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CONTINUOUS MOTION IN *PHYSICS* VIII 8*

Summary: The note discusses Aristotle's arguments concerning continuity of circular motion and shows that, first, they are philosophical in nature and, second, continuity of rectilinear motion can be proven in Aristotelian physics.

Key words: Aristotle, *Physics*, continuous motion.

In *Physics*, Aristotle makes two important statements: first, eternal motion is continuous (259a16–17), second, continuous motion is circular (261b27–28), from which he implies that eternal motion is circular and that non-circular motion is discontinuous; in particular, rectilinear motion cannot be continuous (261b31–32, 262a13). In *Physics* viii 8, Aristotle attempts to prove that continuous motion is circular. To him, circular motion is perfect because it has no limit, no beginning or end, and can be executed without interruption infinitely, and thus it is the most fitting motion for the heavens (*De caelo* 284a2–11). That is, there is a lot at stake to show that only circular motion can be eternally continuous. In this note, I would like to discuss the validity of his argument and show that it is far from convincing and that, ultimately, the statement concerning continuity of circular motion alone is a philosophical assumption, not a thesis to be proven in Aristotelian physics.

Aristotle takes for granted that an infinite locomotion along a finite line interval is impossible because the moving body eventually must turn back (261b31–33). This statement can be refuted by pointing to the function $y = \tan^{-1} t$ that for any nonnegative t, $0 \leq y < \pi/2$, that is, for any instance of time, the position of y is limited to the interval $[0, \pi/2]$, and y moves from 0 toward $\pi/2$, never reaching it. Another function is used in the radioactive decay formula, $y = y_0 e^{-\lambda t}$, which for nonnegative t renders $y_0 \leq y < 0$, when the number y of atoms of a given radioactive element changes with time from y_0 to 0, never becoming 0 (λ is a decay constant). If these examples seem to be too contrived and unsuitable as a counterargument in the

* This stimulating paper treats the topic from the point of view of physics. The editors would be glad to get contributions also from the philological side.

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context of Aristotelian physics, we can use a form of Zeno's argument, which, after all, is discussed by Aristotle.

Assume that the first half of the interval [0, 2] is traversed with velocity $v_0 = 1$, the first half of the remaining half, [1, 3/2], with velocity 1/2, the first half of the last quarter, [3/2, 7/4], with velocity 1/4, etc., so that the subinterval [0, 1] is traversed in time t_0, the subinterval [1, 3/2] in time t_0, the subinterval [3/2, 7/4] in time t_0, so that an infinite time is needed to traverse [0, 2] according to the formula[1]

$$y = 2 + \frac{1}{2^{int(t)-1}}\left(\frac{t - int(t)}{2} - 1\right)$$

Velocity $v = y' = 2^{-int(t)}$ abruptly changes at instances t_0, $2t_0$, …, and yet the overall motion is still one and continuous, although this is continuity of "a lesser degree" (ἧ ττον, 229a1).[2]

However, even granting the validity of the argument concerning the impossibility of infinite locomotion along a finite interval, the argument used to show that infinite motion back and forth along an interval is discontinuous is not convincing. Aristotle says that rectilinear motion cannot be continuous because "the motion must stop before turning back," which is supposedly supported by observation and logical argument (262a12-19). Observation certainly shows that the motion is turning back, but does it really prove that at the point of turning the motion is brought to a halt? In his logical argument, Aristotle states that for a continuous motion from *A* to *B*, there must not be any point *C* in between at which the motion stops because

(1) *C* would turn from a potential point to an actual point, and

(2) stopping at *C* takes a time interval, not an instant; that is, stopping at *C* involves arriving at *C* at one instant and then restarting motion from *C* at another instant,

and, at the same time, there is an implication (2) ⇒ (1) (262a22–25). In this way, continuous uniform motion from *A* to *B* requires less time than uniform motion at the same velocity with a stop at a point in between. Moreover, because there are potentially infinitely many points between *A* and *B*, there may be an infinite number of stops between these two points and, therefore, motion might not ever be completed.[3] In this argument, Aristotle simply states that a stop changes the status of a point from potential to actual and that a stop requires a time interval. Let us observe first, that being at rest at a certain point, i.e., moving with velocity $v = 0$, does not necessarily entail that the moving spends more than an instant at the point of rest. Consider the function $y = (t - 1)^3 + .5$. The velocity $v = y' = 3(t - 1)^2$, which is zero at $t = 1$, which is the function's inflection point. Moreover, this is the only point, for which velocity equals zero; that is, there is no time interval necessary for a momentary stop at $y = .5$.

[1] The function $int(t)$ returns the integer part of t; for instance, $int(2.1) = int(2.9) = 2$.

[2] Abrupt changes can be removed by using the function $y = 1 - v/2^t$.

[3] This is true for some time intervals needed for stopping, e.g., if each stop takes the same amount of time. But if, for example, the first stop takes x time units, the second $x/2$ time units, the third $x/4$ units, etc., then motion would be completed, after all, notwithstanding the infinity of stopping points.

More importantly, Aristotle seems to commit a logical fallacy by implicitly turning the implication (2) $\Rightarrow$ (1) into equivalence (2) $\Leftrightarrow$ (1) because he uses the implication (1) $\Rightarrow$ (2). He says that in the process of moving from *A* to *B* and then back to *A* along the same line, the point *B* is at the same time the starting point of motion in one direction and finishing point of motion in the opposite direction which means that the motion stopped at *B* (262b22–26). The reasoning seems to be that each line interval is limited by two actual points, in this case, points *A* and *B*. Because the point *B* is actual, the motion may be thought of as aiming at it and then leaving it, that is, stopping at it. Therefore, the implication is that if a point is actual, then each motion has to stop at it, whereas the implication (2) $\Rightarrow$ (1) says that if motion stops at a point, then the point becomes actual. The problem is that by singling out a point (making it actual), the expressions "arriving to" and "departing from" are seemingly automatically applicable and the possibility of using these expressions makes the motion actually stop at the point. But this is merely a verbal solution to a physical problem. However, another solution can be offered. Motion can be said to be arriving to a point *P* when it is approaching the point and having arrived at the point when the motion has stopped at the point; that is, motion is a process before reaching the point and after reaching it. But continuous motion *is* or *occurs* at an instance *t* at *P*, although up to the time *t* it is in the process of arriving at *P*, and after *t* it is in the process of leaving it. Aristotle himself allows for such an interpretation. He says that a point *A* when it moves continuously cannot be said to arrive at nor to depart from *C* that is a potential middle point between *A* and *B*: "it can only have been there (ἐν τῷ) at the moment, not in a time interval" (262a28–31). But by singling out the point *C*, the point becomes actual. Even if we agree that this singling out is only hypothetical, cannot an actual point be treated as a "there" of the same category when it comes to continuous motion? Similarly, Aristotle writes that circular motion is continuous because "that which is in motion from *A* will be at the same time in motion to *A*" (264b10–11). However, a mere mentioning of a point *A* on a circle makes it actual by pointing to it, whereby the motion is not continuous any more. If this, however, is just *modus loquendi* in the case of potential points, why not extend it also to actual points? A commentator remarks that a mention of a point is done in this case *par abstraction*, because point *A* really (that is, actually) does not exist; it exists only in the mind of the one who distinguishes *A* from other points.[4] But if the abstraction process may be applied to the potential existence of *A*, why not apply it to its actual existence when analyzing continuous motion by not making a distinction between actual and potential existence? From the perspective of the motion, points just exist; it is not that by passing potential points the motion is passing nothingness. After all, mathematicians abstract from sensory qualities and retain only the quantitative and continuous (*Met.* 1061a28–34). Moreover, because physics investigates being as "sharing in movement" (1061b8), the distinction between potential and actual points can be suspended by investigating the motion of points *qua* points through points.

[4] René MUGNIER: *La théorie du premieur moteur et l'évolution de la pansée aristotélicienne*, Paris: Vrin 1930, p. 161.

Although the implication (2) ⇒ (1) is acceptable, the implication (1) ⇒ (2), allegedly inferred from the former, is not, even on linguistic grounds. It is not acceptable either on physical grounds because a continuous motion oscillating between two points *A* and *B* is, of course, possible. For example, the function $y = \sin t$ gives the position of point *y* in the interval [–1, 1] for any value of *t*. The motion has velocity $v = 0$ for instances $t = \pi/2 + n\pi$ and any integer *n*, because $v = \cos t$. In the case of the uniform motion, as discussed by Aristotle (262b13), velocity $v = c$ when *y* is moving from *A* to *B*, including *A* and excluding *B*, and $v = -c$ when *y* is moving from *B* to *A*, including *B* and excluding *A* (*Figure* 1). Because $y = \int v dt = \pm c + y_0$, for any y_0, we find

$$y = \begin{cases} ct - 2n(B - A) + A & \text{if } 2nt_0 \le t < (2n+1)t_0 \\ -ct + 2n(B - A) + A & \text{if } (2n+1)t_0 \le t < 2(n+1)t_0 \end{cases}$$

for $t_0 = (B - A)/c$, and the motion is both continuous and uniform, including the limit points *A* and *B* (*Figure* 2).

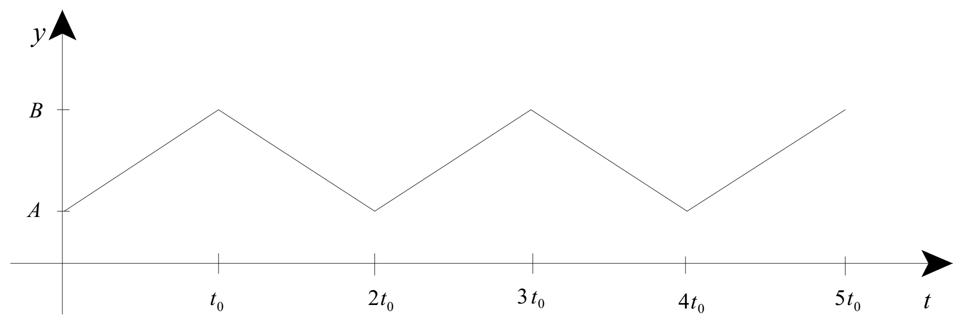

Figure 1

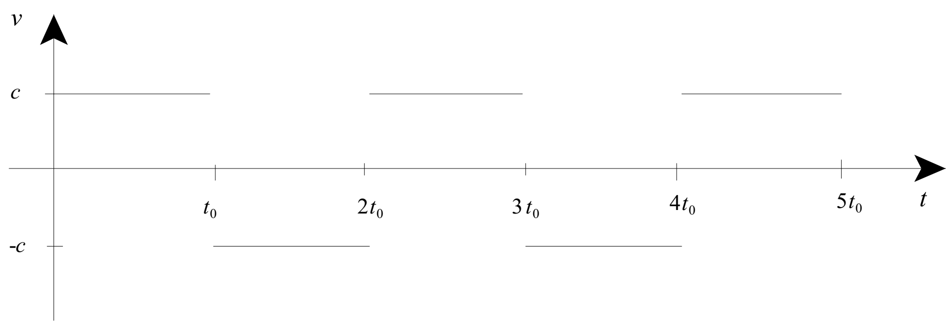

Figure 2

Note that in this example, velocity is measured at the limit points only in one direction; for instance, velocity at *A* is found for the upward movement and so it equals *c*. It could not be found for both directions because $c \neq -c$ except for a trivial case when $c = 0$. It seems that Aristotle allows for such a solution since he says that a point of time separating what is earlier in the process of change from what is later "is common to both times, the earlier as well as the later … but so far as the thing is concerned, it belongs to the later stage of what happens to it" (263b12–15). That is, at least from the perspective of the concerned thing (the point being in motion), the motion is always taking place in one direction, not in two directions at the same time.[5]

Another argument Aristotle uses against continuous rectilinear motion is that if a point moves from *A* to *B* and then from *B* to *A* along the same interval, then the point is undergoing "at the same time (ἅμα) two contrary motions since the two motions that follow the same straight line are contrary" to each other (264a18–19), and such motions cancel each other. Therefore, the motion must stop at *C*. This is, however, playing on the equivocation of the phrase "at the same time." On the one hand, it can be claimed that the motion as a whole, the round trip from *A* back to *A* via *B*, has two stretches with two opposite directions.[6] "At the same time" means here, during the whole process of motion, during the time interval needed to move from *A* to *A*. However, this by itself is no argument against continuity of the motion as a whole because what moves is a point, and this point moves either in one direction or in another, which does not at all mean that the point is "at the same time" in two contrary motions, in which "at the same time" means at the same time instant, at the same point of time.

This brief discussion indicates that arguments offered by Aristotle against continuous rectilinear motion are not decisive. Whether decisive or not, the arguments seem to be secondary to the fact that Aristotle is deeply convinced that continuous motion can only be circular because the circle is an ideal geometric figure. The conviction was very firmly rooted in the Greek mind since at least the time of Pythagoras. For Aristotle, it was obvious that the circle is first among planar figures (*De caelo* 286b17–18). This is just an expression of the conviction that circularity saturates the whole of universe, and this aspect is a manifestation of the divine side of the universe. "The circular motion of the first heaven imitates immobility of the first mover. The cycle of seasons imitates the motion of celestial spheres. Circular generation of living beings imitates the eternal return of the seasons."[7] Aristotle's system is thus characterized by "a veritable obsession for uniform circular motion" and an "ob-

[5] In this example, velocity is never zero, notwithstanding Aristotle's apparent claim that, generally, a motion can change to an opposite motion only through the state of rest (229b18–230a6). This is, however, an arbitrary assumption: why could not motion change into an opposite motion without a mediating state of rest (as claimed in 263a32–33)?

[6] See commentary in Aristoteles, *Physikvorlesung*, Berlin: Akademie Verlag 1979, pp. 694–695. See also Hippocrates G. APOSTLE'S remarks in Aristotle, *Physics*, Bloomington: Indiana University Press 1969, pp. 334–335.

[7] Pierre AUBENQUE: *Le problème de l'être chez Aristote*, Paris: Presses Universitaires de France 1962, p. 498.

session in regard to the sphere as a perfect shape."[8] Circularity is the closest thing to the divine in the universe; circular motion is the cause of perpetuity of coming-to-be (*De gen. et corr*. 336b30–337a1), thus it is not surprising that in his physics Aristotle gives precedence to circular motion over rectilinear motion and that only by imitating circular motion can rectilinear motion be also continuous (337a6–7).

This conviction is seen also in Aristotle's intricate astronomical system. The system is an elaboration of Eudoxus' scheme to which Aristotle added a number of new spheres. The outermost first heaven was the only self-moving sphere, and the uniform rotary motions of the remaining fifty-five spheres depended on the motion of the first heaven. To account for irregularities of motions of heavenly bodies, each sphere rotated about its own axis. The system was not sufficient to explain all the irregularities, which led later to the use of epicycles and eccentrics. It is interesting to see that epicycles can be used to explain an oscillating motion on the interval AB.[9]

Let us consider a circle (deferent) with the center C at the point $(R, (B - A)/2 + A)$ and a diameter $r = (B - A)/4$ (*Figure* 3). Point P_1 moves on the circumference of this circle with a constant angular velocity. P_1 is the center of another circle (epicycle) with the diameter r, and on its circumference there is a point P_2 moving in the opposite direction than P_1 but with the same angular velocity as P_1. The position of the point P_1 is found with the equations

$$x_1 = R + r\sin\alpha$$
$$y_1 = (B - A)/2 + r\cos\alpha + A$$

and the position of P_2 with the equations

$$x_2 = x_1 + r\sin(-\alpha) = R$$
$$y_2 = y_1 + r\cos(-\alpha) = (B - A)/2 + 2r\cos\alpha + A = 2r(1 + \cos\alpha) + A.$$

According to these equations, the coordinate x_2 of P_2 is always the same and the coordinate changes between A and $4r + A$, which means that P_2 is moving up and down the vertical interval with endpoints (R, A) and (R, B). Importantly, P_1 is at the endpoints only momentarily, not for a time interval, however brief, because the angular velocity is constant; that is, the motion never stops at the circumference of the moving circle. Had motion at any of the two endpoints of the interval (R, A)–(R, B) stopped, then the motion on the circumference would have to stop as well because P_1 is moving *at the same time* on the circumference. In this way, continuity of the circular motion can be used as an argument supporting continuity of rectilinear mo-

[8] Herbert BUTTERFIELD: *The origins of modern science*, New York: Collier Books 1962, p. 43. Butterfield refers here to Copernicus who was more consistent than Ptolemy in applying uniform circular motion (thereby renouncing the use of the equant) and in this way aligning himself closer with Aristotle. The "obsession with circular motion" begins to fade away only in the seventeenth century giving primacy to the rectilinear motion, Michel BLAY: *Reasoning with the infinite*, Chicago: The University of Chicago Press 1998, p. 40.

[9] The argument presented here was used by Copernicus in *Commentariolus* and in *De revolutionibus* iii 4 ("How an oscillating motion or libration is compounded of circular motions").

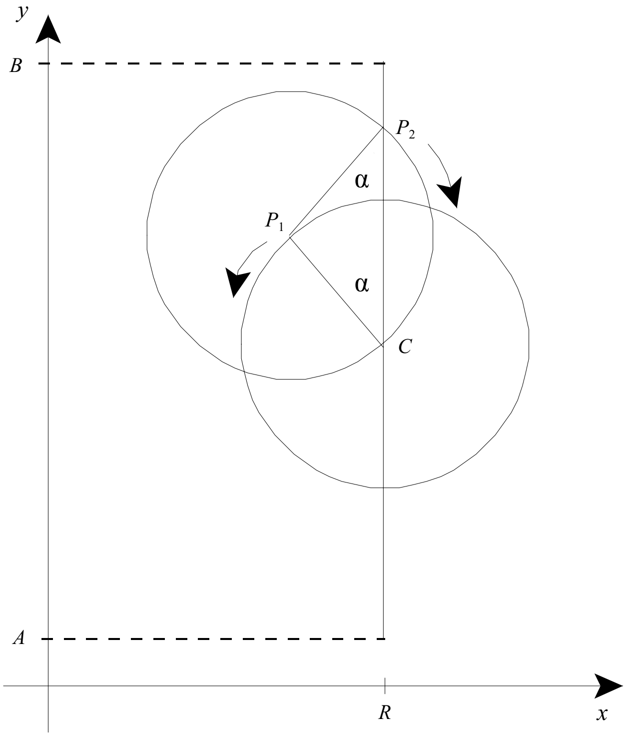

Figure 3

tion.[10] Admittedly, epicycles were introduced after Aristotle's times, but this argument shows how fragile his system is and that it can be undermined from within.

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[10] The motion of P_1 on circumference is uniform (because angular velocity on both circles is uniform and their diameters are the same), therefore, the rectilinear motion is not uniform, but it still is continuous. To make the latter motion uniform, the angular movement would have to be made nonuniform by finding such $\alpha = \alpha(t)$, for which $y'_2 = -2r\sin\alpha\alpha' = v_c$ for some constant linear velocity v_c. By solving the differential equation $-2r\sin\alpha\alpha' = v_c$, we obtain $\alpha(t) = \cos^{-1}(\frac{v_c}{2r}t + \alpha_0)$ for any α_0. However, nonuniformity of the rectilinear motion does not change the thrust of the argument, because the argument is about continuity of the two motions (circular and rectilinear), not their uniformity.