JÁNOSSY'S TRENDLINE THEORY IN THE LIGHT OF THE NEW GROWTH THEORY

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Ferenc Jánossy was the most important Hungarian pioneer of surveys on long time series. In the 1960s he devised the famous theory of trendlines, which allowed him to forecast the great world economic recession of the 1970s a decade in advance. The best-known international authority on compiling historical time series is Angus Maddison, who prepared time series of the main demographic and macroeconomic indicators for 56 countries, from 1820 to the present day. Both scientists, whose survey method showed both a historical and a quantitative approach, reached the conclusion that human capital is the most important of production factors for securing long-term economic growth. The main purpose of this paper is to compare their results with the latest development, which is known as the "new growth theory".*

Keywords: Jánossy's trendline theory, new growth theory, Maddison's historical time series, optimisation models, human capital

JEL classification index: B22, C32, C61, E13, F15, J24, O10, O40

INTRODUCTION

The analytical framework of the long-term process for Hungary and East Central Europe to catch up with Western Europe requires a longer perspective. Therefore it is appropriate to start with previous theories and scientific achievements which have constructed and analysed historical statistical time series to define the most important factors of the economic growth:

Ferenc Jánossy was the most important Hungarian pioneer of surveys on long time series who analysed the reconstruction periods of countries following World War II, and devised the famous theory of trendlines in the 1960s. According to

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Jánossy human capital is the real carrier of economic growth. This theory allowed him to forecast the world economic recession of the 1970s a decade in advance (Jánossy, 1966). It is important to give him his due place in the international economic literature, though this is not an easy task since Jánossy has created a coherent and unconventional theory without having cited any related, well-known neo-classical result. Thus each step forward in this direction supports the validity of the neo-classical, and the new growth theory by Jánossy's rich empirical arguments.

The best-known international authority on compiling historical time series is Angus Maddison, who has prepared time series of the main demographic and macroeconomic indicators for 56 countries, from 1820 to the present day (Maddison, 1995). Both scientists reached the conclusion that human capital is the most important production factor for securing long-term economic growth.

It may be interesting to compare the findings of the two theories with the latest models and findings of the so-called "new growth theory" because they are

- 1. inspired by microeconomic approach to the problem of growth,
- 2. promoting human capital to the position of the most important production factor and
- 3. important contributions to developing the necessary mathematical tools of optimisation.

A SHORT REVIEW OF JÁNOSSY'S THEORY

Ferenc Jánossy scrutinised the post-war reconstruction periods of a great number of countries, and stated that the reconstruction period "doesn't stop when the production has reached again the pre-war level, but only ... when the volume of production corresponds again to the trendline of the economic development" (Jánossy, 1966, p. 19). Specifically, reconstruction follows a path as if the war had not happened. The book starts by the schematic diagram of the post-war reconstruction period (*Figure 1* and Jánossy, 1966, p. 18).

In *Figure 1* the straight line AF expresses the undisturbed production growth prevailing in the long run. This line will be called the "trendline of economic development" or simply "trendline".

If the economic development of a country was undisturbed up to the outbreak of the war, the virtual production coincides with the trendline before the outbreak of the war. This is indicated by the section AB. After the beginning of war (point B), the variation of production depends on the war events: however, at the end of the war or a bit later, production falls to the bottom (point C), not only in

the defeated but in the winning countries as well. Since this fall had not been regulated by any generally prevailing law, and for our further considerations merely the resulting low point C has importance, the segment from B to C is denoted by an arbitrarily drawn (dotted) access line only.

The reconstruction period starts at point C. From this point production is growing incessantly and after some years (point D) reaches the level of the last year of peace. However, the reconstruction period does not stop here because the production progresses nearly at the same rapid pace as before point E. The rate of growth breaks down only at this time and returnes – more or less suddenly – to the normal level of growth, which is typical and determined by the rules of economic development prevailing in the long run. After this point in time, the growth of production follows the Jánossy-trendline (segment EF in Jánossy, 1966, pp. 18–19).

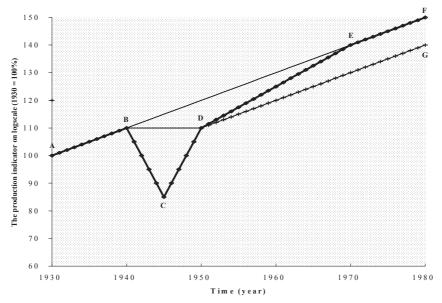


Figure 1. Illustration of the theorem

AF - the Jánossy-trendline of economic growth

AB – the development of production up to the break out of war

BC – the regression of production caused by the war

CE – the development of production during the reconstruction period namely:

CD – up to reaching the pre-war level

DE – up to reaching the Jánossy-trendline

EF – the real development of production after the reconstruction period

DG – the supposed development of production after the reconstruction period

In the first part of his book, Jánossy has examined the most important macro-economic facts of the world from the end of 19th century up to the mid-1960s, laying emphasis on the so-called "economic miracles" like Japan, GFR and Italy. In the mid-60s, he predicted the boom of the 1970s, in spite of the fact that no-body believed him.¹

In the second part, he shows that the reconstruction ends only in point E, lasting until the level of production catches up with its trendline. He concludes that "the process of economic development must imply a crucial factor which remains intact during the war" (Jánossy, 1966, p. 112). He proves that

"... this stable factor is the humanity itself; not the single people whose hundred-thousands have fallen victim to the war, but the human society in its entirety including all its experience, science and knowledge. The nations – in spite of really heavy and hardly assessable losses – up to now not only have survived the wars in the past (even the all-destroying like word war two was) but preserved nearly wholly for posterity the most important heritage the accumulated knowledge and skill and – in some fields – could even enlarge it. ... The workforce, the real carrier of the forces of production though during the war decreases in number but its structure and state of development not only remain but progress unceasingly. ... On the bases of all these it follows objectively that the trendline pendant the war and after increases incessantly. Our latter conclusion in turn implies implicitly the assumption that the steepness of the trendline depends after all on the development of the workforce" (*Ibid.*, pp. 112–113).

Jánossy introduced a new notion: the "professional structure" meaning "the division of manpower of a country into professions according to how many people are in possession of a given profession" (*Ibid.*, p. 234). This is a much more abstract notion than the traditionally well known "employment structure". Using the present terminology it is better to consider the former one as a sort of human capital. At the end of his book the relation between the change of "professional structure" of the manpower of a country and the growth rate of economic development is summed up by Jánossy in the following four points:

- "1. The level of development of a given country even if it is not realised temporary in the real magnitude of production, i.e. it exists just as a realisable possibility depends first of all on the actual professional structure of the whole manpower.
- 2. The economic development is related inseparably to the change of professional structure. The preliminary condition of a faster economic development rate is the faster change of professional structure.
- Then the 1970s ushered in the "age of stagflation", in which rising inflation and unemployment were to appear simultaneously as the growth of living standards slowed sharply. This development was on no scholar's timetable not seen in the crystal ball of Spengler, Toynbee, Marx, Shumpeter or Galbraith. We live in a world no prophet ever predicted (Samuelson–Nordhaus, 1989, p. 853).

- 3. The barriers which limit the speed of change of professional structure limit for long run the economic development rate, too.
- 4. The inertia stabilising the change of professional structure i.e. the influence of changes in the past on the changes of the future years or even decades, determines profoundly the persistency of the trendline of the economic growth" (Jánossy, 1966, p. 245).

Jánossy was among the first economists who based the explanation of the rate of economic growth on the human capital, even if he referred to it in a way different from the mainstream approach.

ANGUS MADDISON'S HISTORICAL TIME SERIES AND THE ROLE OF HUMAN CAPITAL IN ECONOMIC GROWTH

In 1993–94, based on the course set by Jánossy and on the basis of the OECD GDP per capita data, the question was raised whether the empirical facts of economic developments having happened during the preceding almost 30 years – after the birth of Jánossy's trendline theory – confirm or falsify the validity of his theory (Tarján, 1995).

The OECD data have strongly confirmed Jánossy's predictions. No secret is made of considering the transition of Hungary and the ECE countries as Jánossy's reconstruction period in order to forecast their prospects in catching up with Western Europe.² A surprising result is Hungary's predicted position relative to Austria in 2025.

In 1996 – on the basis of Maddison's data (1995) – after having checked my results obtained three years before, it allowed:

- to study the two characteristic turning points (of the reconstruction period described first by Jánossy) by fitting a broken line with two points of inflexion to Maddison's time series after 1945,
- to study by Jánossy's method the development of any groupings of the OECD countries such as the group of the Major Seven, G7 (the best developed seven OECD countries), OECD-22 (all the OECD countries up to 1993, except Iceland and Luxembourg), OECD-EU (the European OECD countries among the OECD-22) (Figure 2).

The results obtained from Maddison's data are summarised in the *Annex Table*. Maddison, studying nearly two hundred years of development of the world economy, has also stressed the importance of the human capital (Maddison, 1995,

See also Bekker (1995) as an other application of Jánossy's theory for the transition into the market economy.

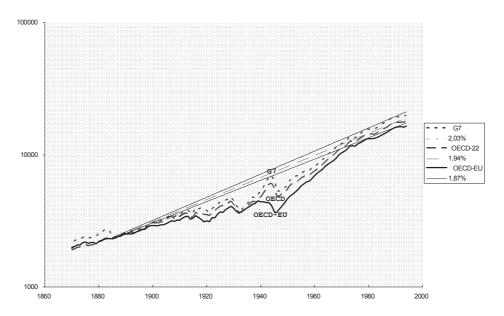


Figure 2. Jánossy's trendline of three major groups of countries

p. 37). His "human capital" is measured by the total stock of education in the age group 15–64, and is illustrated in *Table 1*, with primary education given a weight of 1, secondary 1.4 and higher education 2, to provide a rough correction for the remuneration which these different levels attract.

Table 1

Years of education per person aged 15–64, six countries, 1820–1992
(average for both sexes)

	USA	France	Germany	The Netherlands	UK	Japan
1820	1.75	n.d.	n.d.	n.d.	2.00	1.50
1870	3.92	n.d.	n.d.	n.d.	4.44	1.50
1913	7.86	6.99	8.37	6.42	8.82	5.36
1950	11.27	9.58	10.40	8.12	10.60	9.11
1973	14.58	11.69	11.55	10.27	11.66	12.09
1992	18.04	15.96	12.17	13.34	14.09	14.87

Source: Maddison (1995, p. 37).

Table 1 shows an enormous increase in the average level of education from 1820. In 1820, the majority of the population in all countries was illiterate. In the

advanced capitalist countries, universal enrolment in primary education became obligatory in the 19th century, and the proportion of population receiving secondary and higher education has risen steadily in the 20th century. In Japan and in the USA the average person's human capital by the above yardstick increased tenfold from 1820 to 1992.

Applying the weights of Maddison for Hungary's 1995 data by Kovács–Molnár (1997), our own estimation is that Hungary has 12.9 years. This means that on the basis of this calculation, Hungary has a much better position compared to its actual economic performance. We suggest – on the basis of comparative statistics on education – that the other ECE countries are in a similar situation.

The expansion of education took place for a variety of reasons, cultural and recreational, as well as economic, but the economic impact has been substantial. It was first stressed by Schultz (1961), incorporated in Denison's growth accounts in 1962, and rediscovered more recently by the new growth theorists. The increases in educational level helped to embody technological progress because the content of education changed over time to accommodate to the growing stock of knowledge. There has been a proliferation of specialised intellectual disciplines to facilitate the absorption of knowledge and to promote its development through research.

The education stock is, of course, only a rough measure of changes in human capital. It is better than enrolment ratios, which are often used as a crude proxy in the new growth literature, but it should be adjusted for differences of efficiency of education systems in transmitting cognitive skills, and supplemented with information on less formal types of skill acquisition.

THE MOST RECENT RESULTS OF THE NEW GROWTH THEORY

Jánossy, in the second part of his book, starts with the following:

"The factual material analyzed in Part I made it unmistakably clear that the end point of a reconstruction period is determined by the trendline of economic development. The production-level line raises sharply from its law point after the war, but breaks, after reaching the trendline, as abruptly as if hitting a wall. The fast growth tempo characteristic of the reconstruction period slows down to the degree that the production-level line once again follows the trendline" (Jánossy, 1971a, p. 97).

This concise wording implies actually two laws:

1st law: The curve expressing the increase of production after the war reaches the trendline and follows it further on (i.e. drafting it in terms of *Figure 1:* its path arrives at point E and follows the segment EF further on).

2nd law: The curve expressing the increase of the post-war production – starting from the lowest point – expands steeply but after having reached the trendline breaks sharply if it had clashed with a wall (i.e. drafting it in terms of *Figure 1*: its path breaks sharply at point E).

For making a decision on the validity or invalidity of the above two laws, we applied three models for Maddison's data from World War II to the present (Tarján, 1997, 1998).

We shall show that the 1st law above is well proven both by the neo-classical and the new growth theory, while the 2nd law is proven only by the new growth theory that endogenises the technological change and the saving rate, and applies human capital as the most important production factor.

The three growth models are as follows:

- (I) Solow–Swan model with labour-augmenting technological progress.
- (II) Solow–Swan model with human capital developed by Mankiw, Romer and Weil (1992).
- (III) One-sector model with physical and human capital Barro-Sala-i-Martin (1995).

All three models prove Jánossy's 1st law, but the 2nd is proved only by model (III). Though model (II) applies human capital but does not yet endogenise the saving rate and this is not enough to prove Jánossy's 2nd law. On the basis of Maddison's data we have verified for all the most important OECD countries whether the two laws of Jánossy were satisfied or not, but only the result of model (III) and for Japan will be shown in this paper later on in *Figure 3*.

The neo-classical growth model of Solow and Swan

Jánossy's 1st law will be well proven by this model, stemming from the late 1950s.

Even if the most important message of Jánossy is that we have to look for the mainspring of economic growth, it is worthwhile to raise the question whether the neo-classical "augmented" model of Solow and Swan – i.e. which allows the technological progress – is capable of explaining the theory of Jánossy, which is well confirmed by the empirical facts for almost the last four decades.

For this purpose let us review briefly the Solow–Swan model, for which the production function is:

$$Y = K^a (A L)^{1-a} \tag{A}$$

where the output and the physical capital are Y and K respectively, the level of the technology is A and the quantity of labour is L. The parameter is positive and

 $\alpha < 1$ holds. L and A are growing by a yearly rate of n and x. The production turns to both consumption and capital formation. The capital has a yearly depreciation rate of δ . We suppose that gross physical investment is a part s_k of the production.

The development of the economy is described by the equations as follows:

$$\dot{k} = s_k y - (n + x + \delta)k \tag{B}$$

where y = Y/AL and k = K/AL are expressed by a unit of the effective labour. The production function (i) is expressed also in the following intensive form:

$$y = k^{\alpha}$$
. (C)

Let us put *y* of (C) into (B). Dividing them by *k*, we obtain the rates of growth γ_k of *k*:

$$\gamma_k \equiv \dot{k} / k = s_k \cdot \frac{1}{k^{1-\alpha}} - (n + x + \delta)$$
 (D)

If k^* denotes the equilibrium (steady) state of $\gamma_{k^*} = 0$, then the explicit forms of k^* is as follows

$$k^* = \left(\frac{s_k}{n+x+\delta}\right)^{\frac{1}{1-\alpha}} \tag{E}$$

Let us suppose now that the economy follows an equilibrium path and after an external shock such as a war, a great part of the physical capital is destroyed and k^* reduces to k_1 ($k_1 < k^*$). Thus γ_k becomes positive in (D) and k is increasing from k_1 towards k^* , i.e. followed by a transition period reaches again its equilibrium state.

Referring to the above review of the Solow–Swan model, let us assume that an economy of a given nation before a war had already been in its stable equilibrium state and its labour productivity – thanks to the technological progress – had grown by a yearly rate of x% (Barro–Sala-i-Martin, 1995, pp. 39–41).

Thus the stable equilibrium state coincides with Jánossy's trendline (plotted on logarithm scale), where the per capita macro indicators k, y, c (capital, output and consumption respectively) are growing by the exogenous rate x of technological progress year by year. If we suppose now that the level of capital, output and consumption drops suddenly because of a war, then the above Solow–Swan model ensures that the economy followed by a certain transitory period reaches its stable state, i.e. its Jánossy-trendline. We may thus conclude that the Solow–Swan model with labour-augmenting technological progress explains well the

most important and most surprising law of Jánossy. Specifically, that the warreconstruction period will only be finished when the extrapolated pre-war development is reached and followed just as if the war had not happened, independently of the fact that the country was winner or loser (Jánossy, 1966, p. 19).

It's important to remark that the theory of Jánossy is supported by the Solow—Swan model independently of the extent and the duration of the war. This proves to be important when we want to extend the validity of Jánossy's law not only for a war-reconstruction period but also for the transition that the Central and Eastern European countries are now experiencing.

In the original Solow–Swan model, a Cobb–Douglas-type production function is used, which satisfies all the conditions of the neo-classical production functions. All the statements up to now are valid for all the functions satisfying the neo-classical conditions, but we did our empirical research with a Cobb–Douglas-type production function only.³ This type of model approach doesn't consider the other important character described by Jánossy's second law, that at the end of the reconstruction period the growth rate suffers an abrupt fall.

On the basis of Maddison's data we have examined the most important OECD countries with respect to the two laws of Jánossy, and we have found that only the 1st law has been satisfied.

The model of Mankiw, Romer and Weil (M-R-W)

This model is an extended version of the previous one with human capital, but neither the saving rate nor the technological progress has yet been endogenised.

Let us start from the model of Mankiw, Romer and Weil (Mankiw et al., 1992) i.e. from a generalised Solow–Swan model of which the production function is:

$$Y = K^{\alpha} H^{\beta} (AL)^{1-\alpha-\beta}$$
 (1a)

where the output, the physical capital and the human capital are Y, K and H respectively, the level of the technology is A and the quantity of labour is L. The parameters α and β are positives and $\alpha + \beta < 1$ holds. L and A are growing by a yearly rate of n and x. The production turns to both consumption and one form of capital formation. Both forms of capital have yearly depreciation rate of δ . We suppose that gross physical investment is a part s_k of the production, while the gross investment turned to human capital is a part s_k of the production.

Since this model is a special case of the Mankiw-Romer-Weil model, which will be presented in the following section, we can dispense with the mathematical discussion of the model here.

The production function (1) is expressed also in the following intensive form:

$$y = k^{\alpha} h^{\beta}. \tag{1b}$$

The development of the economy is described by the equations

$$\dot{k} = s_k \ y - (n + x + \delta)k \tag{2a}$$

$$\dot{h} = s_h \ y - (n + x + \delta)h \tag{2b}$$

where y = Y/AL, k = K/AL, and h = H/AL are expressed by a unit of the effective labour.

Let us put y of (1b) into (2a) and (2b). Dividing them by k and h respectively, we obtain the rates of growth γ_k and γ_k of k and k respectively:

$$\gamma_k \equiv \dot{k}/k = s_k \cdot \frac{h^{\beta}}{k^{1-\alpha}} - (n+x+\delta)$$
 (3a)

$$\gamma_h \equiv \dot{h}/h = s_h \cdot \frac{k^{\alpha}}{h^{1-\beta}} - (n+x+\delta). \tag{3b}$$

If k^* and h^* denote the equilibrium (steady) state of $\gamma_{k^*} = 0$ and $\gamma_{h^*} = 0$, then the explicit forms of k^* and h^* are as follow (Mankiw et al., 1992, p. 417):

$$k^* = \left(\frac{s_k^{1-\beta} s_h^{\beta}}{n+x+\delta}\right)^{1/(1-\alpha-\beta)}$$
 (4a)

$$h^* = \left(\frac{s_k^{\alpha} s_h^{1-\alpha}}{n+x+\delta}\right)^{1/(1-\alpha-\beta)}.$$
 (4b)

Let us suppose now that the economy follows an equilibrium path, and after an external shock like a war a great part of the physical capital is destroyed and k^* reduces to $k_1(k_1 < k^*)$ while h^* doesn't change at all. (This is of course only a rough abstraction of a post-war situation when we suppose that h "remains intact" – or has slightly changed – compared to k which falls sharply.) Thus γ_k becomes positive in (3a) while γ_k turns to negative in (3b). In other words k is increasing from k_1 towards k^* while k is decreasing from k_2 . As a final conclusion we may say that even if during the shock k has remained intact, it has to suffer a decrease during the transition period of returning to the equilibrium path.

On the basis of Maddison's data we have verified for all the most important OECD countries if the two laws of Jánossy were satisfied or not. We have found that the fitted curves here deviated much greater from the empirical data than in model (I), having not yet applied human capital among its production factors.

One-sector model of Barro and Sala-i-Martin⁴

This model is one with human capital where both the saving rate and the technological progress are endonegised, proves Jánossy's 2nd law.

In this section we discuss a model in which both the physical and human capital are produced by one production function (the name "one-sector model" originates from this). The final output of the production can equally be turned to the consumption and both to physical and human investments. The quite trivial assumption that neither the physical nor the human capital can become negative has a decisive impact on the process of growth when an imbalance sets in between the stock of physical and human capital. The growth rate of production is higher the more the share of physical and human capital differ from the equilibrium state.

The Cobb-Douglas-type of production function of the physical and human capital K and H is as follows:

$$Y = A \cdot K^{\alpha} H^{1-\alpha} \tag{5}$$

where $0 \le \alpha \le 1$. We suppose that the final output can be used for consumption or investment in physical or human capital. We assume that the stocks of physical and human capital depreciate at the rates δ_K and δ_H , respectively. This is the only difference from the model of Barro and Sala-i-Martin (1995, p. 173), where the physical and the human capital have the same depreciation rate, i.e.: $\delta = \delta_K = \delta_H$. The depreciation of human capital includes losses from skill deterioration and mortality, net of benefits from experience.

The economy's resources constraint is:

$$Y = A \cdot K^{\alpha} H^{1-\alpha} = C + I_K + I_H, \tag{6}$$

where C denotes the consumption, while I_{K} and I_{H} are gross investments in physical and human capital, respectively. The changes in the two capital stocks are given by

⁴ This part of our paper is based on the one-sector model of Barro and Sala-i-Martin (1995).

$$\dot{K} = I_K - \delta_K K \tag{7a}$$

$$\dot{H} = I_H - \delta_H H. \tag{7b}$$

The Hamilton expression is

$$J = u(C) \cdot e^{-\rho t} + v \cdot (I_K - \delta_K K) + \mu \cdot (I_H - \delta_H H) +$$

$$+\omega \cdot \left(A \cdot K^{\alpha} H^{1-\alpha} - C - I_K - I_H\right) \tag{8}$$

where v and μ are shadow prices associated with K and H, respectively, and ω is the Lagrange multiplier associated with the budget constraint from equation (6). We use the usual specification utility,

$$u(C) = (C^{1-\theta} - 1)/(1-\theta).$$

Suppose that we neglect, for the moment, the inequality restrictions $I_K \ge 0$ and $I_H \ge 0$. Then the first-order conditions can be obtained in the usual manner by setting the derivatives of J with respect to C, I_K and I_H to 0, equating $\dot{\boldsymbol{v}}$ and $\dot{\boldsymbol{\mu}}$ to $-\partial J/\partial K$ and $-\partial J/\partial H$, respectively, and allowing for the budget constraints in equation (6). If we simplify these conditions, then we obtain the familiar result for the growth rate of consumption:

$$\gamma_C = \dot{C}/C = (1/\theta) \cdot \left[A\alpha \cdot (K/H)^{-(1-\alpha)} - \delta_K - \rho \right], \tag{9}$$

where $A\alpha \cdot (K/H)^{-(1-\alpha)} - \delta_K$ is the net marginal product of physical capital.

The second condition is that the net marginal product of human capital: $A \cdot (1-\alpha)(K/H)^{\alpha} - \delta_H$ equals to the net marginal product of physical capital. This equality,

$$A\alpha \cdot (K/H)^{-(1-\alpha)} - \delta_K = A \cdot (1-\alpha)(K/H)^{\alpha} - \delta_H.$$
 (10)

implies that the ratio of the two of capital stocks can be given. Let ω^* denote the proportion K/H which satisfies the equation (10).

This implies that the net rate of return to physical and human capital is given by

$$r^* = A\alpha \cdot (\omega^*)^{-(1-\alpha)} - \delta_K. \tag{11}$$

This rate of return is constant because the production function in equation (5) exhibits constant returns with respect to broad capital, K and H. Therefore, diminishing returns do not apply when K/H stays constant in equation (10), that is, when K and H grow at the same rate.

If K/H is constant, then equation (9) implies that γ_C is constant and equal to

$$\gamma_C = \gamma^* = \dot{C}/C = (1/\theta) \cdot \left[A\alpha \cdot (\omega^*)^{-(1-\alpha)} - \delta_K - \rho \right], \tag{12}$$

where we substituted ω^* for K/H of equation (10). We assume that the parameters are so that $\gamma^* > 0$.

Suppose that an economy begins with the two capital stocks, K(0) and H(0). If the ratio K(0)/H(0) deviates from ω^* having prescribed by equation (10), then the solution that we have just found dictates discrete adjustments in the two stocks to attain the value ω^* instantaneously. This adjustment features an increase in one stock and a corresponding decrease in the other stock, so that the sum, K+H, does not change instantaneously. We have to suppose that the investments are irreversible i.e. neither the old units of physical nor human capital can be converted into the other type of capital, i.e. we should impose the inequality restrictions $I_K \geq 0$ and $I_H \geq 0$. On the basis of these restrictions we have to rethink the solution of our model.

If $K(0)/H(0) < \omega^*$ – that is, if H is initially abundant relative to K – then the previous solution dictates a decrease in H and an increase in K at time zero. The desire to lower H by a discrete amount implies that the inequality $I_H \ge 0$ will be binding at time zero (and for a finite interval thereafter). When this restriction is binding, the household chooses $I_H = 0$; hence the growth rate of H is given by $\dot{H}/H = -\delta_{IP}$, and H follows the path

$$H(t) = H(0) \cdot e^{-\delta_H \cdot t} \quad (t = 0,...).$$
 (13)

The agents realise that they have too much H in relation to K, but since it is infeasible to have negative gross investment in H, they allow H to depreciate at the exogenously given rate δ_H .

If $I_H = 0$, the household's optimisation problem can be written in terms of the simplified Hamiltonian expression,

$$J = u(C) \cdot e^{-\rho t} + v \cdot \left(A \cdot K^{\alpha} H^{1-\alpha} - C - \delta_K K \right), \tag{14}$$

where v multiplies for \dot{K} (when $I_H = 0$) and $u(C) = (C^{1-\theta} - 1)/(1-\theta)$. The conditions of first order $\partial J/\partial C = 0$ and $\dot{v} = -\partial J/\partial K$, are conducting the usual way to the growth rate of consumption

$$\gamma_C = \dot{C}/C = (1/\theta) \cdot \left[A\alpha \cdot (K/H)^{-(1-\alpha)} - \delta_K - \rho \right], \tag{15}$$

where $A\alpha \cdot (K/H)^{-(1-\alpha)} - \delta_K$ the net marginal product of physical capital. This condition and the following budget restraint

$$\dot{K} = A \cdot K^{\alpha} H^{1-\alpha} - C - \delta_K K, \tag{16}$$

as well as the equation

$$H(t) = H(0) \cdot e^{-\delta_H \cdot t} \,. \tag{13}$$

altogether determine the paths of C, K and H.

We are looking for the transitory growth path which satisfies the Hamiltonian expression (8) with the constraint of $I_H \ge 0$. Let us start from the following equations:

$$Y = A \cdot K^{\alpha} H^{1-\alpha}. \tag{5}$$

$$\dot{C}/C = (1/\theta) \cdot \left[A\alpha \cdot (K/H)^{-(1-\alpha)} - \delta_K - \rho \right], \tag{15}$$

$$\dot{K}/K = A \cdot (K/H)^{-(1-\alpha)} - C/K - \delta_K , \qquad (16)$$

$$H(t) = H(0) \cdot e^{-\delta_H \cdot t}. \tag{13}$$

Let us denote by T the necessary time of the returning back to the stable state and by κ the extent of the contraction of the output Y caused by the war, i.e.

$$Y(0) = \kappa \cdot Y(T) \cdot e^{-\gamma^* T} \tag{17}$$

because the extent of the output in time 0 must have been just $Y(T) \cdot e^{-\gamma^* T}$ if there had not been war. In the stable state satisfying equation (10)

$$K(T) = \omega^* \cdot H(T). \tag{18}$$

We shall have K(0) if we write (5), (18) and (13) respectively into (17):

$$K(0) = H(0) \cdot \omega^* \kappa^{1/\alpha} \cdot e^{-(\gamma^* + \delta_H)^{T/\alpha}}. \tag{19}$$

If we denote by s_t the proportion of physical capital investment in time $t \in [0,T]$, then for C(t) we have:

$$C(t) = A \cdot K(t)^{\alpha} \cdot H(t)^{1-\alpha} (1 - s_t), \tag{20}$$

while for C/K:

$$C(t)/K(t) = A \cdot \left[K(t)/H(t)\right]^{\alpha-1} (1-s_t). \tag{21}$$

Substituting \dot{K}/K into (16) we have that

$$\dot{K}(t)/K(t) = A \cdot \left[K(t)/H(t) \right]^{-(1-\alpha)} s_t - \delta_K. \tag{22}$$

The rate of growth of the output in time $t \in [0,T]$ is:

$$\dot{Y}(t)/Y(t) = \alpha \cdot \dot{K}(t)/K(t) - (1-\alpha)\delta_H =$$

$$= \alpha \cdot A \cdot (K(t)/H(t))^{-(1-\alpha)} s_t - \alpha \cdot \delta_K - (1-\alpha) \cdot \delta_H. \tag{23}$$

Putting its value in time t = 0 into (19) while its value in t = T into (23) we get that

$$\dot{Y}(0)/Y(0) = \alpha \cdot A \cdot \left(\omega^*\right)^{(\alpha-1)} \kappa^{(\alpha-1)/\alpha} \cdot e^{-\left(\gamma^* + \delta_H\right)T(\alpha-1)/\alpha} \cdot s_0 - \alpha \cdot \delta_K - (1-\alpha) \cdot \delta_H$$
(24)

$$\dot{Y}(T)/Y(T) = \alpha \cdot A \cdot (\omega^*)^{(\alpha-1)} s_T - \alpha \cdot \delta_K - (1-\alpha) \cdot \delta_H. \tag{25}$$

From this we may easily formulate a necessary and sufficient condition that at the two ends of the interval the rate of growth of the output should be equal. This can be put as follows:

$$s_T / s_0 = \kappa^{(\alpha - 1)/\alpha} \cdot e^{-(\gamma^* + \delta_H)T(\alpha - 1)/\alpha}. \tag{26}$$

We must remark here that in the original one-sector model of Barro and Salai-Martin α is given as an exogenous constant parameter while here we suppose (instead of α having been given in advance) the condition (26) that implies (i.e. determines unambiguously) α for us.

Thus for α we have got from equation (26) the explicit formula:

$$\alpha = 1 + \frac{\ln(s_T/s_0)}{\ln \kappa - \ln(s_T/s_0) - (\gamma^* + \delta_H)T}.$$
 (27)

After all this by numeric calculus we may easily determine the path of solution of the Hamiltonian expression (8). The path thus obtained reflects well the character of the reconstruction period, that the formula of the transition is "quick—slow—quick" followed by an abrupt break in growth rate when returning to its original Jánossy-trendline.

SUMMARY

Since for Jánossy the "economic miracle" of Japan had played the most important role, only the numerical fitting to the Japanese data of Maddison for the post-war reconstruction period will be shown (*Figure 3*).

On the basis of Maddison's data we have verified in case of model (III) for all the most important OECD countries if the two laws of Jánossy were satisfied or not, but only model (III) could prove both of them.

After having applied the first two models, (I) and (II) which are both developed versions of the original Solow–Swan model, we see evidence conflicting with Jánossy at first glance.

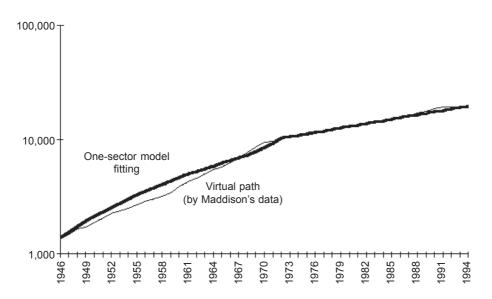


Figure 3. The GDP per capita of Japan after the war-reconstruction period by Maddison's data (in Geary-Khamis Dollar)

Source: Maddison (1995).

As we have already pointed out, Jánossy concluded that there must exist a decisive factor of development which remains intact during the war. He states that this stable component is labour, the crucial factor of production of which both the structure and the level of development has not only been preserved but is also continuing to improve during the war. Jánossy explains all his theory by his own notion, the "professional structure" which could correspond to a certain form of human capital in the light of the modern growth theory.

The two evidences conflicting with Jánossy can be formulated as follows:

- 1. The Solow–Swan model (I) does not imply human capital and nevertheless it gives us some qualitative explanation of the theory of Jánossy (i.e. satisfies the 1st law). After a period of transition the economy regains its trendline (stable state) even if both the physical capital *K* and labour *L* change during the war.
- 2. In spite of the fact that the Solow–Swan model (II) already implies human capital *H*, the fitted curve of model (II) deviates much greater from the empirical data than it happened in the case of model (I).

The solution to the first conflict consists of supposing the existence of a simple production function, which determines the flow of final output Y as the function of the two production factors like the physical capital K and labour L, and satisfies the traditional neo-classical conditions of a production function. We have to suppose only that the production function (with its type and parameters) remains intact during the war. In this case if any country has already found itself in its stable state before the war, and its level of physical capital K and labour L has been changing even sharply during the war, the Solow–Swan model (I) makes us confident that after a certain transitional period, it would reach again its stable state, i.e. its Jánossy-trendline.

Thus, instead of supposing that one of the factors of production (labour or human capital) remains intact during the war, we suppose – on a higher level – their function, the production function, does not change during and after the war, we have the same result on the existence of the Jánossy-trendlines. In other words we have obtained a more general theory than Jánossy.

Let us remark that none of the Solow–Swan models have verified Jánossy's 2nd law, which describes a sharp fall in growth rate at the end of the reconstruction period. This fall in the OECD countries were just one half, while for Japan nearly a third (see the *Annex Table* in row of the OECD-22 from 3.96% to 1.93% and in row of Japan from 8.54% to 3.03%, both before and after the year 1972). This phenomenon has been well described by Jánossy and played an important role in his theory. Related to this phenomena we have to mention here the generally prevailing empirical fact that the fall of growth rate at the end of the recon-

struction period is highly correlated to a preceding abrupt fall of the investments, while both Solow–Swan models worked with constant rate of savings (or investment).

The second conflict can only be solved by the one-sector model (III), for which the solution is a Hamiltonian expression's path. This solution has already reflected the character of the reconstruction periods well described by Jánossy and the production function implies only physical and human capital. It is a model where human capital plays an important role and the path of the solution is split in two, well-separated intervals. Since there is negative investment neither in physical nor in human capital, the restriction $I_H \geq 0$ plays a determinant role in designing the path of solution during the reconstruction period while it is becoming superfluous after the end of reconstruction.

We have found thus a simple model,⁵ where the path of solution breaks like the empirical facts. In other words we may say that the broken character of the post-war path became endogenous by this model.

If we try to confront the recent endogenous models of the new growth theory with the theory of Jánossy, we may conclude that these models prove well not only the most important and most surprising law of Jánossy, namely that Jánossy's war-reconstruction period will only be finished when the extrapolated pre-war development will again be reached and followed just as if the war had not happened. They also prove the well-described character of the reconstruction period that it would return back to its original Jánossy-trendline followed by an abrupt fall in growth rate.

Instead of supposing that the production factors like labour, professional structure or human capital remain intact, we suppose that their function, the production function, does the same during and after the war, and we obtain the same law as Jánossy. In other words, we have got a much more general theorem, which allows us to apply Jánossy's theory not only for post-war-reconstruction periods, but also for the transitions of Eastern Europe. The Hamiltonian expression's path of the output is a slightly cyclic S-shape of which the amplitude and points of inflexion may help us to forecast the next 20–25 years of transition.

⁵ It is an important and unwritten law at Jánossy's scientific approaches to strive for working with as simple models as possible.

ANNEX

For the 22 OECD countries (i.e. all the OECD countries, except Iceland and Luxembourg) I fitted broken lines of 1, 2 and 3 points of inflexion from the Second World War up to now to the GDP per capita time series (measured at the price levels and exchange rates of US dollars adjusted for PPP for 1990). Since – on the basis of Jánossy's theory – the post-war period is characterised by two points of inflexion, the parameters of the broken lines with two points of inflexion are bold-faced type and framed at all the countries or groupings of countries in the *Annex Table*. (It has purely mathematical and technical reason that all the three types of broken lines do not start in 1945 but the broken lines of 1, 2 and 3 points of inflexion start in 1947, 1946 and 1945, respectively. For such a long period of time one year has no importance but in this way all the three cases can be treated uniformly such as broken lines of 3 points of inflexion.)

On the basis of Maddison's data we may study the validity of Jánossy's law not only for 13 OECD countries as we did in Tarján (1995), but nearly for all the OECD countries. Moreover, for the common GDP per capita of the OECD countries (OECD-22), for the European OECD countries and for the group of Major Seven (G7), too. We show these results for groups of countries in italics and hatched rows that are represented in *Figure 2*.

We can state that for these three groups of countries the law of Jánossy is satisfied with high precision (in the *Annex Table* the parameters of the broken lines of 2 points of inflexion and the rate of growth of the Jánossy-trendline, which are bold-faced type and framed). The calculations made for the broken lines of 1 and 3 points of inflexion confirm well the above statement, too, except the group of European OECD countries. In all three groups of countries the last point of inflexion happens in 1972–73 and the average growth rate of the post-war period agree exactly with the rate of growth of the Jánossy-trendline for the 20th century, except the group of European OECD countries, for which the last striking break of the best fitted broken line of 3 points of inflexion happens in 1991, nearly 20 years later. Of course the average growth rate from this break point deviates from the rate of growth of the Jánossy-trendline for the 20th century, since without exception for all countries or groups of countries the average growth rate of the 1990s is much moderate than the rate of growth of the Jánossy-trendline for the 20th century. Here the rate of growth of the last analysed period (from 1951 to 1973) coincides with the rate of growth of the Jánossy-trendline.

Annex Table

Country	No. of break- points	Year	Rate of growth (per cent)	Year	Rate of growth (per cent)	Year	Rate of growth (per cent)	Year	Rate of growth (per cent)	Year	Jánossy- trendline	Standard deviation
Canada	1					47	2.82%	88	-1.41%	94		0.035
Canada	7			46	2.09%	59	3.01%	88	-1.86%	94	2.07%	0.030
Canada USA	ж	45	1.87%	61	3.59%	76	2.16% 2.26%	87	-1.61% 1.56%	94		0.021
USA	2			46	1.66%	62	3.80%	89	1.71%	94	1.78%	0.027
USA Japan	8 1	45	-22.4%	46	1.66%	62 47	3.80%	68 72	1.71%	94		0.027
Japan	2			46	7.42%	59	8.54%	72	3.03%	94	2.97%	0.026
Japan Germany	8 1	45	8.14%	53	6.13%	58 47	9.01%	71 59	3.14% 2.50%	94		0.019
Germany	2			46	11.22%	55	3.82%	72	1.99%	94	2.02%	0.027
Germany Italy	8 1	45	40.0%	46	11.22%	55 47	3.82% 5.47%	72	1.99% 2.44%	94		0.027
Italy	7			46	7.93%	51	5.20%	70	2.40%	94	2.24%	0.023
Italy France	3	45	24.20%	47	5.42%	68 47	3.01% 4.19%	80 74	2.01%	94		0.020
France	2			46	7.65%	20	4.06%	74	1.66%	94	1.85%	0.016
France United Kingdom	s 1	45	49.61%	46	7.65%	50	4.06% 2.31%	74	1.66%	94		0.016
United Kingdom	2			46	2.29%	69	2.01%	06	0.35%	94	1.39%	0.022
United Kingdom	3	45	2.22%	79	-2.13%	81	3.35%	68	-0.27%	94		0.020

											Annex T	Annex Table (cont.)
Country	No. of break- points	Year	Rate of growth (per cent)	Year	Rate of growth (per cent)	Year	Rate of growth (per cent)	Year	Rate of growth (per cent)	Year	Jánossy- trendline	Standard deviation
G7	1					47	3.55%	73	2.06%	94		0.021
G7	2			46	3.30%	61	3.89%	72	2.05%	94	2.03%	0.020
G7 Austria	e -	45	-14.7%	46	3.30%	61	3.89%	72	2.05%	94		0.020
Austria	2			46	19.39%	20	4.77%	47	2.00%	94	1.90%	0.030
Austria Belgium	£ 1	45	16.63%	50	6.38%	57	4.26%	76	1.93%	94		0.023
Belgium	2			46	2.79%	61	4.29%	74	1.82%	94	1.54%	0.019
Belgium Denmark	e -	45	4.52%	51	1.97%	59 47	4.27% 3.30%	74 72	1.83%	94		0.015
Denmark	2			46	2.64%	57	3.90%	69	1.90%	94	1.91%	0.020
Denmark Greece	s -	45	5.47%	50	1.59%	56 47	3.97% 5.84%	69	1.89%	94		0.019
Greece	2			46	5.37%	62	%98.9	74	1.31%	94	2.65%	0.031
Greece Ireland	s 1	45	32.70%	47	4.96%	62	7.38% 2.83%	73	1.45%	94		0.027
Ireland	2			48	3.01%	80	1.10%	98	4.81%	94	2.18%	0.029
Ireland Finland	3	47	2.07%	58	3.39%	78	1.20%	98	4.79%	94		0.020
Finland	2			46	4.01%	74	2.88%	68	-2.34%	94	2.60%	0.026

Finland	ю	45	3.85%	89	5.61%	72	2.68%	06	-3.52%	94	Annex T	0.023 Annex Table (cont.)
Country	No. of break- points	Year	Rate of growth (per cent)	Year	Rate of growth (per cent)	Year	Rate of growth (per cent)	Year	Rate of growth (per cent)	Year	Jánossy- trendline	Standard deviation
The Netherlands	1					47	3.49%	74	1.33%	94		0.025
The Netherlands	2			46	10.83%	48	3.45%	74	1.35%	94	1.49%	0.024
The Netherlands Norway	8 1	45	67.10%	46	10.83%	48	3.45%	74	1.35%	94		0.024
Norway	2			46	3.11%	62	3.58%	98	1.13%	94	2.71%	0.020
Norway Portugal	8 1	45	8.82%	48	2.68%	59 49	3.58%	86	1.13%	94		0.016
Portugal	2			48	3.67%	61	7.08%	72	2.35%	94	2.33%	0.028
Portugal Spain	3	47	3.52%	61	7.32%	73	-2.23% 5.45%	75	2.58%	94		0.024
Spain	7			46	3.41%	09	7.30%	72	1.83%	94	2.00%	0.035
Spain Sweden	3	45	3.36%	09	7.08%	74 47	0.22%	83	2.91% 1.24%	94		0.028
Sweden	2			46	3.12%	73	1.61%	06	-1.66%	94	1.99%	0.020
Sweden Switzerland	3	45	2.72%	69	3.75%	70 47	1.67% 2.90%	90	-1.77% $1.06%$	94 94		0.016
Switzerland	2			46	2.94%	74	-7.94%	75	1.33%	94	2.07%	0.023
Switzerland OECD-Europe	8 1	45	2.97%	74	%99.6-	75 47	1.63%	90	-0.69% 1.88%	94		0.020
OECD-Europe	2			46	2.96%	51	3.96%	73	1.85%	94	1.87%	0.013

OECD-Europe	8	45	5.34%	52	3.94%	73	1.92%	91	0.61%	94	Annex 1	0.013 Annex Table (cont.)
Country	No. of break- points	Year	Rate of growth (per cent)	Year	Rate of growth (per cent)	Year	Rate of growth (per cent)	Year	Rate of growth (per cent)	Year	Jánossy- trendline	Jánossy- Standard trendline deviation
Australia	1					47	2.48%	75	1.64%	94		0.026
Australia	2			46	1.90%	62	3.49%	71	1.60%	94	1.64%	0.017
Australia New Zealand	3	45	-2.75%	46	1.90%	62 47	3.49%	71 73	1.60% 0.81%	94		0.017
New Zealand	2			46	2.02%	75	-1.85%	77	1.03%	94	1.38%	0.032
New Zealand Turkey	3	45	2.03%	75	-2.02%	78 47	2.04% 3.37%	85	0.31% 2.44%	94 94		0.030
Turkey	7			46	3.38%	78	-1.86%	80	2.63%	94	2.29%	0.048
Turkey OECD-22	3	45	27.74%	46	3.38%	78 47	-1.86% 3.58%	80	2.63%	94		0.048
OECD-22	7			46	3.29%	61	3.96%	72	1.93%	94	1.94%	0.018
OECD-22	3	45	-11.2%	46	3.29%	61	3.96%	72	1.93%	94		0.018

Columns Year, the starting point, the break points and 1994, show the last year of the time series, while the rates of growth between them in columns Rates of growth.

Column *Jánossy-trendline* shows the rate of growth of Jánossy-trendline for the 20th century in middle row for each country.

Column *Standard deviation* indicates the dispersion of the difference between the real data and the fitted trendline. It is obvious that the dispersion of trendlines of 1, 2 and 3 points of inflexion form a decreasing sequence.

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