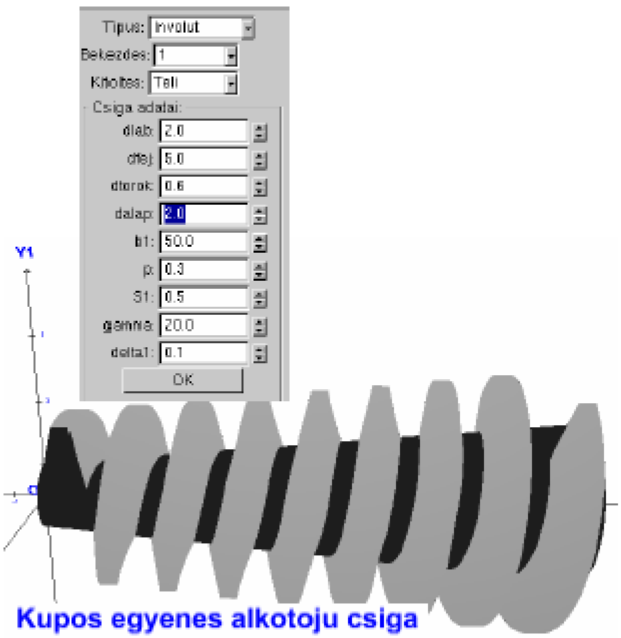

***Fig. 7** The involute conical worm*

Conditions that are more advantageous exist for circular profile worms in axial section. The bearing pattern analysing was introducing in [1]. The new modeling can give more profitable results [1] whereas equation the involute conical worm is in the form:

$$\mathbf{r}_{1F} = \begin{bmatrix} -B_1 \cdot \sin\vartheta + r_a \cdot \cos\vartheta \\ B_1 \cdot \cos\vartheta + r_a \cdot \sin\vartheta \\ u \cdot \sin\beta + p_a \cdot \vartheta \\ 1 \end{bmatrix} \qquad (6)$$

where $r = p_a \cdot ctg\beta - p_t$, $B_1 = u \cdot \cos\beta + p_t \cdot \vartheta$, $p_t = p_a \cdot tg\,\delta_1$ and the $\delta_1 \geq 0$ is the half angle of the cone and in case $\delta_1 = 0$ the worm is cylindrical.

The generated equation of the conical worms is

$$\mathbf{r}_{1F} = \begin{bmatrix} -B_1 \cdot \sin\vartheta + r \cdot \cos\vartheta \\ B_1 \cdot \cos\vartheta + r \cdot \sin\vartheta \\ u \cdot \sin\beta + p_a \cdot \vartheta \\ 1 \end{bmatrix} \qquad (7)$$

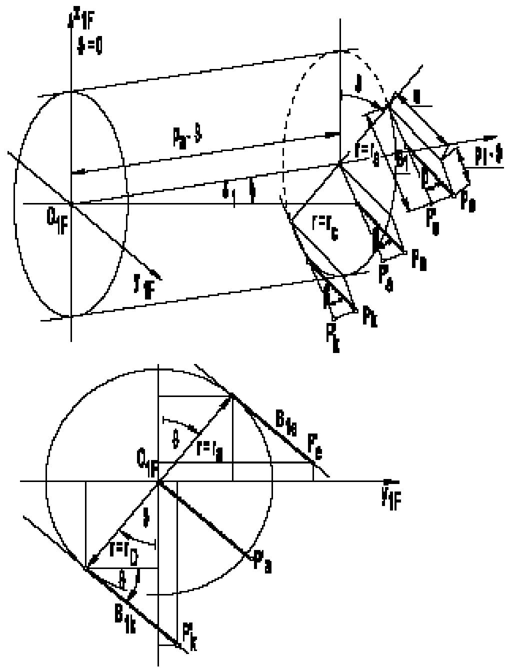

***Fig. 8** Generated summary description of the original of the conical worms*

The spiroid worm with circle profile in axial section is determined the showed geometrical parameters on Fig. 9.

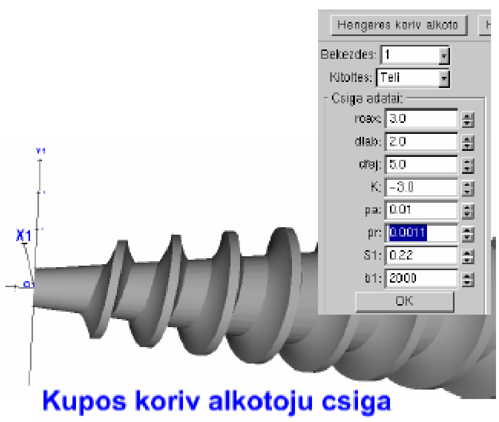

***Fig. 9** The conical worm with arched (circle) profile in axial section*

4 Application function of the mathematical model to determine the curvature of the profile

The selected points on the axial section of convolute of worm are $\mathbf{p}_0, \mathbf{p}_1, \ldots \mathbf{p}_n$, and let them parameters u_i, $i=0,1,\ldots,n$, $u_0=0$, $u_n=1$, and they are proportional length of chord. A curve is needed to find to the given points and parameters while one of the solutions of the problem is the interpolated Bézier curve [7]. The $\mathbf{b}_0, \mathbf{b}_1, \ldots, \mathbf{b}_n$, control points are wanted to determine the Bézier curve to the $\mathbf{p}_0, \mathbf{p}_1, \ldots \mathbf{p}_n$ points.

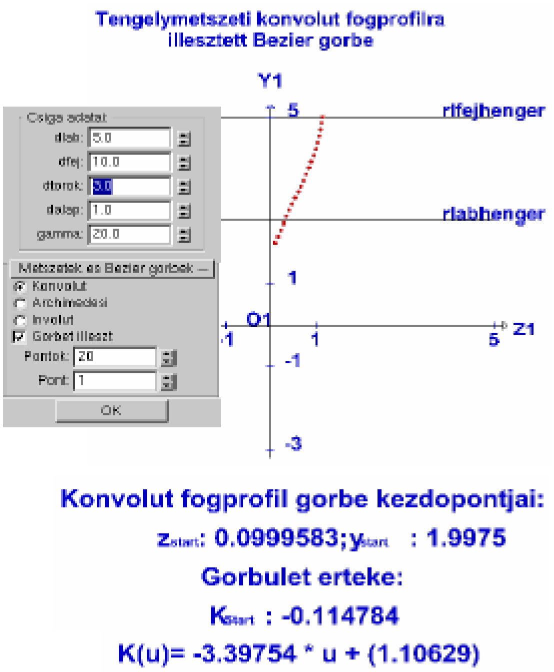

***Fig. 10** The interpolation Bézier curve on the convolute profile with the curvature function*

The equation of the Bézier curve is determined in the next formula:

$$\mathbf{b}(u) = \sum_{j=0}^{n} B_j^n(u)\mathbf{b}_j \quad (i=0,1,\ldots,n), \tag{8}$$

where $B_j^n(u) = \binom{n}{j} u^j (1-u)^{n-j}$ are Bernstein polynoms.

The next linear inhomogeneous equation systems gives the control points as:

$$\begin{bmatrix} \mathbf{p}_0 \\ \mathbf{p}_1 \\ \vdots \\ \mathbf{p}_n \end{bmatrix} = \begin{bmatrix} B_0^n(u_0) & B_1^n(u_0) & \cdots & B_n^n(u_0) \\ B_0^n(u_1) & B_1^n(u_1) & \cdots & B_n^n(u_n) \\ \vdots & \vdots & \ddots & \vdots \\ B_0^n(u_n) & B_1^n(u_n) & \cdots & B_n^n(u_n) \end{bmatrix} \begin{bmatrix} \mathbf{b}_0 \\ \mathbf{b}_1 \\ \vdots \\ \mathbf{b}_n \end{bmatrix}$$

(9)

The $u_i \neq u_j$ condition results the unambiguous solution to $\mathbf{b}_i$. So one receives the $\mathbf{b}_i$ control points of the interpolation Bézier curve to the are $\mathbf{p}_0, \mathbf{p}_1, \ldots \mathbf{p}_n$ points. In case n = 4, the third degree Bézier curve is fitted to the worm profile and it gives the κ(u) first degree function of the curvature as shown in Fig. 9. The direct of the curvature function show the concave or convex form of the profile of the worm in the axial section.

5 Conclusions

Mathematical model approached with projective geometry can be a base of a new source in research work. The new aspect of the modelling in manufacturing can give results of higher accuracy [7]. Program has a few practical functions. Rotation controls allow the user to input rotations into an application by manipulating of an arc ball control. The control displays as a checkerboard-textured sphere, which the user manipulates directly. The current rotation of the control is passed to the application as an array of 16 floats, representing a 4´4 rotation matrix. This array can be passed to OpenGL directly to rotate an object, using the function glMultMatrix(). Rotations control deals with pure rotations only (no translation or scaling), the transpose of the 4´4 matrix is the same as its inverse. Using a suitable equation in projective geometry must be harmonized to the arched profiles and write it in a new function.

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