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NEW RESULTS IN PENSION MODELING

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In a former paper (Simonovits, 1999), I have discussed the problems of the new Hungarian pension system verbally. In this paper I will present some new results obtained by others and myself with mathematical models, which are related to the Hungarian pension reform (see e.g. Palacios and Rocha, 1998). (1) How can one model a pension system with the life-cycle theory? (Of course, this is introduction rather than new result.) (2) How is the model of a funded system modified if volatility of yields and operating costs are taken into account? (3) What would the actuarially fair model be in an unfunded pension system with flexible age of retirement, and how much saving (and damage) is to be expected from replacing the indexation of pensions in progress to earnings by the combined indexation? (4) How is the efficiency of the pension system affected if the unfunded system is replaced by a partially or fully funded system?

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INTRODUCTION*

Pension systems and pension reforms are very topical nowadays. Chile replaced its ailing unfunded system with a fully funded system. With important modifications, Latin American (e.g. Argentina), and East and Central European countries (e.g. Hungary) have just completed a transition to a mixed pension system and similar pension reforms are in the making in other ex-socialist countries. Britain,

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Italy and Sweden have been going through a whole series of reforms. The possibility of the reform of the US Social Security System stands in the centre of discussion among the academics.

In a former paper (Simonovits, 1999) I have discussed the new Hungarian pension system verbally. In this paper I will present some new results obtained by others and myself with mathematical models, which are related to the Hungarian pension reform. (1) How can one model a pension system with the life-cycle theory? (Of course, this is an introduction rather than a new result.) (2) How is the model of a funded system modified if volatility of yields and operating costs are taken into account? (3) What would the actuarially fair model be in an unfunded pension system with flexible age of retirement, and how much saving (and damage) is to be expected from replacing the indexation of pensions in progress to earnings by the combined indexation? (4) How is the efficiency of the pension system affected if the unfunded system is replaced by a partially or fully funded system?

1. LIFE-CYCLE AND INSURANCE

In economics, life-cycle refers to an approach where the consumption and saving decisions depend on the age of the decision-maker. In this section we present a life-cycle model with certain and uncertain life-spans, respectively.

Life-cycle models

The simplest *life-cycle model* works with a deterministic life-span (Modigliani and Brumberg, 1954; Ando and Modigliani, 1963). Suppose that the individual is born at the beginning of year 0, enters the labour force at the beginning of year L, retires at the end of year R and dies at the end of year D, $0 < L < R < D$. For further use we shall introduce the *years of normal service*: $T = R - L + 1$. (Note that we neglect the problems connected with the fine structure of events within the year.) The individual's earning at age i is denoted by w_i, and his consumption at age j is denoted by c_j. For the sake of simplicity, we neglect taxation and social contributions (including the division between the employer and the employee). Earnings mean full earnings, i.e. *total wage costs*. Consumption refers to consumption of non-durable goods, since we do not want to discuss the possibilities of producing a good while our individual is young and consuming it when old.

Neo-classical economics prefers the individual to the family or the household. For the time being, we shall also assume the just born baby takes up loans already at birth, financing his childhood. In the first part of his active stage he repays them and in the second part he accumulates a capital just to deplete it while he is retired. (This assumption is excessive, nevertheless, we shall use it.)

Assume that there is no inflation, and no interest margin (between lending and borrowing). To simplify the forthcoming formulas, we shall work with *factors* rather than *rates*; the interest factor is equal to 1 plus the interest rate. We shall introduce the following simplifying assumptions: the factors of productivity growth and of interest are time-invariant. (Since these factors change in time, we should have written products rather than powers.)

In the case of flows distributed in time, the so-called *present value* is a very useful aggregator. Let $\{c_j\}_{j=0}^{D}$ be the *consumption path*, r be the interest factor used at discounting, then the present value of the consumption path discounted to the birth date is

$$\mathrm{PV} = \sum_{j=0}^{D} c_j r^{-j}.$$

Under our assumptions, the balance conditions can be described rather simply. The present value of the consumption path is equal to that of the earning path:

$$\sum_{i=L}^{R} w_i r^{-1} = \sum_{j=0}^{D} c_j r^{-j}. \tag{1}$$

We frequently assume that the individual consumption increases according to a time-invariant factor, the earning increases according to the product of the productivity growth factor (g) and the seniority growth factor (Γ): $\Omega = g\Gamma$, but the growth factor of the consumption (γ) can be different.

We can now present the earning and consumption paths in terms of the initial values and the growth factors:

$$w_i = w_L \Omega^{i-L}, \quad i = L,\ldots,R, \tag{2}$$

$$c_j = c_0 \gamma^j, \quad j = 0,\ldots,D. \tag{3}$$

Since the sum of the geometric series with $n + 1$ terms will be frequently used, we shall introduce the following notation:

$$I_n(x) = \sum_{i=0}^{n} x^i = \frac{x^{n+1} - 1}{x - 1}.$$

Inserting (2)—(3) into (1), yields

Theorem 1. *(Replacement rate.) Under our assumptions of exponential growth, the following relation holds between the initial values of consumption and of earnings:*

$$c_0 = w_L \frac{I_{T-1}(\Omega / r)}{r^L I_D(\gamma / r)}. \tag{4}$$

Example 1. No growth: $\Gamma = 1$ and $\Omega = 1$ and no interest: $r = 1$. In this simple case (4) reduces to

$$c_0 = \frac{T}{D+1}. \tag{4'}$$

Remark. In the neo-classical economics, the consumer maximises the *utility function* under the budget constraint. In our case, utility refers to the 'value' of a consumption path comprising $D + 1$ years. It can be shown that in the derived optimisation model the growth factor of the consumption is time-invariant, it depends on the so-called discount factor, the interest factor and the intertemporal elasticity of substitution.

Life-insurance with life-annuity

We have seen that a basic feature of human life is that the consumption path is much smoother than the earning path. Another, equally important characteristic is the so-called *longevity risk,* i.e. the date of death of any person is not known to him or to others in advance. Indeed, there are people who die before reaching their first birthdays and there are others, who can live for a hundred years. We should not forget this fact when speaking of the life-expectancy at birth and its secular increase. Furthermore, this uncertainty makes self or family financing much more difficult than would be in a deterministic case (Walliser, 1999).

First we shall examine the ideal system of life-insurance with life-annuity and then turn to the complications arising in real life.

We shall introduce the following notions from probability theory. Let q_i be the probability of the event that a person dies at age i (more precisely, just before reaching his i + 1-th birthday): $q_i \geq 0$ and $\sum_{i=0}^{D} q_i = 1$. We shall also need aged i's *survival probability*: $l_i = \sum_{j=i}^{D} q_j$. Finally, we introduce the *remaining life-expectancy at age i:*

$$E_i = \frac{\sum_{j=i}^{D} q_j (j - i + 1)}{l_i}.$$

It is easy to prove that

$$E_i = \frac{\sum_{j=i}^{D} l_j}{l_i}.$$

Figure 1 presents the age-specific Hungarian survival data of 1997 (CEO, 1998). A historical analysis would demonstrate that the Hungarian age-specific male mortality decreased up until the 1960s, and it has been increasing since then, especially in the age group 40–60. We emphasise that all over the world, male's mortality is much higher than women's.

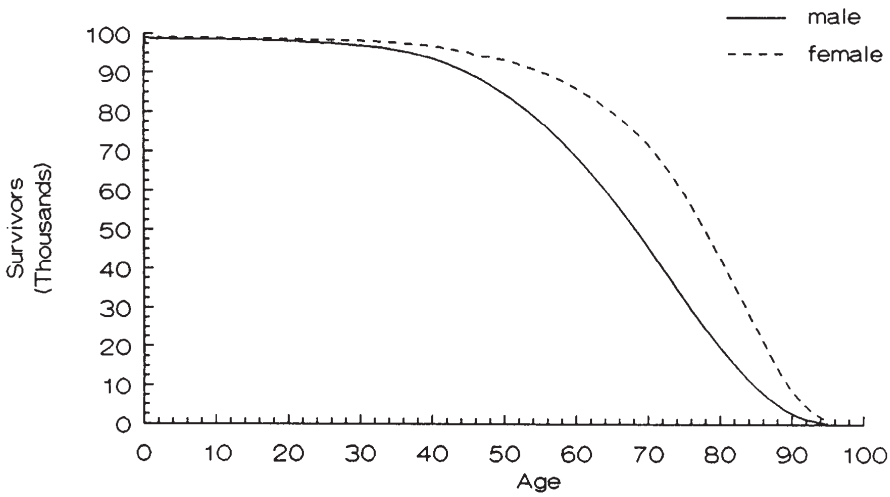

Figure 1. Survival probability: Hungary

For the time being, we shall assume that life-insurance with life-annuity can be bought without paying for the risk. (This assumption will be dropped in Section 2.) Then our earlier equilibrium condition will hold in expected value; the expected present value of the earning path is equal to that of the consumption path:

$$\sum_{i=L}^{R} l_i w_i r^{-i} = \sum_{j=0}^{D} l_j c_j r^{-j}. \tag{5}$$

In a general model, Yaari (1965) studied the optimal life-cycle path under death risk. Here we confine our attention to constant consumption.

Theorem 2. *(Consumption under uncertainty.) a) If there is life-insurance and life-annuity without additional costs, then the constant consumption is equal to*

$$c_0^{\mathrm{I}} = \frac{\sum_{i=L}^{R} l_i w_i r^{-1}}{\sum_{j=0}^{D} l_j r^{-j}}. \tag{6}$$

b) If there is no life-insurance and life-annuity, then the constant consumption is equal to

$$c_0^{N} = \frac{\sum_{i=L}^{R} w_i r^{-i}}{\sum_{j=0}^{D} r^{-j}}. \tag{7}$$

c) If insurance does not cover childhood (L = 0*) and the growth factor of earnings is less than the interest factor (*$\Omega < r$*), then consumption without insurance is lower than with insurance:*

$$c_0^{N} < c_0^{\mathrm{I}}. \tag{8}$$

Remark. The assumptions of c) are not necessary, but some assumptions are needed. For example, in the unrealistic case $R = D$, $\Omega < r$ would be necessary.

Proof. a)–b) From (5).

c) Inserting (6)–(7) into (8) and removing the denominators, yields

$$\sum_{i=0}^{R}\sum_{j=0}^{D} l_i \Omega^i r^{-i-j} > \sum_{i=0}^{R}\sum_{j=0}^{D} l_j \Omega^i r^{-i-j}.$$

Theorem 2 can be illustrated with

Example 2. If there are no children: $L = 0$, there is no interest: $r = 1$ and no growth: $\Omega = 1$, $\gamma = 1$, then

$$c_0^{\mathrm{I}} = w_L\left(1 - \frac{l_{R+1}E_{R+1}}{l_L E_L}\right). \tag{6'}$$

The no-insurance case is given by (4'). Using the 1997 male life-table for Hungary, we have $L = 20$, $R = 61$, $l_L = 0.982$, $l_{R+1} = 0.646$, $E_L = 47.47$ years and E_{R+1} =13.84. Then (6') yields $c_0^{\mathrm{I}} / w_L = 1 - 0.6578 \cdot 0.2916 = 0.808$. For comparison, the no-insurance case, $D = 99$ and $c_0^{N} / w_L = 0.42$.

Complications

It is to be underlined that in most countries there is no developed private market for life-annuities. This probably results from the fact that if the purchase of life-annuity is not mandatory, then only those people who have longer life-expectancy and are richer than the average, buy them (Friedmann and Warshawski, 1990; Alier and Vittas, 1999). This phenomenon, the so-called *adverse selection* was already emphasised by Arrow (1963) in his classic article on health insurance. Following Mitchell et al. (1999) we shall outline the complications arising in the real world.

To evaluate life-annuities, it is important to note that, in the US, the price offers of various firms are very dispersed: the average of the best offers is higher by about 20% than that of the worst offers, but this difference also depends on the gender and age. It should be emphasised that different life-tables give different results. It might be plausible to work with a life-table of general population, but Mitchell et al. (1999) demonstrates that the life-table of annuitants differs significantly from that of the general population. Each cohort of the annuitants has a much lower mortality rate than the others. For example, considering the members of the cohort born 1930 to 1995, the mortality rates of the general population and the annuitants were 22.5 and 11.5 per thousands, respectively. (The numbers referring to the older population are presumably estimates, since among the 1930-cohort nobody was older than 65 in 1995.)

Other tables (e.g. Mitchell et al. 1999, 1308, Table 3), display the differences in the annuity premiums resulting from the differences between the life-tables of the general population and of the annuitants. The basis of comparison is the so-called *money's worth of annuity*: the expected present discounted value of annuity payments per premium dollar in 1995, using the treasury yield curve for a discounting.

There is a strong dependence of money's worth of annuities on gender and age for the general population and its lack for the annuitant population. These numbers confirm the well-known fact that the money's worth is much higher for the annuitants than for the general public. For example, a 55-year-old man from the street receives \$88.2 from a hundred, while a 75-year-old only \$78.3. For male annuitants, this number is around \$92. It is also noteworthy that between 1980 and 1995 the payout value-per-premium has risen by roughly 13 percent points.

These calculations however, gloss over the utility provided by insurance. For simplicity, Mitchell et al. (1999) studied that case, when the individual of age 65 buys a nominally fixed annuity. The question is, if the individual has a CRRA utility function, then is it worthwhile buying an annuity? To avoid the complications with the cardinality of the utility function, they examined what share of the original wealth yields the same utility with insurance as the original share without insurance. If the annual inflation rate is time-invariant (and is equal to 3.2%), then "individuals would accept a reduction of between 30 and 38 percent in their wealth at age 65 if they were able to purchase actuarially fair nominal annuities rather than pursue an optimal consumption strategy without annuity contracts....

[I]f half of an individual's wealth at retirement were held in an annuitized form (similar to Social Security), the share of nonannuatized wealth that he would be prepared to relinquish would [be] between 23 and 31 percent" (Mitchell et al., 1999, 1314–1315).

There are economists who claim that the *increasing annuatization* of the old-age benefits makes people save less and less (Gokhale et al., 1996). These experts suggest that if it were possible to diminish the 'security' provided by the Social Security, then the saving rate would increase: see Theorem 2. Already the commentators of the Brookings-lecture emphasised that it is welfare diminishing to deny insurance from people. Nevertheless, this misunderstanding has a rational core: insurance and efficiency partially contradict each other and a delicate trade-off exists between them.

Voluntary or mandatory insurance

We have almost arrived to the *pension systems*. Of course, the pension insurance differs from the combined life-insurance and life-annuity in several aspects. For our purpose, the most important difference is the following: life-insurance and life-annuity are matters of individual choice, while genuine pension insurance is mandatory and the state (government) significantly regulates its functioning. Even among those economists, who prefer the minimal state, a typical one would not allow somebody who does not care for his old age, and abuses social solidarity, receive an undeserved pension. Perhaps this fact explains why one can buy a life-insurance without purchasing a life-annuity, but the genuine pension systems provide life-insurance with life-annuity rather than lump-sum payments at retirement.

In our approach, *voluntary pensions* are long-term investment rather than pensions.

From now on we shall speak almost exclusively on mandatory pension systems. The costs of child-raising will be neglected, we drop the first stage and set $c_i = 0$, $i = 0,\dots, L-1$.

2. PROBLEMS WITH FULLY FUNDED SYSTEMS

We start our discussion of pension systems with the so-called *fully funded*, for short, *funded system*, or capital reserve system, which is closest to life-insurance. We shall discuss the complications arising with such a system: operating costs, volatility, annuatization.

Basic model

A funded pension system pays a life-annuity to a member (or his survivors) who accumulated capital on an individual account from which the possibly age-dependent annuity b_j is paid. In the case of mandatory systems, it is customary to formulate the *pension contributions* as a percentage of earnings: ω_w, which applies within certain bounds. (From now on the subindex often refers to the variable on which the proportionality is based.) For the sake of completeness, we shall reformulate the balance condition (5) as the equality of the present values of contributions and of benefits:

$$\omega_w \sum_{i=L}^{R} l_i w_i r^{-i} = \sum_{j=R+1}^{D} l_j b_j r^{-j}. \tag{9}$$

In essence, there are three types of funded systems: a) the provident funds, managed by the government, b) occupational pension systems, managed by the firms and c) the personal saving plans (or individual accounts), which are run by private pension funds. By now, there is an emerging consensus that among the three types, the individual accounts are the most advantageous (World Bank, 1994).

We shall confine our attention to *individual accounts*, showing the accumulation of the contributions as pension wealth or capital. A lot of people admire them, since they promise fabulous yields. According to the most optimistic calculations (Feldstein, 1996), these funds, paying 5–9% annual real returns, can provide decent pensions for contribution rates as low as 2–3%. Here is a part of the corresponding calculations from World Bank (1994, Table 6.1, 205).

We concentrate on the simplest form, namely, an annuity with a constant real value, given contribution span (40 years) and payment span (20 years). It is customary to express the value of annuity in terms of final earning:

$$b_{R+1} = \hat{\beta}_w w_R, \tag{10}$$

where $\hat{\beta}_w$ is called the *individual closing replacement rate*.

For illustration, we assume that the individual closing replacement rate is equal to 40% and determine the contribution rates for various real interest rates and real wage growth rates. Using the formulas from Section 1, we obtain the data shown in *Table 1*.

Note the sensitivity of the result to the interest rate and wage dynamics (and also to the ratio of lengths of active and of passive periods, not reported here). The authors of the table also emphasised that for increasing real wages the pension at death relative to the then prevailing average earnings is much lower (27%) than at retirement (40%). Furthermore, if the annual inflation rate is 5% and the benefits are unindexed, then the real and the relative pensions at death may fall to the depressing 15 and 10%, respectively. Finally, these calculations do not consider survivor and disability benefits, a recurrent theme of the pension literature.

Table 1

Wage growth, interest rate and contribution rate

	Growth rate of real wages $100(\Omega - 1)$	
	0	2
Real interest rate $100(r - 1)$		
0	20	29
2	11	16
5	5	7

We shall illustrate our findings in two figures. In the optimistic case the interest rate is equal to 5%, the growth rate of earnings is only 0, and the corresponding contribution rate is rather modest: 4.1%. Observe also, how small the accumulated capital remains, only amounting to 5 years final earnings. In the pessimistic case the interest rate is only equal to 0%, the growth rate of earnings is a healthy 2%, and the corresponding contribution rate is rather high: 28.7%. Note the impressive accumulated capital, amounting to 18 years final earnings. (There is a small discrepancy between these numbers and those of *Table 1*, probably due to a slightly different formulation and rounding errors.)

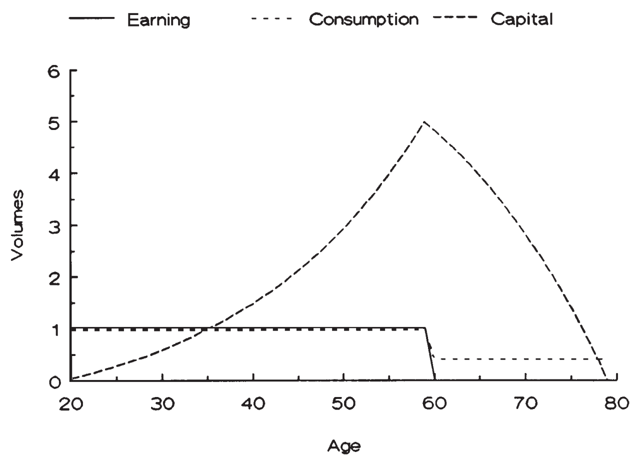

Figure 2. Private pension, optimistic case

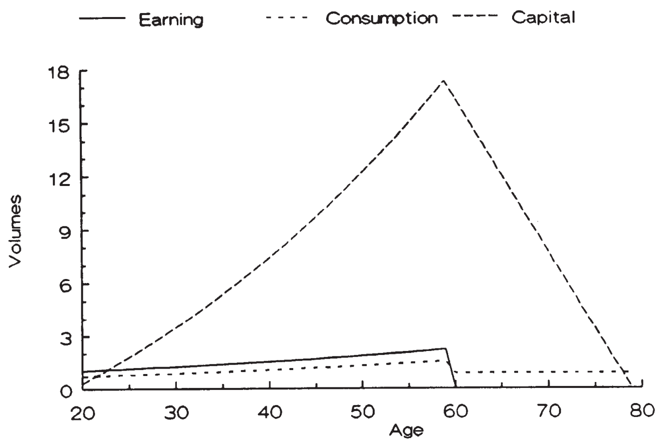

Figure 3. Private pension, pessimistic case

In the sequel we shall discuss problems arising with insurance, volatility and fees.

Insurance

Perhaps the most widespread illusion is connected to the *heritability* of the remaining pension capital. It is true (at least in Hungary) that if the contributor to a private fund dies during the accumulation period, his arbitrarily designed survivors (not necessarily the widow or the orphans) can inherit the accumulated savings. This is very advantageous for the survivors if the contributor dies at the end of the accumulation period when his children are grown up and the pension wealth is significant. What happens, however, if the contributor dies at the beginning of the accumulation period when his children are not yet grown, and the pension wealth is modest? Furthermore, the volume of the total inheritance is independent of the number of survivors: the more orphans remain, proportionally less support flows to each of them. Is it not more advantageous to have a life-insurance in force like in the unfunded systems? (See Section 3 below.) Similar conclusions are valid for the decumulation stage. If the circle of survivors can be made very wide, then the annuity will be rather low.

Many people may find it attractive to relax the fixed amount of life-annuity and allow for *accelerated withdrawal*. The idea is the following. If somebody has already retired and at the end of age $j-1$ has an individual asset a_{j-1}, then the capital is divided into E_{j-1} equal parts and he can withdraw one part: a_{j-1}/E_{j-1}. Assuming that the individual lives until age D', the dynamics is as follows:

$$c_j = \frac{a_{j-1}}{E_{j-1}}, \quad a_j = ra_{j-1} - c_j, \quad j = R+1,\dots,D'$$

and the remaining wealth is distributed among the survivors, either within or outside the family.

Volatility

Business volatility influences both the capital and the life-annuity of the pension wealth. This risk can be diminished if (i) the individual portfolio is made more and more risk-free as the owner approaches the time of retirement and (ii) the purchase of annuity is distributed in time (Alier and Vittas, 1999). While the volatility can be mitigated, it is rather costly to insure the real value of the life-annuity (Barr, 1987; James and Vittas, 1999). There are experts who see the solution in providing inflation-proof government bonds, others (identifying insurance with redistribution) prefer unindexed annuities – inflation is to be eliminated in the first place.

Operating costs

The computations reported above neglect the very significant *operating costs* characterising such funds. We now turn to this issue (cf. Diamond, 1998).

Individual pension assets at the end of year *i* are denoted by a_i. Operating costs will be modelled as proportional to the contributions and to the assets, respectively. We shall denote the pro rate costs of *assets management* as $1 - \theta_a$ and of *contribution management* as $1 - \theta_w$.

Then the individual assets satisfy the following difference equation:

$$a_i = \theta_a ra_{i-1} + \theta_w \omega_w w_L \Omega^{i-L}, \quad i = L,\dots,R, \quad a_{L-1} = 0. \tag{11}$$

Next is the net interest factor $\bar{r} = \theta_a r$, which is to be used at the calculation of the present value of the net contributions paid during the accumulation period:

$$\bar{r}^{1-T} a_R = \theta_w \omega_w w_L \sum_{k=0}^{T-1} \Omega^k \bar{r}^{-k}, \quad T = R - L + 1.$$

We have now

Theorem 3. *The capital accumulated for retirement is*

$$a_R = \bar{r}^{T-1} \theta_w \omega_w w_R I_{T-1}(\Omega / \bar{r}). \tag{12}$$

It is also interesting to know what annuity can be financed from the accumulated capital. In addition to previous operating costs, we also consider the *money's worth ratio of annuity* and denoted here as θ_b, showing the ratio of the actual to the ideal annuities. As a complement to Theorem 3, we have

Theorem 4. *The individual closing replacement rate is*

$$\hat{\beta}_w = \frac{\theta_b r a_R}{w_R I_{D-R-1}(1/r)} \quad . \tag{13}$$

We shall illustrate the effects of these operating costs with the following figures. $L = 20$, $R = 59$, $D = 79$. Growth factor of earnings: $\Omega = 1.02$, contribution rate: $\omega_w = 0.1$, money's worth ratio of annuity: $\theta_b = 0.9$ and interest factor: $r = 1.05$.

Table 2 shows that it is the operating costs on assets rather than on contributions, which matter (the effect of the money's worth ratio on the annuity is simple). For example, if there are no operating costs, then the replacement rate is 53.7%, which is reduced to 48.4% if there is no fee on assets but there is a 10% fee on contributions. On the other hand, if there are no operating costs on contributions but there is a 2% fee on assets, then the replacement rate drops to 34.5%.

Table 2

Operating costs, retirement capital and annuity

Operating costs of		Capital	Annuaty
assets $100(1-\theta_a)$	contribution $100(1-\theta_w)$	at retirement a_R/w_R	$100\,\hat{\beta}_w$
0	0	16.1	53.7
	5	15.3	51.0
	10	14.5	48.4
1	0	12.8	42.8
	5	12.2	40.7
	10	11.5	38.5
2	0	10.3	34.5
	5	9.8	32.7
	10	9.3	31.0

Continuing the illustration started in Example 2, we have

Example 3. If there is no interest: $r = 1$, then at accelerated withdrawal the ratio of consecutive consumptions is equal to

$$\frac{c_{j+1}}{c_j} = \frac{E_{j-1} - 1}{E_j}, \quad j = R+1, \ldots, D' - 1.$$

Because of decreasing survival probabilities $E_{j-1} - 1 < E_j$, i.e. the R.H.S. is a decreasing function of the age. Using the data of Example 2, c_{j+1}/c_j drops from 0.967 at age $j = 62$ to very low positive values.

Funded pension systems are often called *defined contribution* systems, because the pension is determined by the contributions. We have already seen that there is some deviation from this principle, stemming from the non-negligible cost of life-insurance, inflation-insurance, etc.

In normal circumstances, the individual accounts are somehow insulated from political influence. But Diamond (1997, 38) rightly argues that this insulation is far from perfect. On the one hand, the government can levy an additional tax on them; on the other hand, it can allow the owners to tap them in case of extraordinary events like unemployment and serious health problems.

3. UNFUNDED SYSTEMS WITH FLEXIBLE RETIREMENT

In this Section we outline the model of proportional (earnings-related) pension system and then discuss the issue of flexible retirement age.

Basic model

The pension systems, appearing by 1900, by and large started publicly run funded systems. Since life-insurance was born much earlier, there would not have been any special difficulty with funded pension systems, if two World Wars and the Great Depression had not destroyed them. The new pension systems had to start from zero.

The newly introduced, *unfunded* pension systems are called *pay-as-you-go* systems. Their basic principle is as simple as follows: if each generation pays the pension of its parents' generation, then the first generation need not contribute to its own pension. (Before the reader would envy too much this 'free-rider' generation, think about its earlier contributions and their difficulties in restarting after the wars and depression.) After presenting the basic model of an unfunded system, we shall discuss the proportional system with flexible retirement and the method of indexation.

Contrary to a funded system, an unfunded system should be mandatory and publicly managed. Indeed, the implicit social contract between subsequent generations needs government coercion. (In fact, there are privately managed unfunded pension systems in the USA, but their number and membership have been dwindling, World Bank (1994, Box 5.4, 189).)

How does an unfunded system function? Assume that our hero (born in year 0) or his employer pays every year (in fact, month) a prescribed part of his full earnings to the Social Security. For a somewhat mysterious reason, this contribution is generally distributed into two parts, the employer's contribution and the employee's contribution. (In some countries, the government contributes a third part.). The difference between the full earning and the first contribution is called *gross earnings*, notation: v, and everything is calculated on this basis rather than on the full earnings.

At retirement an individual is entitled to an *entry pension* which is an increasing (more precisely, non-decreasing) function of his past contributions, or equivalently, of past earnings. (To avoid notations for calendar years, wages and benefits are also indexed by age rather than calendar year, at least at this stage):

$$b_{R+1} = h(v_L, \ldots, v_R).$$

From a year after retirement till death, the individual receives a *continued pension* or pension in progress, which is a function of the previous pension:

$$b_{j+1} = H(b_j), \qquad j = R + 1, \ldots, D - 1.$$

The simplest measure for the efficiency of the unfunded system was introduced by Aaron (1966), the so-called *internal factor of return* (notation: ρ) which is used as a discount factor, to equalise the expected contributions and benefits (cf. (9)). In formula:

$$\omega_w \sum_{i=L}^{R} l_i w_i \rho^{-i} = \sum_{j=R+1}^{D} l_j b_j \rho^{-j}.$$

As a rule, the continued benefits have the same value: $b_{R+1} = \cdots = b_D$.

In several countries (The Netherlands, Scandinavian countries, etc.) the unfunded pension is totally independent of the individual contributions, and it is sufficient if the beneficiary had been a citizen or a resident of the country for a long enough period:

$$b_{R+1} = \beta_{\mathbf{v}} \mathbf{v}_{R+1},$$

where $\beta_{\mathbf{v}}$ is the value of benefit in terms of national average gross earning $\mathbf{v}_{R+1}$, the so-called *average replacement rate*. (From the point of view of the individual, however, it is the ratio of benefit to the closing earning, the individual closing replacement rate which matters: $b_{R+1} = \hat{\beta}_v v_R$.)

There are other countries (e.g. Great Britain), where the benefit is proportional to the number of years of service, but not to the earnings: *flat rate system*.

Proportional benefit

In another group of countries (Austria, Germany, see Schmaehl, 1999) the benefit is *proportional to the contributions*, achieving the highest degree of earning-relatedness of pensions. Rather than adding up the real values of contributions, every earning-related pension system *valorizes* the contributions according to the national earning dynamics, i.e. the earning (v_i) at age i is multiplied by the accumulated productivity factors till the date of retirement:

$$b_{R+1} = \alpha_2 \sum_{i=L}^{R} v_i g^{R-i},$$

where α_2 is a positive scalar multiplier.

Example 4. For a person, always earning the average wage, the proportional pension reduces to the flat rate benefit. In formula: for $v_i = \mathbf{v}_{R+1} g^{i-R-1}$, $b_{R+1} = \alpha_2 T \mathbf{v}_{R+1}/g$.

It is mentioned here that most pension systems leave out the last year earnings from valorization. Therefore the real value of entry pension depends on the inflation rate of the previous year: the higher the inflation rate, the lower is the real value of the entry pension. This error (as well as others) could be eliminated without raising the real values of the benefits, but these trials always aborted, at least in Hungary.

Continuing the previous factorisation, we introduce the *average indexed annual (gross) earning*, corresponding to an individual earning path:

$$\bar{v}_R = \frac{\sum_{i=L}^{R} v_i g^{R-i}}{T}.$$

Then using the *individual life-time replacement rate*, $\beta_{\mathbf{v}} = \alpha_2 T$, we obtain

$$b_{R+1} = \beta_{\mathbf{v}} \bar{v}_R.$$

Note that the flat (rate) benefit was defined in terms of the current average earnings, but here the past individual average indexed annual (gross) earning is the base.

We have already mentioned that governments revalue individual earnings by the national averages rather than by the price index. Nevertheless, Gokhale and Kotlikoff (1999) consider price valorization as a discrete method of reducing the unfunded pensions.

It is easy to see that for increasing average earnings, this solution decreases the life-time average earning and (with unchanged replacement rate) also the entry pension w.r.t. the traditional method. It has an unpleasant side-effect, however, the relative value of different contribution paths changes, too. The flatter the individual earning path, the larger is the proportional reduction in the pension.

Continued pensions also follow earnings dynamics:

$$b_j = b_{R+1}\, g^{j-R-1}, \quad j = R + 2,\ldots,D.$$

In the proportional pension system, there is no *intracohort income redistribution* (redistribution is achieved through other channels).

At the end of this Subsection, it is mentioned that in still another group of countries, containing the USA, Hungary and other countries, there is a compromise between flat (rate) and proportional benefits.

Flexible retirement

Until now the retirement age was taken as given. As a matter of fact, the effective retirement age is much lower than the normal retirement age and critics of the unfunded system (e.g. Börsch-Supan, 1998) consider this one of the major errors of the system. Probably the governments prefer early retirement to massive unemployment as a short(?) run solution but in the long run the demographic difficulties may backfire.

This problem can be eliminated by a system introduced in Sweden, called *notional defined contribution*, which extends this capitalisation implicit in valorization also to the calculation of annuities. Each year, the actual value of the annuity is the ratio of the notional capital to the remaining life-expectancy. If the latter increases, annuity decreases proportionally. Logically, a person retiring before/after *pensionable age* at age $R^* + 1$, receives a reduced/increased annuity – using actuarially fair treatment: *flexible retirement*,

$$b_{R^*+1} = \frac{l_{R^*+1} E_{R^*+1}}{l_{R+1} E_{R+1}} b_{R+1}.$$

In contrast, the weak reward/premium system introduced in Germany in 1992 was recently replaced only by a partially 'corrected' system. This is presented in *Table 3,* based on a figure in Börsch-Supan (1998) where the last row contains the 'fair' solution.

Table 3

Flexible retirement: Germany

	Age (years)				
	60	63	65	67	70
Benefit – 1992	100	100	100	114	114
Benefit – 2001	82	89	100	112	130
Neutral	72	85	100	120	160

Unfortunately, the introduction of traditional actuarially fair benefit measure may be unjust as well: (i) denying the principle of insurance, this severely punishes those people who have to retire before pensionable age (Diamond and Mirrlees, 1986); (ii) hurting the principle of actuarial fairness, it falsely identifies the death risks of people retiring before, at and after pensionable age (Gruber and Ország, 1999; Simonovits, 1999; Guegano, 2000). What happens in the probable case if the life-expectancy of late retirees is much higher than that of early retirees? Not only the first group is prized at the cost of the second, but the entire balance is destroyed.

In this study, we generally neglect the differences between males and females. At this point, however, it is impossible not to underline that in public pension systems it is more and more difficult to apply gender discrimination and it was always forbidden to apply it at the calculation of life-annuities. Although women generally retire much earlier than men and the former statistically live much longer than the latter, the second factor is not taken into account at the determination of pensions.

Let us look at the simplest model of flexible retirement. There is no growth. Let L, $R+1$ and D be the *average ages* at which the individual enters the labour force, retires and dies, respectively. Let us assume that these averages stem from two groups in the population: the *healthy* (H) and the *sick* (S), with relative frequencies $q_H > 0$ and $q_S > 0$, $q_H + q_S = 1$. Common entrance age L, but different retirement ages R_H and R_S, and different ages at death D_H and D_S are assumed, respectively. Therefore we have the following relations for the averages: $q_H R_H + q_S R_S = R$ and $q_H D_H + q_S D_S = D$.

Let ω_v be the uniform contribution rate and let β be the average replacement rate, β_H be that of the healthy and β_S be that of the sick. Then $q_H \beta_H + q_S \beta_S = \beta$.

Obviously, the contributions ω_v paid for $R - L + 1$ years finance the relative benefits β received for $D - R$ years, that is

$$\beta = \omega_v \frac{R - L + 1}{D - R}.$$

In a correctly differentiated system, both R and D depend on the type of the person:

$$\beta_{\mathrm{H}} = \omega_v \frac{R_{\mathrm{H}} - L + 1}{D_{\mathrm{H}} - R_{\mathrm{H}}} \quad \text{and} \quad \beta_{\mathrm{S}} = \omega_v \frac{R_{\mathrm{S}} - L + 1}{D_{\mathrm{S}} - R_{\mathrm{S}}}.$$

Observe that the benefit is determined at retirement when the age at death is not known yet. Therefore at the actual pension formula the average rather than the actual age at death appears.

In an incorrectly differentiated system, we have

$$\beta^*_{\mathrm{H}} = \omega_v \frac{R_{\mathrm{H}} - L + 1}{D - R_{\mathrm{H}}} \quad \text{and} \quad \beta^*_{\mathrm{S}} = \omega_v \frac{R_{\mathrm{S}} - L + 1}{D - R_{\mathrm{S}}}.$$

According to the properties of average, $D_{\mathrm{S}} < D < D_{\mathrm{H}}$, hence $\beta^*_{\mathrm{H}} > \beta_{\mathrm{H}}$ and $\beta^*_{\mathrm{S}} > \beta_{\mathrm{S}}$.

Theorem 5. *Under flexible retirement with common assumed life-expectancy, where the late retirees live longer than the early retirees, the former receive more and the latter receive less than actuarial fairness would assign to them.*

We use the following data for illustration: age at entering the labour force: $L = 20$, last year of work: $R = 61$, life-expectancy: $D = 74$.

Assuming that the share of healthy people in the population is $q_{\mathrm{H}} = 1/4$, and that of the sick is $q_{\mathrm{S}} = 3/4$, we shall work with the following specific values: $R_{\mathrm{H}} = 64$, $R_{\mathrm{S}} = 60$ and $D_{\mathrm{H}} = 77$, $D_{\mathrm{S}} = 73$, furthermore $\omega_v = 0.2$. In a correctly differentiated system the replacement rate would be $\beta_{\mathrm{H}} = 0.75$ and $\beta_{\mathrm{S}} = 0.68$, while in an incorrectly differentiated system the values are $\beta^*_{\mathrm{H}} = 1$ and $\beta_{\mathrm{H}} = 0.631$. The correct average replacement rate is $\beta = 0.7$, while the incorrect is $\beta^* = 0.723$.

Of course, in reality, there are more than two groups concerning retirement and the ages at death are not deterministic within the groups. But it is quite probable that there is a strong positive correlation between the age at retirement and that of death.

Macroeffect of combined indexation

For recapitulation, let us record that the indexation of benefits by wages is attractive, because (i) in a stationary population it keeps the contribution constant and (ii) it maintains the relative position of pensioners with respect to the workers. There is an international trend, however, to replace wage indexation with price indexation, or as a compromise, with combined wage-price indexation. We shall show that this change only yields a *temporary slow down* in the total pension expenditures, and results in a step-by-step *permanent* relative deprivation of the older cohorts w.r.t. workers. (János Réti called my attention to this apparently neglected phenomenon and I express my gratitude to him at this place.)

We shall use the following simplifying assumptions.

1. Zero population growth: $\nu = 1$ and time-invariant survival probabilities: l_k. The size of every cohort is originally equal to 1.

2. The age at entering and leaving the labour force is time-invariant and is the same for everybody: $0 < L < R$ ($< D$).

3. For any given age, the age-specific earnings grow according to a time-invariant growth factor: $g > 1$.

According to Assumptions 1–3, in year t the nation-wide average earning $\mathbf{w}_t$ also grows with factor g: $\mathbf{w}_t = \mathbf{w}_0 g^t$.

4. The wages taken into account at the entry pensions also grow at rate g and the delay in valorization has a fixed effect, thus can be neglected.

5. The entry pension is proportional to the average earnings: $b_{R+1,t} = \hat{\beta}_w \mathbf{w}_t$. (Note that this replacement rate differs from the ones discussed in Sections 2–3.)

6. In year 0, the average benefit of k-aged was $b_{k,0}$, $k = R+1,\ldots, D$. (If there was correct wage indexation until year 0, then $b_{k,0} = \hat{\beta}_w \mathbf{w}_0$, $k = R+1, \ldots, D$.)

From the next year a *combined wage-price indexation* with θ, $0 \leq \theta \leq 1$, is introduced, i.e. $b_{k,t} = g^{\theta} b_{k-1,t-1}$, i.e. in year t the age-dependent benefits are as follows:

$$b_{k,t} = g^{\theta(k-R-1)} \hat{\beta}_w \mathbf{w}_0 g^{R+t+1-k}, \quad k = R+1, \ldots, R+t;$$

$$b_{k,t} = g^{\theta t} b_{k-t,0} \left(= g^{\theta t} \hat{\beta}_w \mathbf{w}_0\right), \quad k = R+t+1, \ldots, D.$$

In fact, the official combined indexation rule chose the simpler arithmetic average $(1 - \theta + \theta g)$ rather than the logical geometric average g^{θ}. If there is no great change in the real earnings, then the difference between the two methods is negligible.

We turn now to the macroanalysis. The total benefit of k-aged in year t is equal to $l_k b_{k,t}$, the total benefit of the whole population is given by $B_t = \sum_{k=R+1}^{D} l_k b_{k,t}$. The total wage is $W_t = \mathbf{w}_t \sum_{i=L}^{R} l_i$. The contribution rate is equal to the ratio of total benefits to total earnings: $\omega_{w,t} = B_t / W_t$.

Simple calculation yields

Theorem 6. *During the transition period $t = 0,\ldots,D-R$, (i) the contribution rate decreases slower and slower and (ii) the ratio of the closing benefit to the entry benefit decreases. After the transition is over, both the contribution rate and the closing/entry benefit ratio are stabilised at a lower value, but the total benefits grow again with the rate of productivity.*

For illustration, we shall work with the following data; productivity growth factor: $g = 1.03$, the share of the earning index: $\theta = 0$, the age at entering the labour force: $L = 20$, the age at retirement: $R + 1 = 62$, the age at death: $D = 74$. It is assumed that there was a correct wage indexation prior to the reform. Then the contribution rate decreases from 0.2 to 0.169, and the ratio of closing to entry pension drops from 1 to 0.7. The paths of the 12-year transition period are displayed in *Figure 4*.

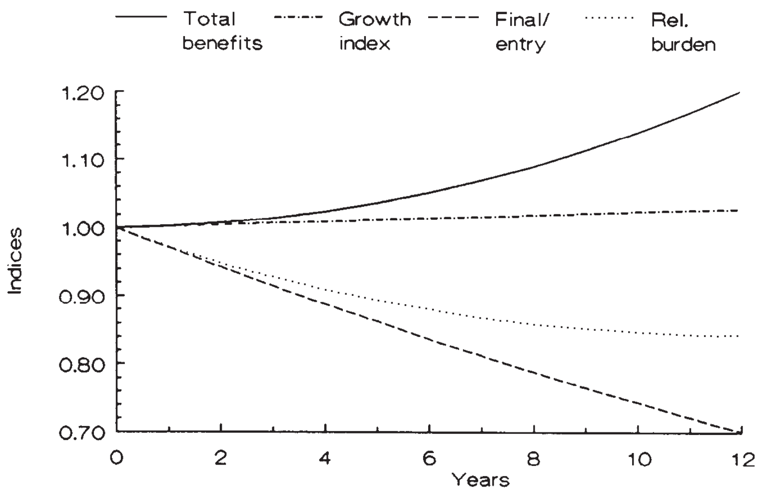

Figure 4. The effects of price indexation

We have chosen the pessimistic rule where pure price indexation is applied, like in the bulk of the countries. Of course, the reality differs from our model. (i) The aging of the population is a permanent process. If it is going on during the transition period, then the slow reduction of the contribution rate can stop and turn into growth. (ii) One of the objectives of any pension system is to defend against the longevity risk. If one calculates correspondingly with death risks, then the closing/entry pension ratio becomes even lower.

On the basis of the German example, I hoped that the wage indexation of pensions can be preserved, even at the price of reducing the initial replacement rate. The latest events occurring in Germany dispel this hope. It seems as if in Germany price indexation replaces wage indexation *and* the replacement rate is also reduced.

4. TRANSITION AND REVIVAL

Because of the worldwide aging and the slowdown of the productivity increase, the bulk of the experts consider the pension problem as menacing or even critical. A majority of them look for solutions and find them in the revival of the funded system, if not fully, then at least in part (e.g. Feldstein, 1974; Feldstein and Samwick, 1997). There are still other economists (e.g. Augusztinovics, 1995; Diamond, 1997; Orszag and Stiglitz, 1999), who disagree with the pessimists and try to mend rather than replace the existing unfunded public pension systems. The author of this study also belongs to this group. This section starts with the general argument, then illustrates the points on the example of Hungary.

The general problems of transition and revival

As was mentioned in Section 3, the transition from a funded system to an unfunded system was relatively simple, because the members of the first generation of the unfunded system need not have paid anything for their pensions, apart from the unimaginable losses (including previously accumulated pension capital) during the World Wars and the Great Depression. (The step-by-step introduction of the unfunded system, i.e. the smooth increase of the entitlement ratio and the replacement rate, made the transition even smoother on the macro level.) The return from the unfunded to the funded system appears much more difficult, because the first generations have to pay the contributions to their parents' pensions and prefund their own pensions. (The revival is especially difficult in a country, where the unfunded system is comprehensive and ensures high replacement.)

Let us make the following thought experiment (Geanakoplos et al., 1998). Assume that at the introduction of the funded system, the government calculates the individual pension entitlements and makes the corresponding *implicit* debt *explicit*. From that moment the unfunded system is closed down, and every participant contributes to or uses up his pension fund, depending on his status (worker or pensioner). For the time being, neglect the following complications, real or imagined: 1) the operating costs of the funded system are much higher than that of the unfunded system, 2) the apparent pension debt inflates the total government debt, 3) the workers contribute much more extensively to the private pension system because they know that the capital is already theirs.

We shall prove

Theorem 7. *(Neutrality.) Under our assumptions 1–3, the return from the unfunded system to the funded one is neutral: it does not change the pensions or the real burden on government debt.*

Proof. The increased government debt does not cause any problem to the government, since from this date, the government ceases to pay any pension. It is true that the additional debt is capitalised at the (higher) interest rate rather than at the (lower) internal rate of return. But the new pensions could be increased much faster than the earlier pensions, just because the interest rate is higher than the internal rate of return, and this difference can be taken away from the pensioners and used for amortisation of the increased debt. Nothing has changed.

But what then is the meaning of a revival of the funded system? There is more than one answer to this question. 1) Certain witch-doctors forget about the burden of transition and only concentrate on the yields. 2) Others (like Feldstein, 1996; Kotlikoff, 1997) do not forget about the burden but they evaluate the yields so highly which make the burden worthwhile to take. Nevertheless, there is a sharp difference in the method of distribution of the burden between the two supporters of funding: Kotlikoff would put the main burden on the shoulders of the old via introducing a value added tax, while Feldstein would make the young pay the bill through temporarily increased social security contributions.

I would like to cite Kotlikoff's following observation: "The weaker the marginal connection between the contributions and the benefits, the greater the probability that the privatisation of social security enhances the efficiency of the system." It is worthwhile to compare this remark with the original endeavour of the World Bank (1994) to weaken the connection between the contributions and the benefits, replacing earning-related pensions with flat benefits. Is it not simpler to retain or even strengthen the link between contributions and the benefits and thus making privatisation superfluous?

We have not reached the end of the story, thus we cannot give a definite answer. We can, however, present a case-study.

Hungary: from 1998

The Hungary government introduced a three-pillar mixed system in 1998. Following the individual's point of view, we distinguish two cases: (i) a person who entered the labour force after July 1, 1998, and who had to start his contributory career in the mixed system; (ii) a person who started to work before July 1, 1998, started his contributory career in the pure system but chose the second pillar voluntarily.

(i) Every starter has to choose a private pension fund, and his employer has to transfer about a quarter (8% of his gross earning) of his total contribution (31% of his gross earning). More precisely, to ease the transition, about a fifth (6 and 7% of his gross earning) of the total contribution was planned to be paid to the second pillar in 1998 and 1999, respectively, but the new government extended the lowest rate (6%) for the coming years, too. The savings accumulated on the individual account earn interest and can be freely transferred among various funds. At retirement the pensioner has to buy a unisex life-annuity, which is to be indexed, the same way as the first pillar benefit. If he worked at least 15 years in the mixed system and his life-annuity does not reach 25% of the first pillar benefit, then a guaranty fund tops up his second pillar annuity to the above value, i.e. 93% of the pure unfunded pension.

(ii) Every insider was allowed to enter the mixed system, but the entrant lost about 1/4 of his entitlement obtained in the old system: he will receive 1.22% rather than 1.65% of the reference earning for every year he contributed to the pure system (as if he had paid and lost his 8% contributions to his private account even before it was set up). The longer somebody participated in the old system, the higher is the loss from entering the new system.

For better understanding, it is underlined that the first pillar will be reformed: distortions between contributions and benefits will be eliminated by 2013, the date when the new system will start paying benefits. Here the transition gain/loss is expressed in terms of the full unfunded pension. It is assumed that the individual will work 40 years, and we are changing the number of years in the old system and the relative interest factor. To underline the significance of government guaranty, we deliberately omit the –7% lower bound for people serving at least 15 years in the mixed system but these cases are denoted by an *. In the last column there are no stars, however, because in these cases our individual will serve only 10 = 40–30 years in the mixed system, excluding the guaranty. Was this the reason that the original upper bound on the age of entry (47 years) was eliminated? Or has the government such a menacing forecast on the benefits in the unfunded system that any transition is advantageous? Anyway, sticking to the actual laws, it can be seen that even after 20 years of past service, even in the case of relative interest rate 4%, the transition loss almost approaches the guaranty.

Table 4

Gain/loss due to entering the mixed system: Hungary

	Years of service in the old system			
	0	10	20	30
Relative interest rate 100 (r/Ω)				
0	0	–6.3	–12.5*	–18.8
2	12.8	4.0	–9.8*	–18.2
4	34.4	10.1	–6.4	–17.5

Another serious problem is that the creators of the mixed system delayed the solution of the survivors and disability benefits in the mixed system (see the subsection Insurance in Section 2). Of course, in a system with a dominant unfunded pillar this is not as serious a problem as it would be in a dominantly funded one. Nevertheless, from the point of view of the much revered actuarial fairness, it is unacceptable that in case of death or disability, the survivors and the disabled are simply returned to the first pillar (Bod, 2000; Réti, 2000).

Retaining the dominance of the first pillar, the government disavowed the bulk of the alleged superiority of the funded system; at the same time, significantly reduced the further accumulation of the visible government debt (cf. Theorem 7). From a macroeconomic point of view, a further favourable deviation from the Chilean solution is that the implicit debt is only partially converted to an explicit debt and the whole process is delayed until retirement of the corresponding cohorts. It is noteworthy that the partisans of the reform hope that the increased government debt will constrain the government.

The pension reformers were very successful in popularising the second pillar (or more realistically, in undermining the credibility of the first pillar). By the final deadline of transition, August 31, 1999, about half the active contributors have opted for the mixed system, much more than originally forecast. (It is true that almost everybody can return to the pure unfunded system by December 31, 2000, but the number of returning people seems negligible.) As could be expected, the share of entrants is decreasing with the age *(Figure 5)*. It is especially disturbing that a significant part of the joining people appear to be sure losers, especially if the contribution rate remains 6% rather than 8% (cf. *Table 4*).

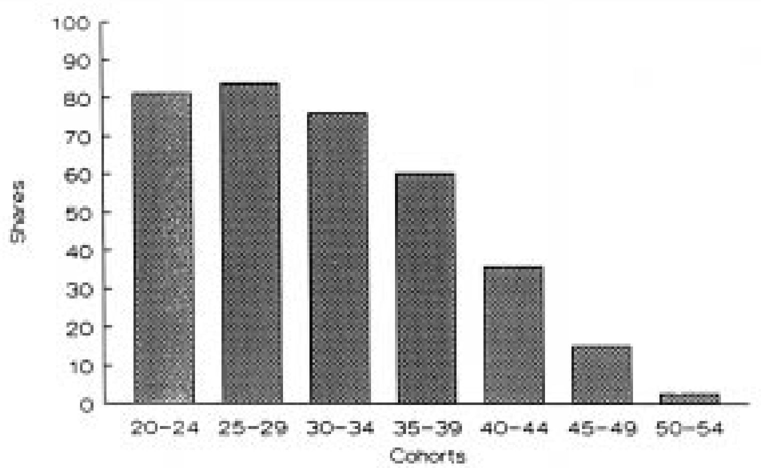

Figure 5. Share of entrants: Hungary

Among others, Augusztinovics (1999), Müller (1999) and Simonovits (1999) have formulated numerous critical remarks. Here we list only the most important ones: 1) The reform of the first pillar (which is planned to remain dominant) is not fast enough: according to the plans, the proportional pension system will only be achieved by 2013. (Incidentally, this delay somehow contradicts a strong recommendation of World Bank (1994, 264): "...the old public plan must be reformed simultaneously with or just before the transition". 2) There remained a lot of uncertainty in the private pillar, planned to be the strong point of the whole system. 3) The private pillar is over-regulated, by all experiments, its operating costs will remain high, and the stock exchange seems to be unreliable. 4) There is a danger that the increasing share of the private pillar will be financed by the compression of the public pillar (decreasing replacement rate, price indexation or *ad hoc* changes). The pretext of the radically restrained pension increase in 1999 was the budget deficit, increased by the introduction of the private pillar. (This is a good example for the central idea of the Generational Accounting (e.g. Auerbach et al., 1994) which tries to compare the net life-time contributions of different cohorts: the budget deficit is not an entirely objective category.)

According to some economists (e.g. Müller, 1999), it is the very complexity of the reform that hides the real changes from the population. Some supporters of the reform also acknowledge that the pensioners of the long transition period will suffer significant losses. It is questionable whether the long-run results (if they will be reached at all), justify the costs (Ország and Stiglitz, 1999).

For the time being, there is no reliable scenario on the transition. On the back of the envelope we can make the following calculation. In 2000 the pension expenditure amounts to about 10% of the GDP. Since about half the workers participate in the mandatory private pillar, and pay about 20% of their contributions to it, about 10% of the total contribution, i.e. about 1% of the GDP flows to the second pillar. This is the upper limit of the pension deficit accepted by the previous government. What will happen in 15 years? Assume that by introducing flexible retirement age and combined indexation, the pension expenditure will be diminished to 8% of the GDP. By then almost everybody, say 80%, will be a member of a pension fund. By the original promise, they can pay about 25% of their contribution to the private pillar. Consequently, about 1.6% of the GDP will flow to the second pillar.

It is an open question if the increased obligation can be distributed in such a way that neither the pensioners, nor the workers become overburdened: the increase of the rates of personal income tax is not more popular than that of the social security contribution; moreover, the cost of servicing the increased government debt can annihilate the advantages of the rising private pensions.

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